Gyromagnetic Ratios of Hydrogen, Tritium, Free Electrons, and Rb^{85}†*

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The spin-exchange optical pumping technique has been used to measure the gyromagnetic ratios of hydrogen, tritium, and free electrons in terms of the electronic gyromagnetic ratio of Rb^{85}. A direct measurement was also made of the ratio of the electronic g factors of tritium and hydrogen. The 60-G magnetic field in which these measurements were made was produced by a precision-wound solenoid which was surrounded by three concentric magnetic shields. The magnetic field varied by 5 parts in 10^6 over the region occupied by a typical absorption flask. The free electrons were produced by ionization resulting from the beta decay of the radioactive tritium atoms. Measurements were made in various sizes of absorption flasks and in flasks containing different pressures of helium, argon, and neon buffer gases. The measured g-factor ratio did not depend upon the nature or pressure of the buffer gas. The results of these measurements were as follows:

$$g_J(\mathrm{Rb})/g(e)=1+(6.3\pm1.0)\times10^{-6};\quad g_J(\mathrm{Rb})/g_J(\mathrm{H})=1+(23.74\pm0.1)\times10^{-6};$$
$$g_J(\mathrm{T})/g_J(\mathrm{H})=1-(0.11\pm0.3)\times10^{-6};\quad g_J(\mathrm{H})/g(e)=1-(17.4\pm1.0)\times10^{-6}.$$

INTRODUCTION

PRECISION measurements of atomic magnetic moments were originally stimulated by interest in the anomalous magnetic moment of the electron. Theoretical calculations of atomic g_J values were made in order to extract a value for the magnetic moment of the free electron from measurements of the electronic magnetic moments of hydrogen and the alkali metals. Recently, Wilkinson and Crane[1] reported a precise, direct measurement of the gyromagnetic ratio of the free electron. They measured the difference between the electron's spin-precession frequency and its cyclotron frequency in the same field. Their results confirmed the quantum-electrodynamic calculation of the anomalous magnetic moment and determined the g factor of the free electron to three parts in 10^8.

It is still of interest, however, to measure the g_J values of hydrogen and the alkali metals in relation to each other and to the g factor of the free electron. These ratios can be used to test the theoretical calculations of atomic g factors; these ratios can be used as an aid in the determination of the magnetic moment of the free electron. A precise absolute determination of the magnetic moment of the free electron could be combined with the result of Wilkinson and Crane to yield a better value for the ratio of the Bohr magneton to Planck's constant.

Margenau[2] calculated g_J for hydrogen by computing the energy of the hydrogen atom in a magnetic field. He employed first-order perturbation theory and used the Dirac hydrogenic wave functions. Perl[3] calculated g_J for the alkalis using a Dirac many-electron equation and approximate wave functions. Phillips[4] considered the effect of configuration mixing and the breakdown of Russell-Saunders coupling upon the g_J values of the alkali atoms. The results of these calculations have been summarized and tabulated in a review article by Hughes.[5]

The first precision measurements of g_J ratios were carried out by the atomic-beam method.[6,7] The estimated errors were generally of the order of one part in 10^5. In 1958, Dehmelt[8] introduced the spin-exchange optical-pumping technique and measured the magnetic moment of the free electron in terms of the magnetic moment of the sodium atom. Dehmelt's initial measurement had an estimated error of thirty parts in 10^6. Hobart[9] repeated this experiment with the intention of improving its precision. Hobart quoted an estimated error of four in 10^6, but systematic errors which he encountered suggest that this estimate is optimistic. Lambe[10] used a paramagnetic-resonance technique to measure the g factor of the electron in the hydrogen atom in terms of the magnetic moment of the proton in a spherical water sample with a precision of one part in 10^7. Recently, Driscoll[11] measured the ratio of the electronic g factor of Rb^{87} to the free-precession frequency of protons in a spherical water sample with a precision of one part in 10^6.

This paper reports a series of spin-exchange optical-pumping experiments which were performed to measure the ratio of the magnetic moments of the free electron and of the bound electron in the ground state of the hydrogen atom to the magnetic moment of the Rb^{85} atom in its ground state. In these experiments rubidium vapor in a magnetic field was polarized by the absorp-

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* This paper is based on material submitted by one of the authors (L.C.B.) to Harvard University in partial fulfillment of the requirements for the Doctor of Philosophy degree.
[1] D. T. Wilkinson and H. R. Crane, Phys. Rev. **130**, 852 (1963).
[2] H. Margenau, Phys. Rev. **57**, 383 (1940).
[3] W. Perl, Phys. Rev. **91**, 852 (1953).
[4] M. Phillips, Phys. Rev. **88**, 202 (1952).
[5] V. W. Hughes, *Recent Research in Molecular Beams*, edited by I. Estermann (Academic Press Inc., New York, 1959), pp. 65-92.
[6] P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948).
[7] S. H. Koenig, A. G. Prodell, and P. Kusch, Phys. Rev. **88**, 191 (1952).
[8] H. Dehmelt, Phys. Rev. **109**, 381 (1958).
[9] J. Hobart, thesis, University of Michigan, 1962 (unpublished).
[10] E. B. D. Lambe, thesis, Princeton University, 1959 (unpublished).
[11] R. L. Driscoll, Phys. Rev. **136**, A54 (1964).

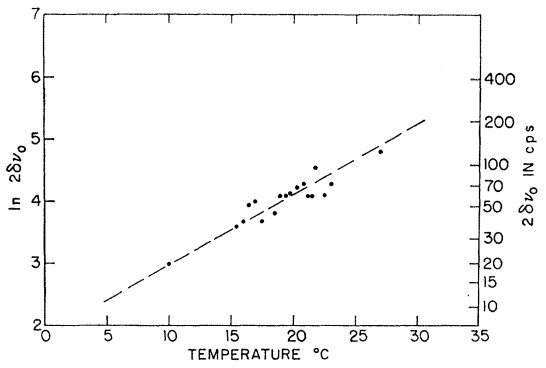

FIG. 1. A plot of the frequency shift versus bulb temperature for free electrons in spin-exchange equilibrium with polarized rubidium atoms.

tion of circularly polarized optical-resonance radiation incident along the direction of the magnetic field. The polarization was monitored by observing the transmission of the resonance radiation through the flask containing the rubidium atoms. Free electrons or hydrogen atoms present in the flask were oriented by spin-exchange collisions with the rubidium atoms. When a radio-frequency field was applied so as to depolarize the electrons, the rubidium atoms were also partially depolarized through spin-exchange collisions and the intensity of the light transmitted by the absorption flask decreased. The rubidium atoms were depolarized directly when a radio-frequency field was applied at one of the ground-state Zeeman frequencies of the rubidium atom. Using this technique, the Zeeman frequencies of rubidium atoms, free electrons, hydrogen atoms, and tritium atoms in the same magnetic field were observed. From the measurements of the various transition frequencies, the ratios of the various g factors were determined. The purpose of these experiments was twofold; first, to improve the precision with which the g-factor measurements were known, and second, to see how well one could measure the magnetic moment of the free electron with a spin-exchange technique.

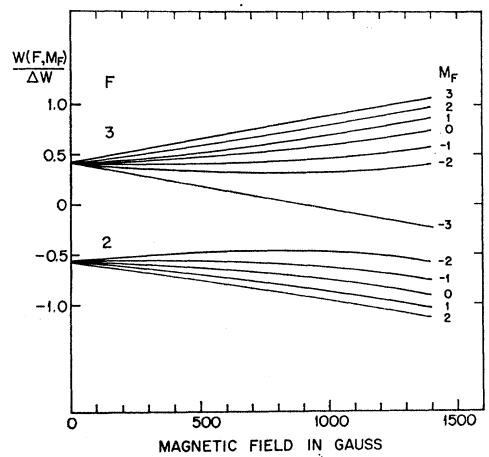

FIG. 2. The energy levels of a Rb^{85} atom in its ground state as a function of the magnetic field.

In the next section, we explicitly write down the dependence of the observed transitions upon the g factors and discuss the frequency shifts associated with the spin-exchange collisions. Then we describe the apparatus, with emphasis on the solenoid which was used to provide a homogeneous magnetic field. Next, the auxiliary physical quantities which enter into the determination of the g_J ratios are discussed. The last section deals with the measurements. The results are compared with the theoretical g_J calculations and with the results of Driscoll and Lambe. A more precise value for μ_0/h is given.

THEORETICAL CONSIDERATIONS

The spin-resonance frequency of the free electron is

$$\nu(e)=|g(e)|\mu_0 H_0/h. \quad (1)$$

Here $|g(e)|$ is the absolute value of the electron g factor, μ_0 is the Bohr magneton, and H_0 is the static magnetic field. The general theory of spin-exchange optical pumping[12,13] predicts that the electron-resonance signal will have a Lorentzian line shape and a center frequency which is shifted by an amount $\delta\nu$ from the true spin resonance frequency $\nu(e)$. The frequency shift is proportional to the rubidium polarization, the density of the rubidium atoms in the bulb, and the velocity of the electrons relative to the rubidium atoms. A full treatment of the theory of the spin-exchange frequency shifts is given in Refs. 12 and 13. The detailed predictions of the theory do not agree quantitatively with experiment, but the predicted dependence of the frequency shift upon the rubidium polarization, rubidium density, and the velocity of the electrons is correct. A logarithmic plot of the frequency shift due to electron-rubidium collisions as a function of bulb temperature is shown in Fig. 1. The sign of the polarization of the rubidium atoms changes when the polarization of the incident pumping light is changed from left to right circular polarization. The frequency shift can be cancelled by measuring the center frequency of the electron signal first with left circular polarization and then with right circular polarization and averaging the two measurements. In principle, the rubidium and hydrogen signals are also subject to spin-exchange frequency shifts. They are, however, expected to be smaller. No shifts were observed in these experiments for either hydrogen or rubidium.

Figures 2 and 3 depict the energy levels of the ground state of Rb^{85} and hydrogen and tritium as a function of the static magnetic field. Just which Zeeman transitions in rubidium and hydrogen were observed was dictated by convenience. The same oscillator was used

[12] L. C. Balling, R. J. Hanson, and F. M. Pipkin, Phys. Rev. 133, A607 (1964).
[13] L. C. Balling and F. M. Pipkin, Phys. Rev. 136, A46 (1964).

to provide the radio-frequency field for the rubidium signal and the fundamental frequencies for the hydrogen, tritium, and electron signals. The transitions chosen minimized the amount of retuning required when one switched from one signal frequency to another.

In Rb^{85}, the $(F=3,\ M=-2\rightarrow F=3,\ M=-3)$ transition was observed. The observed transition in hydrogen and tritium was the $(F=1,\ M=1\rightarrow F=1,\ M=0)$ transition. The energy levels of the ground states of Rb^{85}, H, and T are given by the Breit-Rabi formula. The dependence of the observed rubidium transition upon $g_J(\mathrm{Rb})$ is given by the equation

$$\nu(\mathrm{Rb})_{(F=3,M=-2\rightarrow F=3,M=-3)}=-(\mu_I/I)(H_0/h)+\tfrac{1}{2}[\Delta\nu(\mathrm{Rb})][(1-\tfrac{4}{3}x+x^2)^{1/2}-(1-x)], \quad (2)$$

where

$$x=(|g_J|+g_I)\mu_0 H_0/h\Delta\nu(\mathrm{Rb}), \quad (3)$$

$$\mu_I/I=g_I\mu_0, \quad (4)$$

and

$$\mu_J/J=g_J\mu_0=-|g_J|\mu_0. \quad (5)$$

Here μ_0 is the Bohr magneton; H_0 is the static magnetic field; $\Delta\nu(\mathrm{Rb})$ is the hyperfine splitting of Rb^{85}.

The hydrogen transition frequency is given by the equation

$$\nu(\mathrm{H})_{(F=1,M=1\rightarrow F=1,M=0)}=\mu_I H_0/Ih+\tfrac{1}{2}[\Delta\nu(\mathrm{H})]\times[(1+x)-(1+x^2)^{1/2}], \quad (6)$$

where the meanings of all symbols are the same as in Eqs. (2) through (5) except that they refer to the properties of the hydrogen atom. An identical equation holds for the observed Zeeman transition in tritium.

From these equations it follows that

$$\frac{g_J(\mathrm{Rb})}{g(e)}=\left(1+\frac{g_I(\mathrm{Rb})}{|g_J(\mathrm{Rb})|}\right)^{-1}\frac{6\nu'(\mathrm{Rb})}{\nu(e)}\times\frac{[\nu'(\mathrm{Rb})+\Delta\nu(\mathrm{Rb})]}{[6\nu'(\mathrm{Rb})+\Delta\nu(\mathrm{Rb})]}, \quad (7)$$

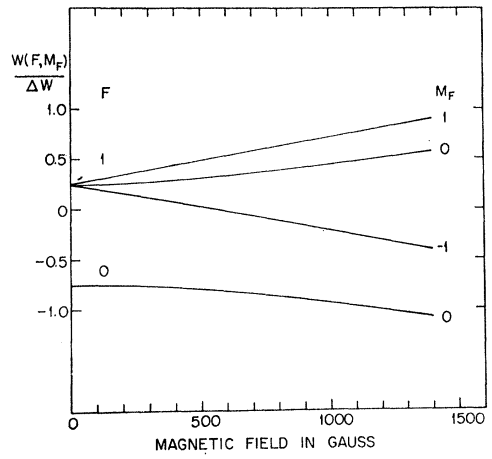

FIG. 3. The energy levels of a hydrogen or tritium atom in its ground state as a function of the magnetic field.

where

$$\nu'(\mathrm{Rb})=\nu(\mathrm{Rb})+(\mu_I(\mathrm{Rb})/I)(H_0/h). \quad (8)$$

Similarly,

$$\frac{g_J(\mathrm{Rb})}{g_J(\mathrm{H})}=\frac{(1+g_I(\mathrm{H})/|g_J(\mathrm{H})|)}{(1+g_I(\mathrm{Rb})/|g_J(\mathrm{Rb})|)}\frac{3\nu'(\mathrm{Rb})}{\nu'(\mathrm{H})}\times\frac{[\nu'(\mathrm{Rb})+\Delta\nu(\mathrm{Rb})]}{[6\nu'(\mathrm{Rb})+\Delta\nu(\mathrm{Rb})]}\frac{[\Delta\nu(\mathrm{H})-2\nu'(\mathrm{H})]}{[\Delta\nu(\mathrm{H})-\nu'(\mathrm{H})]}, \quad (9)$$

with

$$\nu'(\mathrm{H})=\nu(\mathrm{H})+(\mu_I(\mathrm{H})/I)(H_0/h). \quad (10)$$

Also,

$$\frac{g_J(\mathrm{T})}{g_J(\mathrm{H})}=\frac{(1+g_I(\mathrm{H})/|g_J(\mathrm{H})|)}{(1+g_I(\mathrm{T})/|g_J(\mathrm{T})|)}\frac{\nu'(\mathrm{T})}{\nu'(\mathrm{H})}\times\frac{[\Delta\nu(\mathrm{T})-\nu'(\mathrm{T})]}{[\Delta\nu(\mathrm{T})-2\nu'(\mathrm{T})]}\frac{[\Delta\nu(\mathrm{H})-2\nu'(\mathrm{H})]}{[\Delta\nu(\mathrm{H})-\nu'(\mathrm{H})]}. \quad (11)$$

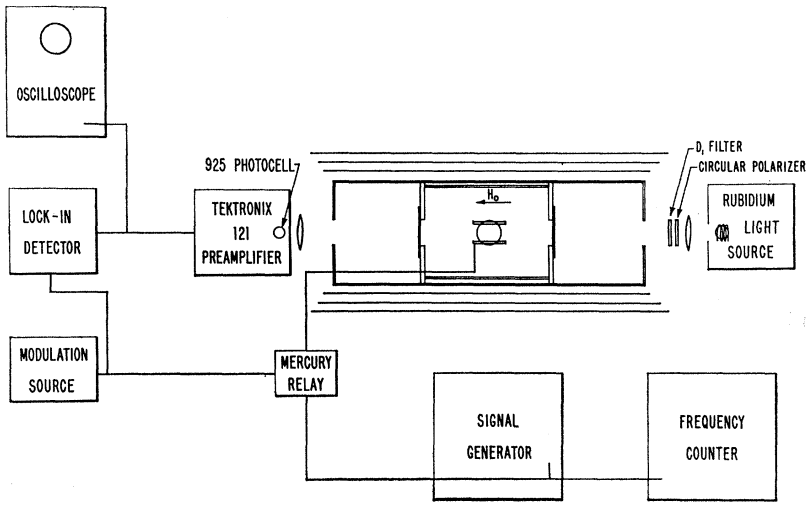

FIG. 4. A block diagram of the optical-pumping apparatus.

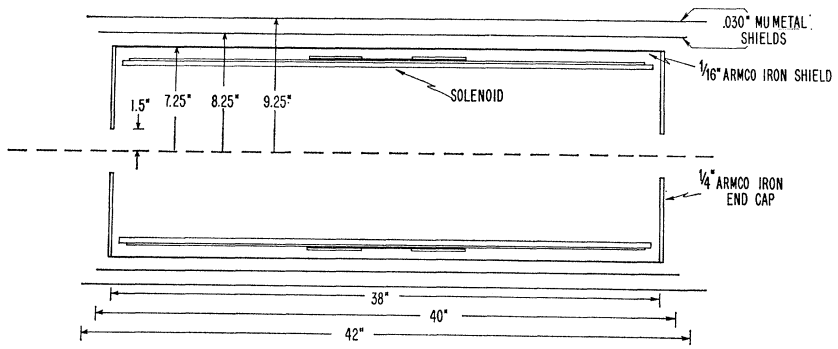

FIG. 5. A diagram showing the solenoid and shield assembly employed to produce the homogeneous magnetic field.

These equations enable one to calculate the g-factor ratios from measurements of the various transitions.

APPARATUS

A block diagram of the optical-pumping apparatus is shown in Fig. 4. The radio-frequency power used to drive the transitions was amplitude-modulated by a mercury relay driven at 10 cps. The fluctuations in the light transmitted by the absorption flask were detected by a photocell, amplified, and displayed on an oscilloscope and a lock-in detector. The principal difference between this arrangement and that used in previous optical-pumping experiments performed in this laboratory[14,15] was the manner in which the magnetic field H_0 was produced.

As the results of this experiment will emphasize, the real limitation on the precision with which one can measure the various g-factor ratios is the homogeneity and temporal stability of the static magnetic field. In order to produce a magnetic field of very high homogeneity, a special solenoid was constructed. This solenoid was equipped with field-correction coils designed to increase the homogeneity of the magnetic field, and it was placed inside three concentric magnetic shields in order to reduce the perturbing effect of external magnetic fields.

Figure 5 shows the solenoid and shield assembly. Since this solenoid and its performance have been described in another publication, we shall only give a brief description of it here.[16] The solenoid was 36 in. long and had an inner diameter of 12 in. The main solenoid winding consisted of two layers of No. 18 Formvar insulated copper wire wound at the rate of 20 turns/in. onto an epoxy-covered aluminum cylinder. The main correction coil was a short solenoid whose center was the same as the center of the main solenoid. The length of this short solenoid was chosen so that the power-series expansion which gave the variation of the field along its axis contained no fourth-power term. Separate leads were provided for the correction solenoid so that the current through it could be adjusted to obtain the most homogeneous field. The magnetic shield immediately around the solenoid was $14\frac{1}{2}$ in. in diameter, 38 in. long, and it was made of $\frac{1}{16}$-in.-thick annealed soft iron. The next shield was $16\frac{1}{2}$ in. in diameter, 40 in. long, and made of 0.030-in.-thick mu metal. The outermost shield was $18\frac{1}{2}$ in. in diameter, 42 in. long, and it was made of 0.030-in.-thick mu metal. The ends of the solenoid were covered with $\frac{1}{4}$-in.-thick soft iron plates which were held firmly against the ends of the innermost shield. A 3-in.-diam hole was cut into the center of each of these iron plates in order to allow passage of the light beam. Each of the shields was provided with a set of demagnetization windings, and the shields were demagnetized by passing a large ($\sim$100 A) sixty-cycle current through these windings. The solenoid field was roughly 20 G/A. When the central field was 60 G and the current in the correction solenoid was adjusted to obtain the most homogeneous field, the magnetic field varied by 5 parts in 10^6 over a spherical region 5 cm in diameter.

For the experiments reported in this paper, the currents of 2.9 A for the solenoid and of 25 mA for the correction coil were provided by a current-stabilized power supply. Current stabilization was achieved by comparing the voltage drop across a 10-Ω reference resistor, in series with the solenoid, with the voltage of a series string of mercury cells, and feeding back the difference voltage to the transistors which controlled the output current of the power supply. The long-term drift of the magnetic field was found to be of the order of a few parts in 10^6 over a period of one hour.

Two types of absorption bulbs were used for these measurements. The first type was used for the electron measurements and consisted of spherical and cylindrical pyrex bulbs approximately 25 cm^3 in volume. They contained a neon, argon, or helium buffer gas at various pressures, rubidium, and approximately 1 Ci of tritium. A drawing of the second type of bulb which was used for the hydrogen and tritium measurements is shown in Fig. 6. Atomic hydrogen was produced in a radio-frequency discharge between glass-covered electrodes

[14] L. W. Anderson, F. M. Pipkin, and J. P. Baird, Jr., Phys. Rev. **116**, 87 (1959); **120**, 1279 (1960); (Erratum) **121**, 1864 (1961); (Erratum) **122**, 1962 (1961).

[15] R. H. Lambert, thesis, Harvard University, 1964 (unpublished).

[16] R. J. Hanson and F. M. Pipkin, Rev. Sci. Instr. **36**, 179 (1965).

and diffused down into the main bulb which was 25 cm^3 in volume. This arrangement makes it easier to control the number of hydrogen atoms in the absorption bulb.

The radio-frequency fields used to drive the transitions were derived from a Gertsch AM-1 frequency multiplier which operated off a standard 1-Mc/sec signal. The time base for the frequency counter was also derived from this 1-Mc/sec reference. The reference frequency was generated by a General Radio 1120 AH 1000 megacycle frequency standard, the oscillator of which was locked to the average frequency of an Atomichron and of a Varian rubidium frequency standard. The General Radio frequency was periodically compared to the Loran C and N.B.S. standard frequency signals. These frequencies are maintained constant each year with reference to atomic standards of frequency, but they are offset to keep the time pulses in close agreement with the UT2 time scale, which is used for navigation. The atomic time scale is such that the Cs^{133} hyperfine splitting is 9192631770 cps. In 1964, the transmitted frequencies were lower than this by 150 parts in 10^{10}. The g-factor ratios depend on the frequency standard through the values of the rubidium and hydrogen hyperfine splitting used in reducing the data.

AUXILIARY PHYSICAL CONSTANTS

It can be seen from Eqs. (7) through (11) that in order to determine the g-factor ratios one must know the hyperfine splittings for Rb^{85}, H, and T, and the nuclear moments of these atoms. The hyperfine splittings vary with the pressure and temperature of the various buffer gases. Bender[17] has measured these shifts for Rb^{87}; his results can be carried over to Rb^{85} if it is assumed that the fractional shift is independent of the isotope. Pipkin and Lambert[18] have measured the pressure shifts for hydrogen and tritium. Because the pressure shifts for rubidium are large and the buffer gas pressures in some of the bulbs were somewhat uncertain, it was decided to measure the rubidium hyperfine splittings in each of the absorption bulbs.

The measurements of the rubidium hyperfine splittings were made in a magnetic field of 25 mG. Microwave power from a Laboratory for Electronics Model 814-A-S-1 microwave oscillator was fed into a crude resonant cavity made from two wavelengths of copper waveguide. Two 2-in.-diam holes were cut in the sides of the cavity to permit the passage of the pumping light, and the absorption bulbs were placed inside the cavity. The magnetic field was modulated by applying a 10-cps sine wave to the second-order correction coil, and the signals were displayed on an oscilloscope. The measured values for the hyperfine splittings were in good agreement with those calculated from Bender's

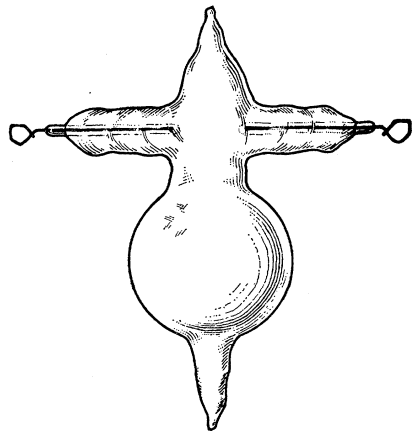

FIG. 6. A drawing of one of the absorption flasks used for the hydrogen and tritium measurements. The atoms were produced by a discharge in the upper part of the bulb and then diffused down into the lower part where they were observed.

measurements and the temperature and pressures of the bulbs. The hyperfine splittings for hydrogen and tritium were calculated from the bulb pressures and the pressure shifts of Pipkin and Lambert. The error in the measurements of the Rb^{85} hyperfine splittings is estimated to be typically ± 500 cps. The estimated upper limit to the error in the bulbs with the highest buffer gas pressures is ± 1000 cps. The error in the calculated hyperfine splittings for hydrogen and tritium is estimated to be only ± 20 cps. This is due to the fact that the pressure shifts for hydrogen and tritium are a factor of 50 smaller than those for Rb^{85}. The estimated maximum error in the g-factor ratios obtained from any one bulb due to an error in the hyperfine splittings used in the calculation of the ratios is one part in 10^8.

The nuclear moments of hydrogen, tritium, and Rb^{85} were obtained from the following experimental numbers[10,11,19–21]

$$g_p'\mu_0/h = 2.67513(1)\times 10^4 \text{ sec}^{-1}\text{ G}^{-1},$$

where g_p' is the g factor for protons in a spherical water sample.

$$|g_J(\text{Rb})|/g_p' = 658.2323(7);$$
$$g_I(\text{Rb})/|g_J(\text{Rb})| = 1.466764(30)\times 10^{-4};$$
$$|g_J(\text{H})|/g_I(\text{H}) = 658.21591(4);$$
$$g_I(\text{T})/g_I(\text{H}) = 1.06663986(11);$$

and the diamagnetic correction[22] for the proton moment in water

$$g_p'/g_p = 1 - 2.7\times 10^{-5},$$

where g_p is the "true" g factor of the proton.

[17] P. L. Bender, E. C. Beaty, and A. R. Chi, Phys. Rev. Letters **1**, 311 (1958).
[18] R. H. Lambert and F. M. Pipkin, Phys. Rev. **127**, 787 (1962).
[19] P. Vigoureux, Nature **198**, 1188 (1963).
[20] S. Penselin, T. Moran, V. W. Cohen, and G. Winkler, Phys. Rev. **127**, 524 (1962).
[21] W. Duffy, Jr., Phys. Rev. **115**, 1012 (1959).
[22] H. Gutowsky and C. Hoffman, J. Chem. Phys. **19**, 1259 (1951).

TABLE I. A summary of the measurements of the ratio of the rubidium electronic g factor to the g factor of the free electron. Each measurement consists of four measurements of the electron frequency and two measurements of the rubidium frequency. Two of the electron measurements were made with left circularly polarized light and two with right circularly polarized light. The standard deviation is the standard deviation of the distribution of the measurements, and it is not the expected standard deviation of the mean. If the set of measurements made on each bulb is called a run, then the unweighted average is the average of the ratios obtained from the seven runs. The weighted average is the average obtained when each of the runs is weighted inversely with the standard deviation of the run. The standard deviation for the distribution of ratios was calculated from the mean ratios of the seven runs. [Unweighted average for $g_J(\mathrm{Rb})/g(e)$—$1+6.23\times10^{-6}$; weighted average for $g_J(\mathrm{Rb})/g(e)$—$1+6.31\times10^{-6}$; standard deviation of distribution of ratios from various runs—1×10^{-6}.]

Buffer gas	Buffer gas pressure at 20°C in mm Hg	Tritium pressure at 20°C in mm Hg	$\Delta\nu$(Rb) in Mc/sec	Number of measurements	$\frac{g_J(\mathrm{Rb})}{g(e)}$	Standard deviation
Helium	43.9	2.3	3035.747605	41	$1+8.43\times10^{-6}$	9.2×10^{-7}
Neon	41.7	2.3	3035.740261	31	$1+6.64\times10^{-6}$	5.9×10^{-7}
Argon	107.9	1.3	3035.730268	11	$1+4.66\times10^{-6}$	18.9×10^{-7}
Helium	88.9	1.5	3035.762547	24	$1+5.97\times10^{-6}$	9.3×10^{-7}
Neon	165.5	2.2	3035.762362	26	$1+5.79\times10^{-6}$	5.5×10^{-7}
Argon	36.2	1.8	3035.732012	25	$1+6.18\times10^{-6}$	5.8×10^{-7}
Neon	99.7	1.5	3035.750659	30	$1+5.95\times10^{-6}$	7.5×10^{-7}

Only the absolute measurement of the precession frequency of the protons in water is sufficiently uncertain to be a possible source of error in the determination of the g-factor ratios. We have used Vigoureux's value which agrees with the number obtained by Bender.[23] Vigoureux estimates his error to be less than 1 in 10^5. His number, however, disagrees with the value obtained by Captuller[24] by 2 in 10^5 and with Willhelmy's[25] value by 1 in 10^4. It seems safe to assume that Vigoureux is not more than 1 in 10^4 off; an error of 1 in 10^4 would result in an error of less than 4 in 10^9 in the g-factor ratios.

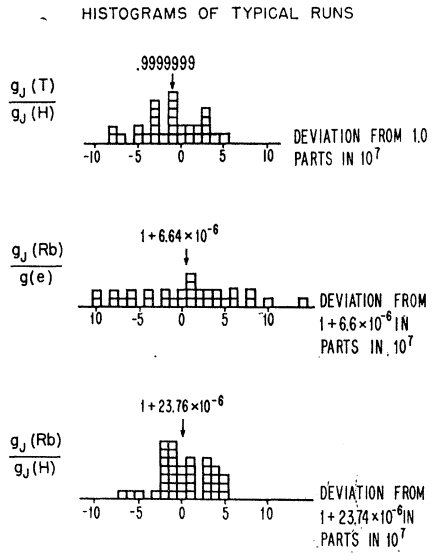

FIG. 7. Histograms showing the distribution of the measured ratios for typical runs. The sharpness and symmetry of these histograms give an indication of the reliability of the measurements.

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[24] H. Captuller, Z. Instrumentenk. **69**, 191 (1961).
[25] W. Willhelmy, Ann. Phys. Leipzig **19**, 329 (1957).

MEASUREMENTS

The procedure for obtaining the g-factor ratio measurements was as follows: The equipment was allowed to warm up for several hours in order to stabilize the field. The shields were demagnetized, and the width of the rubidium signal was measured to determine the homogeneity of the field. The bulb was then moved along the axis of the solenoid, and the resonance frequency for the rubidium was measured as a function of position. The bulb was positioned so that it was in the magnetic center where the field was maximum. Measurements of the electron-resonance frequency were made first with left circularly polarized and then with right circularly polarized light. In between these two sets of measurements, the rubidium frequency was measured. This procedure cancelled out the spin-exchange frequency shift and compensated to first order for the drift in the magnetic field. The hydrogen and tritium measurements were made in the same way.

The shape of the rubidium line was determined by the character of the magnetic field, and the line was slightly asymmetrical. The full width at half-maximum of the rubidium line varied between 150 and 200 cps for the various runs. The center frequency of the rubidium line was approximately 29.5 Mc/sec. The hydrogen, tritium, and electron lines had symmetrical line shapes due to the fact that their natural widths masked the effect of the magnetic field. The center frequencies of the hydrogen and tritium lines were approximately 79.3 and 79.6 Mc/sec, respectively, and the width of the lines was approximately 900 cps. The center frequency of the electron line was approximately 169 Mc/sec, and its width was typically 4500 cps.

Moving the absorption flask along the solenoid axis, away from the center, increased the width and asymmetry of the rubidium line. Such variations were masked in the hydrogen and electron lines. It was discovered at an early stage that the value of $g_J(\mathrm{Rb})/g(e)$ obtained

TABLE II. A summary of the measurements of the ratio of the rubidium electronic g factor to the electronic g factor of the hydrogen atom. The data for the tritium atom has been included in this table. An explanation of what a measurement consists of and of the meaning of the various quantities is given in the caption of Table I. The averages include both the hydrogen and tritium measurements. (Unweighted average—$1+23.76\times10^{-6}$; weighted average—$1+23.74\times10^{-6}$; standard deviation of distribution of ratios—1×10^{-7}).

Buffer gas	Buffer gas pressure at 20°C in mm Hg	Hydrogen or tritium pressure at 20°C in mm Hg	$\Delta\nu$(H) or $\Delta\nu$(T) in Mc/sec	$\Delta\nu$(Rb) in Mc/sec	Number of measurements	$\frac{g_J(\mathrm{Rb})}{g_J(\mathrm{H})}$ or $\frac{g_J(\mathrm{Rb})}{g_J(\mathrm{T})}$	Standard deviation
Neon	17.1	4.1 H_2	1420.405800	3035.735900	14	$1+23.74\times10^{-6}$	2.4×10^{-7}
Helium	199.	2.0 H_2	1420.406971	3035.797300	41	$1+23.76\times10^{-6}$	2.9×10^{-7}
Helium	39	3.3 H_2	1420.405973	3035.746034	55	$1+23.64\times10^{-6}$	1.9×10^{-7}
Neon	42.3	5.9 H_2	1420.405888	3035.741810	19	$1+23.95\times10^{-6}$	4.9×10^{-7}
Helium	43.1	Unknown	1420.406001	3035.747490	20	$1+23.63\times10^{-6}$	2.4×10^{-7}
Helium	43.1	2.7 H_2[3]	1516.701743	3035.747490	66	$1+23.84\times10^{-6}$	2.4×10^{-7}

from the measured frequencies depended on the position of the bulb in the solenoid. The ratio had a minimum at the center of the solenoid and increased monotonically as one moved the bulb along the axis in either direction from the center. A displacement of the bulb of 0.5 in., which was the radius of the bulb, resulted in a fractional increase in the ratio of 2×10^{-6}. Since the magnetic-field-dependent line shape of the electron signal was completely hidden, it was difficult to determine the cause of this variation. It appeared, however, that the electrons, which moved through the bulb at higher velocities than the rubidium atoms, saw a different average magnetic field than the rubidium atoms. The same effect was observed with hydrogen and tritium, but the fractional increase of the ratio $g_J(\mathrm{Rb})/g_J(\mathrm{H})$ was an order of magnitude less than for $g_J(\mathrm{Rb})/g(e)$ with the same bulb displacement. Since in this experiment we were measuring the center of two lines which did not have the same shape and width, various tests were made to see what features of the lines seemed to give the most reliable measurements. The procedure evolved was to carefully center the bulb before each run and then to measure the center of the two lines.

Histograms of typical runs are shown in Fig. 7. The width of the electron line is reflected in the flat character of the electron histogram. The histogram for $g_J(\mathrm{T})/g_J(\mathrm{H})$ is particularly symmetrical. One expects the measurement of $g_J(\mathrm{T})/g_J(\mathrm{H})$ to be very reliable, since the dependence of the line shape on the magnetic field should be identical for the H and T signals. Figure 8 is a plot of the ratio $g_J(\mathrm{Rb})/g(e)$ as a function of buffer gas and buffer gas pressure. There is no strong dependence on the pressure. The average values of the g-factor ratios from each run are listed in Tables I, II, and III. The ratios were calculated from the measured frequencies with the aid of an IBM-7094 computer.

Tables I and II indicate that the standard deviation divided by the square root of the number of measurements is not a valid measure of the reliability of the average g-factor ratio obtained from a given run. The standard deviation, itself, to some extent reflects the homogeneity of the magnetic field, and in this sense it is a measure of the reliability of the result obtained from a given run.

Two additional possible sources of systematic error must be considered. Shifts in the frequency of the rubidium transitions can occur with increasing pumping light intensity.[26] Owing to the length of the solenoid, the intensity of the pumping light reaching the bulb was just sufficiently high to work with, and no trouble from light shifts was expected. A reduction in the light intensity by means of neutral density filters produced no detectable shift in the rubidium transition frequency. The fact that the electrons in the bulb are not at rest, but move with thermal velocities,[27] leads to a correction to the theoretical value of g. This correction amounts to less than 1 in 10^8 and has been ignored.

The quoted errors for the g-factor ratios have been estimated by calculating the standard deviation of the average ratios obtained from each individual run. We have deliberately been conservative and have not divided the resultant standard deviation by the square root of the number of runs.

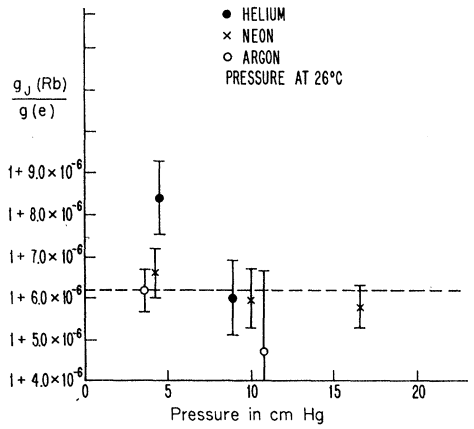

FIG. 8. A plot of the ratio $g_J(\mathrm{Rb})/g(e)$ as a function of the buffer gas pressure for helium, neon, and argon buffer gases. There is no strong dependence of the g-factor ratios on the character or pressure of the buffer gas.

[26] M. Arditi and T. R. Carver, Phys. Rev. **124**, 800 (1961).
[27] H. Mendlowitz and K. M. Case, Phys. Rev. **97**, 33 (1955).

TABLE III. A summary of the measurements of the ratio of the electronic g factor of tritium to the electronic g factor of hydrogen. No hydrogen was deliberately introduced into the bulb. What was there entered via the rubidium or the tritium. An explanation of what a measurement consists of and of the meaning of the various quantities is given in the caption of Table I.

Buffer gas	Buffer gas pressure at 20°C in mm Hg	Tritium pressure at 20°C in mm Hg	$\Delta\nu$(T) in Mc/sec	$\Delta\nu$(H) in Mc/sec	Number of measurements	g_J(T)/g_J(H)	Standard deviation
Helium	43.1	2.7	1516.701743	1420.406001	30	$1-1.1\times10^{-7}$	3.4×10^{-7}

We find the measured ratios

$$g_J(\mathrm{Rb})/g(e)=1+(6.3\pm1.0\times10^{-6}),$$
$$g_J(\mathrm{Rb})/g_J(\mathrm{H})=1+(23.7\pm0.1\times10^{-6}),$$
$$g_J(\mathrm{T})/g_J(\mathrm{H})=1-(0.11\pm0.3\times10^{-6}),$$

and the derived ratio

$$g_J(\mathrm{H})/g(e)=1-(17.4\pm1.0\times10^{-6}).$$

These ratios can be compared with the theoretical values

$$g_J(\mathrm{Rb})/g(e)=1+10.4\times10^{-6},$$
$$g_J(\mathrm{Rb})/g_J(\mathrm{H})=1+28.1\times10^{-6},$$
$$g_J(\mathrm{T})/g_J(\mathrm{H})=1.0,$$

and

$$g_J(\mathrm{H})/g(e)=1-17.7\times10^{-6}.$$

Table IV summarizes the theoretical corrections to the rubidium g factor.

Our results can be compared with the measurements of Driscoll and Lambe. Driscoll obtained for $|g_J(\mathrm{Rb})|/g_p'$ the result

$$|g_J(\mathrm{Rb})|/g_p'=658.2323\pm0.0007.$$

TABLE IV. A summary of the theoretical calculations of the g factors of the free electron, hydrogen, and rubidium. The atomic g_J values differ from g for the free electron by the additive corrections due to relativistic effects (calculated by Perl) and to configuration mixing (calculated by Phillips).

System of interest	Correction calculated by Perl	Correction calculated by Phillips	$g/2$
Free electron	0	0	1.0011596
Hydrogen	-17.7×10^{-6}	0	1.0011419
Rubidium	-11×10^{-6}	$+21.4\times10^{-6}$	1.00117

Lambe has measured $|g_J(\mathrm{H})|/g_p'$ and obtained the number

$$|g_J(\mathrm{H})|/g_p'=658.21591\pm0.0004.$$

These numbers can be combined to give an independent experimental determination of g_J(Rb)/g_J(H). It is

$$g_J(\mathrm{Rb})/g_J(\mathrm{H})=1+(25\pm1)\times10^{-6},$$

which agrees with our number within the quoted errors.

Combining our value for g_J(Rb)/$g(e)$ with Driscoll's value for $|g_J(\mathrm{Rb})|/g_p'$ and the Vigoureux-Bender value for the precession frequency of protons in water, we obtain a new value for μ_0/h. The error is determined by the probable error in the Vigoureux measurement. We conclude that

$$\mu_0/h=1.399613(14)\times10^6\ \mathrm{sec}^{-1}\ \mathrm{G}^{-1}.$$

Agreement between our results and theory is best where it should be since g_J(T)/g_J(H) is one to the precision involved in this experiment, and the calculated ratio g_J(H)/$g(e)$ is probably good to at least 1 in 10^7. If one assumes that Perl's contribution to Δg_J(Rb) is really accurate to one in 10^6, our numbers for g_J(Rb)/$g(e)$ and g_J(Rb)/g_J(H) indicate that Phillips' contribution is the correct order of magnitude but several parts per million too high. It would be useful, at this point, to improve the theoretical calculations.

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