pounds as well as the placement of X_{3c} relative not only to X_5 but also to Γ_{15v}. Furthermore in most cases the $\Gamma_{15v}{}^{\alpha} \rightarrow \Gamma_{15c}$ $(\alpha = \frac{3}{2},\frac{1}{2})$ thresholds are separated, which has not proved possible so far in most optical spectra.

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Stress Effects on Impurity-Induced Tunneling in Germanium*

H. Fritzsche
Department of Physics and Institute for the Study of Metals, The University of Chicago, Chicago, Illinois

AND

J. J. Tiemann
General Electric Research Laboratory, Schenectady, New York

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The effects of uniaxial compression and of hydrostatic pressure on the impurity-induced interband tunneling current in germanium tunnel junctions have been studied experimentally at 4.2°K. The diodes were formed on (100) and (110) faces of arsenic-doped germanium bars. The stress coefficients of the tunnel current were measured at fixed forward and reverse bias voltages. The experiments show that the part of the electron wave function responsible for impurity-induced tunneling is not associated with a particular conduction-band valley. Some structure in the bias dependence of the shear stress coefficients near zero bias remains unexplained. This structure does not appear in the hydrostatic-pressure coefficient.

I. INTRODUCTION

There are three distinctly different interband tunneling processes[1–4] in semiconductors: (i) direct tunneling between states having the same value of the crystal momentum k, (ii) phonon-assisted tunneling between states of different k, and (iii) impurity-induced tunneling. This last tunneling process again occurs between states of different k, but in this case the difference in crystal momenta of the initial and final electron states is supplied by impurities or defects.

All three tunneling processes can be observed in Ge or Si tunnel junctions in different bias ranges.[3–5] In Ge, the relative amount of phonon-assisted and impurity-assisted indirect tunneling depends strongly on the donor element.[6,7] The fraction of impurity-induced tunneling increases with increasing magnitude of the central cell potential of the particular donor element. In Sb-doped germanium, impurity-induced tunneling is almost completely negligible with respect to phonon-assisted tunneling, and the reverse is true for As- and P-doped Ge.

Recently[8,9] many details of the direct and the phonon-assisted indirect tunneling processes have been uncovered by measuring the bias dependence of the effect of pressure and shear stress on the tunneling current in Sb-doped germanium tunnel diodes. The absence of impurity-induced tunneling in these samples made it possible to get some clear answers concerning the other processes.

The work discussed here on As-doped tunnel diodes complements the previous work in that the impurity-induced tunneling current in these samples completely dominates the phonon-assisted components.

There are several questions concerning this mode of tunneling that can be answered by stress experiments. In particular, it has been shown[8] that the presence or absence of a large positive shear stress coefficient for current I along $[1\bar{1}0]$ and compressional stress along

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[110] can be used to decide whether or not an electron tunnels from a particular (111) valley in the conduction band.

This paper reports measurements at 4.2°K of the stress-tunneling coefficients $\Pi=\Delta I/IX$ and $\Pi_p=\Delta I/I3p$ of germanium tunnel junctions containing As concentrations of 1.5×10^{19} cm^{-3}. The direction of the tunnel current was parallel to [001] or [1$\bar{1}$0]; the orientation of the uniaxial compression was along [100] or [110].

II. DEFINITION OF THE STRESS-TUNNELING COEFFICIENTS

We restrict our discussion to the first-order stress effects on tunnel junctions placed (i) on a (001) crystallographic plane so that the tunnel current flows along [001] and (ii) on a (1$\bar{1}$0) plane with the current along [1$\bar{1}$0].

For a junction on a (001) plane of a crystal having cubic symmetry, there are two independent constants which describe the first-order stress dependence of the tunnel current. We shall call these A and B. Here $A=\Delta I/IX$ for a uniaxial compression X lying in the junction plane, as shown in Fig. 1, and $B=\Delta I/I3p$ for hydrostatic pressure p. The relative current change for uniaxial compression normal to the (001) plane is then

$$\Delta I/I=(B-2A)X. \tag{1}$$

For a junction placed in a (1$\bar{1}$0) plane, there are three independent coefficients C, D, and E. C and D are explained in Fig. 1 and E is the hydrostatic pressure coefficient for the tunneling direction [1$\bar{1}$0]. Calling Φ the angle between the compression axis and the [001] direction, the relative current change due to a uniaxial compression lying in the (1$\bar{1}$0) plane is then

$$\Delta I/I=[C+(D-C)\cos^2\Phi]X. \tag{2}$$

The corresponding quantity for a compressional stress normal to the (1$\bar{1}$0) plane is

$$\Delta I/I=(E-C-D)X. \tag{3}$$

III. EXPERIMENTAL DETAILS AND RESULTS

The tunnel junctions were formed by alloying indium dots doped with $\frac{3}{8}\%$ gallium at 540°C on opposite faces of single-crystal germanium bars containing an arsenic concentration of 1.5×10^{19} cm^{-3}. The diameter of a typical diode dot was about 0.05 cm after the etching process which removes the perimeter of the junctions. Uniaxial compressional stress varying between 5×10^7 and 5×10^8 dyn/cm^2 was applied parallel to [110] or [100] at helium temperatures. The stress coefficients averaged over the two opposed diodes (to eliminate the effects of flexure of the bar) were independent of stress in this range. The cryostat and stressing apparatus have been described before.[10] The hydrostatic pressure measurements were extended up to $p=7\times10^7$ dyn/cm^2.

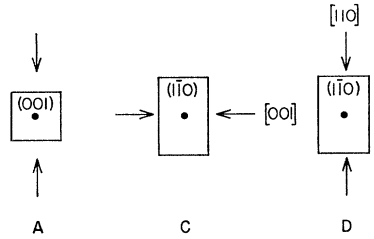

FIG. 1. Definitions of the uniaxial stress coefficients $\Delta I/IX$. The arrows represent uniaxial compressional stress of unit magnitude.

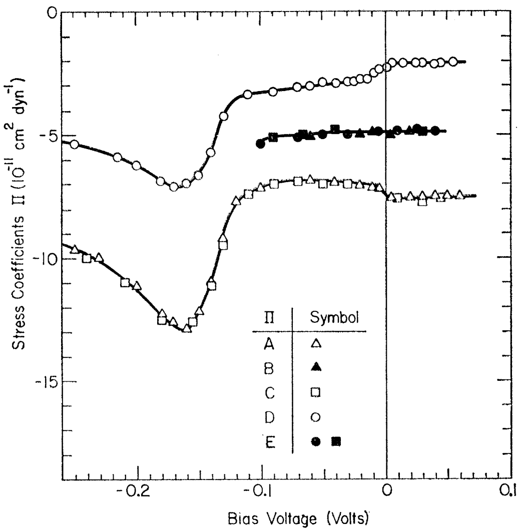

FIG. 2. Stress coefficients for uniaxial compression and hydrostatic pressure for As-doped Ge tunnel junctions as a function of bias voltage at 4.2°K. Note the factor $\frac{1}{3}$ in the definition of the hydrostatic-pressure coefficients B and E.

Figure 2 shows the bias dependence of the various stress coefficients defined in the previous section at 4.2°K. In contrast to the observations on Sb-doped germanium diodes,[8] these As-doped diodes do not exhibit the large positive shear stress contribution to the coefficient D. The stress coefficients A and C are found to be almost identical. The same is true for the pressure coefficients B and E. One further observes that the pure shear component $D-E$ for the orientation I[1$\bar{1}$0], X[110] is nearly identical in magnitude but of opposite sign to the pure shear component $(C-E)$ for this orientation or $(A-B)$ for the orientation I[001], X[100].

The bias dependence of the pressure coefficients B and E of these As-doped functions does not show the structure which was found[9] in the case of Sb-doped junctions. This is to be expected since the structure found in those cases is associated with the threshold voltages for phonon-assisted tunneling, and this process contributes only negligibly to the tunneling current in As-doped junctions. One observes, however, a change of the stress coefficients A and C and an opposite change of D at zero bias which is not present in the bias dependence of the pressure coefficients B and E. Thus

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there is a structure in bias dependence of the coefficients for pure shear [$(A-B)$, $(C-E)$, and $(D-E)$] that does not appear in the coefficients for hydrostatic pressure. This structure is noteworthy because it occurs over a narrow bias range which is symmetric about the origin. All of the pure shear coefficients mentioned above increase in absolute magnitude by $20\pm4\%$ in going from reverse bias to forward bias.

At large negative biases ($V<-0.13$ volts) the stress coefficients show a shallow minimum. In this bias range one expects the onset of direct tunneling into the $k=(000)$ conduction-band states. Since compression increases the direct gap at (0,0,0), and hence the onset voltage for direct tunneling, a negative contribution to the stress coefficients is expected in this range. The obobserved dip in the stress coefficients is less pronounced in As-doped junctions than in Sb-doped junctions because the magnitude of the impurity-assisted current is much more nearly equal to the direct tunneling current in this bias range.

IV. DISCUSSION AND CONCLUSIONS

The absence of a large positive shear contribution to the stress coefficient D in As-doped junctions shows that in this case the tunneling wave functions on the n-type side are not associated with individual (111) conduction-band valleys. This conclusion is not unexpected, since phonon assistance would be very likely for such a case regardless of the details of the tunneling process. Furthermore, the arsenic donors are known to produce a strong localized central cell potential[11] which introduces components into the wave function from large parts of the Brillouin zone including all of the valleys and the region around $k=0$. Since the electron-phonon interaction in germanium is not large, a small admixture of (000) character to the electron wave function should result in a predominant tunneling current which is not assisted by phonons.

At present, there is no theory of impurity-induced indirect tunneling with which the experimental results should be compared. Other measurements on this process have been compared[12] with some success with the expression for direct interband tunneling presented by Kane,[2] but the agreement found in these cases should not be regarded, in our opinion, as evidence that the tunneling process under consideration is the one analyzed by Kane.

However, the form of the exponential factor in the tunneling expression seems to be quite independent of the details of the tunneling process. Therefore, one might tentatively use Kane's expression for direct tunneling with a different interpretation of the effective masses and the band gap for the interpretation of the shear dependence observed in As-doped diodes. The tunneling exponent depends on the projection of the reduced effective-mass tensor in the direction of the electric field. Since the effective-mass tensors are expected to deform under the influence of shear, it is interesting to see if this effect can give rise to the observed differences between the hydrostatic and uniaxial stress coefficients, and if so, how large the changes of the effective masses have to be. The reciprocal reduced effective-mass tensor is defined as

$$\mathbf{m}_r^{-1}=\mathbf{m}_v^{-1}+\mathbf{m}_c^{-1}. \tag{4}$$

In the present case $\mathbf{m}_v^{-1}$ is the reciprocal effective-mass tensor of the light-hole band and $\mathbf{m}_c^{-1}$ is a reciprocal mass tensor for the electrons. Because we are dealing with impurity-induced tunneling it is not certain whether the conduction-band minimum at $k=0$ or an appropriate average of the (111) conduction-band valleys determines $\mathbf{m}_c^{-1}$.

Since the tunneling experiments cannot separate the contributions of the hole and the electron masses, we describe the stress-induced changes of the reduced reciprocal effective-mass tensor by a single fourth-rank deformation-potential tensor $\mathbf{Q}$. It was found experimentally that $B=E$ and $A=C$. These results are consistent with the assumption that $\mathbf{m}_r^{-1}$ is spherically symmetric at zero stress and that cubic symmetry holds for $\mathbf{Q}$. The fact that A and C are more negative than B and E indicates that the reduced effective mass is increased in the plane perpendicular to the [100] compression axis. A [110] compression will deform the reduced effective-mass sphere into a general ellipsoid. The positive shear contribution $(D-E)$ implies a reduction of the mass component along $[1\bar{1}0]$ for shear resulting from a uniaxial compression along [110].

In the following the magnitudes of the deformation potentials Q_{11}, Q_{12}, and Q_{44} will be estimated. We assume that the tunnel current is given by an expression of the familiar form[2,13]

$$I=C\times D\exp(-\alpha)\,, \tag{5}$$

$$\alpha=\lambda E^{3/2}m_{rx}^{1/2}/hF\,. \tag{6}$$

Here, λ is a numerical constant of the order of unity that depends on the particular theory and on the way the average field is calculated but that need not be known for our argument. E is the relevant band gap and F is the average junction field which also does not need to be known.

If one assumes that (similar to the cases of direct and phonon-assisted tunneling[8]) the major effect of pressure results from the exponential factor in Eq. (5), then the coefficient for hydrostatic pressure is given by

$$B=\frac{1}{3}\frac{d(\ln I)}{dp}=-\frac{\alpha}{3}\left[\frac{3}{2}\frac{d\ln E}{dp}+\frac{1}{2}\frac{d(\ln m_{rx})}{dp}-\frac{d\ln F}{dp}\right]. \tag{7}$$

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With the approximate relations

$$\frac{d\ln m_{rx}}{dp}\approx\frac{d\ln E}{dp}, \tag{8}$$

$$\frac{d\ln F}{dp}\approx\frac{1}{2}\frac{d\ln E}{dp}, \tag{9}$$

we obtain approximately

$$B\approx-\frac{\alpha}{2}\frac{d\ln E}{dp}. \tag{10}$$

The factor $\frac{1}{3}$ has been included in the definitions of the hydrostatic pressure coefficients B and E [see Eq. (7)] because a uniaxial stress X produces a dilatation effect equivalent to a hydrostatic pressure of magnitude $p=X/3$. One therefore obtains for the effect of the shear part of the uniaxial stress

$$A-B=-\tfrac{1}{2}\alpha\left[\frac{d\ln m_{rx}}{dX(100)}-\frac{1}{3}\frac{d\ln m_{rx}}{dp}\right]. \tag{11}$$

The combination of Eqs. (10) and (11) yields the effect of pure shear on the reduced effective mass in the tunneling direction in terms of the measured stress coefficients and the effect of pressure on the band gap as

$$\frac{d\ln m_{rx}}{dX(100)}-\frac{1}{3}\frac{d\ln m_{rx}}{dp}=\frac{A-B}{B}\frac{d\ln E}{dp}, \tag{12}$$

with equivalent equations for the other orientations. The components of the deformation-potential tensor **Q** in the coordinate system of the cube axes are then

$$\begin{aligned} Q_{11}&=-\left[\tfrac{1}{3}-\frac{2(A-B)}{B}\right]\frac{d\ln E}{dp},\\ Q_{12}&=-\left[\tfrac{1}{3}+\frac{(A-B)}{B}\right]\frac{d\ln E}{dp},\\ Q_{44}&=\left[\frac{2(D-E)}{E}+\frac{(A-B)}{B}\right]\frac{d\ln E}{dp}. \end{aligned} \tag{13}$$

If we assume the appropriate band gap for impurity-induced tunneling is the indirect gap, then $d\ln E/dp=6.75\times10^{-12}$ cm²/dyn and the components of the deformation-potential tensor are, using the stress coefficients of the positive bias range,

$$\begin{aligned} Q_{11}&=4.5\times10^{-12}\ \mathrm{cm^2/dyn},\\ Q_{12}&=-5.6\times10^{-12}\ \mathrm{cm^2/dyn},\\ Q_{44}&=-4.4\times10^{-12}\ \mathrm{cm^2/dyn}. \end{aligned}$$

The deformation potentials of the reduced-mass tensor and of the appropriate tunneling band gap are found to be of the same order of magnitude. It is still uncertain, however, to which electron masses one refers by writing Eq. (4). The data on Sb-doped tunnel diodes[8] in the bias region where phonon-assisted tunneling dominates yield $(C-E)/E$ and hence a shear-induced mass change of the same order of magnitude as found here. In the bias range of direct interband tunneling those diodes showed a considerably smaller shear effect. This seems to indicate that the shear dependence on the impurity-induced tunneling of the As-doped diodes is not due to a change of the reduced mass of the direct gap at the zone center.

If the shear dependence of the As-doped diodes arises from a reduced mass involving the (111) conduction-band valleys, then the cubic symmetry shown by the data implies that the tunneling electron would have to be equally shared by all four valleys. Although this possibility is not in conflict with the observation[14] of considerable intervalley scattering in As-doped germanium, it is not included in the present theoretical formulations of tunneling.

The data also possess one other feature which remains unexplained. There is found to be some fine structure in the bias dependence of the shear part of the stress coefficient in the region around zero bias. The shear coefficients in the forward bias region are approximately 20% larger in magnitude compared to those of the reverse bias direction.

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