reduction of the force against the pinning centers and leaves the pulse height unchanged.

VIII. CONCLUSIONS

Transmission of pulsed perpendicular magnetic fields through type-II superconducting plates in a perpendicular magnetic field has been observed. For sufficiently large pulses pinning is insignificant except very close to H_{c1} . This pinning near H_{c1} can be reduced if the applied field is changed so that the change ΔH aids the pulse field. The data can be interpreted in terms of an effective resistivity ρ . The data points are somewhat below a line given by $\rho/\rho_n = (H - H_{c1})/(H_{c2} - H_{c1})$.

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L_2 - L_3N Coster-Kronig Transition Probabilities and L-Subshell Fluorescence Yields of Hg and Ta[†]

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The probability for the radiationless transfer of L_2 vacancies to the L_3 subshells of Hg and Ta has been measured directly, by observing the spectra of L x rays from Tl²⁰⁴ and W¹⁸¹ in coincidence with K_{α_2} x rays. The L_{α} component of the coincident spectrum provides a measure of the L_2 - L_3N Coster-Kronig transition probability, L_2 - L_3M transitions being energetically impossible in this region. The following values are found: for Hg, $f_{L_2L_3} = 0.08 \pm 0.02$; for Ta, $f_{L_2L_3} = 0.20 \pm 0.04$. The x-ray coincidence experiments show that the average number of x-ray quanta emitted after creation of a primary vacancy in the L_2 subshell is $\nu_{L_2} = 0.42$ ± 0.02 for Hg and $\nu_{L_2} = 0.31 \pm 0.01$ for Ta. Subshell fluorescence yields are found to be $\omega_{L_2} = 0.39 \pm 0.03$ and $\omega_{L_2} = 0.40 \pm 0.02$ for Hg; $\omega_{L_2} = 0.25 \pm 0.02$ and $\omega_{L_3} = 0.27 \pm 0.01$ for Ta.

I. INTRODUCTION

PRECISE *L*-shell fluorescence yields are needed, particularly in the heavy-element region, to obtain nuclear electron-capture ratios from x-ray intensity measurements. In addition, these yields are of interest in atomic physics. The L-shell fluorescence yield of an element depends upon the relative numbers n_i (i=1, 2, 3) of primary vacancies produced in the three L subshells, on the individual subshell fluorescence yields ω_{L_i} , and on the Coster-Kronig transition probabilities $f_{L_iL_i}$ for the radiationless transfer of vacancies between the subshells. The average numbers ν_{L_i} of L x-ray quanta emitted per primary vacancy in the *i*th subshell are

$$\nu_{L_{1}} = \omega_{L_{1}} + f_{L_{1}L_{2}}\omega_{L_{2}} + (f_{L_{1}L_{3}} + f_{L_{1}L_{2}}f_{L_{2}L_{3}})\omega_{L_{3}},$$

$$\nu_{L_{2}} = \omega_{L_{2}} + f_{L_{2}L_{3}}\omega_{L_{3}},$$

$$\nu_{L_{3}} = \omega_{L_{3}}.$$
(1)

In the shifting of vacancies between L subshells by Coster-Kronig transitions, the difference in binding energies is carried off by an outer shell electron. Consequently, such a transition between two subshells is only possible when the difference in binding energies exceeds the ionization energy of an outer shell. For example, an L_2 - L_3M_5 transition is possible for atomic number Z if $E_{L_2}(Z) - E_{L_3}(Z) > E_{M_5}(Z+1)$, assuming that the M binding energy in an atom of atomic number Z with an L vacancy is approximately equal to the Mbinding energy of a neutral atom of atomic number Z+1. The uncertainty inherent in this assumption and in the various subshell binding energies makes it difficult to predict the exact range in atomic numbers where given Coster-Kronig transitions are energetically possible. Approximate ranges in Z for various types of Coster-Kronig transitions have been listed by several authors.¹⁻³ The probability of energetically possible transitions varies with the phase space available to the ejected electron and with the extent to which its wave function overlaps the initial bound-state wave function.

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This overlap depends on the energy of the ejected electron. There is little overlap when this energy is small or very large. The L_1 - $L_{2,3}M_{4,5}$ Coster-Kronig transitions have been the subject of two recent theoretical investigations,^{4,5} but there is a lack of theoretical work on transitions of the L_2 - L_3 type.

It is apparent from Eqs. (1) that determination of L-subshell fluorescence yields generally requires a knowledge of Coster-Kronig transition probabilities. In most experiments on L fluorescence yields, primary vacancies are found in all three L subshells. Some work has been performed, using an x-ray coincidence method, to obtain one partial L-shell fluorescence yield (ω_{KL}) that results from primary vacancies located in the L_2 and L_3 subshells only.⁶⁻¹¹ Individual subshell fluorescence yields have rarely been measured. The earliest experiments to this effect were performed by Küstner and Arends,¹² and by Stephenson.¹³ With suitable incident radiation, these workers produced vacancies in the L_3 subshell alone, or in the L_2 and L_3 subshells, or in all three subshells, and with an ionization chamber compared the intensity of the radiation falling on a radiator with the fluorescence radiation from it. Kinsey¹⁴ has calculated subshell yields by estimating, from the best available measurements, the radiation width of each subshell energy level and dividing it by the total width of the level. Jopson *et al.*¹⁵ have obtained L_3 subshell fluorescence yields through an x-ray coincidence method in which the K_{α_2} component of the K x rays had been eliminated with the help of secondary radiators.

The majority of currently used subshell fluorescence yields and Coster-Kronig transition probabilities have been derived from experimental results on mean L-shell fluorescence yields and L Auger spectra. A detailed example of such a calculation is given by Ross et al.¹⁶ In calculations of this type, the simplifying assumption is usually made that the Coster-Kronig transition probability $f_{L_2L_3}$ vanishes in the region from Z=30 to Z=90. This assumption is based on the fact that, in this region, the binding-energy difference be-

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¹⁵ R. C. Jopson, J. M. Khan, H. Mark, C. D. Swift, and M. A.
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tween the L_2 and L_3 subshells is insufficient to permit ejection of an M electron, which would be the most probable process otherwise. It remained to be tested experimentally whether the L_2 - L_3N transition rate is indeed negligible and hence, whether the assumption $f_{L_2L_3} \approx 0$ (30 < Z < 90) is valid. Some results to the contrary have been reported. From an extensive analysis of available data on Bi, Ross et al.¹⁶ found $f_{L_2L_3} = 0.06 \pm 0.14$ for Z = 83. Nall, Baird, and Haynes¹⁷ report $f_{L_2L_3}=0.22\pm0.04$ for Hg, and Sujkowski¹⁸ lists $f_{L_2L_3} = 0.25 \pm 0.13$ for Tl. However, these results have been obtained by assuming certain values for one or the other of the L subshell fluorescence yields. In the present investigation, a direct experimental determination of the L_2 - L_3N Coster-Kronig transition probability has been made for Hg and Ta.

II. EXPERIMENTAL METHOD

In principle, the experiment consists in observing the spectrum of L x rays in coincidence with K_{α} , x rays. The latter indicate the formation of L_2 vacancies. If these vacancies are filled by radiative transitions from outer shells, x rays characteristic of the L_2 subshell are emitted $(L_{\beta_1}, L_{\gamma_1}, L_{\gamma_5}, L_{\gamma_6}, \text{ etc.})$. If, however, some L_2 vacancies are transferred to the L_3 subshell by Coster-Kronig transitions and the latter vacancies are radiatively filled, then the L x-ray spectrum will include components characteristic of the L_3 subshell. In particular, L_{α} x rays from the most probable L_3 - $M_{4,5}$ transitions will be emitted, and the relative intensity of these x rays can be used to deduce the Coster-Kronig transition probability.

For the experiment on Hg, a thin vacuum-evaporated source of Tl^{204} on Mylar backing was employed. The K x-ray detector was a scintillation counter consisting of a 5-mm thick Harshaw NaI(Tl) crystal with a beryllium window, mounted on a Philips XP1010 photomultiplier tube. To select K_{α_2} x rays, tungsten was used as critical absorber, since its K absorption edge falls between the Hg K_{α_1} and K_{α_2} x-ray energies. The fluorescent W K x rays were absorbed in a layer of Er, and additional Sn and Cu absorbers were included to obtain clean Hg K_{α_2} x rays. Figure 1 shows the composite, unfiltered K x-ray spectrum from Tl^{204} and the spectrum of K_{α_2} x rays transmitted by the series of absorbers. Residual Hg K_{α_1} x rays in the filtered K_{α_2} x-ray beam do not exceed one percent of the beam intensity. Note added in *proof.* The purity of the K_{α_2} x rays was verified with a Ge(Li) detector of 3.4-keV resolution. The line shape of these x rays is indistinguishable from the line shape produced by monochromatic gamma rays.

In the work on Ta, a source of carrier-free W¹⁸¹ was used, which had been vacuum-evaporated on Mylar. Critical absorption with Er, followed by Sm, Sn, and

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FIG. 1. Spectra of Hg K x rays from Tl²⁰⁴. The $K_{\alpha 1}$ component is removed by critical absorption with W. Fluorescent x rays from W are absorbed in layers of Er, Sn, and Cu.

Cu filters, reduced the intensity of Ta K_{α_1} x rays to less than 3% of the filtered beam intensity.

The $L \ge rays$ were detected in a proportional counter¹¹ filled to 1 atm with a 9:1 mixture of argon and methane. The gas was taken through a cold trap before entering the counter. Good stability was attained over the protracted counting periods which were made necessary by low counting rates from the highly collimated $L \ge ray$ beam and the severely attenuated $K \ge rays$.

Because of the inherent fluctuations in the proportional counter's response time, a slow coincidence resolving time of 2.2 μ sec was chosen, lest true coincidence



FIG. 2. Typical spectra of Hg $L \times \text{rays}$ from Tl²⁰⁴, obtained with a proportional counter. Spectrum (a) is gated by unfiltered $K \times \text{rays}$, and hence arises from primary vacancies in both the L_2 and L_3 subshells. Spectrum (b), gated by $K_{\alpha 2} \times \text{rays}$ only, arises from primary vacancies in the L_2 subshell alone; Coster-Kronig transitions account for the residual L_{α} peak.

events be missed. Hence, the chance coincidence counting rates were considerable. The total and chance coincidence spectra were therefore recorded simultaneously in the two halves of a 512-channel analyzer memory, employing two identical coincidence units provided with identical input signals, but one of them having a 25- μ sec delay introduced in one channel.¹⁹ The coincidence units were interchanged for every other run, in order to cancel out errors from any slight difference between the effective resolving times of the two units.

Typical Hg L x-ray spectra are shown in Fig. 2, obtained in coincidence with all K x rays and in coincidence with K_{α_2} x rays only. The small L_{α} peak in the latter spectrum indicates the presence of L_2 - L_3 Coster-Kronig transitions. The spectra were analyzed into L_{α} , L_{β} , and L_{γ} components, including their associated argon escape peaks, with the aid of line shapes derived from Ge, Br, and Rb K x rays excited by fluorescence and recorded under the same conditions. Corrections for attenuation and counter efficiency were performed as described elsewhere¹¹ in order to obtain the intensity ratios $I_{L_{\alpha}}:I_{L_{\beta}}:I_{L_{\gamma}}$, which are listed in Table I.

TABLE I. Relative intensities of L_{α} , L_{β} , and L_{γ} x rays in coincidence with K_{α} x rays, and average number ν_{L_2} of L x-ray quanta emitted after creation of a primary vacancy in the L_2 subshell.

	$I_{L_{\alpha}}: I_{L_{\beta}}: I_{L_{\alpha}}$ for L x rays in coincidence with				
Ζ	$K_{\alpha 1}$ and $K_{\alpha 2}$	$K_{\alpha 2}$ only ^a	ν_{L_2}		
73	53.4:40.2:6.4	15.0:74.7:10.3	0.31 ± 0.01		
80	52.0:40.0:8.0	6.2:76.9:16.9	0.42 ± 0.02		

^a Corrected for residual $K_{\alpha_1} \mathbf{x}$ rays in filtered $K \mathbf{x}$ -ray beam.

III. DETERMINATION OF v_{L_2}

The average number ν_{L_2} of L x-ray quanta emitted in the de-excitation of an atom with an L_2 primary vacancy can be obtained directly from the coincidence counting rate N_c between K_{a_2} and L x rays, because of the relation

$$N_c = N_{K_{\alpha_2}} \epsilon_L a_L \Omega_L \nu_{L_2}. \tag{2}$$

Here, $N_{K_{\alpha_2}}$ is the net counting rate in the K_{α_2} x-ray detector, ϵ_L and Ω_L are the efficiency and solid angle pertaining to the *L* x-ray detector, and a_L is the correction factor for absorption of *L* x rays between source and detector. In computing this absorption correction, the relative intensities of the L_{α} , L_{β} , and L_{γ} components were taken into account.

Since N_c refers to coincidences with all L x rays, it was not necessary to resolve the L x-ray spectrum in determining this quantity. The L x-ray detecting proportional counter could, therefore, be replaced with a scintillation counter for this phase of the experiment. A 5-mm thick NaI(Tl) crystal with Be window provided increased efficiency and solid angle and permitted shorter coincidence resolving time.

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Results of the determinations of ν_{L_2} are included in Table I.

IV. DETERMINATION OF $f_{L_2L_3}$, ω_{L_2} , AND ω_{L_3}

The calculations of this section are based on three relations. The first of these expresses the fact that atoms with a primary vacancy in the L_2 subshell will, in the course of de-excitation, emit L_2 or L_3 x rays in the ratio

$$I_{L_3}/I_{L_2} = (f_{L_2L_3}\omega_{L_3})/\omega_{L_2}.$$
 (3)

The second relation states that the average number ν_{L_2} of L x-ray quanta emitted after creation of a primary vacancy in the L_2 subshell is

$$\nu_{L_2} = \omega_{L_2} + f_{L_2 L_3} \omega_{L_3}. \tag{4}$$

The third relation defines the partial fluorescence yield ω_{KL} , which corresponds to the distribution of primary L subshell vacancies following K_{α} x-ray emission:

$$\omega_{KL} = n_2 \nu_{L_2} + n_3 \omega_{L_3}. \tag{5}$$

The three equations, (3), (4), (5), can be solved to obtain the Coster-Kronig transition probability $f_{L_2L_3}$ and the subshell fluorescence yields ω_{L_2} and ω_{L_3} , once I_{L_3}/I_{L_2} , ν_{L_2} , and ω_{KL} have been determined.

The ratios I_{L_3}/I_{L_2} have been calculated from the measured intensity ratios $I_{L_{\alpha}}:I_{L_{\beta}}:I_{L_{\gamma}}$ (Sec. II). All L_{α} x rays and a small fraction of the L_{β} x rays originate from the filling of L_3 vacancies. The fraction of $L_\beta x$ rays originating from the L_3 shell has been computed from M. Goldberg's measurements.²⁰

An experimental determination of the partial fluorescence yield $\omega_{KL} = 0.41 \pm 0.02$ for Hg has been reported previously.¹¹ The value of ω_{KL} for Ta has been measured by Jopson et al.,8 who obtained 0.28±0.01. A redetermination of this fluorescence yield was performed as part of the present work, with the identical result. The ratio $n_2:n_3$ of primary vacancies following K x-ray emission is equal to the intensity ratio of K_{α_2} to K_{α_1} x rays, tabulated by Nijgh et al.²¹

TABLE II. Measured L_2 and L_3 subshell fluorescence yields and L2-L3 Coster-Kronig transition probabilities.

Element	ω_{L_2}	ω_{L_3}	$f_{L_2L_3}$
73Ta	0.25 ± 0.02	0.27 ± 0.01	0.20 ± 0.04
80Hg	0.39 ± 0.03	0.40 ± 0.02	0.08 ± 0.02

V. DISCUSSION

The L subshell fluorescence yields of Hg and Ta obtained in the present work differ from values calculated under the assumption that the L_2 - L_3 Coster-Kronig transition probability can be neglected.¹⁵ The subshell yields ω_{L_2} reported elsewhere under the assumption $f_{L_2L_3} \approx 0$ are actually the average number ν_{L_2} of L x-ray quanta emitted after creation of a primary vacancy in the L_2 subshell; this number is greater than ω_{L_3} . However, $\nu_{L_2} \neq \omega_{L_2}$ when $f_{L_2L_3} \neq 0$ [cf. Eqs. (1)]. When the nonvanishing L_2 - L_3 Coster-Kronig probability is taken into account, it is found that ω_{L_2} is smaller than ω_{L_3} in the region under consideration. This result is contrary to the general trend assumed, for example, in the review article by Listengarten,³ but it is not unreasonable: x-ray emission and Coster-Kronig transitions (in addition to the ordinary Auger process) are alternative modes of deexcitation for an atom ionized in the L_2 subshell, while only x-ray and ordinary Auger-electron emission compete when an L_3 vacancy is to be filled.

The L_2 - L_3N Coster-Kronig transition probability is found to be smaller for Hg than for Ta. This fact is in qualitative agreement with the trend to be expected, since the N electron ejected from Hg has higher energy (1.1 to 1.5 keV, depending on the subshell) than the N electron ejected from Ta (0.7 to 1.0 keV), and the wave length of these electrons is, in either case, substantially shorter than the radial distance over which the bound-state N-electron wave functions have an appreciable magnitude.²²

Table II contains a summary of the final results.

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