

Temperature Dependence of Microwave Phonon Attenuation*

M. POMERANTZ

IBM Watson Research Center, Yorktown Heights, New York

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Measurements of the temperature dependence of the attenuation of 9-Gc/sec phonons are presented. They include measurements on crystals of quartz, CdS, GaAs, Ge, Si, CaF₂, Al₂O₃, and MgO in several directions and modes of polarization. The attenuation in most of the materials is proportional to T^n , where the average value of $n=4.1\pm 1.2$ for the fast transverse waves, 4.0 ± 1.7 for the slow transverse waves, 4.8 ± 1.4 for the longitudinal waves. For MgO and Al₂O₃ the attenuation is proportional to the frequency of the sound between 3 and 9 Gc/sec. An empirical correlation of the data is that the attenuation of 9-Gc/sec phonons is 3 dB/cm when $(T/\Theta)\approx 0.1$, where Θ =Debye temperature of the crystal. Values of the third-order elastic constants are used to make absolute comparisons of the theory of attenuation by three-phonon processes with the data. The agreement is good in some cases but not in others.

I. INTRODUCTION

AFTER the method of generating kilomegacycle ultrasonic waves^{1,2} by the surface excitation of quartz was discovered, one of the first measurements made was the temperature dependence of the attenuation of the ultrasound in the quartz itself.^{2,3} Since that time there have been several reports of attenuation measurements in various materials⁴⁻⁹ (the cited references are representative, not exhaustive) at various temperatures and kilomegacycle frequencies, but these have not been sufficiently comprehensive to make possible a thorough intercomparison of the results for various materials. We report here the results of measurements of the temperature dependence of attenuation of longitudinal and transverse 9-Gc/sec phonons in a variety of materials. These results reveal considerable correlation between the attenuation and the Debye temperatures of the crystals studied. This empirical observation may be useful in predicting the attenuation to be expected in other materials.

Theories of the attenuation of ultrasound in the high-frequency, low-temperature region have been extant for many years. The theory for the attenuation of transverse waves was given by Landau and Rumer¹⁰;

the mechanism was three-phonon collisions involving two longitudinal thermal phonons and the transverse ultrasonic wave. The theory of the longitudinal-wave attenuation was given by Pomeranchuk.¹¹ Because of the dispersion of the longitudinal velocity, interactions involving more than three waves were invoked to give allowed transitions of longitudinal ultrasound. It followed that the attenuation of longitudinal waves was predicted to be much smaller than that of transverse waves. It was surprising, therefore, when measurements^{2,3} on quartz showed that the attenuation of the longitudinal waves was comparable in magnitude and temperature dependence to the attenuation of the transverse waves. As a consequence, the theory of the longitudinal wave attenuation was re-examined.^{6,7} When the finite lifetime of the thermal phonons was taken into account, it was found that the presence of dispersion no longer prevented three-phonon collisions, and the longitudinal-wave attenuation then became similar to that of the transverse waves. In Sec. V we shall compare our experimental results on the transverse- and longitudinal-wave attenuations to these theories.

II. EXPERIMENTAL METHOD

The experimental method used to measure attenuation is described in some detail elsewhere.¹² Basically we measure the change in attenuation of several acoustic pulse echoes as the temperature is increased from 4°K. We do not measure the exponential envelope of the echo pattern, as is the practice with lower frequency acoustic measurements, because it is very difficult to obtain a truly exponential decay for X-band phonons. The echo pattern is modified by "beating" as a result³ of inaccuracies of sample orientation and polishing. Our method is estimated to give an average error of ± 1 dB/cm. The attenuation of 9-Gc/sec phonons is generally < 1 dB/cm at 4°K, so that we set the attenuation at 4°K equal to zero and record the change in attenuation from this reference level.

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¹ K. N. Baranskii, Dokl. Akad. Nauk SSSR **114**, 517 (1957) [English transl: Soviet Phys.—Doklady **2**, 237 (1958)].

² H. E. Bömmel and K. Dransfeld, Phys. Rev. **117**, 1245 (1960).

³ E. H. Jacobsen, *Quantum Electronics* (Columbia University Press, New York, 1960), p. 468.

⁴ H. J. Shaw, D. K. Winslow, A. Karp, and R. A. Wilson, Appl. Phys. Letters **4**, 28 (1964). (Al₂O₃, Rutile, 2.8 Gc/sec, room temperature, longitudinal and transverse.)

⁵ R. Nava, R. Arzt, I. Ciccarello, and K. Dransfeld, Phys. Rev. **134**, A581 (1964). (Quartz, 0.5 Gc/sec $\leq f \leq 10$ Gc/sec, $T < 30^\circ\text{K}$, longitudinal and transverse.)

⁶ I. S. Ciccarello and K. Dransfeld, Phys. Rev. **134**, A1517 (1964). (MgO, Al₂O₃, $f=0.5$ and 3 Gc/sec, $T < 100^\circ\text{K}$, longitudinal and transverse.)

⁷ H. J. Maris, Proc. Phys. Soc. (London) **9**, 901 (1964). (Quartz, $0.6\text{Gc/sec} \leq f \leq 1.1\text{Gc/sec}$, $T \leq 40^\circ\text{K}$, longitudinal and transverse.)

⁸ T. M. Fitzgerald, B. B. Chick, and R. Truell, J. Appl. Phys. **35**, 2647 (1964). (Ge, Si, Quartz, Al₂O₃, $f < 3$ Gc/sec, room temperature, Al₂O₃, $f=1.3$ Gc/sec, $T < 125^\circ\text{K}$, longitudinal.)

⁹ E. G. Spencer, R. T. Denton, and R. P. Chambers, Phys. Rev. **125**, 1950 (1962). (Yttrium iron garnet, $f=0.5$ Gc/sec, 1 Gc/sec, $4^\circ\text{K} < T < 300^\circ\text{K}$, transverse waves.)

¹⁰ L. Landau and G. Rumer, Phys. Z. Sowjetunion **11**, 18 (1937).

¹¹ I. J. Pomeranchuk, J. Phys. (USSR) **6**, 237 (1942).

¹² M. Pomerantz, Trans. IEEE Sonics Ultrasonics **SU-11**, 68 (1964).

The transducers used were ferromagnetic films evaporated on the ends of the samples. The samples were single crystals, oriented along symmetry axes to about $\pm 0.5^\circ$. The lengths of the samples were typically 1 cm and the areas were of order 6 mm^2 . The end surfaces were polished to a flatness of less than one wavelength of He light. The temperature was varied above 4°K by starting with the liquid He just below the microwave cavity, but thermally connected to the cavity (and sample) by a cold finger. The cavity was heated via a resistor attached to it. The temperature could thus be varied slowly or equilibrated by adjusting the current in the heater resistor. The temperature was measured by a Au-Co thermocouple attached near the sample. The accuracy of the temperature measurement was $\pm 2^\circ\text{K}$.

III. EXPERIMENTAL RESULTS

The measured attenuations (relative to the attenuation at 4°K , which is taken equal to zero) as functions of temperature for fast transverse, slow transverse, and longitudinal waves are plotted in Figs. 1, 2, and 3, respectively. We define "fast" transverse as the transverse wave which has the highest velocity along the given axis. Along the (100) and (111) axes of cubic crystals the two transverse waves are degenerate and both are thus classified as "fast" waves.¹³ In the (110) direction of cubic crystals there are usually two distinct trans-

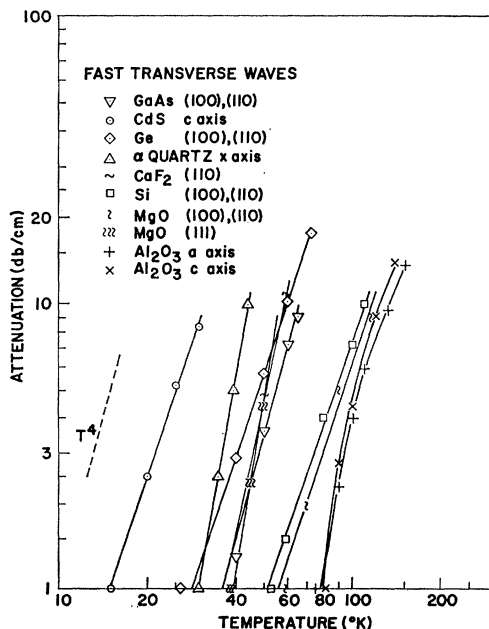


Fig. 1. Attenuation of fast transverse 9-Gc/sec phonons versus temperature.

¹³ This definition leads to the situation that the slowest transverse wave in MgO, that along the [111] axis, is classified as a "fast" wave because it is the fastest (being the only one) along the [111] direction.

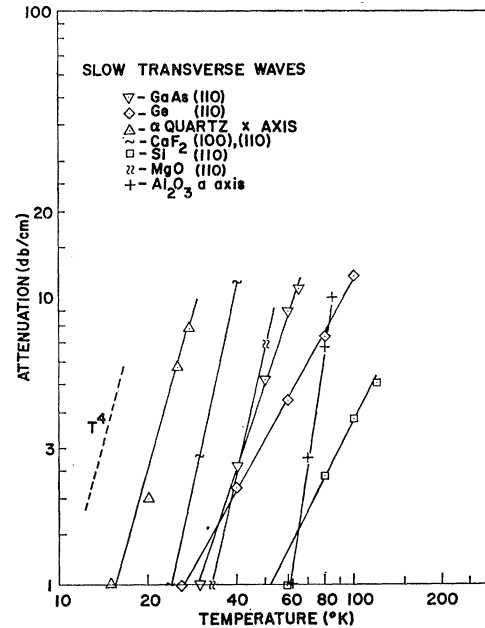


Fig. 2. Attenuation of slow transverse 9-Gc/sec phonons versus temperature.

verse wave velocities, given by $v_1 = (c_{44}/\rho)^{1/2}$ and $v_2 = [(c_{11} - c_{12})/2\rho]^{1/2}$, where ρ = density and c_{ij} are elastic constants. For all the cubic crystals reported here, except CaF_2 , the ratio $2c_{44}/(c_{11} - c_{12})$ is greater than unity. Thus the waves whose velocities are given by v_1 are the "fast" waves for Ge, Si, GaAs, and MgO. For CaF_2 the wave whose velocity is given by v_2 is included as a "fast" wave. In some cases the attenuation of waves along different directions in the same crystals [e.g., fast transverse waves in the (100) and (110) directions] are equal (within experimental error). For reasons of clarity only one curve was drawn for both directions. The key on each graph indicates those curves that represent two directions. We now discuss some of the features of the experimental results.

(i) *Fast transverse waves.* The fact that most of the log-log plots of the data are straight lines means that the attenuations are proportional to T^n , where n is the slope of the lines. For Al_2O_3 the slope is not constant, and average values were used in computations. The average slope of all the curves for fast transverse waves is

$$\bar{n}_{\text{Fast}} = 4.1,$$

and the standard deviation from the average is

$$\sigma_{\text{Fast}} = \left(\sum (n - \bar{n}_{\text{Fast}})^2 / N \right)^{1/2} = 1.2,$$

where N = the number of values of n . The experimental error in determining n is estimated to be ± 1 for a given curve. This result for the temperature dependence is in reasonable agreement with the theory of Landau and Rumer,¹⁰ which predicts that the attenuation of trans-

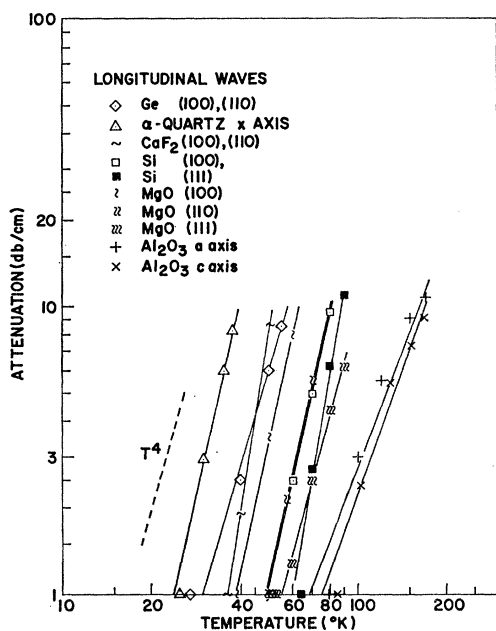


FIG. 3. Attenuation of longitudinal 9-Gc/sec phonons versus temperature.

verse waves is proportional to T^4 when $\omega\tau > 1$, where ω is the acoustic frequency, and τ is the relaxation time of the thermal phonons that scatter the acoustic wave.

The early attenuation measurements for quartz^{2,3} showed that the microwave phonons were strongly attenuated at temperatures much above 40°K. Our results show that this is not typical of most materials. In fact, in some materials (e.g., Si, MgO, and Al₂O₃), the attenuation of 9-Gc/sec phonons is not excessively high at temperatures well above the liquid nitrogen boiling point.

(ii) *Slow transverse waves.* The log-log plots are straight lines. The average slope is $\bar{n}_{\text{slow}} = 4.0$ and the average deviation $\sigma = 1.7$. Similar to fast transverse waves, an average temperature dependence T^4 is found. The deviations from the average, however, are greater in the case of the slow transverse waves.

(iii) *Longitudinal waves.* Again, the linear log-log plots show that the attenuation is proportional to T^n . The average, $\bar{n}_{\text{Long}} = 4.8$, with a standard deviation of $\sigma_{\text{Long}} = 1.4$.

(iv) *Frequency dependence.* Comparing our results for the transverse and longitudinal waves in MgO and the longitudinal wave in *c* axis Al₂O₃ with measurements made⁶ at 3 Gc/sec, we find that the attenuation at 9 Gc/sec is approximately three times larger. The attenuation has thus increased linearly with frequency for these materials in this temperature range. This agrees with the theories involving three-phonon processes^{6,7,10} which predict that the attenuation will be proportional to ω .

IV. CORRELATION OF ATTENUATIONS WITH THE DEBYE TEMPERATURES OF THE CRYSTALS

One of the motivations of this work was to attempt to find a simple means of predicting the attenuation behavior of various crystals. The increase of attenuation with increasing temperature is caused by scattering of the microwave phonons by thermally excited phonons. The attenuation will thus be related to the state of thermal excitation of the lattice. In the Debye theory,¹⁴ the number of thermally excited phonons is proportional to $(T/\Theta)^3$, where Θ is the Debye temperature. The important parameter in determining the thermal excitation is not the temperature, but the ratio of the temperature to the Debye temperature. We thus have redrawn the data with (T/Θ) as the abscissa instead of temperature.¹⁵ Thus Figs. 1, 2, and 3 go over into Figs. 4, 5, and 6, respectively.

It is now pertinent to inquire whether a significant correlation of the data is achieved by plotting attenuation versus (T/Θ) instead of attenuation versus T . In particular, one wonders if α is a universal function of (T/Θ) , as is the specific heat. We examine this by considering the variation of the temperatures and (T/Θ) at which $\alpha = 3$ dB/cm, the "3-dB points." If α is correlated with (T/Θ) , the values of (T/Θ) at $\alpha = 3$ dB/cm should be grouped more closely than are the values of T at $\alpha = 3$ dB/cm.

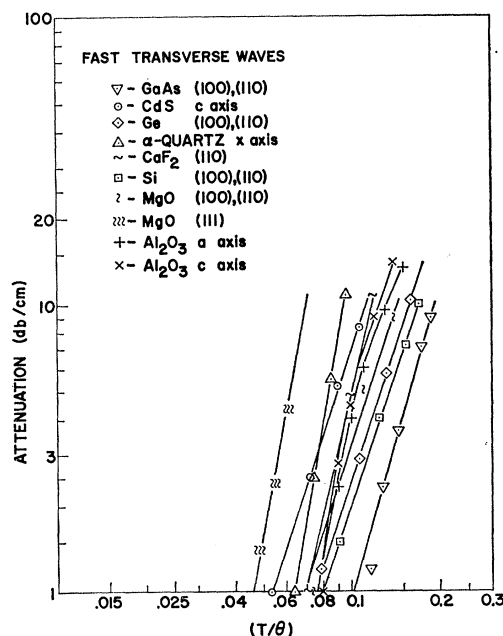


FIG. 4. Attenuation of fast transverse 9-Gc/sec phonons versus (T/Θ) .

¹⁴ J. M. Ziman, *Electrons and Phonons* (Oxford University Press, New York, 1960).

¹⁵ Values of θ were obtained from *American Institute of Physics Handbook*, edited by D. E. Gray (McGraw-Hill Book Company, Inc., New York, 1963), 2nd ed., or calculated using the elastic constants of the materials.

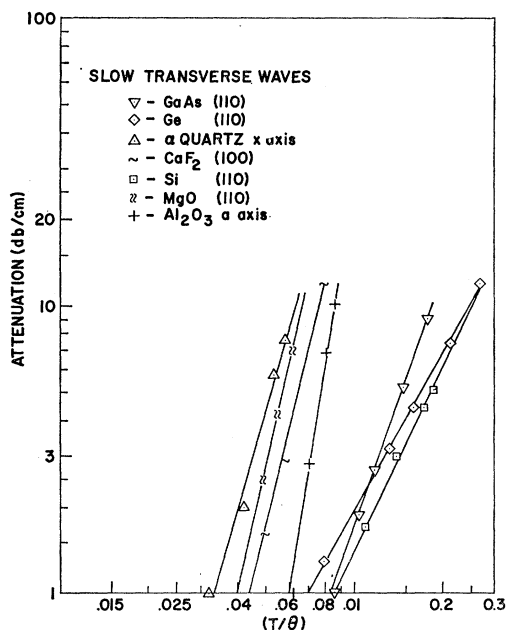


Fig. 5. Attenuation of slow transverse 9-Gc/sec phonons versus (T/Θ) .

(i) *Fast transverse waves.* For fast transverse waves, the temperatures of the 3-dB/cm point vary from 21°K (CdS) to 94°K (Al_2O_3), i.e., by a factor of more than 4. The values of (T/Θ) at the 3-dB/cm point vary from 0.057 [MgO in (111) direction] to 0.135 (GaAs), which is a range of a factor of 2.5. Thus it appears that there is some correlation between the onset of appreciable attenuation and the ratio (T/Θ) .

This correlation can be examined statistically. Thus, the average temperature of the 3-dB/cm points for fast transverse waves is $\bar{T}_{\text{Fast}}=61^\circ\text{K}$. The standard deviation from this average is $\sigma_{T_{\text{Fast}}}=23^\circ\text{K}$. A measure of the spread of 3 dB/cm on a temperature scale is the ratio $\sigma_{T_{\text{Fast}}}/\bar{T}_{\text{Fast}}$:

$$\sigma_{T_{\text{Fast}}}/\bar{T}_{\text{Fast}}=0.37. \quad (1)$$

The average value of (T/Θ) at the 3-dB/cm points is found to be $\langle T/\Theta \rangle_{\text{Fast}}=0.11$ with a standard deviation of $\sigma_{(T/\Theta)_{\text{Fast}}}=0.02$. The distribution has a fractional spread of

$$\sigma_{(T/\Theta)_{\text{Fast}}}/\langle T/\Theta \rangle_{\text{Fast}}=0.18. \quad (2)$$

Equations (1) and (2) show that at $\alpha=3$ dB/cm the values of (T/Θ) are more closely grouped about their average value than the values of T are grouped about their average. Thus, attenuation is correlated more with (T/Θ) than with T .

(ii) *Slow transverse waves.* The same analysis as in (i) above, when repeated for the slow transverse waves, gives the values: $\bar{T}_{\text{Slow}}=49^\circ\text{K}$, $\sigma(T)_{\text{Slow}}=22^\circ\text{K}$ and $\sigma(T)_{\text{Slow}}/\bar{T}_{\text{Slow}}=0.45$. Considering the attenuation as a function of (T/Θ) gives $\langle T/\Theta \rangle_{\text{Slow}}=0.09$, $\sigma_{(T/\Theta)_{\text{Slow}}}=0.04$, and $\sigma_{(T/\Theta)_{\text{Slow}}}/\langle T/\Theta \rangle_{\text{Slow}}=0.44$. It appears

that for slow transverse waves the correlation with temperature is as good as it is with (T/Θ) .

The slow wave attenuation when plotted versus (T/Θ) is peculiar in that there seem to be two groups of curves, one for the hard insulators (MgO , Al_2O_3 , CaF_2 , and quartz) and one for the group of semiconductors Ge, Si, and GaAs. We have at present no explanation for this grouping of the materials or for the particularly weak temperature dependences of the semiconductors.

(ii) *Longitudinal waves.* Analysis of the data gives the values $\bar{T}_{\text{Long}}=61^\circ\text{K}$, $\sigma_{\text{Long}}=23^\circ\text{K}$, and $\sigma_{T_{\text{Long}}}/\bar{T}_{\text{Long}}=0.38$. In terms of (T/Θ) , the average is $\langle T/\Theta \rangle_{\text{Long}}=0.09$, the standard deviation is $\sigma_{(T/\Theta)_{\text{Long}}}=0.016$, which gives a fractional spread of $\sigma_{(T/\Theta)_{\text{Long}}}/\langle T/\Theta \rangle_{\text{Long}}=0.18$. The values for the longitudinal waves are remarkably similar to those of the fast transverse waves. The relative grouping of 3-dB/cm points when considered in terms of (T/Θ) indicates a correlation of longitudinal wave attenuation with the Debye temperatures of the crystals.

The conclusion that we draw from this discussion is that the attenuations of fast transverse and longitudinal waves in a variety of crystalline materials and directions can usefully be related to the Debye temperatures of the crystals. The attenuation of 9-Gc/sec longitudinal and fast transverse phonons is 3 dB/cm when the temperature $T \approx 0.1 \Theta$.

V. COMPARISON OF THEORY AND OBSERVED ATTENUATION

In Sec. III we indicated that the attenuations of both longitudinal and transverse waves were propor-

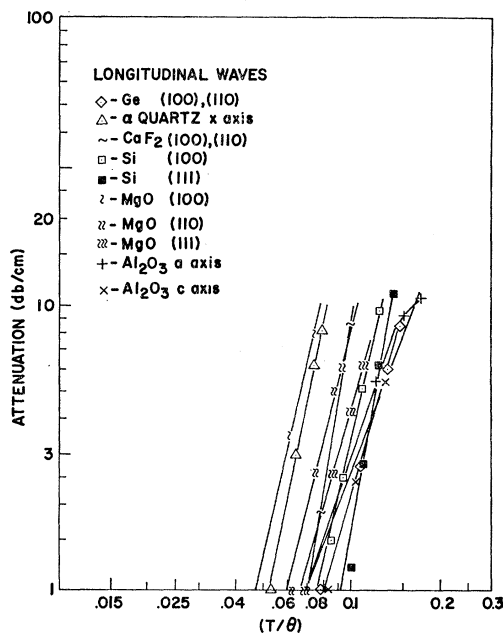


Fig. 6. Attenuation of longitudinal 9-Gc/sec phonons versus (T/Θ) .

tional to T^n , where the average value of $n \sim 4$. For MgO and Al₂O₃, we noted a linear dependence of attenuation on the frequency. These observations are in general agreement with the three-phonon theories of attenuation^{6,7,10} which predict that the attenuation will be proportional to ωT^4 . In this section we shall use measured values of third-order elastic constants to calculate the attenuation of transverse and longitudinal waves by three-phonon processes. This provides an absolute comparison of the theories and experiment. Mason and Bateman¹⁶ have previously used measured third-order elastic constants to calculate the attenuation of ultrasound of frequency $\lesssim 500$ Mc/sec in Ge and Si. Their theory, a modification of Akhieser's¹⁷ theory, presumably does not apply under the conditions of our experiments, $\omega\tau > 1$. (See Note 1, added in proof.)

(i) *Transverse waves.* Landau and Rumer¹⁰ derived the following formula for the attenuation of the energy of transverse waves in an isotropic medium under the conditions that $\omega\tau > 1$:

$$\alpha_t = (\pi k^4 / 80 \rho \hbar^3 V_t^6) \gamma^2 [1 - (V_l/V_t)^2] \omega T^4, \quad (3)$$

where γ = ratio of an appropriate third-order elastic constant to a second-order elastic constant; V_l and V_t are the longitudinal and transverse wave velocities, respectively; ω = acoustic circular frequency; ρ = density; T = absolute temperature; τ = thermal phonon relaxation time. This equation does not apply exactly to crystalline solids; in these the third-order elastic-constant tensor is more complicated than for an isotropic medium. Furthermore, in an arbitrary direction one cannot describe the waves as purely longitudinal or transverse modes. Calculations¹⁸ indicate that the isotropic (pure mode) approximation may not lead to great error for real crystals. A practical problem is the determination of the third-order elastic constants of the material. These are not known with accuracy for many crystals, but recently improved values for some materials have become available.

The appropriate anharmonic elastic constant¹⁹ for transverse waves propagating in the [100] direction is $\gamma = (c_{11} + c_{12} + 4c_{44} + 4c_{166}) / 2c_{11}$. The thermodynamic definition of c_{166} is used here in order to agree with McSkimin and Andreatch,²⁰ whose measurements of the third-order elastic constants for Ge and Si we shall use. The attenuation data for Ge and Si were compared to the theory by solving Eq. (3) for the temperature at which the attenuation = 3 dB/cm, which is denoted by T_3 . The calculated and observed values of T_3 for the

TABLE I. Comparison of three-phonon theory of attenuation with experiment: transverse waves.

Material	Axis	$ \gamma ^a$	T_3 (°K) (calc.)	T_3 (°K) (exptl.)
Ge	[100]	2.8	64	40
Si	[100]	2.1	137	74

^a Values of γ from Ref. 20.

shear waves in the [100] directions of Ge and Si are listed in Table I. It is seen that the theoretical values are higher than the experimental values by as much as about a factor of 2. The origin of this discrepancy may lie in the fact that the anisotropies of the longitudinal velocity and the anharmonic constants have been neglected.

(ii) *Longitudinal waves.* An expression for the attenuation of longitudinal waves by three-phonon collinear interactions has been given by several authors.^{6,7,21} We use the formula derived by Shiren,²¹

$$\alpha_l = \pi^3 \gamma^2 k^4 \omega T^4 / 120 \rho V_l^6 \hbar^3, \quad (4)$$

where $\gamma = (3c_{11} + c_{111}) / c_{11}$ for the (100) direction and $\gamma = [3(c_{11} + c_{12} + 2c_{44}) + 6c_{166} + 3c_{112}/2 + c_{111}/2] / (c_{11} + c_{12} + 2c_{44})$

for the [110] direction (thermodynamic definitions of the third-order moduli are used). The other symbols are defined below Eq. (3). Values of c_{ijk} for Ge and Si have been measured by McSkimin and Andreatch.²⁰ Values of γ for several materials have been measured by Shiren.²¹

We use Eq. (4) for T_3 , the temperature at which the attenuation = 3 dB/cm. The values of T_3 computed using the measured values of γ are compared with experiment in Table II. It may be noted that the agreement is reasonably good for most of the cubic cases but is poor for the trigonal materials (quartz and Al₂O₃), in which the small value of the anharmonic constant leads to a predicted value of T_3 that is much higher than the observed value. The theory also predicts more anisotropy of T_3 than is observed.

The better agreement of theory and experiment for longitudinal waves in Ge and Si, compared to the only rough agreement for the transverse waves, may be related to the fact that anisotropy of the third-order elastic constants will be less important for the longitudinal waves. The thermal and acoustic waves are nearly collinear for the longitudinal case. In the transverse case the longitudinal thermal waves that interact with the acoustic wave are moving in an approximately annular cone with axis along the direction of propagation of the acoustic wave. Anisotropy will thus be more

¹⁶ W. P. Mason and T. B. Bateman, J. Acoust. Soc. Am. **36**, 644 (1964).

¹⁷ A. Akhieser, J. Phys. (USSR) **1**, 277 (1939).

¹⁸ R. Orbach, thesis, University of California, 1960, pp. 126 and 151 (a) (unpublished).

¹⁹ This has been determined by N. Shiren (unpublished) by comparison of the Landau-Rumer expression for the energy and the definitions of the c_{ijk} .

²⁰ H. J. McSkimin and P. Andreatch, Jr., J. Appl. Phys. **35**, 3312 (1964).

²¹ N. S. Shiren, Progress Report No. 4 on Contract DA 36-039 AMC-02280(E), U. S. Army Electronics Research Laboratory, Fort Monmouth, New Jersey (unpublished), and private communications.

TABLE II. Comparison of three-phonon theory of attenuation with experiment: longitudinal waves.

Material	Axis	$ \gamma $	T_3 (°K) (calc.)	T_3 (°K) (exptl.)
MgO ^a	[100]	8.7±0.9	44±2	45
	[110]	3.5±0.4	77±4	62
	[111]	0.4 _{-0.2} ^{+0.4}	230 ₊₉₀ ⁻⁷⁰	70
Ge ^b	[100]	2.5	36	43
	[110]	5.7	27	44
CaF ₂ ^a	[100]	6 ₋₃ ⁺⁶	34 ₊₁₃ ⁻¹⁰	43
	[110]	10 ₋₅ ⁺¹⁰	24 ₊₉ ⁻⁷	40
Si ^b	[100]	2.0	76	62
	[110]	4.6	48	62
α-Quartz ^a	(x)	<0.2	>130	30
Al ₂ O ₃ ^a	(a)	<1	>175	103
	(c)	<1	>180	100

^a Values of γ obtained from Ref. 21.
^b Values of γ obtained from Ref. 20.

important for the transverse attenuation and some deviations from the (isotropic) Landau-Rumer theory may be expected.

VI. CONCLUSION

The attenuations of longitudinal and transverse (both slow and fast) 9-Gc/sec phonons have been measured as a function of temperature. The attenuations in a variety of crystals are proportional to T^n , where the average value of $n \sim 4$. The temperatures at which the attenuation becomes appreciable (say 3 dB/cm) varies over a range from about 20 to 100°K. For longitudinal and fast transverse waves we note an empirical correlation between attenuation and the Debye temperature of the material: the attenuation is 3 dB/cm when $(T/\Theta) \approx 0.1$, for 9-Gc/sec phonons.

This provides a simple empirical rule for guessing the temperature at which a material will become lossy. Thus, for example, this rule predicts that in diamond, for which $\Theta = 2000$ °K, the attenuation will be 3 dB/cm at $0.1 \Theta = 200$ °K. At room temperature the attenuation in diamond is predicted to have a value of about 15 dB/cm, at 9 Gc/sec.

Comparison of the theories of ultrasonic attenuation in the low-temperature, high-frequency range, ($\omega\tau > 1$), with the experiments shows agreement within a factor of 2 for the transverse waves. The agreement between theory and experiment for the longitudinal waves is good for several cubic materials but is poor for the trigonal materials studied. It would be desirable to have accurate values of the third-order elastic constants of more materials so that the origin of the discrepancies may become clearer.

Note added in proof. 1. In Sec. III we applied the theory of three-phonon collisions but did not discuss whether the regime of applicability $\omega\tau > 1$ was realized at the temperatures of interest ($T \approx 0.1\theta$). The thermal-phonon lifetime τ is limited by (1) normal processes, (2) umklapp processes. (1) The normal-process scattering of thermal phonons by each other is dominated by the same three-phonon processes that we have calculated for ultrasonic waves in Sec. III. This scattering is proportional to ω , and since the mean free path L of $\omega = 6 \times 10^{10}$ sec⁻¹ phonons is of order $L \approx 1$ cm at $(T/\theta) \approx 0.1$, the mean free path for normal collisions of the highest frequency thermal phonons ($\omega \approx 10^{13}$ sec⁻¹) will be $L \gtrsim 10^{-3}$ cm. Thus, for normal collisions $\tau = L/v \gtrsim 10^{-9}$ cm/10⁶ cm/sec $\approx 10^{-9}$ sec. Thus $\omega\tau \gg 1$, for normal-process scattering. (2) The umklapp scattering relaxation time τ_u can be estimated from the thermal conductivity $\kappa = \frac{1}{3}c\bar{v}^2\tau_u$ where c = specific heat and v = sound velocity. From Debye theory, at $(T/\theta) = 0.1$, $c \approx 0.2$ J/cc °K for a typical crystal; then $\tau_u \approx 10\kappa/v^2 \approx 10^{-11}\kappa$. Thus $\omega\tau_u \gtrsim 1$ for materials with $\kappa \gtrsim 1$ W/cm deg at $T \approx 0.1\theta$. Thermal conductivity data at low temperatures are available^{14,15} for Al₂O₃, α quartz, diamond, Si, and Ge. In all of these cases $\kappa > 1$ W/cm deg at $T = 0.1\theta$, so that $\omega\tau_u > 1$. κ may be approximately a universal function¹⁴ of (T/θ) at low temperatures so that the condition $\omega\tau_u > 1$ may be satisfied for most materials at $T/\theta = 0.1$, for $\omega \approx 10^{11}$ sec⁻¹. Conversely, the observation that $\alpha \sim \omega T^4$ indicates that $\omega\tau > 1$, and hence gives some measure of the lifetime of the thermal phonons with which the ultrasound is interacting.

Note added in proof. 2. Preliminary measurements of the attenuation of transverse waves in the (100) direction in diamond have been made. The attenuation seems to be lower than in any material reported above, in agreement with the suggestion made in the conclusion.

In addition, some preliminary measurements in YIG and Ga substituted YIG show quite low attenuation for shear waves along the (110) directions. The attenuation is less than would be predicted from the rule that $\alpha = 3$ dB/cm at $T = 0.1\theta$. Low acoustic attenuation in garnets has previously been observed by Spencer *et al.*⁹ The results of our continuing study will be reported later.

ACKNOWLEDGMENTS

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