

Validity of Townsend's Ionization Function in Nonuniform Electric Fields*

D. E. GOLDEN AND L. H. FISHER

Research Laboratories, Lockheed Missiles and Space Company, Palo Alto, California

(Received 5 April 1965)

A number of authors have concluded that experimental values of prebreakdown ionization-gas-current multiplications in nonuniform electric fields are not correctly predicted by generalizing Townsend's ionization function for uniform fields to nonuniform fields. It is shown that the reported failures are mainly due to the use of incorrect values of the first Townsend ionization coefficient.

IONIZATION CURRENTS IN UNIFORM AND NONUNIFORM FIELDS

UNIFORM field dc prebreakdown ionization-current multiplications in non-electron-attaching gases are interpreted in terms of two ionization coefficients η and γ ; η is the average number of ionizations produced in the gas per electron per unit of potential difference applied across the gap, and γ is the average number of secondary electrons released at the cathode per positive ion produced in the gas. The coefficients η and γ are generally functions of E/p_t (E is electric field strength and p_t is gas pressure referred to temperature t). It follows that values of η and γ may be obtained from steady-state prebreakdown currents I measured for various values of the product of p_t and the electrode separation d at a constant value of E/p_t with a constant current of photoelectrons I_0 released from the cathode due to external illumination. The equations often used for obtaining values of η and γ from such experiments are

$$I/I_0 = \exp[\eta(V - V^*)], \quad (\text{if } \gamma = 0) \quad (1)$$

and

$$I/I_0 = \exp[\eta(V - V^*)] / [1 - \gamma \{ \exp[\eta(V - V^*)] - 1 \}], \quad (2)$$

where V is the potential difference across the gap, and V^* is a parameter which allows for the fact that electrons from the cathode must fall through a potential difference comparable to the ionization potential of the gas in order to produce ionization characteristic of the applied value of E/p_t .

It might be expected that in the case of nonuniform fields, where the problem is still one-dimensional, Eqs. (1) and (2) could be generalized as

$$I/I_0 = \exp\left(\int_{V_a}^{V_b} \eta dV\right), \quad (\gamma = 0) \quad (3)$$

and

$$I/I_0 = \exp\left(\int_{V_a}^{V_b} \eta dV\right) / \left[1 - \gamma \left\{ \exp\left(\int_{V_a}^{V_b} \eta dV\right) - 1 \right\} \right], \quad (4)$$

where η is assumed to be known from uniform-field measurements. The limits of integration refer to the values of potential at the two electrodes. Equations (3) and (4) assume that the electron-energy distribution at any point in the field is characteristic of the value of E/p_t at that point and that $(V_b - V_a) \gg V^*$. The introduction of η as a function of E/p_t in Eqs. (3) and (4) is an obvious generalization. However, the proper value of γ to be used in Eq. (4) may not be related to uniform-field values of γ in a simple way because of the wide variation in E/p_t in a nonuniform field.

Morton¹ measured prebreakdown ionization currents in hydrogen at low pressures (10 Torr and below) in concentric-cylindrical geometry in order to test the validity of Eq. (3). Similar tests of Eq. (3) were also carried out in hydrogen by Fisher and Weissler² in confocal paraboloidal geometry at higher pressures. Johnson³ extended Morton's studies in hydrogen to higher pressures. (Johnson also studied air.) All of the above authors concluded that their measured current multiplications are not correctly calculated by Eq. (3). (All of the above authors assumed $V^* = 0$ and $\gamma = 0$.) The existence of such an effect means that Eq. (3), and parenthetically Eq. (4), cannot be used to calculate prebreakdown ionization-current multiplications in Geiger counters, corona onset voltages, and that one cannot solve the problem of space-charge distortion in a direct way even in parallel-plate geometry. Furthermore, any electron transport property depending on the electron energy distribution would also be affected. For example, the transit time of an electron across a Geiger counter would not be given correctly by assuming that the drift velocity is everywhere determined by the local field. Thus, the existence of such an effect has far-reaching consequences.

At the times that the studies of Morton, Fisher and Weissler, and Johnson were carried out, the most reliable values of η in hydrogen were considered to be those of Hale⁴ and all of the above authors used Hale's values to analyze their current-voltage characteristics. Measurements of η in hydrogen subsequent to Hale's

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² L. H. Fisher and G. L. Weissler, Phys. Rev. **66**, 95 (1944).

³ G. W. Johnson, Phys. Rev. **73**, 284 (1948).

⁴ D. H. Hale, Phys. Rev. **55**, 815 (1939); **56**, 1199 (1939). Numerical values for the first of these two papers are given by L. B. Loeb, *Basic Processes of Gaseous Electronics* (University of California Press, Berkeley, California, 1955), p. 691.

* Supported by the Lockheed Independent Research Fund.

work have been carried out by a number of other investigators.⁵⁻¹³ The discussion in Ref. 13 demonstrates that Rose's values of η in hydrogen must be considered to be accurate. Rose⁵ has been able to give a simple analytical expression for η which represents his data to within $\pm 2\%$ for $22 < E/p_0 < 1000$.¹⁴ For $40 < E/p_0 < 300$, Hale's values are considerably lower than those of Rose while for $E/p_0 > 300$ Hale's values are considerably higher than those of Rose. It thus appeared that a reanalysis of the data of Morton, Fisher and Weissler, and Johnson using Rose's values of η was warranted.

We have therefore set before us the task of reanalyzing the data of Morton, Fisher and Weissler, and Johnson using the simple analytical expression of Rose for η and bearing in mind the possibility of a γ mechanism's being active.

FISHER AND WEISSLER'S EXPERIMENTS

Fisher and Weissler illuminated a small area of their blunter paraboloid on the axis of symmetry with ultraviolet light and assumed that the field in which the liberated electrons moved corresponds to the field along the axis. The focal lengths of the paraboloids were 0.005 and 4.00 cm. For these experiments, p_0 ranged from 378 to 625 and voltages ranged from 2.6 to 4.4 kV. At each pressure they measured a current-voltage curve. They then calculated the ratio of the measured current I_{meas} to the calculated multiplication using Eq. (3) and Hale's values of η to find an apparent I_0 . If Eq. (3) is valid, if correct values of η are used, and if back diffusion has a negligible effect on I_0 at the voltages they used, then the apparent I_0 should be a constant independent of voltage. Fisher and Weissler found the apparent I_0 not to be a constant but to increase with voltage, the effect being larger at lower pressures. Since at that time, Hale's values of η were considered to be reliable, the variation of the apparent I_0 was interpreted as a failure of Eq. (3). More ionization was found than was expected.

We have reanalyzed the data of Fisher and Weissler by constructing plots of $\exp(\int V_a^{\gamma} \eta dV)/I_{\text{meas}}$ versus $\exp(\int V_a^{\gamma} \eta dV) - 1$ for the various pressures used. If

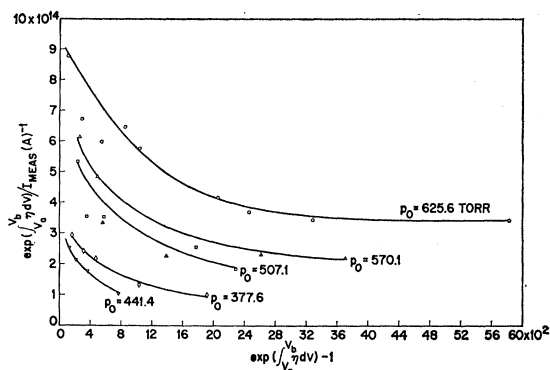


FIG. 1. $\exp(\int V_a^{\gamma} \eta dV)/I_{\text{meas}}$ versus $\exp(\int V_a^{\gamma} \eta dV) - 1$ using Hale's values of η for the data of Fisher and Weissler.

Eq. (4) is valid, and if correct values of η are used, such plots should be straight lines provided that I_0 and γ are constant. The intercepts of such lines should yield values of $1/I_0$ and the slopes values of γ/I_0 . Figure 1 shows such plots using Fisher and Weissler's observed prebreakdown currents and Hale's values of η at different constant pressures. These curves are not straight lines. Figure 2 shows the same function plotted using Rose's values of η instead of Hale's. In contrast to Fig. 1, excellent straight lines are obtained with values of $\gamma < 10^{-6}$. Even if Rose's values of η are used with $\gamma = 0$ a very reasonable fit is obtained. Therefore, Fisher and Weissler's conclusion that Townsend's ionization function fails over the range of variables studied by them was distorted by Hale's values of η .

JOHNSON'S STUDIES IN HYDROGEN

Johnson's prebreakdown ionization currents were measured with concentric cylinder geometry for values of p_{22} of 0.01 to 760. Below $p_{22} = 5$ he found it unnecessary to correct his data for back diffusion; above $p_{22} = 5$, he corrected his raw current data for back diffusion. Johnson found that a plot of $(1/arp_{22}) \ln(I_{\text{meas}}/I_0)$ versus E_{max}/p_{22} yielded a single curve for pressures greater than 0.5. (Here a is the radius of the inner cylinder and E_{max} is the field at the inner cylinder.) He then obtained the value of η at the surface of the small cylinder by graphically differentiating the curve of $(1/arp_{22}) \ln(I_{\text{meas}}/I_0)$ versus E_{max}/p_{22} and using the slope in an appropriate approximate expression. He obtained larger values of η than those of Hale by as much as 80% and concluded that Townsend's ionization function did not correctly predict pre-breakdown ionization currents in nonuniform electric fields. When Johnson's values of η are compared with the values of Rose, rather than to those of Hale, the agreement is somewhat better but still not good. Johnson states that he does not know how much error is inherent in this method of obtaining η .

Using Rose's analytical function for η , the value of

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¹⁴ Units of E will be taken as V/cm, of p as Torr, and t will be referred to degrees centigrade.

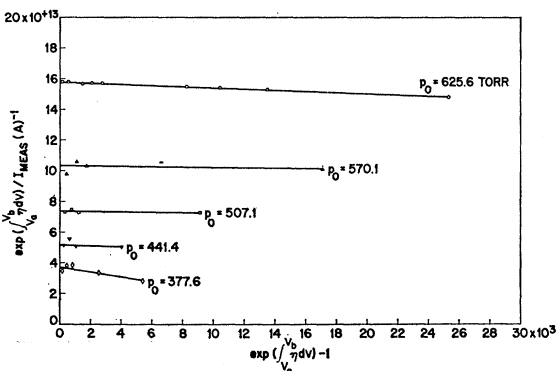


FIG. 2. $\exp(\int V_a^V \eta dV)/I_{\text{meas}}$ versus $\exp(\int V_a^V \eta dV) - 1$ using Rose's values of η for the data of Fisher and Weissler.

$(1/ap_{22})\ln(I/I_0)$ may be calculated from

$$(1/ap_{22})\ln(I/I_0) = [0.0368E_{\text{max}}/p_{22}][\exp(-128.4p_{22}/E_{\text{max}}) - \exp(-128.4p_{22}/E_{\text{min}})], \quad (5)$$

where E_{min} is the field at the outer cylinder. If the second exponential on the right is neglected, the right hand side of Eq. (5) is indeed a function of E_{max}/p_{22} only.¹⁵ We have calculated a curve of $(1/ap_{22})\ln(I/I_0)$ versus E_{max}/p_{22} using Eq. (5) and neglecting the second exponential on the right. The calculated curve deviates a maximum of 7.5% from Johnson's curve of $(1/ap_{22})\ln(I_{\text{meas}}/I_0)$ versus E_{max}/p_{22} at the highest values of E/p_{22} . In actual fact $(1/ap_i)\ln(I/I_0)$ would only be expected to be a function of E_{max}/p_i at sufficiently high pressures (≥ 0.5).¹⁶ Plots of $(1/ap_{22})\ln(I_{\text{meas}}/I_0)$ as given by Johnson for $p \geq 0.5$, as well as calculations of $(1/ap_{22})\ln(I/I_0)$ versus E_{max}/p_{22} using Hale's and Rose's values of η are shown in Fig. 3. The calculations using Hale's values give average deviations from Johnson's curves of 25%. Had Johnson compared his curves with those calculated from Hale's values of η , he might not have concluded that Townsend's ionization function fails. With the use of Rose's values of η , the agreement is so good as to preclude any questioning of the validity of Townsend's ionization function. All of the above discussion has been limited to pressures greater than 0.5, because there is no way to analyze Johnson's curves for $p < 0.5$. This is because Johnson does not present the values of E_{min} used.¹⁶

MORTON'S EXPERIMENTS

In order to avoid corrections for back diffusion, Morton restricted his pressures to below about 4 for the larger of two cathodes used and to below about 10

¹⁵ It may be shown in general that regardless of the functional dependence of η on E/p_i , $(1/ap_i)\ln(I/I_0)$ will be a function of E_{max}/p_i providing the value of η at the anode is sufficiently small.

¹⁶ At low pressures where the second exponential of Eq. (5) cannot be neglected V^* is not negligible compared to $V_b - V_a$ and Eqs. (3) and (4) are not applicable in any case.

for the smaller cathode. His pressures ranged down to about 0.01. His applied voltages ranged up to 320 V, although he did not analyze any data obtained above 180 V. From each current-voltage curve, he evaluated I_0 , and then plotted I_{meas}/I_0 as a function of voltage for each pressure studied. Morton's I_{meas}/I_0 versus voltage curves are concave toward the voltage axis for $p < 0.0806$ and are convex toward the voltage axis for $p > 0.1296$. Morton states that for $p < 0.0806$ there are not enough collisions to apply Townsend's theory even in uniform fields.

From his I_{meas}/I_0 versus voltage curves Morton constructed plots of I_{meas}/I_0 as a function of pressure for various constant voltages and compared these plots with those calculated using Eq. (3) with Hale's values of η . The calculated curves terminate at those pressures corresponding to $E_{\text{max}}/p_{22} \cong 900$.¹⁷ The disagreement between I/I_0 and I_{meas}/I_0 is very severe over a large part of the pressure range being as much as a factor of 5. We have calculated the ionization to be expected using Rose's values of η in conjunction with Eq. (3); using Rose's values of η rather than Hale's makes the values of I/I_0 lie somewhat closer to Morton's I_{meas}/I_0 curves, but the discrepancy is still serious. The calculations

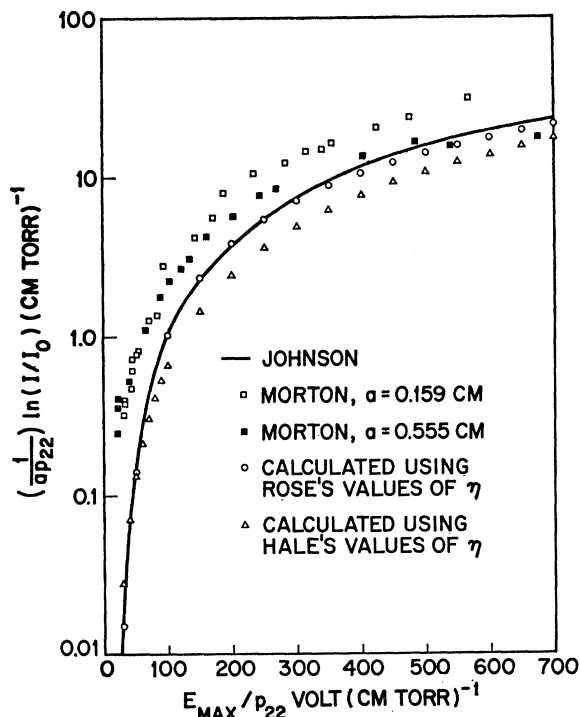


FIG. 3. $(1/σp_{22})\ln(I/I_0)$ versus E_{max}/p_{22} for $0 \leq E_{\text{max}}/p_{22} \leq 700$ from Johnson's and Morton's data, and calculated using Rose's and Hale's values of η .

¹⁷ The data of Hale reported in Phys. Rev. 56, 1199 (1939) goes up to $E/p_{22} \cong 1800$, whereas his data reported in Phys. Rev. 55, 815 (1939) only goes up to $E/p_{22} \cong 900$. Morton did not use nor does he refer to Hale's measurements for $E/p_{22} \geq 900$.

using Hale's or Rose's values are not much different despite the considerable difference in measured η values. Morton's fields are both below and above $E/p_0=300$ and since Hale's values of η are lower than Rose's for $E/p_0 < 300$ and are higher than Rose's for $E/p_0 > 300$, the errors due to Hale's values of η tend to compensate. If one assumes that the primary ionization is correct as calculated by Rose's values of η and that the discrepancy is due to a γ , it is possible to calculate values of γ from the curves of I_{meas}/I_0 versus pressure as given by Morton. This procedure results in values of γ varying from 0.15 to unity and in a few cases even exceeding unity. These values of γ are much too high (and indeed much too variable) to be accepted as reasonable. The above conclusions are equally valid

if one uses Hale's values of η . However, Morton's data for 150 and 180 V when put into the same form as used by Johnson and plotted in Fig. 3 lie considerably higher than Johnson's curve. Thus, there is an experimental discrepancy between Morton's and Johnson's data. This discrepancy is not understood. A further complication is the fact that V^* is not negligible compared to the gap voltages of 100, 150, and 180 V analyzed by Morton. If such an effect were taken into account, the agreement between Morton's measured and calculated current multiplications would be even worse. The basis for doubting the validity of Townsend's ionization function at these low pressures from Morton's data would not seem to be on a firm foundation at the present time.

Angular Distribution and Thickness Dependence of Transition Radiation from Thin Aluminum Foils*

R. J. HERICKHOFF,[†] W. F. HANSON,[‡] E. T. ARAKAWA, AND R. D. BIRKHOFF

Health Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee

(Received 5 April 1965)

The angular distribution of photons emitted from Al foils 320 Å in thickness bombarded with 80-keV electrons, and the dependence of the peak intensity on foil thickness, have been determined experimentally in the wavelength region from 650 to 1100 Å. The absolute efficiency of the optical spectrometer was determined in the visible-wavelength region from a standard lamp and extrapolated into the vacuum ultraviolet-wavelength region using the measured grating efficiency and quantum efficiency of sodium salicylate. Experimental photon intensities were found to be approximately one half of the theoretical photon intensities. The intensity at the peak wavelength had a maximum at 22° and 157° from the foil normal, with zero intensity at 0°, 180°, and 90°. The angles for which the theoretical curves exhibit maximum intensity are 15 and 168°. The wavelength for which the experimental curves exhibit maximum intensity was found to decrease as the angle of observation increased from 0° to 40° or decreased from 180° to 140°. The plasma wavelength and energy, 835 Å and 14.9 eV, respectively, were obtained by extrapolating to 0°. The intensity at the wavelength of maximum emission was found to decrease rapidly as the foil thickness was increased from 180 to 435 Å and then to remain fairly constant out to 700 Å.

INTRODUCTION

THE study by optical methods of plasma oscillations induced by charged-particle excitation was initiated by Ferrell's¹ prediction in 1958 that these oscillations should decay by the emission of monochromatic photons at the plasma frequency. Numerous observations of electron-irradiated Ag have revealed a peak at 3300 Å with a continuum in the longer wavelength region.²⁻⁶ Ritchie and Eldridge⁷ generalized the

transition-radiation theory of Frank and Ginsburg⁸ for an idealized semi-infinite medium to the case of a finite foil characterized by a dielectric constant $\epsilon(\omega)$. A complete description of the Ag spectrum could not be obtained from the simple plasmon-decay picture but required the transition-radiation theory.

A recent paper reported a sharp peak in the optical-emission spectrum of electron-irradiated Al at 15.2 eV which was identified as plasmon radiation.⁹ Similar investigations of Mg, Cd, In, and Zn were reported.¹⁰

* Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

[†] ORINS Graduate Fellow, Vanderbilt University, Nashville, Tennessee.

[‡] AEC Health Physics Fellow, Vanderbilt University, Nashville, Tennessee.

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