Contribution to Internal Friction from a Dislocation Pileup with Application to Deformed Single Crystals

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The contribution to the internal friction resulting from a group of dislocations confined to a single slip plane, and forced against a barrier, is calculated. Application of the result to the long-range-stress theory of work hardening, in conjunction with the Seeger theory of the Bordoni peak, suggests that, in deformed single crystals, an internal friction peak of constant magnitude should be observed throughout stage II of the stress-strain curve.

I. INTRODUCTION

In most problems in dislocation theory, it is usual to treat each dislocation independently and to describe the effect of mutual interactions by an average internal stress proportional to the square root of the dislocation density.[1] Although this procedure has never been justified in detail, intuitively it appears to be reasonable as long as the dislocation density is low and approximately homogeneous, or as long as the correlations in the motion of different dislocations are relatively unimportant. Accordingly, we believe the approximation should furnish an adequate description of the effect of interactions in, say, undeformed single crystals, or in the intracellular regions in deformed polycrystals.[2] By contrast, an obvious case where such a procedure would be unjustified is provided by the classical example of a dislocation pileup.[3]

The above remarks are of immediate relevance here in relation to the change in the internal friction of a solid with increasing plastic deformation. It is well known that cold working of polycrystals produces a characteristic attenuation peak, the Bordoni peak,[4] as a function of temperature. Its origin has been attributed to a specific aspect of dislocation motion by Seeger,[5] while later Paré[6] has pointed out that the action of internal stresses is necessary in order that the Seeger mechanism correctly describe the observed temperature variation of the attenuation. Moreover, it has been shown[7] that one can account qualitatively for the change in magnitude of the peak with increasing deformation, if the square of the internal stress is assumed proportional to an average dislocation density in the manner prescribed above. Consequently it appears that, in deformed *polycrystals* at least, one can obtain a consistent (albeit not necessarily correct) picture of the effect of deformation upon the attenuation.

The same claim cannot be made at present in relation to our understanding of the effect of deformation on the internal friction in *single crystals*, mainly because essentially no systematic experimental work at all[8] has been done on this topic. But of almost equal importance is the fact that even if any experimental data were available, there may be as yet no theory with which it might be compared. To appreciate this remark, it is sufficient to note that essential features of the long-range-stress theory of linear work hardening are the presence of dislocation pileups and the role of their mutual interaction. Thus if the description of the deformed state implied by this theory is correct, any model based upon *isolated* dislocations under an internal stress is clearly inadequate for a complete description of the internal friction.

The purpose of the present paper is to alleviate the above deficiency in the theory. The internal friction of an isolated pileup is investigated in detail and the results are applied to single crystals deformed in stage II of the stress-strain curve. It is shown that the long-range-stress theory, in conjunction with Seeger's mechanism for the origin of the Bordoni peak, leads to the prediction of the existence of a Bordoni peak, in single crystals, whose magnitude is independent of the amount of predeformation in the region of linear work hardening.

Concomitant with the theoretical comparison between the expected attenuation in deformed single crystals and deformed polycrystals, it was also hoped that the work to be presented might further resolve the conflict between the "forest" and the long-range-stress theories of linear work hardening.[9,10] Unfortunately, it appears that such is not the case, for we will show that the former theory also leads to the same predicted independence of the Bordoni peak of the amount of deformation. However, we should emphasize that this does not detract from the desirability of future experimental work on the topic discussed in this paper, since the further data would be of assistance in attempting to

[1] H. G. van Bueren, *Imperfections in Crystals* (North-Holland Publishing Company, Amsterdam, 1961), p. 146.
[2] J. E. Bailey, Phil. Mag. **8**, 223 (1963).
[3] J. D. Eshelby, F. C. Frank, and F. R. N. Nabarro, Phil. Mag. 2, 351 (1951).
[4] D. H. Niblett and J. Wilks, Advan. Phys. **9**, 1 (1960).
[5] A. Seeger, Phil. Mag. **1**, 651 (1956).
[6] V. K. Paré, J. Appl. Phys. **32**, 332 (1961).
[7] A. D. Brailsford, Phys. Rev. **137**, A1562 (1965).
[8] We refer here specifically to the relation between the internal friction and the degree of deformation as contained in a stress-strain curve. Experimental investigation of the internal friction in stage II deformation is planned for the near future [D. O. Thompson (private communication)].
[9] P. B. Hirsch, *Relation Between the Structure and Mechanical Properties of Metals* (H. M. Stationery Office, London, 1963), p. 48.
[10] A. Seeger, see Ref. 9, p. 1.

identify unambiguously the origin of the Bordoni peak itself.

The paper is divided as follows. Section II contains a treatment of the internal friction associated with a single pileup, while the application of the results to single crystals deformed in stage II is presented in Sec. III. Finally, Sec. IV contains a qualitative comparison of this work with the behavior of the internal friction to be expected on the basis of forest theories of work hardening.

II. THEORY

Suppose there are n identical dislocations, in a single slip plane, which are forced against a rigid barrier by the action of a constant external stress. Specifically, a coordinate system is chosen such that the dislocations lie parallel to the y axis in the region, $0<x\leq L$, of the plane $z=0$, with the barrier at $x=0$. In this section the contribution to the internal friction from this array is determined.

In the static case described above, it is clear that the extent of the pileup L_0 will be determined by the magnitude of the constant stress σ_0. Thus under the influence of an additional time-dependent stress σ_1, there will be not only a redistribution of the dislocations within the length L_0, but also a variation in the extent of the pileup itself. To demonstrate this explicitly, we define a continuous density $\rho(x,L,t)$ such that $\rho(x,L,t)\delta x$ is the number of dislocations between x and $x+\delta x$ at time t, when the pileup terminates at a distance L from the barrier. Then, since dislocations are neither generated nor lost, the density ρ is determined by the continuity equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho}{\partial L}\frac{dL}{dt}+\frac{\partial I}{\partial x}=0, \tag{1}$$

where I is the dislocation current. If we define μ and D as an effective mobility and diffusion coefficient, respectively (see Sec. III), then

$$I(x,t)=F\mu\rho-D(\partial\rho/\partial x), \tag{2}$$

where F is the force on a dislocation at x, namely,

$$\frac{F}{L_1b}=A\int_0^L\frac{\rho(x',L,t)}{(x-x')}dx'-(\sigma_0+\sigma_1). \tag{3}$$

Here σ_1 is a small, harmonically time-varying stress, and L_1 is the length of the dislocation segments parallel to the y axis. (As usual, end effects have been neglected.) The remaining parameter A in (3) is given by

$$A\equiv A_s=Gb/2\pi \tag{4}$$

for screw dislocations, and

$$A\equiv A_e=A_s/(1-\nu) \tag{5}$$

for edge dislocations, G being the shear modulus, ν Poisson's ratio, and b the magnitude of the Burgers vector.

Equation (1) is sufficiently complex that it is propitious to examine first the solution when $\sigma_1=0$. In this instance, $L=L_0$, and the distribution is determined by the condition $I=0$. Even then the problem is not trivial. However, on the basis of general arguments by Cottrell,[11] we note that the configurational entropy of a rigid dislocation (such as are assumed here) is negligible compared with its self-energy. Since the latter is also proportional to A, it is reasonable to seek an approximate solution of (2) by neglecting the diffusion current, a procedure equivalent to finding the minimum in the internal energy (i.e., $F=0$) rather than in the free energy. The former problem has been solved previously[3,12] but we will present an alternative treatment here in order to estimate the error involved in the neglect of the configurational entropy.

With $\sigma_1=0$, the condition that the current should vanish may also be written in the form

$$\int_0^\pi\frac{R(\theta')\sin\theta'd\theta'}{(\cos\theta-\cos\theta')}+\gamma+\lambda S(\theta)=0, \tag{6}$$

by substituting $x=(L_0/2)(1-\cos\theta)$, $\rho(x)\equiv L_0^{-1}R(\theta)$, and

$$\gamma=(\sigma_0L_0/A);\quad \lambda=(2\kappa T/AbL_1), \tag{7}$$

$$S(\theta)=R^{-1}\csc\theta(\partial R/\partial\theta), \tag{8}$$

in (3). (We assume the Einstein relation $D=\kappa T\mu$ is applicable.) For $\lambda=0$, the solution of (6) may be found by noting the expansion[13]

$$(\cos\theta-\cos\theta')^{-1}=-2\sum_{r=1}^{\infty}(\sin r\theta\cos r\theta')/\sin\theta. \tag{9}$$

Thus Eq. (6) reduces to

$$\pi\sum_{r=1}^{\infty}A_r\sin r\theta=\gamma\sin\theta, \tag{10}$$

where

$$A_r=(2/\pi)\int_0^\pi R(\theta)\sin\theta\cos r\theta d\theta. \tag{11}$$

These have the general solution

$$R\sin\theta=(2n/\pi)\{1+(\gamma/2n)\cos\theta\}, \tag{12}$$

the constant of integration being determined by the condition that the total number of dislocations be equal to n.

Although (12) is the general solution in the region $0\leq x\leq L_0$ it does not satisfy as yet all the physical

[11] A. H. Cottrell, *Dislocations and Plastic Flow in Crystals* (Clarendon Press, Oxford, 1961), p. 39.
[12] G. Leibfried, Z. Angew. Phys. **6**, 251 (1954).
[13] P. M. Morse and H. Feshbach, *Method of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 556.

constraints on the problem. For since the stress a distance d from a dislocation varies as d^{-1}, if $\rho(L_0)\{\equiv R(\pi)\}$ is nonzero the stress acting in the slip plane tends to infinity in the region near $x=L_0$ immediately exterior to the pileup. As this must be excluded on physical grounds, we must impose the additional general boundary condition

$$\rho(L)=0. \tag{13}$$

Then (13) implies, by virtue of (7) and (12), that

$$L_0=(2nA/\sigma_0), \tag{14}$$

and

$$R(\theta)=(2n/\pi)\cot(\theta/2), \tag{15}$$

or

$$\rho(x,L_0)=(2n/\pi L_0)\{(L_0/x)-1\}^{1/2}. \tag{16}$$

The above results are not new. They are of primary interest here in estimating the error involved in their approximate derivation from (6). From (7) and (8) we obtain $|\lambda S/\gamma|=(\kappa T/nAbL_1)\csc^2\theta$. As expected, this is largest near the barrier (where the density varies most rapidly) and at the termination of the pileup (where the density is small). However, with $n\sim 20$, $L_1\sim 10^{-4}$ cm, the factor $(\kappa T/nAbL_1)$ is only $\sim 10^{-7}$. Thus apart from a small region near the extreme ends of the pileup, we find $|\lambda S/\gamma|\ll 1$. This justifies the procedure leading to (16). By a similar argument one can validate our neglect of the diffusion current in the ensuing analysis.

We now consider the time-dependent equation (1), from which we wish to derive ultimately the dislocation strain to first order in σ_1. The boundary conditions, for general L, are that the total number of dislocations equal the fixed number n, that (13) be satisfied, and that

$$I(0,t)=0, \tag{17}$$

since the barrier is assumed to be rigid. We look for a solution of the form

$$\rho=\rho_0(x,L)+\rho_1(x,L,t), \tag{18}$$

where ρ_0 is defined by (16) and ρ_1 is assumed to be of order (σ_1/σ_0). Since $l=L-L_0$ is at least of first order in σ_1, we obtain then, after linearizing (1), the equation

$$\partial\rho_1/\partial t+\partial(\delta I)/\partial x=-(\partial\rho_0/\partial L)(dl/dt), \tag{19}$$

where

$$\delta I\approx\mu bL_1\rho_0\left[A\int_0^L\{\rho_1/(x-x')\}dx'-\{\sigma_1+(\sigma_0 l/L_0)\}\right], \tag{20}$$

the diffusion current having been neglected. For convenience, we now define an auxiliary function η by the equation

$$\eta(x,L,t)=\int_0^x\rho_1 dx. \tag{21}$$

Thus

$$\eta(0,L,t)=0;\quad \eta(L,L,t)=0, \tag{22}$$

and

$$(\partial\eta/\partial x)_{x=L}=0. \tag{23}$$

Here the first follows from the definition (21), the second from (18) and the fact that the total number of dislocations is fixed, and the third, (23), is a re-expression of Eq. (13). Now since the solution for η (or ρ_1) can only depend upon x through the combination (x/L), after integrating (19) between 0 and x, we set $x=L(1-\cos\theta)/2$ as before, $\eta(x)\equiv N(\theta)$, and find

$$i\omega\tau N(\theta)+\cot(\theta/2)\left[\frac{1}{\pi}\int_0^\pi\frac{[\partial N(\theta')/\partial\theta']d\theta'}{\cos\theta'-\cos\theta}-\alpha(1+\beta)\right]=i\omega\tau\alpha\beta\sin\theta, \tag{24}$$

where

$$\alpha=(\sigma_1 L_0/2\pi A), \tag{25}$$

$$\beta=(\sigma_0 l/\sigma_1 L_0), \tag{26}$$

and

$$\tau=(nA/\mu bL_1\sigma_0{}^2). \tag{27}$$

In obtaining (24) it has been assumed that explicitly time-dependent quantities vary like $\exp(i\omega t)$. The physical significance of τ as a relaxation time will become evident in the further discussion leading to Eq. (39).

In order to solve (24), we note from (22) that a possible expansion of $N(\theta)$ is provided by a series in $\sin(r\theta)$, where r is an integer. It is convenient to write this in the form

$$N(\theta)=\alpha\{\sum_{r=1}A_r\sin(r\theta)+\beta\sin\theta\}. \tag{28}$$

Then, by direct substitution in (24), one finds that the unknown coefficients A_r are determined by the following equations:

$$(1+i\omega\tau)A_1-(i\omega\tau/2)A_2=1, \tag{29}$$

and

$$(r+i\omega\tau)A_r-(i\omega\tau/2)(A_{r+1}+A_{r-1})=0,\quad (r\geq 2). \tag{30}$$

These are independent of β. However, the latter is determined by (23), which in these variables becomes the relation

$$\sum_{r=1}(-1)^r rA_r=\beta. \tag{31}$$

Hence (31) determines the change in length of the pileup once the coefficients A_r are known.

Since we cannot offer a mathematically rigorous solution of (29) *et seq.* it is expedient to remind oneself of the purpose of the above analysis. Fundamentally, we are interested in the strain resulting from the pileup. Apart from a constant this is

$$\epsilon_p=-(b/V)\int_0^L xL_1\rho(x)dx, \tag{32}$$

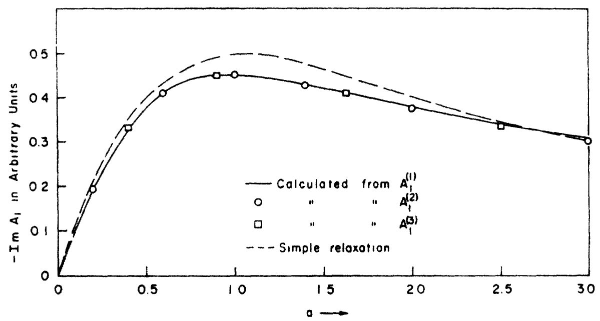

FIG. 1. Comparison between the calculated values of A_1 and the simple relaxation result described in the text.

where V is the sample volume. Moreover, after a little algebra one finds from the definitions (18), (21), and (28) that this is simply

$$\epsilon_p=\epsilon_p{}^0[1-(\pi\alpha/n)A_1], \tag{33}$$

where again, apart from the constant noted above, $\epsilon_p{}^0$ is the strain associated with the pileup for $\sigma_1=0$, viz.,

$$\epsilon_p{}^0=-(nbL_1L_0/4V). \tag{34}$$

[The factor of 4 in the denominator in (34) arises because the center of gravity of the array is at $L_0/4$.] Thus we see from (33) that only A_1 is of interest for the internal friction.

In addition we note that, since the total number of dislocations is finite (~ 20), the upper limit of summation in (28), presently undesignated, should really be fixed at $r=n$. For we cannot have more degrees of freedom (coefficients A_r) than there are dislocations. But if the sequence A_r is terminated, Eqs. (29) and (30) become a finite set of simultaneous linear algebraic equations which can be solved readily. Of course, they should be solved by setting $A_{21}=0$, $A_{22}=0$, and subsequently determining A_1 and hence ϵ_p. However it is less laborious to terminate the sequence at the lowest possible integer, then determine A_1, repeat for the next lowest integer, calculate A_1 again and so on and then compare the results to see if termination errors are significant. This latter procedure is the one we have adopted. For example, the lowest integer for termination is $r=3$, since otherwise one cannot satisfy (29), (30), and (31) simultaneously. Thus with $A_r=0$ for $r\geq 3$ we find a first approximate to A_1, namely $A_1{}^{(1)}$, given by

$$A_1{}^{(1)}=(2+ia)/\{2-(3a^2/4)+3ia\}, \tag{35}$$

where $a=\omega\tau$. The imaginary part of (35), which is proportional to the attenuation produced by the pileup, in this approximation, is illustrated by the full curve in Fig. 1. It yields a result which is not different, in any essential way, from that obtained from the simple relaxation expression

$$A_1\sim(1+ia)^{-1}. \tag{36}$$

Similarly, when the higher approximates $A_1{}^{(2)}$ and $A_1{}^{(3)}$ (given explicitly in the Appendix), obtained by setting $A_r=0$ for $r\geq 4$ or 5, respectively, are used to calculate the internal friction, no significant differences arise, as we show in Fig. 1. Moreover, the same comments are applicable to the real part of A_1, which essentially duplicates that obtained from (36). Hence we believe that, in all its qualitative aspects, the internal-friction parameters for the pileup may be estimated from the approximate relation

$$\epsilon_p-\epsilon_p{}^0\simeq-(\pi\alpha\epsilon_p{}^0/n)(1+i\omega\tau)^{-1}. \tag{37}$$

In like fashion one can obtain successive approximates to the parameter β. For example, to lowest order we obtain

$$\beta^{(1)}=-2/[2-(3a^2/4)+3ia]. \tag{38}$$

For interest, the real and imaginary parts of this expression are shown in Fig. 2. At low frequencies ($a\simeq 0$), $\beta^{(1)}\to(-1)$, which gives the correct result, to first order, for the change in length associated with a *static* stress σ_1, as one may verify from (14) and (26). This feature is retained in the higher approximates we have calculated. In addition we note that β is complex. The significance of the imaginary part is that the function ρ_0 in (18) incorporates just that part of the change in the density of the dislocations with time which can be described by the same distribution function as in the static case. However, the necessary change in length of the pileup cannot take place instantaneously. Hence the imaginary part of β describes that part of the attenuation which is associated with this rearrangement.

The whole burden of this section has been to attempt a rigorous treatment of the internal friction arising from an isolated pileup. The end result is Eq. (37). On the basis of the static properties of a pileup, one might conjecture that the answer should be similar to that obtained from a single dislocation of strength nb. Inasmuch as (37) describes a simple relaxation, this surmise is correct. But, on the other hand, the depend-

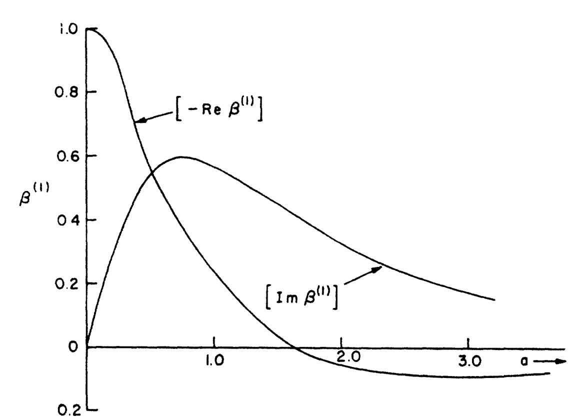

FIG. 2. Variation of the in-phase and out-of-phase components of the extent of the pileup with increasing frequency. The parameter a is defined by $a=\omega\tau$.

ence of τ upon L_0 (or σ_0) indicates that in the present model the relaxation involves simultaneous readjustment over the whole pileup, an effect which is outside the scope of such a simple picture.

III. APPLICATION

In this section we shall investigate the effect of deformation on the internal friction of fcc single crystals. The discussion will be confined completely to the deformation range stage II, where linear work hardening is observed.

Since the long-range-stress theory has already been reviewed extensively,[10,14] we shall not enter into a detailed discussion here. For the present purpose it suffices to note that, in stage II, groups of dislocation rings are assumed to be piled up against barriers formed by Lomer-Cottrell dislocations lying along the close-packed [110] directions in the slip plane.

If the curvature of the dislocation rings is ignored, an assumption which is invariably made in practice, each part of the ring can be considered independently. The physical model is then equivalent to that discussed in Sec. II. We must first determine the effective mobility μ. Now in the early presentation of a theory of the Bordoni peak, Seeger[5] proposed that dislocations lying parallel to close-packed directions were enabled to advance by the generation of double kinks. Thus since Paré's modifications,[6] which are only pertinent to *pinned* dislocations, are not important here, we can construct an effective diffusion coefficient (and hence a mobility) by the following relation:

$$\mu\kappa T = D \simeq \{g_0 L_1 \exp(-2\epsilon_k/\kappa T)\} b^2. \qquad (39)$$

That is, we assume a fixed double-kink generation rate per unit length equal to $g_0 \exp(-2\epsilon_k/\kappa T)$, where ϵ_k is the kink self-energy, and then multiply the total rate for a length L_1 by the square of the jump distance ($\simeq b$). It is tacitly assumed, of course, that once generated the kinks propagate instantaneously to the opposite ends of the length L_1. We have ignored the possibilities that kinks may recombine, by diffusion, after their generation[15] or that, because of entropy considerations, the generation over a fixed length may be other than linear in the length considered.[16] Moreover in suggesting (39), we have tacitly introduced a Peierls' stress through the energy ϵ_k. While it is generally accepted that the latter has negligible influence on the flow stress of fcc metals,[14] nevertheless it will be of prime importance in determining relatively minor dislocation rearrangements, such as are involved here, in an internal-friction experiment.

We can now calculate the relaxation time associated with a pileup. In the relation

$$\tau^{-1} = (g_0 L_1^2 b^3 \sigma_0^2 / nA\kappa T) \exp(-2\epsilon_k/\kappa T), \qquad (40)$$

obtained from (27) and (39), we insert the parameters from the theory[14] of the flow stress σ_f. Thus we identify σ_0 with the long-range stress from different pileups and set

$$\sigma_0 = \sigma_f = \theta_{II}(\epsilon - \epsilon^*), \qquad (41)$$

where θ_{II} is the work hardening in stage II, ϵ^* is some reference strain, and ϵ the actual strain in the sample. Similarly, from slip-line observations,[14]

$$L_1 = \Lambda/(\epsilon - \epsilon^*), \qquad (42)$$

where $\Lambda \sim 10^{-3}$ cm. Thus we obtain finally

$$\tau^{-1} = \omega_A \exp(-2\epsilon_k/\kappa T), \qquad (43)$$

where the "attempt frequency" ω_A is given by

$$\omega_A = (\theta_{II}^2 \Lambda^2 b^2 / nA\kappa T) g_0 b. \qquad (44)$$

The most significant aspect of this result is its independence of the amount of deformation ϵ. To estimate the magnitude of ω_A we require a value of $g_0 b$. In principle the latter quantity could be obtained from the attempt frequency of the Bordoni peak in deformed polycrystals, according to a previous study.[7] Unfortunately, as we have pointed out, present experimental information indicates no more than that, if $g_0 b = \gamma\omega_D$, where ω_D is the Debye frequency, then γ lies between 6×10^{-5} and 1×10^{-9}. Accordingly one can fix (44) only within a very wide range. Specifically, we find that if $(\theta_{II}/G) \simeq 4\times10^{-3}$, $\Lambda \simeq 10^{-3}$ cm, $n \simeq 20$, and $T = 100$°K then, for Cu, ω_A lies within the limits 10^{11} to 10^{16} rad sec^{-1}. This frequency range is significantly higher than that for the Bordoni peak in polycrystals[4] (roughly 10^8 to 10^{13} rad sec^{-1}) but, in view of the assumptions which have been made, it is difficult to assess the uncertainty to attach to these estimates.

We shall now calculate the magnitude of the internal friction. This is defined quantitatively by the decrement Δ, which is the ratio of the energy dissipated per radian to the maximum stored energy.[4] Thus, with the stress-strain law (41),

$$\Delta = (\theta_{II}/2\pi|\sigma_1|^2) \operatorname{Re} \int_0^{2\pi/\omega} \sigma_1^* \dot{\epsilon} dt. \qquad (45)$$

Collecting together then Eqs. (25), (33), and (37), after multiplying by a factor of 4 to include the contribution from all sides of a piled-up set of loops (assumed square-shaped), we find that, for N_p pileups per unit volume, the result is

$$\Delta = (2\theta_{II} b n^2 A N_p L_1/\sigma_0^2)\omega\tau/(1+\omega^2\tau^2). \qquad (46)$$

Hence, substituting for A from (4), and using the alternative[14] approximate form for σ_0,

$$\sigma_0 \simeq (Gbn/2\pi)(N_p L_1)^{1/2}, \qquad (47)$$

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[14] A. Seeger, *Dislocations and Mechanical Properties of Crystals* (John Wiley & Sons, Inc., New York, 1957), p. 243.
[15] J. Lothe and J. P. Hirth, Phys. Rev. **115**, 543 (1959).
[16] D. O. Thompson (private communication).

we arrive at the following estimate Δ_s for the relaxation strength,

$$\Delta_s \simeq 4\pi(\theta_{\mathrm{II}}/G). \tag{48}$$

This again is independent of the amount of deformation, and has the value $\Delta_s \sim 0.05$, or approximately three times the value obtained from the observed Bordoni peak in polycrystalline Cu after 3% tensile deformation.[7,17]

IV. DISCUSSION

The major result of the foregoing analysis may be summarized as follows: If the Seeger-Paré mechanism is the correct description of the origin of the Bordoni peak in deformed polycrystals, and if simultaneously the long-range-stress theory is appropriate to linear work hardening, then a necessary corollary is that a Bordoni peak of constant magnitude should be observed in single crystals deformed in stage II.

The question which immediately springs to mind then is, how does this compare with the internal friction which is predicted on the basis of the forest theory of linear work hardening. Unfortunately this presents a very difficult problem, mainly because the theory is based not upon any specific dislocation configuration but rather upon a general three-dimensional network. Thus, for lack of anything better, we can here only follow convention and *assume* each dislocation segment moves independently, despite an indirect inference to the contrary.[7] However, granted the assumption, it is a straightforward matter to determine the consequence. For we have shown previously that in the Seeger-Paré mechanism of the Bordoni peak, the relaxation strength depends only upon the ratio $\delta = (\sigma_i b^2 L/2\epsilon_k)$, where σ_i is the internal stress. Since, in a three-dimensional network, $\sigma_i \propto L^{-1}$, the parameter δ is constant for all deformations, and we would again surmise that a Bordoni peak of constant magnitude is predicted. Consequently, apart from the inference noted above, there appears no reason to believe that observation of the attenuation in deformed single crystals will distinguish between the two theories of linear work hardening.

We have already indicated in Sec. I that presently there is no experimental evidence with which to compare these predictions. And, in view of the conclusion we have just reached, it is legitimate to inquire what purpose would be served by doing the relevant experiments in the future. In our opinion this lies solely in helping to resolve the origin of the Bordoni peak itself. While so many of the models[5,17–19] for this phenomenon presently relate only to properties of the peak alone, it appears that here at least is one aspect which ultimately encroaches upon other provinces of the theory of the plastic deformation of solids. As such, the data obtained from future experiments would be of great help in attempting to correctly identify the Bordoni-peak mechanism.

To conclude we should mention the experimental setup to which all the foregoing theory is presumed to be applicable. We have applied both the pertinent models as if the sample were under a continuous load. Of course this would present the experimentalist with a very difficult task. However, since there is no appreciable reverse plastic flow upon unloading,[14] we believe the theory should be applicable with only minor numerical modification (to account for some stress relaxation, for example), to the attenuation of deformed single crystals that are subsequently unloaded before measurement.

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APPENDIX

We present here explicit expressions for the approximates $A_1^{(2)}$ and $A_1^{(3)}$ which were obtained by the procedure described in the text. These are

$$A_1^{(2)} = \{6-(3a^2/4)+5ia\}/\{6-5a^2+11ia-i(a^3/2)\}, \tag{A1}$$

and

$$A_1^{(3)} = \frac{\{24-(15a^2/2)+ia[26-(a^2/2)]\}}{\{24-(61a^2/2)+(5a^4/16)+ia[50-(25a^2/4)]\}}, \tag{A2}$$

where $a=\omega\tau$. Calculated values of the real part of these functions are illustrated in Fig. 1.

[17] L. J. Bruner and B. M. Mecs, Phys. Rev. **129**, 1525 (1963).

[18] A. D. Brailsford, Phys. Rev. **122**, 778 (1961).

[19] J. J. Gilman, Phil. Mag. **7**, 1779 (1962).