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Čerenkov Radiation from Charged Particles in a Plasma in a Magnetic Field*

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The Čerenkov radiation from electrons in a plasma in a magnetic field has been calculated by Kolomenskiĭ, who determined the allowed frequency regions and the energy loss. It is shown here that his results are in error. The allowed frequency regions have a much more complicated dependence on the plasma frequency, gyrofrequency, and electron velocity than he indicates, and he overestimates the energy loss for electrons in the earth's ionosphere by a factor of about 100. The total energy loss from slow electrons by Čerenkov radiation is comparable to their synchrotron loss, but is mainly at lower frequencies. For fast electrons, the Čerenkov loss is much smaller than the synchrotron loss.

I. INTRODUCTION

IN this paper we investigate the Čerenkov radiation from an electron, moving uniformly in a collisionless electron plasma in a magnetic field. This radiation was first calculated by Kolomenskiĭ,[1] who derived the allowed frequency regions of the emitted ordinary and extraordinary Čerenkov waves for a given plasma electron-number density N and magnetic field H. Also, he calculated the energy loss of an electron due to the emission of Čerenkov radiation.

We have reinvestigated this problem and have come to conclusions that differ from those of Kolomenskiĭ. In particular, we find a more complex frequency region for the allowed waves, and that the energy loss is of a qualitatively and quantitatively different character than the results of Kolomenskiĭ.

Recent investigation of this problem has been due to the interest in the radiation from fission electrons in the earth's upper atmosphere resulting from nuclear explosions.[2] We show that for relativistic electrons moving in the earth's magnetic field in the ionosphere, the rate of Čerenkov radiation is much less than the radiation from an electron with the same energy moving in a circle (i.e., synchrotron radiation).[3]

In the next section we derive the conditions under which Čerenkov radiation is possible in a plasma. We then go on to calculate the general expression for the energy loss. The allowed frequency regions for Čerenkov radiation for different values of the parameters (ω_H, ω_0, and v) are determined, and the energy loss evaluated. Although the Čerenkov radiation is basically a classical effect, a quantum-mechanical derivation for the energy loss is somewhat simpler.

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[1] A. A. Kolomenskiĭ, Dokl. Akad. Nauk SSSR **106**, 982 (1956) [English transl.: Soviet Phys.—Doklady **1**, 133 (1956)].

[2] "Collected Papers on the Artificial Radiation Belt," J. Geophys. Res. **68**, 605–759 (1963).

[3] Shafranov {V. D. Shafranov, Zh. Eksperim. i Teor. Fiz. **34**, 1475 (1958) [English transl.: Soviet Phys.—JETP **34**, 1019 (1958)]} indicates that, for the case where the gyromagnetic frequency (ω_H) equals the plasma frequency (ω_0), the Čerenkov radiation is larger by a factor of (c/v) than the synchrotron radiation. We show that this is not the case.

We are using Gaussian units with $\hbar=c=1$; then $e^2\cong 1/137$. Four-vectors are denoted by small lightface letters [e.g., $k=(\omega,\mathbf{K})$]. The dot product of two four-vectors is taken as $a\cdot b=a_tb_t-\mathbf{a}\cdot\mathbf{b}$. Also, the notation $\not{a}=a_t\gamma_t-\mathbf{a}\cdot\boldsymbol{\gamma}$ is used.

II. CALCULATION OF ČERENKOV RADIATION

It is well known that Čerenkov radiation is possible if the following condition is satisfied:

$$n>1/v\,, \tag{1}$$

where n is the index of refraction and v the velocity of the incident electron. In the case of a collisionless isotropic electron plasma with no magnetic field

$$n_0^2=1-\omega_0^2/\omega^2\,, \tag{2}$$

where ω is the frequency of the wave and the plasma frequency is given by

$$\omega_0^2=4\pi Ne^2/m\,. \tag{3}$$

(N is the number of electrons per cm^3; and e and m are the charge and mass of the electron, respectively.) Since $v<1$, it follows from Eqs. (1) and (2) that Čerenkov radiation is not possible in this case.

In the case of a magnetic field the index is given by[4]

$$n^2=1-X\left(1-\frac{Y^2\sin^2\theta}{2(1-X)}\pm\left[\frac{Y^4\sin^4\theta}{4(1-X)^2}+Y^2\cos^2\theta\right]^{1/2}\right)^{-1}, \tag{4}$$

where $X=\omega_0^2/\omega^2$, $Y=\omega_H/\omega$, $\omega_H=eH/m$, and θ is the angle between the direction of the magnetic field and the direction of propagation of the wave. The plus and minus sign in Eq. (4) refers to the ordinary and extraordinary waves, respectively. For the case $\theta=0$ we obtain from Eq. (4) that

$$n^2=1-\omega_0^2/\omega(\omega\pm\omega_H)\,. \tag{5}$$

Note that in this case ($\theta=0$) Čerenkov radiation is

[4] J. A. Ratcliff, *The Magneto-Ionic Theory* (Cambridge University Press, Cambridge, England, 1959).

possible only for the extraordinary wave. Furthermore, the expression under the square root is never less than the second term in the denominator. Hence, for the plus sign (ordinary wave), the denominator is always positive, and the index less than 1. Therefore, there is never Čerenkov radiation associated with ordinary waves, and only extraordinary waves need be considered.

In the presence of collisions n^2 is complex. The imaginary part of n^2 determines directly the polarization loss while the real part of n^2 plays the corresponding role in the Čerenkov loss. We shall assume that the imaginary part of n^2 is negligible in the following. In the earth's atmosphere this assumption restricts the investigation to altitudes above 100 km.

For simplicity we confine ourselves to the case of electron motion along the direction of the magnetic field. The general case of an arbitrary angle between **v** and **H** is unnecessarily complicated (algebraically) to warrant a full quantitative analysis in view of the fact that the Čerenkov radiation turns out to be much smaller than other forms of radiation (e.g., synchrotron radiation). We have no reason to expect orders of magnitude enhancement of the Čerenkov radiation for arbitrary angle between **v** and **H**. The method that we use to calculate the Čerenkov radiation for **v** parallel to **H** can be applied directly to an arbitrary angle between **v** and **H**.

To obtain the energy loss per unit time dW/dt we first calculate the probability per second that the incident electron of energy E (momentum $[E^2-m^2]^{1/2}$) makes a transition to a state of energy $E-\omega$ in range $d\omega$ by emitting a photon of energy ω and momentum $K(\omega n=K)$. Calling this quantity $d\Gamma$, the energy loss per unit time dW/dt is given by

$$\frac{dW}{dt}=\int_0^\infty \omega d\Gamma=\int_0^\infty \omega\left(\frac{d\Gamma}{d\omega}\right)d\omega. \qquad (6)$$

To calculate the decay rate $d\Gamma/d\omega$ we use the result that the total decay rate is twice the imaginary part of the self-energy of the electron in the medium.[5] The Feynman diagram for the self-energy of an electron is shown in Fig. 1. The probability amplitude M for this process to first order in e^2 is given by [see Eq. (87) of Ref. 5]

$$M=ie^2\int \frac{d^4k}{(2\pi)^4}\bar{u}_p\left(\gamma_\nu \frac{1}{p-k-m+i\epsilon}\gamma_\mu\right)u_p\pi_{\mu\nu}; \qquad (7)$$

u_p is the free-particle spinor of four-momentum p (normalized to $\bar{u}_p u_p=2m$) and $\pi_{\mu\nu}$ is the photon propagator in a medium and is given in Ref. 5. Only transverse waves will be propagated. The transverse photon propagator is given by

$$\pi_{tt}=4\pi/(\omega^2n^2-K^2+i\epsilon), \qquad (8)$$

[5] H. T. Yura, The RAND Corporation, Santa Monica, California, Report No. R-410, 1963 (unpublished).

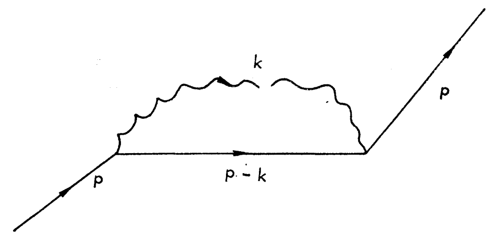

FIG. 1. Diagram depicting the self-energy of an electron.

where n^2 is given by Eq. (4) with the minus sign and ϵ is an infinitesimally small positive number.

With the normalization used, the relation between M and ΔE (the self energy) is given by

$$\Delta E=M/2E. \qquad (9)$$

The decay rate Γ is related to ΔE by

$$\Gamma=-2\,\mathrm{Im}\Delta E. \qquad (10)$$

Hence from Eqs. (6) through (10) we have

$$\frac{dW}{dt}=\mathrm{Im}\left\{-i\left(\frac{e^2}{E}\right)\int\frac{\omega d\omega d^3K}{(2\pi)^4}\left(\frac{4\pi}{\omega^2n^2-K^2+i\epsilon}\right)\times\frac{[\bar{u}_p\gamma_{\mathrm{tr}}(p-k+m)\gamma_{\mathrm{tr}}u_p]}{k^2-2p\cdot k+i\epsilon}\right\}. \qquad (11)$$

In Eq. (11) γ_{tr} are the appropriate transverse gamma matrices.

In the terms in the square brackets in Eq. (11) we may neglect k compared to $p+m$ and in the denominator we may neglect k^2 compared to $2p\cdot k$.[6] Noting that $\bar{u}_p\gamma_{\mathrm{tr}}(p+m)\gamma_{\mathrm{tr}}u_p=-4P_{\mathrm{tr}}{}^2=-2P^2\sin^2\theta$ where θ is the angle between **P** and **K**, and using the well-known rule to obtain the imaginary part of Feynman amplitudes[7] we obtain

$$\frac{dW}{dt}=\left(\frac{e^2}{2E}\right)\int\frac{\omega d\omega d^3K}{(2\pi)^4}(4\pi)(2\pi)^2 \times\delta(\omega^2n^2-K^2)\delta(2p\cdot k)2P^2(1-\cos^2\theta)$$

$$=e^2v\int \omega d\omega K dK d(\cos\theta) \times\delta(\omega^2n^2-K^2)\delta\left(\cos\theta-\frac{1}{nv}\right)(1-\cos^2\theta)$$

$$=\frac{e^2v}{2}\int_0^\infty \omega d\omega\int_1^{-1} d(\cos\theta)\delta\left(\cos\theta-\frac{1}{nv}\right)(1-\cos^2\theta). \qquad (12)$$

Note, if n is a function of frequency only Eq. (12) reduces to the familiar Čerenkov loss expression in an

[6] We are interested in the radiation of soft photons. That is, $\omega\ll m$. For $\omega\gtrsim m$, $n^2=1$ and hence there is no Čerenkov radiation.
[7] See Ref. 5.

isotropic medium:

$$\frac{dW}{dt}=\frac{e^2v}{2}\int' \omega d\omega\left[1-\frac{1}{n^2(\omega)v^2}\right], \tag{13}$$

where the prime on the integral indicates integration only over positive values of ω such that $n>1/v$.

In the case at hand n^2 is given by Eq. (4). The delta function in Eq. (12) indicates that we must obtain the roots ($\cos\theta_\pm$) of

$$\cos^2\theta=1/n^2(\theta)v^2. \tag{14}$$

From Eqs. (4) and (14) we find

$$\frac{1}{n_\pm^2(\omega)v^2}=\cos^2\theta_\pm=\frac{s[2v^2(s-1)^2-\alpha^2[2v^2s+1-v^2]\pm\alpha\{4v^2(s-1)+\alpha^2(1-v^2)^2\}^{1/2}]}{2v^2\{v^2(s-1)^3-\alpha^2s[sv^2+1-v^2]\}},$$

where

$$s=\omega^2/\omega_0^2;\quad \alpha=\omega_H/\omega_0. \tag{15}$$

The frequency regions ω corresponding to Čerenkov radiation are determined by requiring

$$0\leq\cos^2\theta_\pm\leq 1. \tag{16}$$

III. ALLOWED REGIONS

In this section we shall consider the locations of the allowed regions in the frequency plane as the parameters ω_0, ω_H, and v are varied. Since the parameter ω_0 only appears in the ratios s and α, it may be used to normalize the variables. Therefore, the problem is to ascertain for what values of s the inequality (16) is satisfied as α varies arbitrarily, and v varies between 0 and 1. For the earth's ionosphere, typical values of α are 0.1 [F_2 layer, winter day, sunspot maximum], 0.28 [F_2 layer, sunspot minimum], and 0.5 [E layer, winter day, sunspot minimum]. These correspond to plasma frequencies of 13, 5, and 2.8 Mc/sec, with the variation of the earth's magnetic field taken into account.

Kolomenskiĭ asserts that there are two different propagation modes, depending on whether or not ω is less than ω_0. We shall show that the behavior is considerably more complicated, and there are 17 different cases to consider.

The solutions of Eqs. (15) with the plus sign will be referred to as P waves, those with the minus sign as Q waves. It can be shown that the only values of s for which the numerator can vanish are $s=0$ and $s=1+\alpha^2$, equivalent to $\omega^2=\omega_0^2+\omega_H^2$. The latter root is a P wave. A consideration of the values of the right-hand side of (15) for s near $1+\alpha^2$ shows that $\cos^2\theta_+$ is positive for $s<1+\alpha^2$, and is negative for $s>1+\alpha^2$. Hence, there will always be an allowed mode for frequencies slightly below $\omega^2=\omega_0^2+\omega_H^2\equiv\omega_M^2$.

For the zero of the numerator at $s=0$ to be reachable from finite values of frequency, corresponding to propagation of low-frequency waves, the square root expression in the numerator of (15) must be positive for $s=0$. This corresponds to the condition

$$\alpha>2v/(1-v^2). \tag{17}$$

If this condition is satisfied, there will be two waves (P and Q) capable of propagation at very low frequencies in directions close to $\theta=90°$. If the condition is not satisfied, there will be a frequency ω_L for which the square root vanishes, and no propagation is possible below ω_L. The frequency ω_L is

$$\omega_L=\omega_0\left[1-\frac{\alpha^2(1-v^2)^2}{4v^2}\right]^{1/2}. \tag{18}$$

The other end point of the inequality (16), $\cos^2\theta=1$, has three roots. One is at ω_0, which is a singular point of the defining equation (4) for the refractive index. The others are at the frequencies ω_+ and ω_- given by

$$\omega_\pm=\frac{\omega_H}{2}\left[1\pm\left\{1-\frac{4v^2}{\alpha^2(1-v^2)}\right\}^{1/2}\right]. \tag{19}$$

For the roots $\omega_\pm$ to be real, the square root in (19) must be positive, giving the inequality

$$\alpha>\frac{2v}{(1-v^2)^{1/2}},\quad \omega_\pm\text{ real}. \tag{20}$$

Since the right-hand member of the inequality (20) is always smaller then the right-hand member of (17), it follows that if (20) is not satisfied, the only possible mode of propagation is a P wave, which exists between ω_0 and ω_M. If (20) is satisfied, it is necessary to consider the relative values of ω_0, ω_+, ω_-, and ω_L if the latter exists.

The roots ω_0, ω_+, and ω_- are found by equating the numerator and denominator of (15) and performing squarings and other algebraic manipulations. It is therefore necessary to ascertain whether the roots correspond to P or Q modes. Direct substitution shows that ω_0 and ω_+ are always P modes. However, ω_- may be either P or Q, as determined by the inequalities

$$\alpha<\frac{2v}{(1-v^2)(1+v^2)^{1/2}},\quad \omega_-\to P, \tag{21}$$

$$\alpha>\frac{2v}{(1-v^2)(1+v^2)^{1/2}},\quad \omega_-\to Q. \tag{22}$$

The expression on the right of (21) and (22) is always greater than that of (20), which is, of course, necessary.

The relative positions of ω_+, ω_-, and ω_0 will now be considered. By definition, $\omega_+ > \omega_-$, and it may be shown easily that $\omega_+ < \omega_M$, $\omega_- > \omega_L$ when the latter exists. The values of ω_+ and ω_- become equal when (20) becomes an equality.

The values of ω_+ and ω_- may cross ω_0 if α and v vary. The conditions take a rather complicated form:

$$v^2 < \tfrac{1}{2}, \quad \omega_+ \gtreqless \omega_0, \quad \text{if} \quad \alpha \gtreqless 1/(1-v^2), \tag{23}$$

$$v^2 < \tfrac{1}{2}, \quad \omega_- < \omega_0, \tag{24}$$

$$v^2 > \tfrac{1}{2}, \quad \omega_+ > \omega_0, \tag{25}$$

$$v^2 > \tfrac{1}{2}, \quad \omega_- \gtreqless \omega_0, \quad \text{if} \quad \alpha \lesseqgtr 1/(1-v^2). \tag{26}$$

The inequality (25) shows that for $v^2 > \frac{1}{2}$, there is a P mode extending from ω_+ to ω_M. The relative values of the inequalities (21), (22), and (23)–(26) depend on v. They are equivalent if

$$\frac{1}{1-v^2} = \frac{2v}{(1-v^2)(1+v^2)^{1/2}}, \tag{27}$$

$$v^2 = \tfrac{1}{3}.$$

Hence, there is a change in character of the modes for $v^2 < \frac{1}{3}$ and $v^2 > \frac{1}{3}$. Also, the relative values of the inequalities (17) and (23)–(26) change at $v^2 = \frac{1}{4}$. Therefore, the position of the roots and mode types is different for four different regions, viz., $0 < v^2 < \frac{1}{4}$, $\frac{1}{4} < v^2 < \frac{1}{3}$, $\frac{1}{3} < v^2 < \frac{1}{2}$, $\frac{1}{2} < v^2 < 1$.

For the three ionospheric values of α, (0.1, 0.28, 0.5), the squared velocities corresponding to the inequality (20) are 0.0025, 0.0192, and 0.0588, which correspond to electron energies of 6, 50, and 150 keV. These are all

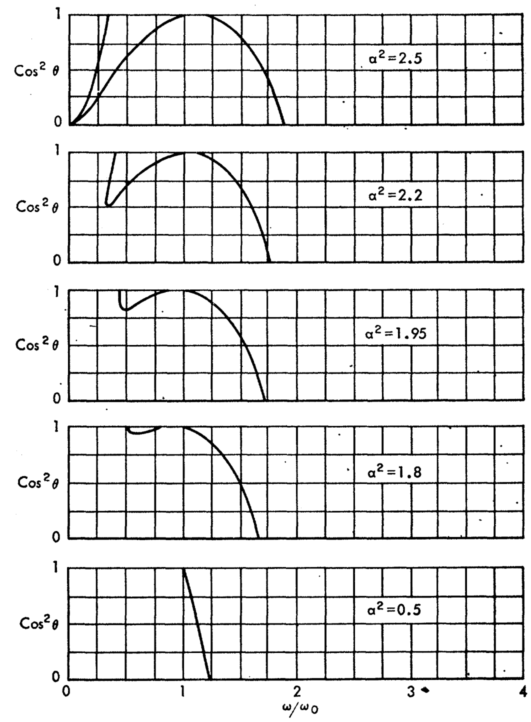

FIG. 3. Allowed regions for $v^2 = 0.3$.

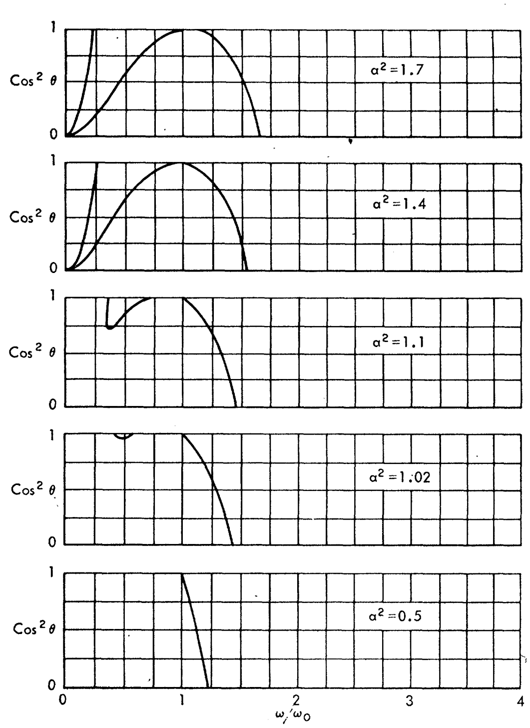

FIG. 2. Allowed regions for $v^2 = 0.2$.

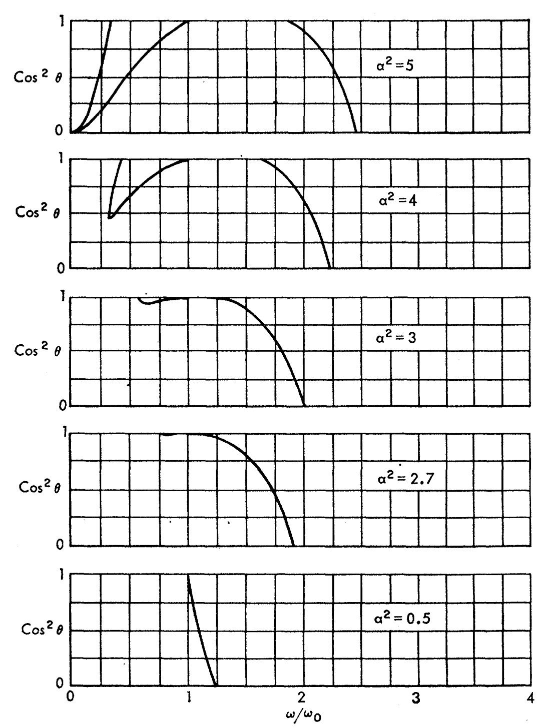

FIG. 4. Allowed regions for $v^2 = 0.4$.

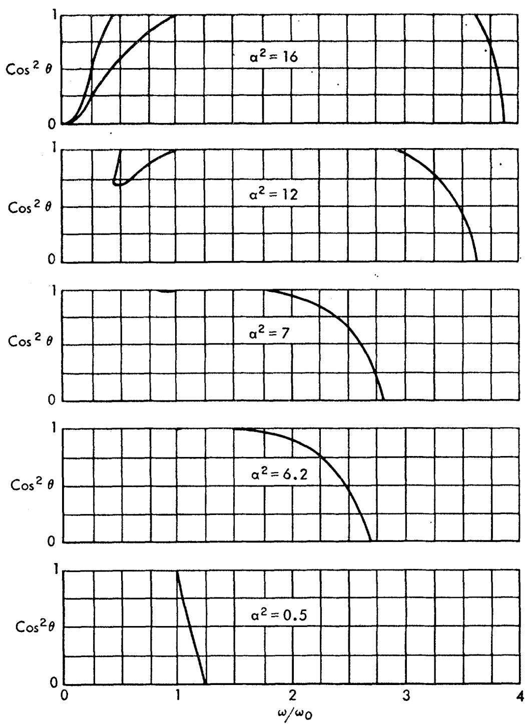

FIG. 5. Allowed regions for $v^2=0.6$.

relatively slow compared with the electrons produced in nuclear-bomb processes. Therefore, for most of the electrons produced in nuclear explosions, the inequality (20) will not be satisfied, and the only possible Čerenkov radiation will come from the narrow region between ω_0 and ω_M.

Figures 2 through 5 show the characteristic pattern of allowed modes for $v^2=0.2, 0.3, 0.4, 0.6$, covering the four regions and displaying the numerous changes of type. The abscissa is ω/ω_0, the ordinate is $\cos^2\theta$. When there are both P and Q waves present, the Q wave always has the larger value of $\cos^2\theta$. While 20 curves are shown, the single-branched patterns at the bottom of each set, corresponding to the failure of inequality (20), are essentially common to the four sets, so there are really 17 changes of type, all corresponding to extraordinary waves, instead of the two described by Kolomenskiĭ.

Kolomenskiĭ presents an inequality similar to (23)–(26). He asserts that for magnetic fields less than a critical value H_1, there will be only extraordinary waves (our P waves), and for $H>H_1$, also ordinary waves (our Q waves), where, in his notation

$$H_1=\frac{8\pi Nec\beta}{(1-\beta^2)^{1/2}} \quad \text{if} \quad \beta<\frac{1}{\sqrt{2}},$$

$$H_1=\frac{4\pi Nec}{1-\beta^2} \qquad \beta>\frac{1}{\sqrt{2}}.$$

This result is manifestly incorrect, since it does not check dimensionally. If the equation is multiplied by e/mc, the left side is a frequency, the right is the square of a frequency. The correct value for the critical field, derived from (23)–(26), is:

$$H_1=(4\pi Nm)^{1/2}c/(1-\beta^2).$$

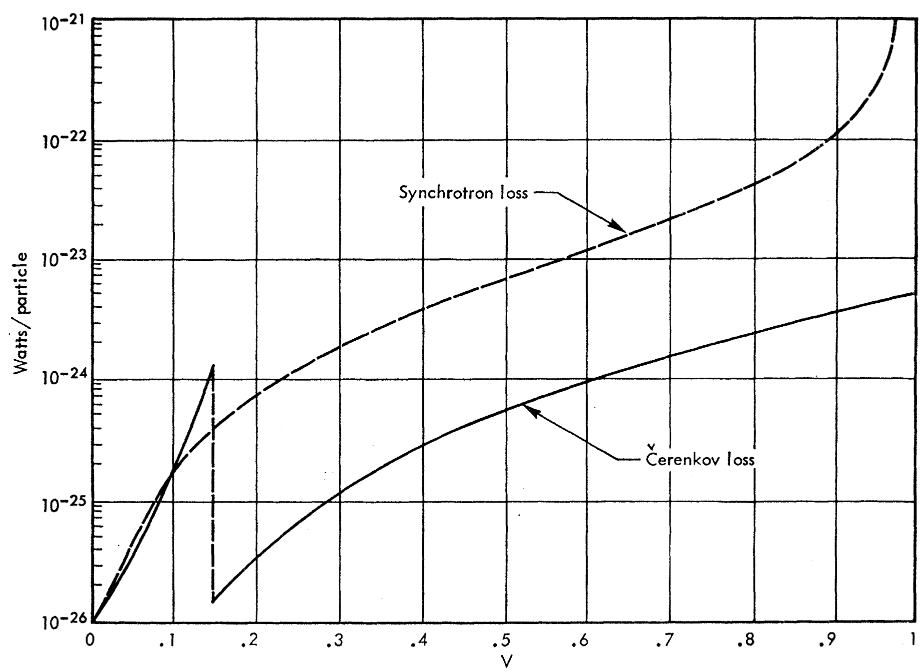

FIG. 6. Energy loss per particle.

IV. ENERGY LOSS

Returning now to the expression for the Čerenkov loss [Eq. (12)], we have

$$\frac{dW}{dt}=\frac{e^2v}{2}\int_0^\infty \omega d\omega\int_1^{-1} d(\cos\theta)\delta\left(\cos\theta-\frac{1}{n(\theta)v}\right)\times(1-\cos^2\theta). \quad (28)$$

By using the relation $\delta(f(x))=\delta(x-x_0)/|df(x)/dx|_{x=x_0}$, where $f(x_0)=0$ we obtain, with $s=\omega^2/\omega_0^2$,

$$\frac{dW}{dt}=\frac{e^2\omega_0^2v}{4}\int' \frac{ds(1-\cos^2\theta_+)}{A(s)},$$

where

$$A(s)=\left|1+\frac{\alpha^2(n^2-1)^2(1-n_0^2)}{2n_0^2n^2v^2(n^2-n_0^2)-\alpha^2(n^2-1)(n^2v^2-1)(1-n_0^2)}\right|, \quad (29)$$

$$n_0^2=1-\omega_0^2/\omega^2, \qquad n^2=1/v^2\cos^2\theta_+,$$

and $\cos^2\theta_+$ is given by Eq. (14) with the plus sign in front of the radical sign. The prime on the integral in Eq. (29) indicates that the range of integration is restricted to values of s such that $0\leq\cos^2\theta_+\leq 1$.

We write the Čerenkov loss as

$$dW/dt=\tfrac{1}{4}e^2\omega_0^2 I(\alpha,v), \quad (30)$$

where

$$I(\alpha,v)=v\int' ds\frac{(1-\cos^2\theta_+(s))}{A(s)}. \quad (31)$$

We note that the upper limit on the integral in Eq. (31) is at $\omega_M^2=\omega_0^2+\omega_H^2$. This value of ω corresponds to emission at $\theta=\frac{1}{2}\pi$. According to the relations $\cos^2\theta=1/n^2v^2$ and $n^2=\omega^2/K^2$ we see that at $\theta=\frac{1}{2}\pi$, $K=\infty$ (i.e., $\lambda=0$) where the results of the macroscopic theory cannot be applied. Therefore, for the calculation of the integral in Eq. (31) we have used as an upper limit that value of $\omega(\omega_{\text{lim}})$ which corresponds to wavelengths about 10 times the mean distance between particles of the medium. To determine ω_{lim} we substitute $K\sim 1/10\lambda_d$ (where λ_d is the Debye length) into the relation $n^2=K^2/\omega^2$. That is, ω_{lim} is determined by the equation

$$n^2(\omega_{\text{lim}}^2)=(1/10\lambda_d)^2/\omega_{\text{lim}}^2. \quad (32a)$$

Or, equivalently,

$$n^2(s_{\text{lim}})=(10^{-2}/s_{\text{lim}})(m/kT), \quad (32b)$$

where $s_{\text{lim}}=\omega_{\text{lim}}^2/\omega_0^2$, k is the Boltzman constant and T is the temperature of the plasma. In this paper we are interested mainly in the evaluation of the Čerenkov loss from free electrons in the earth's ionosphere. In this

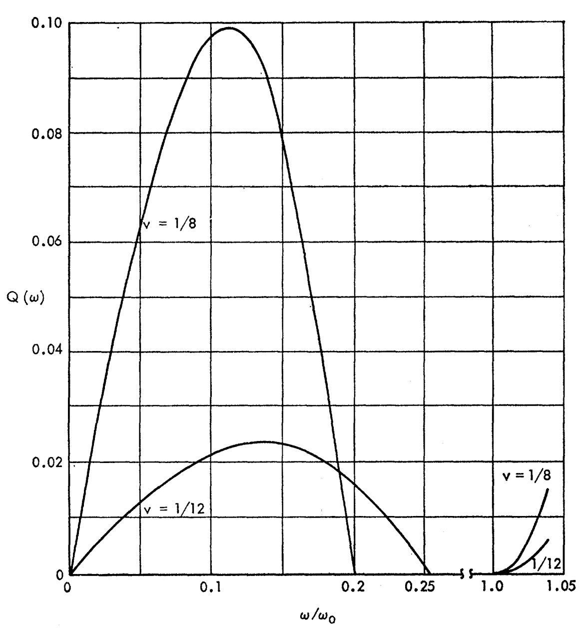

FIG. 7. Normalized power spectrum, $\alpha=0.28$, slow electrons.

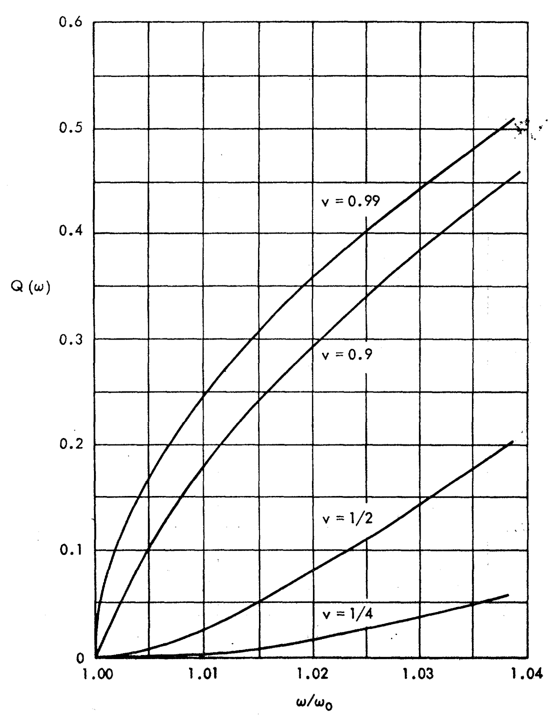

FIG. 8. Normalized power spectrum, $\alpha=0.28$, fast electrons.

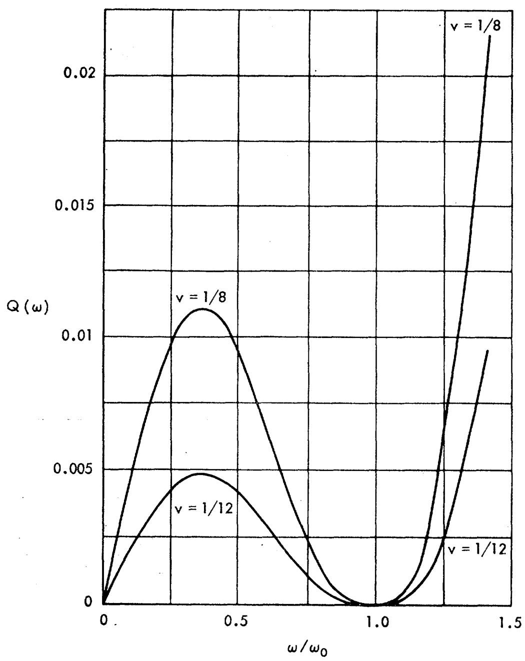

FIG. 9. Normalized power spectrum, $\alpha=1$, slow electrons.

case[8] $\omega_H \simeq 2\pi \times 1.4$ Mc/sec, $\omega_0 \simeq 2\pi \times 5$ Mc/sec (i.e., $\alpha \simeq 0.28$), and $T \sim 1000°\mathrm{K}$,[9] Eq. (32b) becomes

$$n^2(s_{\mathrm{lim}}) = 10^5 / s_{\mathrm{lim}}. \qquad (32\mathrm{c})$$

We have solved Eq. (32c) graphically for $\alpha=0.28$ and various values of v, with the result that ω_{lim}^2 is extremely close to $\omega_M^2 = \omega_0^2 + \omega_H^2 = 1.078\omega_0^2$. That is, negligible error is introduced by integrating out to ω_M^2.

In the next section we give numerical results and compare the Čerenkov loss to the synchrotron loss.

V. NUMERICAL RESULTS AND DISCUSSION

We have evaluated $I(\alpha,v)$ for various values of v and $\alpha=0.28$ $[\omega_H = 2\pi \times (1.4 \text{ Mc/sec}),\ \omega_0 = 2\pi \times (5 \text{ Mc/sec})]$. This value of α corresponds to the case of the earth's magnetic field and ionosphere ($\sim$150 km). Figure 6 shows the total Čerenkov loss as a function of v for $\alpha=0.28$. Equation (20) shows that there will be low-frequency radiation emitted only for $v \lesssim 1/7$. For larger values of v, the radiation will be in the range from ω_0 to $\omega_M = (\omega_0^2 + \omega_H^2)^{1/2} \cong 1.04\omega_0$. The integral $I(0.28, v)$ is of the order 10^{-4} to 10^{-2}. By contrast, Kolomenskiĭ, who incorrectly associates the Čerenkov loss with the polarization losses in a plasma without a magnetic field (i.e., the Coulomb loss), obtains a result that is logarithmically-dependent on a long-wavelength cutoff. He takes the integral to be on the order of unity, and thereby greatly overestimates the energy loss.

It is also of interest to determine the power spectrum $P(\omega)$ (energy radiated per unit time per unit frequency interval). Now, $dW/dt = \int P(\omega)d\omega$, hence from Eq. (29)

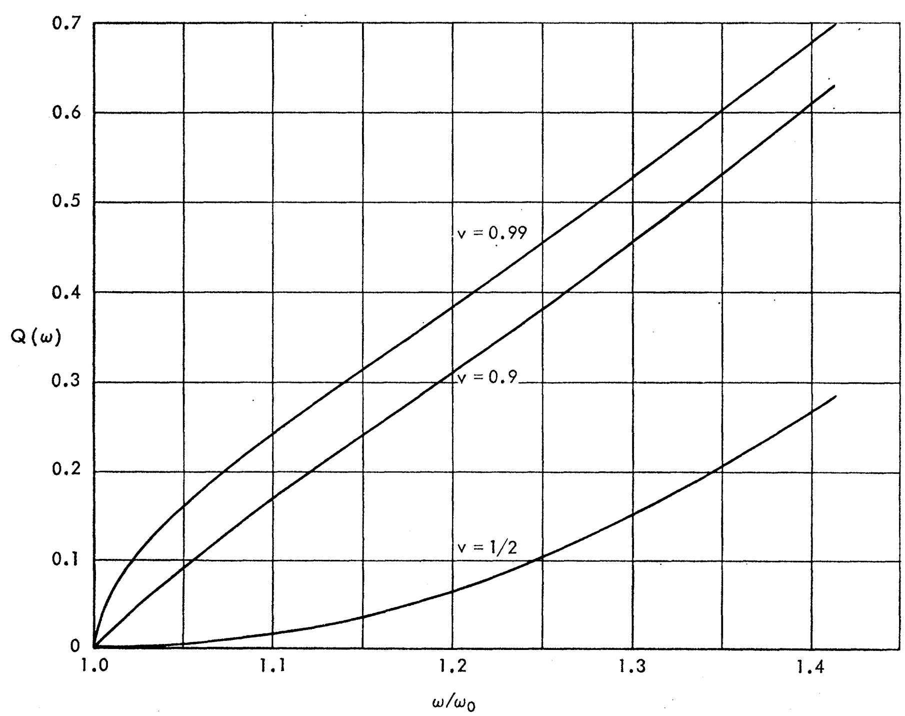

FIG. 10. Normalized power spectrum, $\alpha=1$, fast electrons.

[8] H. K. Kallman-Bijl *et al.*, *Cospar International Reference Atmosphere* (North-Holland Publishing Company, Amsterdam, 1961).
[9] This value of T corresponds to temperature of the ionosphere $\sim$150 km.

we obtain

$$P(\omega)=e^2v\omega(1-\cos^2\theta_+)/2A(\omega)\,,$$
$$\equiv\tfrac{1}{2}e^2vQ(\omega)\,. \qquad (33)$$

Figures 7 through 10 show the normalized power spectrum $Q(\omega)$ for $\alpha=0.28$, 1.0 and various values of v.

We now compare the Čerenkov loss to the synchrotron loss of an electron of the same energy moving in a circle perpendicular to the magnetic field. The synchrotron dW_s/dt loss is given by[10]

$$\frac{dW_s}{dt}=\frac{2e^2\omega_H{}^2v^2}{3(1-v^2)}\,. \qquad (34)$$

The synchrotron loss for $\alpha=0.28$ is also plotted in Fig. 6 as a function of v. At relativistic velocities the synchrotron loss dominates the Čerenkov loss owing to its $(1-v^2)^{-1}$ dependence on the velocity. At nonrelativistic velocities we see that the Čerenkov and synchrotron loss are of the same order of magnitude with the Čerenkov loss being at most ~ 3 times as large as the synchrotron loss for $v\simeq 1/7$.

[10] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1958).

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Enhancement of Plasma Density Fluctuations by Nonthermal Electrons*

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In a plasma in thermal equilibrium, the spectrum of electron density fluctuations that have a wavelength longer than the Debye length has a sharp maximum near the electron plasma frequency. In this paper, the effect of a non-Maxwellian electron velocity distribution on the spectrum of electron density fluctuations is computed for frequencies near the electron plasma frequency. The electron velocity distribution is assumed to be isotropic but not necessarily Maxwellian and the effects of electron-ion collisions are included. The results show how the presence of a small number of energetic electrons can enhance the intensity of the fluctuations near the plasma frequency, provided the Landau damping resulting from these energetic electrons is greater than both the collision damping and the Landau damping caused by the ambient electrons. The results are applied to the ionosphere radar-backscatter experiments, where the energetic electrons are photoelectrons produced by solar uv radiation. In the case of the Arecibo radar experiments, the intensity of the fluctuations near the electron plasma frequency is estimated to be enhanced at plasma frequencies greater than about 4 or 5 Mc/sec.

1. INTRODUCTION

RADAR backscatter from sufficiently high levels in the ionosphere is mainly "incoherent scatter," i.e., scattering from random electron density fluctuations which exist because the electrons are discrete particles. Such experiments single out the spatial Fourier transform of the electron density with wave vector $q=4\pi\lambda^{-1}$, where λ is the radar wavelength. The experiments measure the total backscattered power which is proportional to the mean-square value of the spatial Fourier transform and also the distribution of backscattered power with frequency which is related to the time dependence of the Fourier transform.

A number of authors[1–4] have calculated the theoretical frequency spectrum $I(\omega)$ for a given wave vector. These calculations in general assume (1) that the dynamics of the plasma can be described by the Vlasov equation which neglects charged-particle collisions, and (2) that the electrons and ions have Maxwellian velocity distributions which are not necessarily at the same temperature. The charged-particle collision frequency can be expressed in terms of the parameter, Λ:

$$\Lambda=4\pi nD^3=DK\langle T\rangle e^{-2}=(K\langle T\rangle)^{3/2}(4\pi n)^{-1/2}e^{-3}\,, \quad (1)$$

where D is the electron Debye length. D is defined by

$$D^{-2}=4\pi ne^2K^{-1}\langle T\rangle^{-1} \quad (2)$$

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[1] E. E. Salpeter, Phys. Rev. **120**, 1528 (1960) hereafter referred to as I.
[2] E. E. Salpeter, Phys. Rev. **122**, 1663 (1961).
[3] J. P. Dougherty and D. T. Farley, Proc. Roy. Soc. (London) **A259**, 79 (1960).
[4] J. A. Fejer, Can. J. Phys. **39**, 716 (1961).