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Electron Density Dependence of Cyclotron Harmonic Radiation from a Plasma

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Simultaneous measurements of electron density and radiation emitted near harmonics of the electron cyclotron frequency have been made with a microwave cavity coaxial with a low-pressure argon discharge in a magnetic field. The dependence of the line position and width on electron density is in reasonable agreement with recent calculations made of the spectrum of electrostatic waves excited by fast electrons. The intensity of the radiation increased rapidly with electron density, and no radiation was observable unless the maximum hybrid frequency in the column was close to or greater than the frequency of observation. It is shown that axial variations in electron density account for nonuniform emission of radiation along the discharge.

INTRODUCTION

ENHANCED microwave radiation at high harmonics of the electron cyclotron frequency from a plasma in a magnetic field was first observed by Landauer[1] with a Penning-ion-gauge discharge. Bekefi *et al.*[2] later observed the radiation from the positive column of an arc discharge. In both experiments, the intensity of the radiation was several orders of magnitude too great to be accounted for on the basis of single-particle emission from the plasma. Subsequently, it was suggested by Canobbio and Croci[3] that the harmonic radiation may arise from the excitation of electrostatic waves propagating perpendicularly to the magnetic field. This hypothesis has been supported by the work of Mitani *et al.*,[4] who observed a change in the resonance frequency with discharge current in qualitative agreement with the behavior of solutions of the dispersion relation for these waves.[5]

This paper describes simultaneous measurements of emitted radiation and electron density made with a microwave cavity surrounding the positive column of a low-pressure argon discharge. In general, the observed spectrum consisted of three lines near each harmonic of the electron cyclotron frequency (Fig. 1). The relative intensity of the lines depended on the locations of the cathode and the cavity along the axis of the magnetic-field solenoid, and it was possible to arrange conditions so that any of the three lines was the most intense. Two of these lines (1 and 2) occurred very close to the exact cyclotron harmonic frequency, one at a higher and one at a lower frequency ($\Delta\omega/n\omega_c \sim \pm 0.01$). The third line (3) was below the exact harmonic frequency and considerably displaced from it.

No significant change in the positions of lines 1 and 2 could be detected as the electron density was varied, and the origin of these lines is not known. With the cathode of the discharge located outside the magnetic field, line 3 was very much more intense than the other two, enabling its position, width, and intensity to be measured. The dependence of the position and width of this line on electron density is compared with calculations by Stone and Auer[6] for the spectrum of electrostatic waves excited by fast electrons, and reasonable agreement is found. A strong dependence of the radiated power on electron density, not predicted by the theory for an infinite plasma, was also observed.

EXPERIMENTAL

A 9-mm-i.d. quartz discharge tube with a hot oxide-coated cathode passed coaxially through a cylindrical microwave cavity 12.2 cm in diameter and 6.9 cm long, equipped with four coupling loops (Fig. 2). The power which coupled into the TE_{011} mode at 3.714 Gc/sec was used as a measure of the emission from the plasma, while the frequency shift of the TM_{020} mode at 4.20 Gc/sec was measured to determine the electron density. The power from the cavity was measured with a superheterodyne receiver with a noise figure of 10 dB and a bandwidth of 1 Mc/sec, which was equal to the width of the TE_{011} mode of the loaded cavity. Absolute values

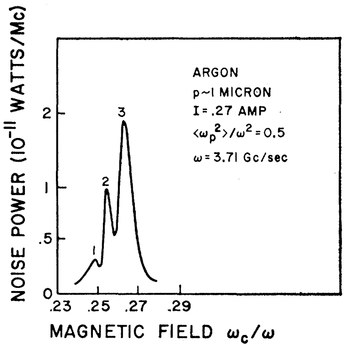

FIG. 1. Noise resonance lines near 4th harmonic.

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[1] G. Landauer, *Proceedings of the Fifth International Conference on Ionization Phenomena in Gases* (North-Holland Publishing Company, Amsterdam, 1962), Vol. 1, p. 389; J. Nucl. Energy **4**, 395 (1962).

[2] G. Bekefi, J. D. Coccoli, E. B. Hopper, Jr., and S. J. Buchsbaum, Phys. Rev. Letters **9**, 6 (1962).

[3] E. Canobbio and R. Croci, *Proceedings of the Sixth International Conference on Ionization Phenomena in Gases* (Paris, 1963), Vol. 3, p. 269.

[4] K. Mitani, H. Kubo, and S. Tanaka, J. Phys. Soc. Japan **19**, 211 (1964).

[5] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).

[6] P. M. Stone and P. L. Auer, Phys. Rev. **138**, A695 (1965).

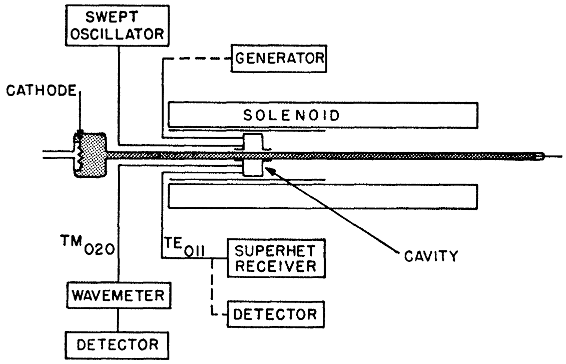

FIG. 2. Diagram of the apparatus. The dashed connections were made for absorption measurements.

for the power were obtained by inserting calibrated attenuators between the cavity and the receiver until a direct comparison with the output from a standard noise tube could be made. Absorption measurements were also made in the TE_{011} mode. The cavity could be moved along the length of the discharge tube inside the magnetic field, over a distance of 15 in. All quantitative measurements were made with the cathode of the discharge situated 10 in. outside the magnetic-field solenoid, whose construction has been described elsewhere.[7] The experiments were carried out with argon at a pressure of about 1 μ, with currents from 0.08 A to 0.35 A, and with axial electric fields of roughly 3 V/cm.

The electron density $\langle\omega_p^2\rangle$ measured with the cavity is an average over the cross section of the discharge. An estimate of the electron density on the axis was made by measuring the onset of the absorption resonance of the plasma column.[8] The low-magnetic-field onset occurs when the frequency of observation ω equals the hybrid frequency on the axis, and represents a maximum plasma frequency in the column given by

$$\omega_{p0}^2=\omega^2-\omega_c^2. \qquad (1)$$

The average value of $\omega_{p0}^2/\langle\omega_p^2\rangle$ was 2.9 for the 4th–6th harmonics, on which our measurements were made. Neglecting variations in the electron-density profile with discharge current, we have throughout multiplied the average densities $\langle\omega_p^2\rangle$ by 2.9 to obtain the axial densities.

In order to relate the measured power output from the cavity to the noise power radiated by the plasma, voltage standing-wave-ratio measurements were made on cavity resonance with the discharge on and off.[9] The effective coupling between the plasma and the receiver was almost independent of the plasma conditions, and at high electron densities the cavity output was 14% of the noise power radiated by the plasma. In all of the figures in this paper, the noise power shown is the output from the cavity.

Figure 3 shows the power output from the cavity as a function of magnetic field with the cathode located outside the magnetic field. The line shape is similar to that observed by Lustig *et al.*[7] in experiments over a wide frequency interval at a fixed magnetic field.[10] The large peaks correspond to line 3 in Fig. 1. Line 2 was not observable for this cathode and cavity position, although line 1 was detected when the receiver sensitivity was increased.

LINE POSITION AND WIDTH

The position of the peak of the emission line, the width at half power, and the electron density were measured near the 4th–6th harmonics as the discharge current was varied. Since the intensity of radiation increased rapidly with the electron density (see Fig. 6), the variation of the position of the peak as a function of the maximum electron density on the axis of the tube seemed the most appropriate for comparison with theory.

In Fig. 4, the positions of the peaks of the lines as a function of ω_{p0}^2 are compared with solutions of the dispersion relation[5] for electrostatic waves propagating in an unbounded plasma nearly perpendicularly to the magnetic field, as computed by Stone and Auer.[6] It is assumed that the conversion of electrostatic waves to electromagnetic waves has not affected the line shapes. The experimental results are similar to those of Mitani *et al.*[4] The theoretical curves in Fig. 4 are solutions of the dispersion relation for fixed values of λ, where

$$\lambda=(k_\perp u_T/\omega_c)^2. \qquad (2)$$

$k_\perp$ is the wave number for the electrostatic waves and u_T is the thermal velocity of the plasma electrons,

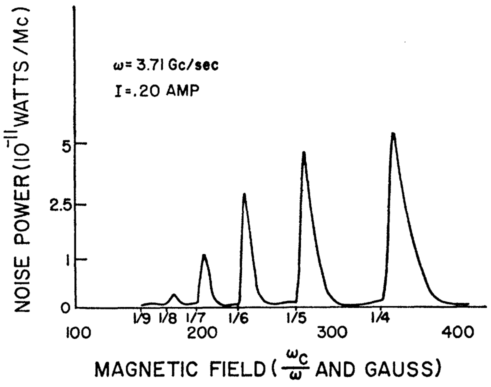

FIG. 3. Noise resonance spectrum with the cathode outside the magnetic field.

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[7] C. D. Lustig, W. D. McBee, and A. Kalisky, Rev. Sci. Instr. 35, 869 (1964).

[8] S. J. Buchsbaum and A. Hasegawa, Phys. Rev. Letters 12, 685 (1964). Absorption curves similar to those shown in this reference were observed in our experiments.

[9] L. W. Davies and E. Cowcher, Australian J. Phys. 8, 108 (1955). E. F. Labuda and E. I. Gordon, J. Appl. Phys. 35, 1647 (1964).

[10] The small peaks on the high-frequency side of nf_c in Ref. 7 are equivalent to line 1 in Fig. 1.

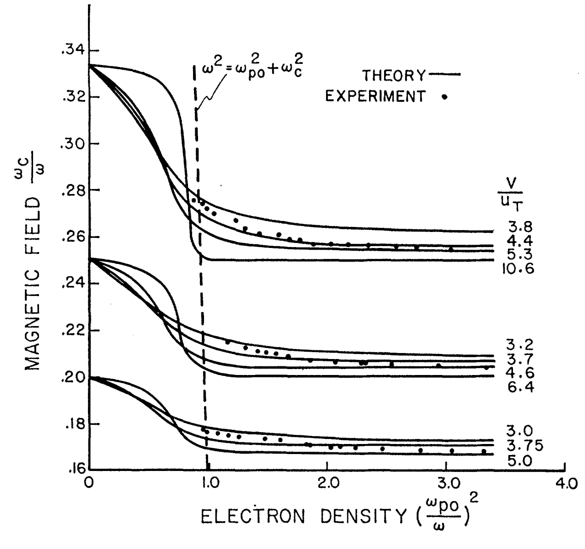

FIG. 4. Dependence of resonance line position near 4th, 5th, and 6th harmonics on axial electron density. Experimental values of ω_{p0}^2 are 2.9 times the density $\langle\omega_p^2\rangle$ measured with the cavity.

defined as $(kT/m_e)^{1/2}$. Assuming that the electrostatic waves are generated by fast electrons with velocity components u and v parallel and perpendicular to the magnetic field, Stone and Auer[6] have calculated the following expression for the intensity near the sth harmonic:

$$\left(\frac{dw}{dtd\omega}\right)_{\omega\simeq s\omega_c}=\frac{e^2}{8\pi^2}\frac{\omega}{u}J_s^2\left((\lambda)^{1/2}\frac{v}{u_T}\right). \tag{3}$$

If u and v/u_T are assumed not to vary with the discharge current, the curves $\lambda=$constant predict the positions of the line peaks for those values of v/u_T which make J_s^2 a maximum. These values of v/u_T have been used to label the theoretical curves in Fig. 4.

Reasonable agreement between theory and experiment is obtained for v/u_T roughly equal to 4. The corresponding value for λ indicates that for a plasma temperature of 10 eV, the wavelength of the electrostatic waves near the line peaks is about 0.13 cm. This value satisfies the requirement of being greater than the Debye length but less than the tube diameter. It has been assumed that the boundaries of the system impose no restriction on the possible values of λ.

In Fig. 5, the measured linewidths as a function of axial electron density are compared with calculations by Stone and Auer for fixed values of v/u_T. The value $v/u_T\sim4$ deduced from Fig. 4 is seen to be consistent with the line-width data. One would expect the experimental widths to be somewhat larger than the theoretical results, which are for monoenergetic electrons in a uniform-density plasma, and the difference should be greater at low densities; at the higher densities, radial density gradients are not expected to increase the linewidth because the line positions and widths become practically independent of the density. In view of the differences between the theoretical model and the experimental conditions, it is probably not meaningful to compare in detail the theoretical and experimental line shapes. However, it was observed that the line shapes become more symmetrical as ω_p^2/ω^2 approaches unity, in agreement with Fig. 4 of Stone and Auer.[6] Part of the discrepancy between theory and experiment in Figs. 4 and 5 can be accounted for by assuming that v/u_T increases slightly with increasing discharge current.

Fast electrons can acquire the necessary transverse energy if they pass through the magnetic-field-gradient region without being scattered. If the field gradient is indeed responsible for the high transverse velocity v,

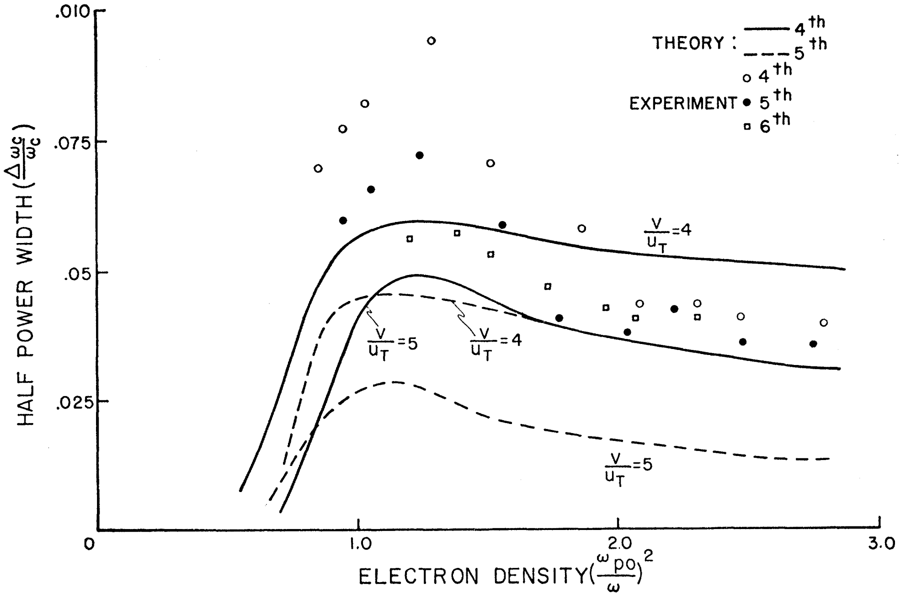

FIG. 5. Dependence of linewidth on axial electron density. Experimental values of ω_{p0}^2 are 2.9 times the density $\langle\omega_p^2\rangle$ measured with the cavity.

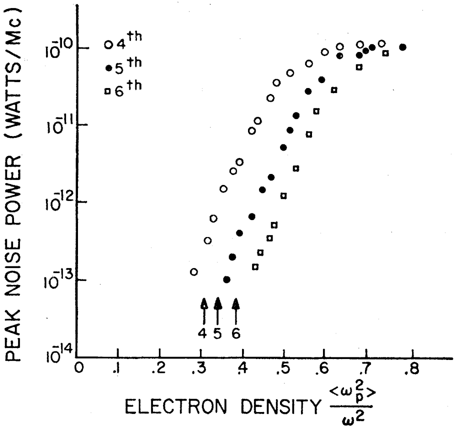

FIG. 6. Dependence of peak noise power near cyclotron harmonics on electron density. The arrows indicate the values of $\langle\omega_p^2\rangle$ for which $\omega^2=\omega_{p0}^2+\omega_c^2$ for each harmonic.

one would expect the displacement of the line from the exact harmonic frequency to increase as v is reduced by moving the cathode into a region where the magnetic field is appreciable. This effect was observed. However, comparison with theory suggests that under these conditions v/u_T increases rather rapidly with current.

It is interesting to examine how far it is possible to explain lines 1 and 2 in Fig. 1 on the basis of the model used above for line 3. In principle, electrostatic waves can be excited in the low-density plasma near the tube walls in a magnetic field just below the exact cyclotron harmonic field (line 1). However, in order to account for line 2, it is apparently necessary to make the unlikely assumption of a second group of fast electrons with transverse velocity higher than that deduced for line 3.

RADIATION INTENSITY

With the location of the cavity kept fixed at 19 in. from the cathode, simultaneous measurements were made of the radiated power and the electron density for different currents. The radiated power depended strongly on the average electron density in the plasma column, as shown in Fig. 6. The arrows indicate the values of the average density $\langle\omega_p^2\rangle/\omega^2$ when the frequency of observation was equal to the hybrid frequency on the axis of the column, determined from the onset of the absorption resonance, as previously described.

Within the accuracy of the measurements, Fig. 5 shows that power is radiated near the harmonics at frequency ω only if $\omega_{p0}^2+\omega_c^2>\omega^2$, or $\omega_{p0}^2>\omega^2$ since $\omega_c^2\ll\omega^2$. This critical dependence on electron density is not predicted by the theory for the excitation of electrostatic waves in an infinite plasma.[6] It may, however, be related to the conversion of electrostatic waves to electromagnetic radiation, or it may result from the presence of the electron-density profile in the column. Our observations support the recent suggestion by Stix[11] that conversion occurs in the plasma in the vicinity of the hybrid resonance. The volume of the plasma for which $\omega_p>\omega$ increases rapidly with $\langle\omega_p^2\rangle$ when $\omega_{p0}\sim\omega$, which may in part account for the observed large rate of increase of radiated power. The intensities of lines 1 and 2 in Fig. 1 also increased rapidly with electron density, although accurate measurements were not made.

It is difficult to estimate reliably the number of fast electrons which is necessary to give the observed radiation. If it is assumed that the electrostatic waves are excited incoherently,[6] and that the power radiated by the plasma is seven times the output from the cavity (see above), and if conversion losses are neglected, roughly 6% of the electrons would have to be fast electrons to account for the saturation level which the detected noise power reaches at high electron densities. This percentage must be increased when the conversion of electrostatic to electromagnetic waves is taken into account. It seems likely that coherent excitation of electrostatic waves may have to be assumed to account for the radiation intensity observed at high densities.

The theory for the excitation of electrostatic waves by fast electrons predicts only a small decrease in intensity with increasing harmonic number [see Eq. (3)]. The rather rapid decrease observed in Fig. 3 is due to a reduction in electron density as the magnetic field was reduced to record the higher harmonics. At a fixed magnetic field,[7] the intensity of the high harmonics falls off because $\langle\omega_p^2\rangle/\omega^2$ decreases with increasing ω.

AXIAL VARIATIONS IN ELECTRON DENSITY AND RADIATION

The electron density and the noise radiation were measured for different positions of the cavity along the length of the discharge, with the current kept constant.

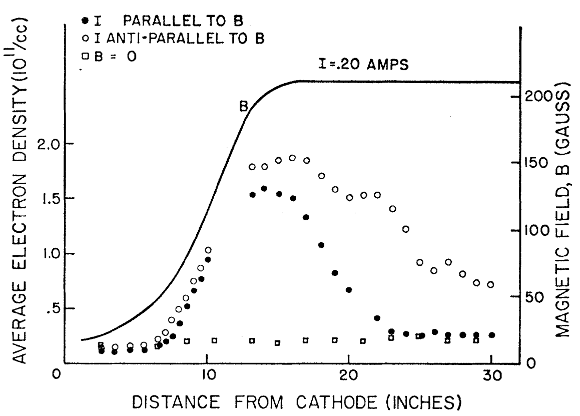

FIG. 7. Variation of electron density along the discharge in the vicinity of the magnetic-field gradient. The left-hand set of data was taken further from the cathode than indicated, but for the magnetic-field gradient shown (see Ref. 12).

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[11] T. H. Stix, Bull. Am. Phys. Soc. **10**, 230 (1965).

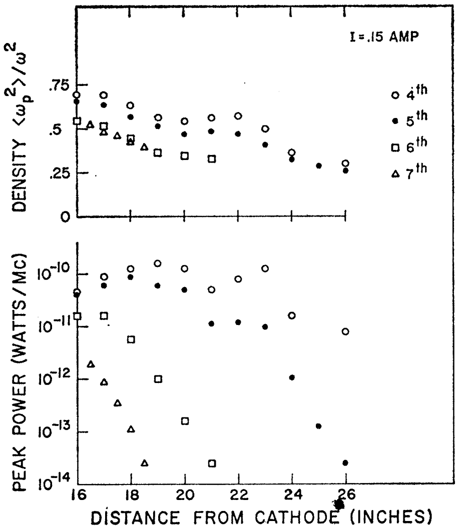

FIG. 8. Variation of electron density and peak power radiated near electron cyclotron harmonics along the discharge.

A large axial electron-density gradient was found in the vicinity of the magnetic-field gradient. This is shown in Fig. 7.[12] The electron density measured with the cavity represents the total number of electrons within a given length of the discharge and does not reflect radial variations in density. The buildup of the number of electrons to a value many times larger than that which exists when no magnetic field is applied arises because the cathode and part of the discharge are located outside the magnetic field. Those electrons with energies which can provide enough ionization to overcome diffusion losses outside the field drift into the field region where the losses are lower. Since the current along the tube is continuous, the increase in the number of electrons must be accompanied by a reduction in their drift velocity. Part of this comes about because electrons entering the magnetic-field gradient have some of their longitudinal energy converted to transverse energy.

The dependence of the electron density on the magnetic-field direction, which becomes most pronounced in the uniform-field region, is not understood. No changes in the magnitude of the magnetic field or its axial gradient were measurable when the current through the solenoid was reversed to reverse the field. It is believed that the effect may be due to a slight misalignment of the tube with the magnetic field, although the mechanism which causes the observed asymmetry is not known. It should be emphasized that the behavior of the noise spectrum as a function of electron density was not significantly different for the two directions of magnetic field, although larger currents were required to provide electron densities high enough for the radiation to be observable when the magnetic field was parallel to the current. All the radiation data presented in this paper were taken with the magnetic field antiparallel to the discharge current.

The electron-density variations shown in Fig. 7 decreased as the gas pressure was increased, probably because of the decrease in the electron mean free path. When the cathode was located in the uniform-field region inside the solenoid, a dependence of the electron density on magnetic-field direction was still observed, which shows that this effect is not due to some property imparted to the plasma by the magnetic-field gradient.

Since the radiated intensity near the electron cyclotron harmonics decreases rapidly with the electron density (Fig. 6), and since the density decreases with distance away from the magnetic-field gradient, the radiation is generated most strongly in a short length of the plasma column near the field gradient. This effect was first observed with a discharge tube which was coaxial with a waveguide,[13] but much better axial resolution was provided in the present experiment, in which a relatively short microwave cavity could be moved along the discharge tube.

Figure 8 shows the peak power radiated near the 4th–7th harmonics, with simultaneous measurements of the average electron density, for different positions of the cavity along the tube and with the discharge current kept fixed. A different electron-density curve was obtained for each harmonic because of the difference in magnetic field.

It is believed that the decrease in intensity along the tube is due principally to the observed decrease in electron density. It is apparently not due to a reduction, through scattering, of the number of fast electrons which are considered to excite the electrostatic waves, and which may in fact acquire the necessary transverse energy in passing through the magnetic-field gradient. Radiation near high harmonics was observable at large distances from the magnetic-field gradient, provided that the current was increased to produce the necessary electron density.

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[12] The movement of the cavity was restricted to the range from 13 to 30 in., and the data to the left of this region were obtained by disconnecting coils from the solenoid to produce the required magnetic-field gradient near the cavity.

[13] C. D. Lustig, Bull. Am. Phys. Soc. 9, 313 (1964).