we obtain

$$P(\omega)=e^2v\omega(1-\cos^2\theta_+)/2A(\omega)\,,$$
$$\equiv\tfrac{1}{2}e^2vQ(\omega)\,. \qquad (33)$$

Figures 7 through 10 show the normalized power spectrum $Q(\omega)$ for $\alpha=0.28$, 1.0 and various values of v.

We now compare the Čerenkov loss to the synchrotron loss of an electron of the same energy moving in a circle perpendicular to the magnetic field. The synchrotron dW_s/dt loss is given by[10]

$$\frac{dW_s}{dt}=\frac{2e^2\omega_H{}^2v^2}{3(1-v^2)}\,. \qquad (34)$$

The synchrotron loss for $\alpha=0.28$ is also plotted in Fig. 6 as a function of v. At relativistic velocities the synchrotron loss dominates the Čerenkov loss owing to its $(1-v^2)^{-1}$ dependence on the velocity. At nonrelativistic velocities we see that the Čerenkov and synchrotron loss are of the same order of magnitude with the Čerenkov loss being at most ~3 times as large as the synchrotron loss for $v\simeq1/7$.

[10] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1958).

PHYSICAL REVIEW VOLUME 139, NUMBER 1A 5 JULY 1965

Enhancement of Plasma Density Fluctuations by Nonthermal Electrons*

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(Received 11 February 1965)

In a plasma in thermal equilibrium, the spectrum of electron density fluctuations that have a wavelength longer than the Debye length has a sharp maximum near the electron plasma frequency. In this paper, the effect of a non-Maxwellian electron velocity distribution on the spectrum of electron density fluctuations is computed for frequencies near the electron plasma frequency. The electron velocity distribution is assumed to be isotropic but not necessarily Maxwellian and the effects of electron-ion collisions are included. The results show how the presence of a small number of energetic electrons can enhance the intensity of the fluctuations near the plasma frequency, provided the Landau damping resulting from these energetic electrons is greater than both the collision damping and the Landau damping caused by the ambient electrons. The results are applied to the ionosphere radar-backscatter experiments, where the energetic electrons are photoelectrons produced by solar uv radiation. In the case of the Arecibo radar experiments, the intensity of the fluctuations near the electron plasma frequency is estimated to be enhanced at plasma frequencies greater than about 4 or 5 Mc/sec.

1. INTRODUCTION

RADAR backscatter from sufficiently high levels in the ionosphere is mainly "incoherent scatter," i.e., scattering from random electron density fluctuations which exist because the electrons are discrete particles. Such experiments single out the spatial Fourier transform of the electron density with wave vector $q=4\pi\lambda^{-1}$, where λ is the radar wavelength. The experiments measure the total backscattered power which is proportional to the mean-square value of the spatial Fourier transform and also the distribution of backscattered power with frequency which is related to the time dependence of the Fourier transform.

A number of authors[1–4] have calculated the theoretical frequency spectrum $I(\omega)$ for a given wave vector. These calculations in general assume (1) that the dynamics of the plasma can be described by the Vlasov equation which neglects charged-particle collisions, and (2) that the electrons and ions have Maxwellian velocity distributions which are not necessarily at the same temperature. The charged-particle collision frequency can be expressed in terms of the parameter, Λ:

$$\Lambda=4\pi nD^3=DK\langle T\rangle e^{-2}=(K\langle T\rangle)^{3/2}(4\pi n)^{-1/2}e^{-3}\,, \qquad (1)$$

where D is the electron Debye length. D is defined by

$$D^{-2}=4\pi ne^2K^{-1}\langle T\rangle^{-1} \qquad (2)$$

* This research was sponsored by the Advanced Research Projects Agency as part of Project DEFENDER and technically monitored by the U. S. Air Force Office of Scientific Research under Contract No. AF 49(638)-1156.

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[1] E. E. Salpeter, Phys. Rev. **120**, 1528 (1960) hereafter referred to as I.

[2] E. E. Salpeter, Phys. Rev. **122**, 1663 (1961).

[3] J. P. Dougherty and D. T. Farley, Proc. Roy. Soc. (London) **A259**, 79 (1960).

[4] J. A. Fejer, Can. J. Phys. **39**, 716 (1961).

and $\langle T\rangle$ is an average temperature defined below in Eq. (7). The parameter Λ is extremely large compared with unity in all practical cases. The order of magnitude of the charged-particle collision frequency [see Eqs. (29) and (33)] is $\nu\sim\omega_p \ln\Lambda/\Lambda$ where $\omega_p=(4\pi ne^2/m)^{1/2}$ is the electron plasma frequency in rad/sec.

For most purposes, the charged particle collisions are indeed unimportant. However, in the problem under consideration here, charged particle collisions may be important. Consider the backscatter from regions where the parameter α is larger than unity:

$$\alpha=(qD)^{-1}=\lambda(4\pi D)^{-1}=\lambda(ne^2/4\pi KT)^{1/2}. \quad (3)$$

When α is larger than unity, most of the backscattered intensity resides in the "ion component" which has a frequency width of the order of Doppler shifts corresponding to the ion thermal velocities. The intensity of the electron component is only of order α^{-2} compared to the ion component, but is of practical interest since (in the absence of a magnetic field) it is a pair of sharp lines displaced from the transmitted frequency by an amount $\nu_p=\omega_p/2\pi$. This is interpreted as backscattering from the longitudinal, electrostatic plasma waves that a plasma can maintain when the wavelength is longer than the Debye length, and is called the plasma line. In the absence of collisions, the frequency width of this line is due only to Landau damping and it is of order $\omega_p\alpha^3 e^{-\alpha^2/2}$. Although Λ is very large, in some cases

$$\alpha^2/2>\ln(\Lambda\alpha^3)=\ln(4\pi nq^{-3})=\ln[n\lambda^3(4\pi)^{-2}] \quad (4)$$

and the width of the plasma line is due to charged particle collisions. In the discussion above, collisions of electrons with neutrals were neglected and the electron velocity distribution was assumed to be Maxwellian.

The effect of charged-particle collisions on the plasma line was computed by Ron, Dawson, and Oberman[5] for an equilibrium model of the electron plasma. The assumption of equilibrium is necessary because these authors use a generalized Nyquist theorem to relate the power spectrum of electron density fluctuations to the conductivity of the plasma which had been calculated previously.[6–8] The results of this calculation are as expected. The width of the plasma line is given by the electron-ion collision frequency when this is greater than the Landau damping and the power returned in the plasma line is not affected by collisions—in agreement with the predictions of equilibrium statistical mechanics.[1]

The plasma in the ionosphere (and in many laboratory experiments) is not accurately in thermal equilibrium, and we are interested in how departures from thermal equilibrium affect the spectrum and intensity of electron density fluctuations. In this paper the (zero-order) normalized distribution function, $F_0(\mathbf{v})$, will be taken to be isotropic but not necessarily Maxwellian. The corresponding one-dimensional velocity distribution, $F_0^{(1)}(u)=\int_{|u|}^{\infty} F_0(\mathbf{v})2\pi v dv$, will also be used extensively.

It will be convenient to measure the departure from thermal equilibrium in terms of two "velocity-dependent temperatures" defined in terms of the logarithmic derivatives of F_0 and $F_0^{(1)}$:

$$mv/KT(v)=-d\ln F_0(\mathbf{v})/dv, \quad (5)$$

$$mu/KT_{||}(u)=-d\ln F_0^{(1)}(u)/du. \quad (6)$$

Two different average temperatures will be required:

$$\langle T\rangle^{-1}=\int F_0(\mathbf{v})T^{-1}(v)d\mathbf{v};\quad 3K\bar{T}=\int mv^2F_0(\mathbf{v})d\mathbf{v}. \quad (7)$$

It will be shown that $\langle T\rangle$ is the appropriate average temperature to use in the formula for the Debye length.

In the ionosphere, the departure from thermal equilibrium results from the production of very energetic photoelectrons which are only slowly thermalized. Hence, for energies considerably above the average energy, there are many more electrons than would be predicted by an equilibrium distribution. On the other hand, the majority of the electrons in the plasma have had many collisions and are thermalized so that $T(v)\approx\langle T\rangle\approx\bar{T}$ except at velocities much greater than the average, where the recently produced photoelectrons contribute a high-energy tail to the distribution function and increase $T(v)$ greatly [see Sec. 6].

In a stable but nonequilibrium plasma, the intensity of electron density fluctuations can be enhanced above the equilibrium value and an example has been given by Rosenbluth and Rostoker.[9] In the case of the plasma line, the collisionless theory states that the intensity can be enhanced by a ratio $R=T_{||}(v_\phi)/T$, where $v_\phi=\omega_p q^{-1}$ is the phase velocity of the plasma wave. As an example, suppose $F_0^{(1)}(u)\propto u^{-s}$ in the high-energy tail. Then $R=\alpha^2 s^{-1}=(qD)^{-2}s^{-1}$ can become very large provided s is of order unity. Physically, the collisionless theory describes the excitation of plasma waves by fast electrons[10] and the subsequent Landau damping of these waves. The random electron-ion collisions can also excite plasma waves and cause damping. These latter processes become predominant at very long wavelengths and do not cause an enhancement of electron density fluctuations because collisions involve particles principally of average energy. Thus R cannot become arbitrarily large simply by making q arbitrarily small.

The body of this paper will be concerned only with the electron component or plasma line so that the effects of ion motion are negligible and the large ion-to-electron mass ratio can be replaced by infinity. A brief review of

[5] A. Ron, J. Dawson, and C. Oberman, Phys. Rev. **132**, 497 (1963).
[6] J. Dawson and C. Oberman, Phys. Fluids **5**, 517 (1962).
[7] C. Oberman, A. Ron, and J. Dawson, Phys. Fluids **5**, 1514 (1962).
[8] J. Dawson and C. Oberman, Phys. Fluids **6**, 394 (1963).
[9] M. N. Rosenbluth and N. Rostoker, Phys. Fluids **5**, 776 (1962).
[10] D. Pines and D. Bohm, Phys. Rev. **85**, 338 (1952).

the collisionless theory will be given in Sec. 2 for a general isotropic $F_0(\mathbf{v})$. In Sec. 3, we shall derive a Fokker-Planck equation for the electrons using the method described by Berk.[11] This method accounts for electron-ion collisions, but ignores electron-electron collisions. The electron-electron collisions do not change the momentum of the electron gas on scales q^{-1} which are much larger than the scale D of a typical collision. Thus electron-electron collisions cannot damp or excite plasma waves when $\alpha \gg 1$.[12] Section 4 describes a Green's function method of solving the Fokker-Planck equation and gives two important approximations to the Green's function. The principal results of this paper, the power spectrum in the vicinity of the plasma line and intensity of the plasma line, are given in Sec. 5. The application of this work to the incoherent scatter experiments at Arecibo is discussed in Sec. 6.

2. GENERAL CONSIDERATIONS AND THE COLLISIONLESS THEORY

The power spectrum of electron density fluctuations will be calculated by a generalization of the method used in I. The calculations will be carried out for the case of a uniform plasma consisting of N electrons and N/Z ions of charge Z contained in a large box of volume Ω. Periodic boundary conditions will be used. We shall work with the Fourier-Laplace transform of the electron density

$$Q(\mathbf{q},\omega)=\sum_{i=1}^{N}\int_0^\infty dt\,\exp\{-i\mathbf{q}\cdot\mathbf{x}_i(t)+(i\omega-\gamma)t\}\,, \quad (8)$$

where the summation is the spatial Fourier transform of point electrons and γ is a small positive number. The power spectrum is then given by

$$I(\omega)=\lim_{\gamma\to 0}(\gamma/\pi N)\,|Q(\mathbf{q},\omega)|^2\,, \quad (9)$$

which is normalized so that $\int I d\omega$ would be unity for completely randomly distributed electrons.

We shall describe the dynamics of the electrons in terms of a distribution function $F(\mathbf{x},\mathbf{v},t)$ which satisfies the Vlasov equation

$$\partial F/\partial t+\mathbf{v}\cdot\partial F/\partial\mathbf{x}+em^{-1}\nabla\Phi\cdot\partial F/\partial\mathbf{v}=0\,,$$
$$\nabla^2\Phi=4\pi e\left[n\int F d\mathbf{v}-Z\sum_{j=1}^{N/Z}\delta(\mathbf{x}-\mathbf{R}_j)\right], \quad (10)$$

where Φ is the electrostatic potential and $\mathbf{R}_j$ is the position of the jth ion. Since the ion thermal velocities are much smaller than any relevant electron velocities, we shall assume the $\mathbf{R}_j$ to be time-independent and shall subsequently perform statistical averages over these ion positions.

In this limit of infinite ion/electron mass ratio, the electron distribution function can be uniquely divided into three parts:

$$F=F_0(\mathbf{v})+F_1(\mathbf{x},\mathbf{v})+f(\mathbf{x},\mathbf{v},t)\,.$$

Here F_0 is the zero-order uniform isotropic distribution function which produces no electrostatic field at all. F_1 is the time-independent part of the first-order distribution function which is directly correlated with the fixed discrete ions and which (together with the ions) produces a time-independent potential, Φ_1. Finally f is the time-dependent part of the first-order distribution function and is the quantity of main interest in this paper.

We shall find it convenient to work with the Fourier-Laplace transform of f:

$$f(\mathbf{q},\mathbf{v},\omega)=\int d\mathbf{x}\int_0^\infty dt e^{-i\mathbf{q}\cdot\mathbf{x}+(i\omega-\gamma)t}f(\mathbf{x},\mathbf{v},t)\,. \quad (11)$$

For nonzero ω, the quantity Q defined in Eq. (8) is related to f by

$$Q(\mathbf{q},\omega)=n\int d\mathbf{v}f(\mathbf{q},\mathbf{v},\omega)\,. \quad (12)$$

The Fourier-Laplace transform for $\omega\neq 0$ of the potential Φ is then $(-4\pi e/q^2)Q(\mathbf{q},\omega)$. We shall also need a Fourier transform of the initial conditions

$$f(\mathbf{q},\mathbf{v},0)=n^{-1}\sum_i e^{-i\mathbf{q}\cdot\mathbf{x}_i}\delta(\mathbf{v}-\mathbf{v}_i)\,, \quad (13)$$

where $\mathbf{x}_i$, $\mathbf{v}_i$ are the initial position and velocity of the ith electron.

A linearized equation for f, the time-dependent part of the electron distribution function, can be obtained from the time-dependent part of Eq. (10). This is accomplished by dropping all terms of both zero and second order in f and/or Q. In its Fourier-Laplace transform, this linearized equation is

$$(\gamma+i\mathbf{q}\cdot\mathbf{v}-i\omega)f(\mathbf{q},\mathbf{v},\omega)-i(\omega_p^2/nq^2)Q(\mathbf{q},\omega)\mathbf{q}\cdot\partial F_0/\partial\mathbf{v}$$
$$-i(\omega_p^2/n\Omega)\sum_{\mathbf{k}}\left\{k^{-2}Q(\mathbf{k},\omega)\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}F_1(\mathbf{q}-\mathbf{k},\,\mathbf{v})\right.$$
$$\left.-(4\pi e)^{-1}\Phi_1(\mathbf{q}-\mathbf{k})(\mathbf{q}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}f(\mathbf{k},\mathbf{v},\omega)\right\}=f(\mathbf{q},\mathbf{v},0)\,, \quad (14)$$

where Q is related to f by Eq. (12). Although the quadratic terms in f (or Q) are missing, note that all terms bilinear in f (or Q) and Φ_1 (or F_1) are still present in the sum over $\mathbf{k}$. In Sec. 3, we shall return to these bilinear terms which take account of electron-ion collisions.

The collisionless theory is obtained by simply omitting the sums over $\mathbf{k}$ (which is of higher order in the small parameter Λ^{-1}) in Eq. (14). Let $\eta(\mathbf{q},\mathbf{v},\omega)$ be the approxi-

[11] H. L. Berk, Phys. Fluids 7, 257 (1964).
[12] See, for example, Eq. (68) in P. L. Bhatnager, E. P. Gross, and M. Krook, Phys. Rev. 94, 511 (1954). When this equation is made dimensionally correctly, it shows that, for their collision model, the importance of electron-electron collisions decreases as α becomes large.

mation to f and let $\psi(\mathbf{q},\omega)$ be the approximation to Q obtained from the collisionless theory;

$$\eta(\mathbf{q},\mathbf{v},\omega)=(\mathbf{q}\cdot\mathbf{v}-\omega-i\gamma)^{-1}\times\{-if(\mathbf{q},\mathbf{v},0)+(\omega_p^2/nq^2)\psi(\mathbf{q},\omega)\mathbf{q}\cdot(\partial F_0/\partial\mathbf{v})\}. \quad (15)$$

Equation (12) gives $\psi=n\int\eta d\mathbf{v}$ in the collisionless approximation, and one easily obtains

$$\psi(\mathbf{q},\omega)=[-in/\epsilon_0(\mathbf{q},\omega)]\times\int f(\mathbf{q},\mathbf{v},0)(\mathbf{q}\cdot\mathbf{v}-\omega-i\gamma)^{-1}d\mathbf{v}, \quad (16)$$

where

$$\epsilon_0(\mathbf{q},\omega)=1-\omega_p^2q^{-2}\int d\mathbf{v}(\mathbf{q}\cdot\mathbf{v}-\omega-i\gamma)^{-1}\mathbf{q}\cdot(\partial F_0/\partial\mathbf{v}). \quad (17)$$

The integral in Eq. (16) consists of a sum of N terms and, according to Eq. (9), we must find the modulus squared of this sum in the limit $\gamma\to 0$. As discussed in paper I, only the $i=j$ terms in this double sum contribute to the γ^{-1} divergence and survive in the limit. This single sum can be replaced by an integral and one has

$$\lim_{\gamma\to 0}(\gamma/\pi N)\sum_{ij}e^{-i\mathbf{q}\cdot(\mathbf{x}_i-\mathbf{x}_j)}(\mathbf{q}\cdot\mathbf{v}_i-\omega-i\gamma)^{-1}(\mathbf{q}\cdot\mathbf{v}_j-\omega+i\gamma)^{-1}=\lim_{\gamma\to 0}\gamma\pi^{-1}\int F_0(\mathbf{v})[(\mathbf{q}\cdot\mathbf{v}-\omega)^2+\gamma^2]^{-1}d\mathbf{v}=q^{-1}F_0^{(1)}(\omega q^{-1}). \quad (18)$$

The power spectrum in the collisionless theory is then given by

$$I(\omega)=q^{-1}F_0^{(1)}(\omega q^{-1})|\epsilon_0(\mathbf{q},\omega)|^{-2}. \quad (19)$$

The integral in Eq. (17) can be evaluated for simple enough zero-order distribution functions, and this has been done for the Maxwell distribution. However, in this paper we shall be interested only in cases where $\mathbf{q}\cdot\mathbf{v}\ll\omega_p$ for all values of $\mathbf{v}$ for which $F_0(\mathbf{v})$ is appreciable. In this case, $\epsilon_0(\mathbf{q},\omega)$ can be approximated by

$$\epsilon_0(\mathbf{q},\omega)=1-\omega_p^2\omega^{-2}(1+3q^2K\bar{T}m^{-1}\omega^{-2})+i\pi\omega_p^2q^{-2}(dF_0^{(1)}/du)_{u=\omega q^{-1}}, \quad (20)$$

where $\bar{T}$ is defined in Eq. (7). The power spectrum can then be approximated by two sharp lines of Lorentzian shape (the plasma lines) which occur when the real part of ϵ_0 is zero, i.e., when $\omega=\pm\omega_r$ where ω_r is given by $\omega_r=(\omega_p^2+3q^2K\bar{T}m^{-1})^{1/2}$. In the neighborhood of the plasma line which occurs at $\omega=\omega_r$, the power spectrum can be written as

$$I(\omega)=\frac{\omega_r^2F_0^{(1)}(\omega_r/q)}{4q\left\{(\omega-\omega_r)^2+\left[\frac{\pi}{2}\frac{\omega_r\omega_p^2}{q^2}\left(\frac{d}{du}F_0^{(1)}(u)\right)_{u=\omega_rq^{-1}}\right]^2\right\}}. \quad (21)$$

The numerator in Eq. (21) is proportional to the zero-order distribution function of electrons at the phase velocity of the plasma oscillation with wave vector $\mathbf{q}$. Physically, the numerator represents the excitation of plasma waves by the fast electrons (the "Čerenkov" wake discussed by Pines and Bohm[10]). The width of the plasma line is determined by Landau damping and so is proportional to the derivative of the distribution function at the phase velocity. The integrated power in a single plasma line is

$$I_p=\frac{q\omega_rF_0^{(1)}(\omega_rq^{-1})}{2\omega_p^2[dF_0^{(1)}(u)/du]_{u=\omega_rq^{-1}}}=\frac{T_{||}(\omega_rq^{-1})}{2\alpha^2\langle T\rangle}. \quad (22)$$

If the collisionless theory is valid, then Eq. (22) shows that the integrated power of the plasma line depends only on the logarithmic derivative of $F_0^{(1)}$ and not on its absolute value. The presence of a high-energy tail in $F_0^{(1)}$ could lead to a substantial enhancement in the total power (and also to a broadening) of the plasma line.

3. FOKKER-PLANCK EQUATION

The summation over $\mathbf{k}$ in Eq. (14) represents the effects of electron-ion collisions, and we shall need explicit expressions for both time-dependent terms and the time-independent factors in order to evaluate the sum. The time-independent terms of interest are the spatial Fourier transform $F_1(\mathbf{k},\mathbf{v})$ of the time-independent part of the first-order electron distribution function and the corresponding potential $\Phi_1(\mathbf{k})$. These are easily obtained by taking the Fourier transform of Eq. (10), linearizing and neglecting any time dependence. The result can be written in the form

$$F_1(\mathbf{k},\mathbf{v})=e\Phi_1(\mathbf{k})\mathfrak{F}(\mathbf{v})/K\langle T\rangle,\quad \Phi_1(\mathbf{k})=4\pi eZ(k^2+D^{-2})^{-1}\sum_j e^{-i\mathbf{k}\cdot\mathbf{R}_i}, \quad (23)$$

where D is defined in Eq. (2) and $\mathfrak{F}$ is a modification of the zero-order distribution function:

$$\mathfrak{F}(\mathbf{v})=\langle T\rangle T^{-1}(v)F_0(\mathbf{v}). \quad (24)$$

We shall assume the ions have correlations appropriate to a kinetic temperature T_i so that

$$|\Phi_1(\mathbf{k})|^2=\frac{(4\pi)^2e^2NZ}{(k^2+D^{-2})(k^2+k_i^2+D^{-2})};\quad k_i^2=\frac{4\pi ne^2Z}{KT_i}. \quad (25)$$

In our approximation, which replaces the ion thermal velocity v_i by zero, the direct contribution of F_1 to the power spectrum is a delta function at zero frequency. In reality, this contribution, which contains most of the power when $\alpha\gg1$, has a nonzero frequency width of the order of qv_i. In this paper, we are interested only in the indirect effect F_1 has in Eq. (14).

At first sight, one is tempted to approximate the time-dependent terms in the sum over $\mathbf{k}$ by the results of the collisionless approximation. Indeed, η is a good approximation to f for most purposes, especially in the important region $D^{-1}\lesssim k\lesssim K\langle T\rangle e^{-2}$. If the collisionless approximation is used, then the sum on $\mathbf{k}$ will accurately describe the excitation of the $\mathbf{q}$ mode produced by electron density fluctuations in the $\mathbf{k}$ mode interacting with the $(\mathbf{q}-\mathbf{k})$ mode of the static electron-ion correlations. However, since $\eta(\mathbf{k},\mathbf{v},\omega)$ is completely independent of $f(\mathbf{q},\mathbf{v},\omega)$, the damping of $\mathbf{q}$-mode plasma oscillations due to scattering from the $\mathbf{q}$ mode to the $\mathbf{k}$ mode would not be obtained. For this reason, in the equation for $\eta(\mathbf{k},v,\omega)$ a source term must be added that is proportional to the amplitude of the $\mathbf{q}$ mode. This source term can be obtained by rewriting Eq. (14) for $f(\mathbf{k},\mathbf{v},\omega)$, (the summation then being over $\mathbf{k}'$), and then neglecting all terms except the $\mathbf{k}'=\mathbf{q}$ term in the summation. The equation for $f(\mathbf{k},\mathbf{v},\omega)$ is then

$$\begin{aligned}(\mathbf{k}\cdot\mathbf{v}-\omega-i\gamma)f(\mathbf{k},\mathbf{v},\omega)&\\ -(\omega_p{}^2/nq^2)Q(\mathbf{k},\omega)\mathbf{k}\cdot(\partial F_0/\partial\mathbf{v})+if(\mathbf{k},\mathbf{v},0)&\\ =(\omega_p{}^2/n\Omega q^2)[Q(\mathbf{q},\omega)\mathbf{q}\cdot(\partial F_1(\mathbf{k}-\mathbf{q},\,\mathbf{v})/\partial\mathbf{v})&\\ -(q^2/4\pi e)\Phi_1(\mathbf{k}-\mathbf{q})(\mathbf{k}-\mathbf{q})\cdot(\partial f(\mathbf{q},\mathbf{v},\omega)/\partial\mathbf{v})].&\quad(26)\end{aligned}$$

When the solution to (26) is used in Eq. (14), one obtains several sums over $\mathbf{k}$ containing either $Q(\mathbf{q},\omega)$ or $f(\mathbf{q},\mathbf{v},\omega)$. Only one of these sums is logarithmically divergent. We shall keep just the logarithmically divergent sum and this will be cutoff at $k=K\langle T\rangle e^{-2}$. In this way, terms of order unity compared with $\ln\Lambda$ have been neglected. The result of this procedure is a Fokker-Planck equation for $f(\mathbf{q},\mathbf{v},\omega)$:

$$\begin{aligned}(\mathbf{q}\cdot\mathbf{v}-\omega-i\gamma)f(\mathbf{q},\mathbf{v},\omega)+i\nu(v)\frac{\partial}{\partial v_\alpha}(v^2\delta_{\alpha\beta}-v_\alpha v_\beta)\frac{\partial}{\partial v_\beta}f(\mathbf{q},\mathbf{v},\omega)&\\ =S(\mathbf{q},\mathbf{v},\omega)+(\omega_p{}^2/nq^2)Q(\mathbf{q},\omega)\mathbf{q}\cdot(\partial F_0/\partial\mathbf{v}),&\quad(27)\end{aligned}$$

where

$$\begin{aligned}S(\mathbf{q},\mathbf{v},\omega)=-if(\mathbf{q},\mathbf{v},0)+(\omega_p{}^2/q^2n\Omega)\sum_{\mathbf{k}}\Phi_1(\mathbf{q}-\mathbf{k})&\\ \times\{[e\psi(\mathbf{k},\omega)/K\langle T\rangle]\mathbf{k}\cdot[\partial\mathfrak{F}(v)/\partial\mathbf{v}]&\\ -(q^2/4\pi e)(\mathbf{q}-\mathbf{k})\cdot[\partial\eta(\mathbf{k},\mathbf{v},\omega)/\partial\mathbf{v}]\},&\quad(28)\end{aligned}$$

and

$$\nu(v)=\frac{Z\omega_p{}^4\ln\Lambda}{8\pi nv^3};\quad\bar{\nu}=\int\frac{\nu(v)mv_z{}^2}{KT(v)}F_0(\mathbf{v})d\mathbf{v}.\quad(29)$$

The quantity $\nu(v)$ is a velocity-dependent collision frequency and $\bar{\nu}$ is an average collision frequency which will be used later. Equation (27) is identical with Berk's[11] equation except that the rhs describes how the $\mathbf{q}$ mode is excited by random collisions instead of by an external field. In the application under consideration, the frequency and velocity dependence of the logarithm given by Berk are not important. Equation (27) agrees with the Fokker-Planck equation of Rostoker and Rosenbluth[13] in the limit where the field particle mass is infinite and the velocity of the test particle is much greater than the velocity of the field particles.

[13] N. Rostoker and M. N. Rosenbluth, Phys. Fluids 3, 1 (1960).

4. A GREEN'S FUNCTION SOLUTION

The approximations needed to solve Eq. (27) will be better understood if a Green's function method of solution is introduced. We shall use a Green's function that physically describes the propagation of a test electron in a medium consisting of the infinitely massive discrete ions shielded by the static electron correlations. Even with such a simplified problem, an exact expression for the Green's function is not obtained, but two approximate forms that describe the essential physics are found.

The Green's function, $G(\mathbf{q},\mathbf{v},\mathbf{v}',\omega)$ is chosen to satisfy the equation

$$\left[\mathbf{q}\cdot\mathbf{v}-\omega-i\gamma+i\nu(v)\frac{\partial}{\partial v_\alpha}(v^2\delta_{\alpha\beta}-v_\alpha v_\beta)\frac{\partial}{\partial v_\beta}\right]G(\mathbf{q},\mathbf{v},\mathbf{v}',\omega)=-i\delta(\mathbf{v}-\mathbf{v}').\quad(30)$$

In terms of the Green's function, $f(\mathbf{q},\mathbf{v},\omega)$ is given by

$$\begin{aligned}f(\mathbf{q},\mathbf{v},\omega)=i\int d\mathbf{v}'G(\mathbf{q},\mathbf{v},\mathbf{v}',\omega)&\\ \times\{S(\mathbf{q},\mathbf{v}',\omega)+(\omega_p{}^2/q^2n)Q(\mathbf{q},\omega)\mathbf{q}\cdot(\partial F_0/\partial\mathbf{v}')\}.&\quad(31)\end{aligned}$$

We shall also introduce the function $H(\mathbf{q},\mathbf{v}',\omega)$ which will be used often:

$$H(\mathbf{q},\mathbf{v}',\omega)=\int d\mathbf{v}G(\mathbf{q},\mathbf{v},\mathbf{v}',\omega).\quad(32)$$

The quantity $Q(\mathbf{q},\omega)$ can then be expressed in terms of H:

$$Q(\mathbf{q},\omega)=[ni/\epsilon(\mathbf{q},\omega)]\int d\mathbf{v}'H(\mathbf{q},\mathbf{v}',\omega)S(\mathbf{q},\mathbf{v}',\omega),\quad(33)$$

where

$$\epsilon(\mathbf{q},\omega)=1-i\omega_p{}^2q^{-2}\int d\mathbf{v}'H(\mathbf{q},\mathbf{v}',\omega)\mathbf{q}\cdot(\partial F_0/\partial\mathbf{v}').$$

In the collisionless theory, $\nu=0$ and

$$H=-i(\mathbf{q}\cdot\mathbf{v}'-\omega-i\gamma)^{-1},$$

so that $\epsilon(\mathbf{q},\omega)$ becomes $\epsilon_0(\mathbf{q},\omega)$. Thus $\epsilon(\mathbf{q},\omega)$ is the appropriate generalization of $\epsilon_0(\mathbf{q},\omega)$ when electron-ion collisions are taken into account. Equation (33) has the advantage that the two roles of H are now separated. The denominator of (33), $\epsilon(\mathbf{q},\omega)$, gives the damping of plasma waves whereas the numerator is associated with the excitation. We shall obtain two simple approximations to H, one suitable for use in the numerator, and the other suitable for use in the denominator.

The Fourier-Laplace transform equation for G, Eq. (30), can be readily inverted to give

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{x}}-\nu\frac{\partial}{\partial v_\alpha}(v^2\delta_{\alpha\beta}-v_\alpha v_\beta)\frac{\partial}{\partial v_\beta}\right]G(\mathbf{x},\mathbf{v},\mathbf{v}',t)=0,\quad(34)$$

with the initial condition $G(\mathbf{x},\mathbf{v},\mathbf{v}',0)=\delta(\mathbf{x})\delta(\mathbf{v}-\mathbf{v}')$. Thus G describes the propagation of a test electron which started at zero time with velocity $\mathbf{v}'$ and position $\mathbf{x}=0$. Note that Eq. (34) conserves the energy of the test electron, so that $\nu(v)$ can be regarded as a constant for any given electron. By taking the appropriate moments of Eq. (34), we can obtain the exact result $\langle\mathbf{x}\rangle=\int\mathbf{x}Gd\mathbf{x}d\mathbf{v}=\mathbf{v}'(2\nu)^{-1}(1-e^{-2\nu t})$.

The important physical process in the damping of plasma waves is the randomization of the momentum of the electron gas by electron-ion collisions. Since q^{-1} is much larger than D which is the scale of a typical collision, it will be sufficient to approximate G by a delta function in position and velocity space that slows down along the direction of motion and that gives $\langle\mathbf{x}\rangle$ correctly. Accordingly, in the denominator of Eq. (33), we approximate H by

$$H(\mathbf{q},\mathbf{v}',\omega)\approx H^D(\mathbf{q},\mathbf{v}',\omega)$$

$$=\int_0^\infty \exp\{(i\omega-\gamma)t-i\mathbf{q}\cdot\mathbf{v}'(2\nu)^{-1}(1-e^{-2\nu t})\}dt. \quad (35)$$

We are interested in H^D only for the case $\omega\approx\omega_p$ and $\alpha\gg1$. In this case there are two important regions where we must find more explicit expressions for H^D. In the first region where F_0 is appreciable and $\mathbf{q}\cdot\mathbf{v}'\ll\omega$, H^D can be obtained by an asymptotic expansion. In the second region, $\mathbf{q}\cdot\mathbf{v}'\approx\omega$ and a good approximation to H^D can be made by the method of stationary phase. When these methods are used, one finds that $\epsilon(\mathbf{q},\omega)$ is well approximated by

$$\epsilon(\mathbf{q},\omega)=1-\omega_p^2\omega^{-2}(1+3q^2K\bar{T}/m\omega^2)$$
$$-i[2\omega_p^2\bar{\nu}\omega^{-3}-\pi\omega_p^2q^{-2}(\partial F_0^{(1)}/\partial u)_{u=\omega q^{-1}}], \quad (36)$$

where $\bar{T}$ is given by Eq. (7) and $\bar{\nu}$ by Eq. (29). Since the integrals defining $\bar{T}$ and $\bar{\nu}$ depend principally on particles of average energy, we can replace $F_0(\mathbf{v})$ by a Maxwellian in these integrals and obtain

$$\bar{T}\approx T, \quad \bar{\nu}\approx\omega_pZ\Lambda^{-1}\ln\Lambda(2\pi)^{1/2}/6. \quad (37)$$

The excitation of plasma oscillations depends on the deflections in velocity space being given correctly. For short times ($\nu t\ll1$), the sideways deflections in velocity space are larger than the slowing down along the direction of motion, and we shall approximate the Green's function used in the numerator of Eq. (33) by the solution to an approximate Fokker-Planck equation which gives the sideways deflections correctly. The restriction to short times is not important because for $\nu t\sim1$, $G(\mathbf{x},\mathbf{v},\mathbf{v}',t)$ propagates any fluctuation in the electron density into a very smooth density distribution which contributes negligibly to the incoherent scatter. The sideways deflections also account satisfactorily for this smoothing which occurs in a time $t\sim\nu^{-1}(\alpha\ln\Lambda/\Lambda)^{2/3}$ which is short compared with ν^{-1} provided $q^{-1}\ll D\Lambda/\ln\Lambda$. Physically, this means that the radar wavelength must be much less than the electron mean free path, which is true in all the ionosphere experiments.

The approximate Fokker-Planck equation can be obtained from Eq. (34) by replacing the variable tensor, $v^2\delta_{\alpha\beta}-v_\alpha v_\beta$ by a constant tensor, $v'^2\delta_{\alpha\beta}-v_\alpha'v_\beta'$. Let $H^N(\mathbf{q},\mathbf{v}',\omega)$ denote the approximation to $H(\mathbf{q},\mathbf{v}',\omega)$ obtained by using the solution to the approximate Fokker-Planck equation in Eq. (32). In Sec. 5, we shall need only those components of $\partial H^N/\partial\mathbf{v}'$ which are perpendicular to $\mathbf{v}'$. These denoted by $(\partial H^N/\partial\mathbf{v}')_\perp$ and are given by

$$(\partial H^N/\partial\mathbf{v}')_\perp=-\mathbf{q}_\perp\int_0^\infty dt[it-\tfrac{2}{3}\nu t^3(\mathbf{q}\cdot\mathbf{v}')]$$
$$\times\exp\{(i\omega-i\mathbf{q}\cdot\mathbf{v}'-\gamma)t-\tfrac{1}{3}\nu t^3[v'^2q^2-(\mathbf{q}\cdot\mathbf{v}')^2]\}. \quad (38)$$

The important regions of $(\partial H^N/\partial\mathbf{v}')_\perp$ are (1) the region where $F_0(\mathbf{v}')$ is appreciable and $\omega\gg\mathbf{q}\cdot\mathbf{v}'$ and (2) the neighborhood of $\mathbf{q}\cdot\mathbf{v}'=\omega$ [denoted by $\mathfrak{N}(\mathbf{q}\cdot\mathbf{v}'=\omega)$] where $(\partial H^N/\partial\mathbf{v}')_\perp$ is large. In the first region an asymptotic expansion can be obtained.

$$(\partial H^N/\partial\mathbf{v}')_\perp\sim i\mathbf{q}_\perp\omega^{-2}[1+O(\mathbf{q}\cdot\mathbf{v}'/\omega)]. \quad (39)$$

In the second region, the only important information is the fact that $|(\partial H^N/\partial\mathbf{v}')_\perp|^2$ is sharply peaked and the value of the integral,

$$\int_{\mathfrak{N}(\mathbf{q}\cdot\mathbf{v}'=\omega)}|(\partial H^N/\partial\mathbf{v}')_\perp|^2d\mathbf{q}\cdot\mathbf{v}'=\pi[v'^2\nu(v)]^{-1}. \quad (40)$$

5. POWER SPECTRUM NEAR THE PLASMA LINE

There are two simplifying procedures that allow us to evaluate the power spectrum in the vicinity of the plasma line. The first procedure is that only those terms in $|Q(\mathbf{q},\omega)|^2$ which diverge as γ^{-1} survive the limit in Eq. (9) and contribute to the power spectrum. Physically, the limit $\gamma\to0$ represents an average over long times. As a result, the contributions of the initial conditions to $S(\mathbf{q},\mathbf{v},\omega)$ [the first term on the right-hand side of Eq. (28)] do not affect the power spectrum because the initial electron density fluctuations are propagated into completely smooth density distributions by the diffusion in coordinate and velocity space. The second simplifying procedure is that only the logarithmically dominant sums over $\mathbf{k}$ will be considered (the $\ln\Lambda$ terms). When the formulas for η and ψ, Eqs. (15) and (16), are substituted into Eq. (28), there will be one term that is much larger than the others for large $\mathbf{k}$. We shall keep only this term since it is the one that leads to a term in $\ln\Lambda$. Thus, for the purposes of obtaining the power spectrum, the numerator of Eq. (33) can be approximated by

$$ni\int d\mathbf{v}'H^N(\mathbf{q},\mathbf{v}',\omega)=\sum_i \mathcal{S}^i(\mathbf{q},\mathbf{x}_i,\mathbf{v}_i,\omega), \quad (41)$$

where

$$\mathcal{S}^i=(-e/m\Omega)\sum_{\mathbf{k}}\Phi_1(\mathbf{q}-\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}_i}(\mathbf{k}\cdot\mathbf{v}_i-\omega-i\gamma)^{-1}(\mathbf{q}-\mathbf{k})\cdot(\partial H^N(\mathbf{q},\mathbf{v}_i,\omega)/\partial\mathbf{v}_i). \quad (42)$$

The power spectrum is proportional to the double sum $\sum_{ij}\mathcal{S}^i(\mathcal{S}^j)^*$ and the situation here is the same as discussed in paper I and in Sec. 2: Only the $i=j$ terms contribute to the γ^{-1} divergence. However, in the present case, a double sum over $\mathbf{k}$ and $\mathbf{k}'$ remains after the $i\neq j$ terms have been dropped. One can show that only the $\mathbf{k}=\mathbf{k}'$ terms contribute to the γ^{-1} divergence with the result that

$$\lim_{\gamma\to 0}(\gamma/\pi N)[\sum_{ij}\mathcal{S}^i(\mathcal{S}^j)^*]=\lim_{\gamma\to 0}\gamma\pi^{-1}\int d\mathbf{v}F_0(\mathbf{v})\left(\frac{\partial}{\partial v_\alpha}H^N\right)T_{\alpha\beta}(v)\left(\frac{\partial}{\partial v_\beta}H^{N*}\right), \quad (43)$$

where

$$T_{\alpha\beta}(v)=\int\frac{d\mathbf{k}e^2|\Phi_1(\mathbf{k})|^2k_\alpha k_\beta}{(2\pi)^3m^2\Omega[(\mathbf{k}\cdot\mathbf{v}+\mathbf{q}\cdot\mathbf{v}-\omega)^2+\gamma^2]}. \quad (44)$$

In Eqs. (43) and (44), the sums over i and $\mathbf{k}$ have been replaced by integrals. The integral over $\mathbf{k}$ is logarithmically divergent and is cut off at $k=K\langle T\rangle e^{-2}$. Only the logarithmically dominant part of this integral is kept.

In a Cartesian coordinate system with one axis in the direction of $\mathbf{v}$, the only nonzero elements of $T_{\alpha\beta}$ are the two diagonal elements which correspond to directions perpendicular to $\mathbf{v}$ and these elements have the value $\nu(v)v^2\gamma^{-1}$. Thus we obtain the result

$$\lim_{\gamma\to 0}(\gamma/\pi N)[\sum_{ij}\mathcal{S}^i(\mathcal{S}^j)^*]=\pi^{-1}\int d\mathbf{v}F_0(\mathbf{v})|(\partial H^N/\partial\mathbf{v})_\perp|^2\nu(v)v^2$$
$$=q^2\omega^{-4}\pi^{-1}\int d\mathbf{v}F_0(\mathbf{v})(\mathbf{v}_\perp)^2\nu(v)+q^{-1}F_0^{(1)}(\omega/q). \quad (45)$$

The remaining integral is not sensitive to the high-energy tail and can be evaluated by replacing $F_0(\mathbf{v})$ by a Maxwellian:

$$\int d\mathbf{v}F_0(\mathbf{v})(\mathbf{v}_\perp^2)\nu(v)=2\bar{\nu}KT/m. \quad (46)$$

Equations (45) and (46), together with Eq. (36) for ϵ, can be combined to give an explicit formula for the power spectrum. Again there are two sharp lines occurring at $\omega=\pm\omega_r$. In the neighborhood of $\omega=\omega_r$, the power spectrum is well approximated by a narrow line of Lorentzian shape.

$$I(\omega)=|\epsilon(\mathbf{q},\omega)|^{-2}\{2\bar{\nu}KTm^{-1}+q^{-1}F_0^{(1)}(\omega q^{-1})\}$$
$$=\frac{\omega_r^2\{F_0^{(1)}(\omega_rq^{-1})+2q^3KT\bar{\nu}\pi^{-1}\omega_r^{-4}m^{-1}\}}{4q\{(\omega-\omega_r)^2+\omega_p^4\omega_r^{-4}[\bar{\nu}-(\pi/2)\omega_r^3q^{-2}(\partial F_0^{(1)}/\partial u)_{u=\omega_rq^{-1}}]^2\}}. \quad (47)$$

The power spectrum is seen to have the properties which were expected. The width of the plasma line is the sum of the collision damping and Landau damping widths and both collisions and fast electrons can excite plasma waves. The intensity I_p for a single plasma line is given by

$$I_p=\frac{\chi(v_\phi)+Z\Lambda^{-1}\alpha^{-3}(2/3\pi)\ln\Lambda}{2\alpha^2\{T\chi(v_\phi)T_{||}^{-1}(v_\phi)+Z\Lambda^{-1}\alpha^{-3}(2/3\pi)\ln\Lambda\}}, \quad (48)$$

where

$$\chi(u)=(2\pi KT/m)^{1/2}F_0^{(1)}(u);\quad v_\phi=\omega_rq^{-1}.$$

The quantity χ is a dimensionless, one-dimensional, velocity distribution function and v_ϕ is the phase velocity of the plasma wave. $T_{||}(u)$ was defined in Eq. (6) in terms of the logarithmic derivative of the one-dimensional velocity distribution function.

When the first terms in the numerator and denominator of (48) are the important terms, the excitation of plasma waves by fast electrons and Landau damping are the important physical processes and the results of the collisionless theory are recovered. The electron-ion collisions are the important process when the second terms predominate and the intensity of the plasma line has the value appropriate for an equilibrium gas.

6. APPLICATION TO THE IONOSPHERE RADAR EXPERIMENTS AT ARECIBO

In the daytime ionosphere, the electron density has a maximum of the order of 10^6 electrons/cm^3 at an altitude near 250–300 km. The corresponding plasma frequency ν_p is about 10 Mc/sec, and the electron temperature at these altitudes and above is close to 2000°K ($kT\approx 0.2$ eV).[14] A large fraction of the solar ultraviolet and x rays is absorbed near these heights, producing energetic photoelectrons in the energy region from 1 to 30 eV. The production spectrum for photoelectron energies (see Mariani[15]) decreases slowly as the energy increases from 1 to 30 eV and falls off steeply at higher energies. We shall limit our considerations to altitudes above 250 km where the photoelectrons are slowed down by electron-electron collisions and inelastic elec-

[14] J. V. Evans and M. Loewenthal, Planet. Space Sci. **12**, 915 (1964).

[15] F. Mariani, Preliminary Report, Goddard Space Flight Center, Greenbelt, Maryland (unpublished).

tron-neutral collisions can be ignored. In this region, the photoelectron mean free path is very long and is an important consideration in estimating the contribution χ_{ph} of the photoelectrons to the dimensionless velocity distribution χ, defined following Eq. (48). Near 250–300 km, we find $\chi_{ph}(v)\sim 10^{-6}$ for $E(v)=mv^2/2<30$ eV. The photoelectron contribution will dominate the Maxwellian contribution when $E(v)>14KT\sim 2.8$ eV. Hence, in the neighborhood of $E(v)=2.8$ eV, the quantity $KT(v)$ changes quite rapidly from the value $KT(v)\approx 0.2$ eV for the thermal electrons to the value $KT(v)\sim 10$ eV which is appropriate for the photoelectrons.

Photoelectrons of energy $\gtrsim 10$ eV produced at altitudes greater than 300 km have a good chance of escaping upwards without collisions.[16] (They are returned by the earth's magnetic field to the other hemisphere.) At higher altitudes the electron density decreases with a scale height of 150–200 km and the photoelectrons produced directly per unit volume decrease with the neutral particle scale height which is approximately 100 km. Therefore, most of the photoelectrons present at the higher altitudes were produced at lower altitudes and the photoelectron density is independent of altitude. It is evident then that χ_{ph} is inversely proportional to local electron density. For example, at an altitude of 700 km where $n\approx 5\cdot 10^4$ ($\nu_p\approx 2.0$ Mc/sec), χ_{ph} dominates the Maxwellian contribution when $E(v)>2.2$ eV.

In a radar backscatter experiment, the energy of the electrons which are traveling at the phase velocity of the plasma oscillations is given by

$$E(v_\phi)=(0.35\text{ eV})(\nu_p/1.0\text{ Mc/sec})^2(\lambda/70\text{ cm})^2. \quad (49)$$

The wavelength of the Arecibo radar is 70 cm (430 Mc/sec) and the plasma frequency in the ionosphere varies between 1 and 10 Mc/sec so that the photoelectrons are in just the right energy range to enhance the plasma line in the Arecibo radar experiments. The plasma line can be fully enhanced if the ratio of the Landau damping resulting from the photoelectrons to the collision damping is greater than unity. Since both χ_{ph} and $(\Lambda\alpha^3)^{-1}$ are inversely proportional to the electron density and since χ_{ph} is not very velocity-dependent for $E(v_\phi)<30$ eV, this ratio is independent of the electron density and is

$$\frac{T\chi_{ph}(v_\phi)\Lambda\alpha^3 3\pi}{2T_{||}(v_\phi)\ln\Lambda}\sim 30\gg 1 \quad \text{if} \quad E(v_\phi)<30\text{ eV} \quad (50)$$

when $\lambda=70$ cm. Hence collisions (and the main results of this paper) can be neglected in the Arecibo experiments. For a 7.5-m (40-Mc/sec) radar experiment, $E(v_\phi)$ lies in the range 35 eV$<E(v_\phi)<$3.5 keV, where χ_{ph} falls off sharply and electron-ion collisions are probably important.

The presence of the earth's magnetic field can change the Landau damping from the field-free expression and make it orders of magnitude larger for the wavelength of the Arecibo experiments.[2,17] The reason for this is that the gyroradius of the slow thermal electrons is less than q^{-1} and they can also contribute to the Landau damping when a magnetic field is present. The Landau damping caused by the photoelectrons will be less affected by the earth's magnetic field since their gyroradius is greater than q^{-1}. Therefore, we will approximate the Landau damping by a sum of the Landau damping caused by the thermal electrons including the effects of a magnetic field and the Landau damping caused by the photoelectrons calculated according to the field-free theory.

If the earth's magnetic field could be neglected, then the plasma line enhancement would start at a plasma frequency of 2.4 Mc/sec and be essentially complete at $\nu_p=2.9$ Mc/sec, for the Arecibo experiments. The magnetic field modifies this result so that the enhancement is estimated to start at $\nu_p=3.6$ Mc/sec (corresponding to an altitude of approximately 400 km) and to be complete at $\nu_p=4.5$ Mc/sec (altitude about 300 km). The enhancement is predicted to be a factor of about 50.

The 7.5-m ionosphere radar experiments will probably not be enhanced because of electron-ion collisions except for plasma frequencies near 1 Mc/sec where the theory is complicated by the presence of the electron gyro frequency. At wavelengths shorter than about 20 cm, the plasma frequencies are not large enough to make $E(v_\phi)>14KT$, the criterion needed for enhancement.

Note added in proof. Incoherent scatter from plasma oscillations in the ionosphere has been observed at Arecibo, [see F. Perkins, E. E. Salpeter, and K. O. Yngvesson, Phys. Rev. Letters **14**, 579 (1965)]. The results of these experiments agree with the theory developed in this paper.

ACKNOWLEDGMENT

We are indebted to M. Nicolet for stimulating discussions.

[16] W. B. Hanson, *Space Research III*, edited by W. Priester (North-Holland Publishing Company, Amsterdam, 1963), p. 282.

[17] F. W. Perkins, Research Report No. 145-I, Cornell University, 1963 (unpublished).