thus

$$\kappa_n=\frac{K_{\text{expt}}''}{K_r}=\frac{K_{\text{expt}'}/\nu}{K_r}=\frac{\kappa}{\nu}, \qquad \text{(A12)}$$

but from Eqs. (A4) and (A10), $\alpha^2=\nu$, and so

$$\kappa_n=\kappa_a=\kappa', \qquad \text{(A13)}$$

where κ' is the ratio given in line 8 of Table II.

Note added in proof. The assumption of incompressible gas flow with zero slip is implicit in Eq. (A1). In analyzing the relative gas-flow measurements, the validity of this assumption does not affect the evaluation of κ_n; approximate calculations for κ_a show that the inclusion of slip corrections does not appear to eliminate the "length effect."

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Position- and Time-Dependent Diffusion Modes for Electrons in Gases

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The pure-diffusion (field-free) distribution function for electrons in a gas enclosed by absorbing walls is obtained. The distribution is described in terms of spatial modes, each of which decays exponentially with time. Special emphasis is placed on the lowest modes which explicitly display the "diffusion cooling" effect for electrons.

I. INTRODUCTION

This paper deals with the pure-diffusion (field-free) energy distributions for electrons in a gas and enclosed by absorbing walls. The present study is an extension of previous work[1] by the author on the spatial dependence of distribution functions for electrons in the presence of electric fields. The purpose of the present work is to describe the manner in which the distribution function for electrons, which are initially *not* in thermal equilibrium with the gas, changes with time and position owing to collisions between the electrons and the gas atoms and to diffusion of the electrons to the walls. This descritpion is given in terms of spatial modes, each of which decays exponentially with time. It will be shown that in general the electrons never come into equilibrium with the gas but have a terminal energy that is lower than the thermal energy of the gas. This effect, which has been called "diffusion cooling,[2]" is displayed by the lowest modes, i.e., the modes with the longest decay times. For simplicity the interaction between the electrons and atoms is taken as elastic and only the plane-parallel geometry is considered. The entire mode system is given for the case of an energy-independent collision frequency while only the lowest modes are obtained for the case of an energy-independent collision cross section.

II. CONSTANT COLLISION FREQUENCY

The starting point for the present discussion is the Boltzmann equations for $f^0(\mathbf{r},v,t)$ and $\mathbf{f}^1(\mathbf{r},v,t)$. These functions represent, respectively, the isotropic and nonisotropic parts of the electron distribution function, i.e.,

$$f(\mathbf{r},\mathbf{v},t)=f^0(\mathbf{r},v,t)+\mathbf{f}^1(\mathbf{r},v,t)\cdot\hat{v}$$

where $\mathbf{r}$ is the position vector and $\mathbf{v}$ is the electron velocity. The Boltzmann equations for f^0 and $\mathbf{f}^1$, with the collision frequency independent of energy, are[3]

$$\frac{\partial f^0}{\partial t}=\frac{2m\nu}{M\epsilon^{1/2}}\frac{\partial}{\partial\epsilon}\left[\epsilon^{3/2}\left(f^0+kT\frac{\partial f^0}{\partial\epsilon}\right)\right]-\frac{1}{3}\left(\frac{2\epsilon}{m}\right)^{1/2}\boldsymbol{\nabla}_{\mathbf{r}}\cdot\mathbf{f}^1, \qquad (1)$$

and

$$\frac{\partial\mathbf{f}^1}{\partial t}+\nu\mathbf{f}^1=-\left(\frac{2\epsilon}{m}\right)^{1/2}\boldsymbol{\nabla}_{\mathbf{r}}f^0, \qquad (2)$$

where m is the electron mass, M the atomic mass, ϵ the electron kinetic energy, ν the momentum-transfer collision frequency, and T the gas temperature. It will be assumed that f^0 varies sufficiently slowly so that Eq. (2) can be written as

$$\nu\mathbf{f}^1\approx-(2\epsilon/m)^{1/2}\boldsymbol{\nabla}_{\mathbf{r}}f^0. \qquad (3)$$

The conditions under which this approximation is valid will be discussed at the end of this section. When Eq. (3) is combined with Eq. (1), the resulting equation for f^0 is

$$\epsilon^{-3/2}\frac{\partial}{\partial\epsilon}\left[\epsilon^{3/2}\left(f^0+kT\frac{\partial f^0}{\partial\epsilon}\right)\right]+\frac{M}{3m^2\nu^2}\nabla_{\mathbf{r}}{}^2f^0=\frac{M}{2m\nu\epsilon}\frac{\partial f^0}{\partial t}. \qquad (4)$$

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[1] J. H. Parker, Jr., Phys. Rev. **132**, 2096 (1963).

[2] M. A. Biondi, Phys. Rev. **93**, 1136 (1954).

[3] The detailed derivation of these equations along with a discussion of the approximations used in obtaining them is given by W. P. Allis, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21. Also see T. Holstein, Phys. Rev. **70**, 367 (1946).

It is convenient to introduce dimensionless variables into Eq. (4) given by

$$\mathbf{R}=mv\left(\frac{3}{MkT}\right)^{1/2}\mathbf{r},$$

$$\tau=\frac{2mv}{M}t,$$

and

$$u=\epsilon/kT.$$

Then Eq. (4) becomes

$$u^{-3/2}\frac{\partial}{\partial u}\left[u^{3/2}\left(f^0+\frac{\partial f^0}{\partial u}\right)\right]+\nabla_{\mathbf{R}}{}^2 f^0=\frac{1}{u}\frac{\partial f^0}{\partial\tau}. \qquad (5)$$

Equation (5) can be separated into three ordinary differential equations, i.e., with $f^0=T(\tau)\mathcal{R}(\mathbf{R})F(u)$, then

$$\frac{1}{T}\frac{dT}{d\tau}=-\theta, \qquad (6)$$

$$\nabla_{\mathbf{R}}{}^2\mathcal{R}=-K^2\mathcal{R}, \qquad (7)$$

and

$$u\frac{dF^2}{du^2}+(\tfrac{3}{2}+u)\frac{dF}{du}+(\tfrac{3}{2}+\theta-K^2u)F=0, \qquad (8)$$

where θ and K are the separation constants.

The solution of Eq. (6) is

$$T(\tau)=\exp(-\theta\tau).$$

The solution to Eq. (7) depends on the geometry of the enclosure. For plane parallel it is

$$\mathcal{R}(X)=\sin KX \quad\text{or}\quad \cos KX,$$

where X is the Cartesian coordinate perpendicular to the plane. The extension to other simple geometries such as spherical or cylindrical is straightforward. For electrons in a perfectly absorbing enclosure the condition which sets the allowed values of K is

$$f_{\text{boundary}}{}^0=0.$$

With one plane at $X=0$ and the other at $X=X_0$, $\mathcal{R}(X)$ becomes

$$\mathcal{R}=\sin KX,$$

with

$$K=\beta\pi/X_0,$$

where $\beta=1, 2, 3 \cdots$.

Equation (8) can be transformed into a recognizable form by letting

$$F(u)=\exp[-\tfrac{1}{2}(1+\alpha)u]h(u),$$

where

$$K^2=\tfrac{1}{4}(\alpha^2-1).$$

Then Eq. (8) becomes

$$u\frac{d^2h}{du^2}+(\tfrac{3}{2}-\alpha u)\frac{dh}{du}+[\tfrac{3}{4}(1-\alpha)+\theta]h=0. \qquad (9)$$

The solution to this equation which is regular at $u=0$ is a confluent hypergeometric function,[4] i.e.

$$h=F\left(-\frac{\frac{3}{4}(1-\alpha)+\theta}{\alpha}\middle|\tfrac{3}{2}\middle|\alpha u\right).$$

Since K (or α) is a parameter, Eq. (8) becomes an eigenvalue equation for θ. It is easily shown that the corresponding eigenfunctions are orthogonal with respect to the density function $u^{1/2}e^u$. Inspection of the asymptotic behavior of the confluent hypergeometric function[4] as $u\to\infty$ leads to the allowed spectrum for θ and to the corresponding eigenfunctions. The complete set of eigenfunctions is discrete and is given by

$$F_l=(1/N_l)\exp[-\tfrac{1}{2}(1+\alpha)u]L_l{}^{(1/2)}(\alpha u),$$

where $L_l{}^{(1/2)}$ is a Laguerre polynomial[4] and N_l is the normalization constant given by

$$N_l{}^2=\Gamma^3(\tfrac{3}{2}+l)/l!\alpha^{3/2},$$

with $l=1, 2, \cdots, \infty$. The eigenvalues for θ are

$$\theta_l=l(1+4K^2)^{1/2}+\tfrac{3}{4}[(1+4K^2)^{1/2}-1]. \qquad (10)$$

Then the general solution to Eq. (5) is

$$f^0(u,X,\tau)=\sum_{\beta,l}A_{\beta,l}\exp(-\theta_{\beta,l}\tau)F_{\beta,l}(u)\sin\left(\frac{\beta\pi X}{X_0}\right). \qquad (11)$$

This solution can be used to describe the decay of electron energy and density for an arbitrary initial distribution in the space between the plates. It is not the purpose of this paper to discuss any specific initial-condition problem in detail but to point out some of the general features of this modal description.

The case of a uniform spatial distribution is obtained by letting $K\to 0$. In this limit

$$\theta_l=l,$$

and

$$F_l=(1/N_l)\exp(-u)L_l{}^{(1/2)}(u).$$

These results for a uniform spatial distribution have been derived by Bayet, Delcroix, and Denisse[5] and further discussed by Levine and Uman.[6]

It is seen that each of the terms or modes in Eq. (11) decays with a time constant, given by $M/2mv\theta$, that decreases as K and l increase. For the lowest modes ($l=0$) and with $K^2<1$, $\theta_0\tau$ and F_0 can be expressed as

$$\theta_0\tau\approx\tfrac{3}{2}K^2(1-K^2)\tau,$$

and

$$F_0\approx(1/N_0)e^{-(1+K^2)u},$$

where

$$L_0{}^{1/2}(u)=1.$$

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[4] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), Chaps. 5 and 6.

[5] M. Bayet, J. Delcroix, and J. Denisse, J. Phys. Radium **17**, 923 (1956).

[6] N. E. Levine and M. A. Uman (to be published).

If these relations are expressed in terms of t and ϵ, we obtain

$$\theta_0\tau\approx\left(\frac{\beta\pi}{d}\right)^2\frac{kT}{m\nu}\left[1-\left(\frac{\beta\pi}{m\nu d}\right)^2\frac{MkT}{3}\right]t \tag{12}$$

and

$$F_0\approx\frac{1}{N_0}\exp\left\{-\left[1+\left(\frac{\beta\pi}{m\nu d}\right)^2\frac{MkT}{3}\right]\frac{\epsilon}{kT}\right\}, \tag{13}$$

where d has been taken as the actual plate spacing. From Eq. (13) it is clear that even when the higher modes have disappeared the mean energy of the electrons is not equal to the thermal energy of the gas. Equation (13) represents a Maxwell distribution with a mean energy ϵ_{av} of

$$\epsilon_{\mathrm{av}}\approx\frac{3kT}{2}\left[1-\left(\frac{\beta\pi}{m\nu d}\right)^2\frac{MkT}{3}\right].$$

This expression explicitly displays the diffusion cooling of the electrons. This effect is due to the higher loss of high-energy electrons than low-energy electrons to the walls by diffusion. If we define a characteristic energy decay distance as

$$d_\epsilon=(\pi/m\nu)(MkT/3)^{1/2},$$

then the fractional deviation of the electron energy from that of the gas can be written as

$$\delta\epsilon/\epsilon_{\mathrm{gas}}=-(\beta d_\epsilon/d)^2.$$

Equations (12) and (13) show, as expected, that as $\nu\rightarrow\infty$ the time constant and the mean energy of the lowest mode approach values characteristic of electrons in equilibrium with the gas. That is, the time constant goes to

$$\text{time constant}\rightarrow(d/\beta\pi)^2(1/D_\nu),$$

where D_ν is the equilibrium diffusion coefficient for electrons given by $kT/\nu m$, and the average energy approaches $\frac{3}{2}kT$.

The inequality that must be satisfied in order that Eq. (3) be valid is

$$\partial\mathbf{f}^1/\partial t<\nu\mathbf{f}^1 \quad\text{or}\quad (2\epsilon/m)^{1/2}\nabla_r f^0.$$

This inequality can be expressed for a single mode of Eq. (11) as

$$1/\theta>2m/M,$$

or in terms of the time constant $M/2m\nu\theta$ of that mode as

$$\text{time constant}>1/\nu.$$

This implies that Eq. (11) is valid only after a time that is long in comparison to a collision period. This also means that the present solution cannot ascribe to $\mathbf{f}^1$ an initial condition independent of f^0 and that the solution therefore ignores any initial transient behavior of $\mathbf{f}^1$. A similar approximation has been used by Holstein.[3]

III. CONSTANT CROSS SECTION

For the case of an energy-independent cross section the equation governing f^0 is[3]

$$\frac{1}{\epsilon}\frac{\partial}{\partial\epsilon}\left[\epsilon^2\left(f^0+kT\frac{\partial f^0}{\partial\epsilon}\right)\right]+\frac{M\lambda^2}{6m}\nabla_r{}^2f^0 =(2\epsilon m)^{-1/2}\frac{\lambda M}{2}\frac{\partial f^0}{\partial t}, \tag{14}$$

where λ is equal to $1/N\sigma$, with σ the momentum-transfer cross section and N the gas density. As in the case of constant collision frequency we have assumed that f^0 varies sufficiently slowly so that an equation of the type (3) is valid. With a change to dimensionless variables given by

$$u=\epsilon/kT,$$

$$\tau=(2mkT)^{1/2}(2/\lambda M)t,$$

and

$$\mathbf{R}=(6m/M)^{1/2}(1/\lambda)\mathbf{r},$$

Eq. (14) becomes

$$\frac{1}{u}\frac{\partial}{\partial u}\left[u^2\left(f^0+\frac{\partial f^0}{\partial u}\right)\right]+\nabla_{\mathbf{R}}{}^2f^0=u^{-1/2}\frac{\partial f^0}{\partial\tau}.$$

As before, a separation of variables leads to

$$T(\tau)=\exp(-\theta\tau)$$

and

$$\mathcal{R}(X)=\sin KX \quad\text{or}\quad \cos KX$$

for the case of plane-parallel geometry. The energy equation is

$$u\frac{d^2F}{du^2}+(2+u)\frac{dF}{du}+(2+u^{-1/2}\theta-K^2)F=0. \tag{15}$$

With K^2 as a parameter, Eq. (15) becomes an eigenvalue equation for θ. The density function for Eq. (15) is $u^{1/2}e^u$. The exact analytic form of the solutions of this equation could not be obtained because the behavior of this equation near its irregular singular point at infinity indicated the need for a more complicated solution than the usual exponential.[7] However, an approximate expression for the lowest mode for $K^2<1$ can be obtained by a perturbation calculation.

The lowest mode eigenfunction $F_0(K^2,u)$ and eigenvalue $\theta_0(K^2)$ can be expanded in powers of K^2, i.e.,

$$F_0(K^2u)=F_0{}^{(0)}+K^2F_0{}^{(1)}+K^4F_0{}^{(2)}+\cdots$$

and

$$\theta_0(K^2)=\theta_0{}^{(0)}+K^2\theta_0{}^{(1)}+K^4\theta_0{}^{(2)}+\cdots.$$

If the following transformation is made,

$$F(u)=e^{-u}h(u),$$

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[7] E. L. Ince, *Ordinary Differential Equations* (Dover Publications, Inc., New York, 1944), Chap. 17.

then Eq. (15) becomes

$$u\frac{d^2h}{du^2}+(2-u)\frac{dh}{du}+\theta u^{-1/2}h=K^2h. \tag{16}$$

Up to now we have considered θ to be the eigenvalue which is a function of K^2. However in the perturbation calculation it is more convenient to reverse this viewpoint and to consider K^2 the eigenvalue which is now a function of θ. Then the term $\theta u^{-1/2}h$ in the above equation is considered as a perturbation. It is convenient to put Eq. (16) into a standard quantum-mechanical form[8] so that the usual perturbation formulas may be applied. Equation (16) can therefore be expressed as

$$H_0\Psi_0+\theta H'\Psi_0=W_0\Psi_0,$$

where

$$H_0=ud^2/du^2+(2-u)d/du,$$

$$H'=u^{-1/2},$$

$$\Psi_0=h_0,$$

and

$$W_0=K^2.$$

When θ is equal to zero the complete set of discrete orthonormal solutions is[4]

$$V_n=L_n^{(1)}(u)/(n+1)!(n+1)^{1/2},$$

where $L_n^{(1)}$ is a Laguerre polynomial.[4] The corresponding eigenvalues are

$$E_n=0,\ -1,\ -2,\ -3,\ \cdots,\ -n.$$

These functions are orthogonal with respect to the density function ue^{-u}.

If the usual perturbation formulas are used, Ψ_0 and W_0, to second order in perturbation theory, are

$$W_0=\theta H_{00}'-\theta^2\sum_{n=1}^{\infty}\frac{(H_{n0}')^2}{E_n}+\cdots \tag{17}$$

and

$$\Psi_0=V_0-\theta\sum_{n=1}^{\infty}\frac{H_{n0}'}{E_n}V_n,+\cdots. \tag{18}$$

We have explicitly put $E_0=0$ in Eqs. (17) and (18) and H_{n0}' is given by

$$H_{n0}'=\int_0^{\infty}u^{1/2}e^{-u}V_0V_ndu.$$

TABLE I. Constants for the perturbation calculation.

n	$2H_{n0}'/\pi^{1/2}$	a_n
0	1	-0.165
1	$\frac{1}{2}\sqrt{2}$	$\frac{1}{8}$
2	$\frac{3}{8}\sqrt{3}$	1/96
3	15/96	5/4608
4	$105/384\sqrt{5}$	7/61440

The values of H_{n0}' were evaluated with the help of the generating function for the Laguerre polynomals.[4] The values of H_{n0}' for n up to four are given in Table I.

We can now obtain $\theta(W)$ and $\Psi_0(W)$ from Eqs. (17) and (18) and these relations are

$$\theta_0=\frac{W}{H_{00}'}+\frac{W^2}{(H_{00}')^3}\sum_{n=1}^{\infty}\frac{(H_{n0}')^2}{E_n}+\cdots$$

and

$$\Psi_0=V_0-\frac{W}{H_{00}'}\sum_{n=1}^{\infty}\frac{H_{n0}'}{E_n}V_n+\cdots.$$

It should be noted that $V_0=1$. The function Ψ_0 which is normalized with respect to ue^{-u} is now renormalized with respect to $u^{1/2}e^{-u}$. Then θ_0 and h_0 can be written in terms of K^2 as

$$h_0=\left(\frac{4}{\pi}\right)^{1/4}(1+K^2\sum_{n=0}^{\infty}a_nL_n^{(1)}(u))$$

and

$$\theta=(2/\pi^{1/2})K^2(1+a_0K^2).$$

The constants a_n can be expressed in terms of H_{n0}' as

$$a_0=-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(H_{n0}')^2}{n}$$

and

$$a_n=\frac{2H_{n0}'}{\pi^{1/2}n(n+1)!(n+1)^{1/2}}\quad\text{for}\quad n>0.$$

Values for a_n are given in Table I for n up to four. Therefore the lowest mode for Eq. (15) can be written as

$$F_0(K^2,u)=\left(\frac{4}{\pi}\right)^{1/4}e^{-u}(1+K^2\sum_{n=1}^{\infty}a_nL_n^{(1)}(u)+\cdots), \tag{19}$$

with the eigenvalue for θ as

$$\theta_0=\frac{2}{\pi^{1/2}}K^2(1+a_0K^2+\cdots).$$

These expressions can now be used to display the "diffusion cooling" of the electrons. The average energy for this mode is

$$\epsilon_{\text{av}}(K^2)=\frac{\int_0^{\infty}\epsilon^{3/2}f^0d\epsilon}{\int_0^{\infty}\epsilon^{1/2}f^0d\epsilon}=kT\frac{\int_0^{\infty}u^{3/2}F_0du}{\int_0^{\infty}u^{1/2}F_0du}.$$

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[8] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), Chap. 7.

The average energy, which was evaluated by using the expression for F_0 as given in Eq. (19) and using the generating function for the Laguerre polynomials, can be expressed to first order in K^2 as

$$\epsilon_{\mathrm{av}}=\tfrac{3}{2}kT(1-0.30K^2).$$

The accuracy of the numerical constant appearing in this expression is about 1%. As in the case for constant collision frequency, we again see that the average energy is lower than the thermal energy of the gas. The deviation of the electron energy away from the thermal value can be put in the following form

$$\delta\epsilon/\epsilon_{\mathrm{gas}}=-(\beta d_\epsilon/d)^2,$$

where d_ϵ, the characteristic energy decay distance, is

$$d_\epsilon=\pi\left(\frac{M}{6m}\right)^{1/2}\frac{1}{N\sigma}(0.55),$$

and d is the distance between the planes. For example, d_ϵ for He can be expressed as

$$d_\epsilon{}^{\mathrm{He}}=\left(\frac{3.7}{Pd}\right)^2,$$

where σ is taken as 5.0×10^{-16} cm^2 [9] and P is the pressure expressed in mm Hg at 300°K. If we take $d=1$ cm, $P=10$ mm, and $\beta=1$ (lowest spatial mode), then

$$\delta\epsilon/\epsilon_{\mathrm{gas}}=-0.14.$$

We again see, as in the case for constant ν, that as $\lambda\rightarrow0$ the expressions for the time constant and the average energy approach the equilibrium values with

$$\text{time constant}\rightarrow(d/\beta\pi)^2(1/D_\sigma),$$

where

$$D_\sigma=(2kT/m\pi)^{1/2}(2\lambda/3),$$

and

$$\epsilon_{\mathrm{av}}\rightarrow\tfrac{3}{2}kT.$$

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[9] L. S. Frost and A. V. Phelps, Phys. Rev. **136**, A1538 (1964).

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Nanosecond-Pulse Breakdown in Gases*

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A theory describing the formative period of breakdown in gases following the sudden application of a dc electric field has been developed and applied to the design of experiments to measure lag times in nine gases. It is shown analytically that under certain conditions pulsed-dc and pulsed-microwave breakdown are directly comparable. A pulsed-dc experimental system is described which permits measurements of the formative period over a wide range of applied field, gas pressure, and gap space. For those gases where sufficient basic data are available, theoretical and experimental results are in good agreement.

INTRODUCTION

THE electrical breakdown of a gas is, in general, characterized by the net buildup of ionization from processes within the gas and from a host of secondary processes at walls and electrodes of the discharge vessel. Under certain conditions it has been possible to investigate breakdown with cw microwave techniques[1–3] where the electron generation and loss mechanisms are confined to the gas alone. The applicable diffusion theory then accounts for the balance between impact ionization as the generation process and attachment and diffusion as the loss processes. The validity of the diffusion theory[3] is subject to requirements on field uniformity, electron mean free path, and the amplitude of electron oscillation in the alternating field. The same theory has been applied successfully to the investigation of the formative time for breakdown under pulsed-microwave conditions.[4] An additional simplification can be achieved in this case since it is possible to design experiments in which the diffusion loss of electrons is negligible.

The research described in this paper was undertaken with the premise that the restriction of breakdown processes to those taking place in the gas could also be achieved under pulsed-dc breakdown conditions. Under these conditions the formative processes of breakdown can be described within the same theoretical framework as employed in microwave breakdown, and the experimental results for the two types of break-

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