the model gives good agreement for the low-temperature second virial coefficients of both isotopes.[5,25]

The calculated diffusion coefficients seem to be in agreement with experiment, as nearly as it is possible to assess the experimental uncertainties.

Thermal diffusion in He^3-He^4 shows a distinct discrepancy at higher temperatures, due to the steepness of the repulsion term in the (12-6) model. This is not surprising, since it has been known for some time[1a] that the high-temperature viscosity clearly indicates that an r^{-12} repulsion is too "hard."

We conclude, therefore, that there is no justification for trying to improve the agreement between calculation and experiment at low temperatures by adjustment of the (12-6) potential parameters. However, it is known that the (12-6) model is not as good at high temperatures, and it is then of interest to ask how the model should be altered to obtain agreement at high temperatures without spoiling the agreement at low temperatures. Although use of the (exp-6) or (exp-6-8) models will "soften" the repulsion and thereby improve matters at high temperatures, as well as improve the agreement for α_T, it has previously been shown, [1(b),6] at least for the (exp-6) model, that the low-temperature agreement is then worse. The reason is that these particular three-parameter models are insufficiently flexible; that is, that the repulsive and attractive parts cannot be varied independently. It is not clear how to alter the usual models in other ways to achieve this without at the same time introducing more adjustable parameters and allowing the whole procedure to degenerate into physically meaningless curve fitting.

---

[25] W. E. Keller, Phys. Rev. **97**, 1 (1955); **98**, 1571 (1955).

PHYSICAL REVIEW VOLUME 139, NUMBER 4A 16 AUGUST 1965

Temperature and Frequency Dependence of Ultrasonic Absorption in Liquid Helium below 1°K*

W. A. JEFFERS, JR.,† AND W. M. WHITNEY‡

Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 24 March 1965)

From measurements of the temperature dependence of the amplitude of ultrasonic pulses propagated over a fixed path in liquid helium, the attenuation coefficient has been determined at the six frequencies 1.00, 2.02, 3.91, 6.08, 10.2, and 11.7 Mc/sec, at temperatures extending down to 0.2°K. Below 0.6°K, the observed variation of the absorption coefficient α (cm^{-1}) with frequency f (Mc/sec) and temperature T (°K) can be represented by the empirical equation $\alpha=0.11\ f^{3/2}\ T^3$. This behavior differs from that predicted by several existing theories that are based upon the three-phonon or four-phonon interaction between longitudinal acoustic quanta and thermal phonons. Over the temperature interval 0.6–0.8°K, the measured attenuation is somewhat greater than would be expected from an extrapolation of the results below 0.6°K into this temperature region. A comparison of the data with some calculations by Khalatnikov suggests that the additional absorption arises from thermal conduction in the normal fluid. An equation that is analogous to the classical Kirchhoff expression adequately accounts for this contribution to the total absorption, even at temperatures for which the lifetime of the thermal phonons exceeds the period of the sound wave.

I. INTRODUCTION

At temperatures just below 1°K, the absorption of ordinary sound in liquid helium goes through a maximum. The behavior of the absorption on the high-temperature side of the peak has been quite well defined experimentally.[1] Chase[2] has made careful measurements of the (amplitude) attenuation coefficient α at the frequencies 2, 6, and 12 Mc/sec at temperatures above 0.85°K, and has established a close correspondence between his results and a microscopic theory of the absorption developed by Khalatnikov.[3] The pressure dependence of the attenuation has been examined by Newell and Wilks.[4] Although it is not feasible to make a detailed quantitative comparison with the

---

* This work was supported in part by the U. S. Army Signal Corps, the U. S. Air Force Office of Scientific Research, and the U. S. Office of Naval Research; additional support was received from the National Science Foundation. Concluding phases of the work were performed at the Jet Propulsion Laboratory, California Institute of Technology, supported by the National Aeronautics and Space Administration under Contract No. NAS 7-100. Some of the results described in the paper form the basis of a Ph.D. thesis submitted by W. A. Jeffers, Jr., to the Department of Physics, MIT, 1962. A preliminary account of the research has been given at a meeting of the American Physical Society [W. A. Jeffers, Jr., and W. M. Whitney, Bull. Am. Phys. Soc. **7**, 472 (1962)].

† Present address: Battelle Memorial Institute, Columbus, Ohio.

‡ Present address: Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California.

[1] For accounts of previous studies of sound propagation in liquid helium, see the review article by J. Wilks, Z. Physik. Chem. **16**, 372 (1958); and K. R. Atkins, *Liquid Helium* (Cambridge University Press, Cambridge, England, 1959), Chap. 5.

[2] C. E. Chase, Proc. Roy. Soc. (London) **A220**, 116 (1953).

[3] I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. **20**, 243 (1950).

[4] J. A. Newell and J. Wilks, Phil. Mag. **1**, 588 (1956).

theory, Dransfeld, Newell, and Wilks[5] (DNW) have shown that these results are qualitatively in accord with it. Measurements of the absorption of sound in dilute solutions of ^{3}He in ^{4}He[6–8] can also be accounted for by making appropriate modifications[9] in Khalatnikov's original model. It therefore seems likely that the principal microscopic mechanisms contributing to the absorption in this temperature region have been correctly identified, and that it is primarily analytical complications that stand in the way of a complete quantitative description of the experimental results.

For the temperature region below the peak, however, there is much less experimental information available, and moreover there is as yet no general agreement about the ideas that must be invoked to explain the existing data.

The first study of ultrasonic propagation in liquid helium at temperatures below 0.8°K was that of Chase and Herlin.[10] Their measurements, which were conducted at the frequency 12.1 Mc/sec and at vapor pressure, showed that the absorption does in fact pass through a maximum, and falls asymptotically to zero as the temperature approaches 0°K. Over the temperature range 0.3–0.8°K, the measured values of α were found to be proportional to $T^{2.8}$; near 0.3°K a change in the power law occurred, and from this point down to 0.15°K, the lowest temperature reached, the descent was more rapid.

Subsequent measurements of α in pure ^{4}He at vapor pressure have been reported by Newell and Wilks[4] at 14.4 Mc/sec, by Whitney[11] at 11.8 Mc/sec, by DNW at 6.0 Mc/sec, and by Harding and Wilks[8] at 14.0 Mc/sec. From the results of Whitney, which extend down to 0.2°K, it is found that $\alpha \propto T^{2.9}$ over the range 0.4–0.8°K, with a change in power law occurring near 0.4°K. The Oxford results do not go below 0.4°K. No temperature dependence is stated for them, but from tabulated or plotted values of α it can be ascertained that they are compatible with the power law T^n, with $n \sim 2.5$ for all three sets of measurements. The values of α below the peak at 6.0 and 14.4 Mc/sec were stated to be roughly consistent with a linear frequency dependence.

The absorption coefficient has also been measured in pure ^{4}He below 1°K as a function of pressure by Newell and Wilks[4] at 14.4 Mc/sec, and by Whitney[11] at 11.8 Mc/sec. The decrease in α with increasing pressure is greater than can be accounted for by making allowances for increased density and sound velocity in the classical phenomenological equations for sound absorption. It was reported by Whitney that $\alpha \propto T^{2.8}$ at 8.0 atm; the temperature variation of the values tabulated by DNW for the absorption at 8.4 atm is compatible with the same exponent. From both sets of measurements there is indication that the temperature dependence is considerably more rapid than T^3 at higher pressures. No systematic study of the frequency dependence of the absorption in liquid helium under increased pressure has been carried out in any temperature region.

Experiments by Harding and Wilks[8] with dilute solutions of ^{3}He in ^{4}He have shown that, on the low-temperature side of the peak as well as above, the absorption is substantially reduced by the impurity atoms. Below 0.9°K, the temperature dependence of α in a solution containing 0.32% ^{3}He can be characterized by the exponent $n \sim 2.6$. The results at two higher concentrations (1.6 and 5.2% ^{3}He) appear not to be compatible with a simple power-law representation. Nothing at all is known about the frequency dependence of the absorption in ^{3}He-^{4}He mixtures in the low-temperature region.

Calculations by Arkhipov[12] and by Chase[13] have demonstrated that Khalatnikov's theory of the sound absorption is not adequate to describe the observed behavior of the attenuation coefficient in the temperature region below the peak. The predicted values fall off very rapidly with decreasing temperature, and below 0.6°K are more than one order of magnitude smaller than the measured ones. Alternative theories of the absorption have been put forward by DNW, Kawasaki,[14] Dransfeld,[15] Woodruff,[16] Kawasaki and Mori,[17] and Khalatnikov.[18] This work will be referred to later in the paper. Although several different points of departure are represented in these investigations, all but one are based upon the three-phonon scattering process, and lead to the prediction that the absorption coefficient will be proportional to ωT^4 at temperatures below 0.5°K (ω is the angular frequency of the sound wave). Khalatnikov, who considers the four-phonon process, obtains $\alpha \propto \omega T^6$. The experiments referred to above indicate that the temperature dependence is more likely T^3 than T^4; however, it is shown by DNW (see Fig. 11 of their paper) that the original experimental points of Chase and Herlin in the temperature interval 0.15–0.5°K are also reasonably consistent with T^4 behavior. Unfortunately, it is in this temperature region that the experimental scatter, when the results are plotted

---

[5] K. Dransfeld, J. A. Newell, and J. Wilks, Proc. Roy. Soc. (London) **A243**, 500 (1958).

[6] E. W. Guptill, A. M. R. van Iersel, and R. David, Physica **24**, 1018 (1958).

[7] G. O. Harding and J. Wilks, Phil. Mag. **3**, 1469 (1958).

[8] G. O. Harding and J. Wilks, Proc. Roy. Soc. (London) **A268**, 424 (1962).

[9] I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. **23**, 265 (1952); A. F. Andreev, *ibid.* **40**, 1705 (1961) [English transl.: Soviet Phys.—JETP **13**, 1201 (1961)].

[10] C. E. Chase and M. A. Herlin, Phys. Rev. **97**, 1447 (1955).

[11] W. M. Whitney, Phys. Rev. **105**, 38 (1957).

[12] R. G. Arkhipov, Dokl. Akad. Nauk SSSR **98**, 747 (1954).

[13] C. E. Chase, Am. J. Phys. **24**, 136 (1956).

[14] K. Kawasaki, Progr. Theoret. Phys. (Kyoto) **26**, 793, 795 (1961).

[15] K. Dransfeld, Phys. Rev. **127**, 17 (1962).

[16] T. O. Woodruff, Phys. Rev. **127**, 682 (1962).

[17] K. Kawasaki and H. Mori, Progr. Theoret. Phys. (Kyoto) **28**, 784 (1962).

[18] I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. **44**, 769 (1963) [English transl.: Soviet Phys.—JETP **17**, 519 (1963)].

logarithmically, is greatest; from the same data, support can also be claimed for a T^6 dependence below 0.3°K.[18]

From the brief description of previous experimental work given above, it is quite clear that our knowledge of the behavior of the absorption coefficient, in particular with respect to its frequency and temperature dependence, is by no means definitive for the temperature region below the attenuation maximum. The existing data at vapor pressure represent essentially only three different frequencies: 6, 12, and 14 Mc/sec, and only two of these are significantly different. Furthermore, only the results at 12 Mc/sec extend below 0.4°K; because of the scatter of the data in this region, the temperature dependence is uncertain, and the change in power law may in fact reflect a systematic error.[11]

In this paper we present the results of measurements of the absorption coefficient in liquid helium at vapor pressure at six frequencies in the range 1 to 12 Mc/sec, and at temperatures from approximately 0.2 to above 1°K. In interpreting these results, we attempt specifically to ascertain the frequency and temperature dependence of the attenuation on the low-temperature side of the maximum. It will be shown that, below 0.6°K, the data are consistent with the relationship $\alpha \propto \omega^{3/2}T^3$. The apparent change in power law reported in earlier measurements is attributed to the manner in which signal-amplitude measurements were converted to attenuation values. It is concluded that none of the theories of the absorption so far proposed gives an adequate description of the observed behavior in this temperature region.

It is also shown that a small contribution to the total absorption in the neighborhood of the attenuation peak, but at slightly lower temperatures, can be identified with the mechanism of thermal conduction in the normal fluid, described by Khalatnikov's phenomenological coefficient of thermal conductivity κ.[19,20]

II. EXPERIMENTAL METHOD

Cryostat

Our experimental techniques, both cryogenic and electronic, are similar to those used by Chase and Herlin[10] and others in previous investigations. The demagnetization cryostat consists of two coaxial cylindrical brass cans, the outer one immersed in liquid helium, the inner one holding the ultrasonic chamber and the paramagnetic salt (iron ammonium alum) used for cooling and for thermometry. The space between the two cans is evacuated. The contents of the inner can are placed in thermal contact with, and isolated from, the liquid-helium bath by means of an Ashmead conical valve,[10,21] which also supports the inner can within the outer one. The valve consists of two closely fitting, thin-walled, truncated stainless-steel cones, the inner of which can be raised or lowered from outside the Dewar system by means of a long thin-walled tube. When the cones are separated, the liquid helium filling the space between them carries the heat of magnetization away from the paramagnetic salt in a few seconds. To isolate the inner can, the upper cone is seated firmly in the lower one. After demagnetization, the inner can warms up from the lowest temperature reached, approximately 0.2°K, to 0.9°K in periods of time ranging from a few minutes (if the valve is not properly closed) to one-half hour or more.

The cones in our apparatus are 0.020 in. thick, and have been carefully lapped together to produce a snug fit. In time, the fit deteriorates, and relapping is required. The top and bottom of the hollow inner cone are soft-soldered in place; air trapped inside freezes out when the apparatus is cooled. All other permanent joints have been brazed in a hydrogen furnace to ensure nonporous, vacuum-tight seals and to minimize oxidation and thermal distortion of the parts during assembly.

Coaxial electrical connections to the chamber from the bath are made through two thin-walled, stainless-steel tubes, each with glass-to-metal seals soldered into its two ends.

Thermometry

Temperatures below approximately 1.4°K are measured with an ac mutual-inductance thermometer,[22] using a Hartshorn mutual-inductance bridge operated at 33 cps. Temperature changes of 0.1% can be resolved with the bridge, but errors in the absolute temperatures are an order of magnitude larger. Below 0.4°K, there are uncertainties which arise in the process of making corrections for departures from the Curie-Weiss law.[22,23] The data on which such corrections are based are usually obtained in experiments carried out with spherical or spheroidal salt pills, whose internal magnetization is uniform in a uniform external magnetic field. The internal magnetization will not be uniform within our cylindrical samples, but we nevertheless treat them as if they were ellipsoids, estimating their Weiss temperatures from their shape and density. Since, at our lowest temperatures, the correction for departures from the Curie-Weiss law amounts to less than 15% of the absolute temperature for a sphere of iron-ammonium alum,[24] our corrected temperatures in this region are probably in error by no more than a few percent. An additional error, which is more important at the higher temperatures, is introduced by a slow drift in the fre-

---

[19] I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. **23**, 8 (1952).

[20] I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. **23**, 21 (1952).

[21] D. V. Osborne, Phil. Mag. **1**, 301 (1956).

[22] D. de Klerk, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 15, p. 38 ff.

[23] G. K. White, *Experimental Techniques in Low-Temperature Physics* (Oxford University Press, London, England, 1959), Chap. VIII.

[24] D. de Klerk, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), p. 99.

quency of the 33-cps signal supplied to the bridge. Since the bridge balance is somewhat frequency-sensitive, there is a small shift in one of the calibration constants for the mutual-inductance thermometer during the time required for the inner can and its contents to warm up after a demagnetization. Resulting errors in the absolute temperature T at a given time are proportional to T^2, and can amount to as much as 0.05°K above 1°K, if no allowance is made for the drift.

It was found that the mutual inductance required for balance was shifted by a small constant amount when the upper cone was moved up or down between the limits of its vertical range, a distance of approximately one-half inch. A small correction for this effect was applied in calculating temperatures from the bridge settings.

Ultrasonic Measurements

Sound pulses 15–50 μsec in duration are produced in the liquid helium by exciting an x-cut crystal, 1 cm in diameter, at its resonant frequency or at an odd multiple thereof. The crystal holders are patterned after those of Chase and Herlin.[10] Two crystals are employed, one as transmitter and one as receiver. With a single crystal and reflector, there is danger that the tail of the transmitted pulse will interfere coherently with the first returning echo to produce a spurious change in amplitude of the echo. Such an effect apparently was responsible[11] for the double maximum observed by Chase and Herlin at the top of the principal attenuation peak. Even when two separate crystals are used, the electrical oscillations that accompany prolonged ringing of the transmitting crystal, after the exciting pulse has stopped, can interfere with the signal at the receiving crystal as the result of direct electrical pickup. It was found necessary to back up both crystals with conical brass end-pieces in order to damp out these oscillations.

The signal obtained from the receiving crystal is amplified and displayed on an oscilloscope screen. Pulse-amplitude measurements are made with the aid of a second pulse of the same frequency and duration as the first, obtained from a gated signal generator with a calibrated variable output. The comparison pulse is injected into the amplifier shortly before or after the arrival of the ultrasonic pulse, and its amplitude is adjusted until the two pulses appear to have the same height on the oscilloscope screen. By this method pulse amplitudes can be measured to within 1 or 2%.

The frequency of the carrier-wave component of the pulses exciting the transmitting crystal is determined by counting zero crossings on the calibrated sweep of an oscilloscope, or by switching the pulsed oscillator to cw operation and measuring its frequency with a wavemeter or a frequency counter. Values quoted for the frequencies are considered to be accurate to ±1%.

It is the variation of pulse height with temperature that is measured directly in the experiment and not the attenuation, since the distance d over which the sound pulses are propagated remains constant during a given run. However, if the attenuation α_0 is known at some reference temperature T_0, the attenuation α at any other temperature T can be calculated from a knowledge of the signal amplitudes at the two temperatures:

$$\alpha=\alpha_0+d^{-1}\ln(S_0/S). \qquad (1)$$

The attenuation measurements of Chase[2] above 0.85°K, which were carried out in an apparatus with a variable path, can be used to convert measurements of relative signal amplitude to values of the attenuation at lower temperatures. It was by this procedure that Chase and Herlin established originally that the attenuation at 12 Mc/sec falls to zero as absolute zero is approached. Alternatively, therefore, we can set $\alpha_0=0$ and use the pulse height observed at the lowest temperature reached as the reference amplitude, or determine the amplitude S_0 at $T=0$ by extrapolation. For reasons given in the following section, extrapolation is used in the adjustment of the data presented in this paper.

The values substituted for d in Eq. (1) are the room-temperature crystal spacings reduced by 0.4%[25] to correct for the thermal contraction of brass down to liquid-helium temperatures. The numbers to be quoted are considered accurate to within ±0.5% or less.

III. RESULTS

By exciting a single pair of quartz crystals resonant at 2.02 Mc/sec on their first, third, and fifth harmonic, it was possible to carry out measurements at the three frequencies 2.02, 6.08, and 10.2 Mc/sec without opening the inner can. (Loading of the crystals by the brass holders produced a small displacement of the optimum operating frequency from the true harmonic.) The crystal spacing was 1.90 cm. Warm-up times to 0.9°K were 20 to 40 min for the demagnetizations at 2 and 6 Mc/sec, 10 to 15 min for those at 10 Mc/sec. The amplitude of the transmitted pulse was changed from one demagnetization to the next to ensure that the final results were characteristic of small-amplitude sound waves.

The method by which signal-amplitude measurements are converted to attenuation values is illustrated with data from two demagnetizations at the frequency 2.02 Mc/sec, results from which are shown in Fig. 1. Also shown are smoothed values given by Chase[26] for the attenuation at 2.0 Mc/sec. The data from our two runs are in good agreement with each other, but they lie nearly 0.015 cm^{-1} above a curve drawn through Chase's points. Since a reduction of our values by this amount would make the attenuation negative below 0.35°K, the disagreement must arise in part from systematic errors in the temperature scales used in the two experiments.

---

[25] R. J. Corruccini and J. J. Gniewek, Natl. Bur. Std. (U. S.), Monograph 29 (1961).

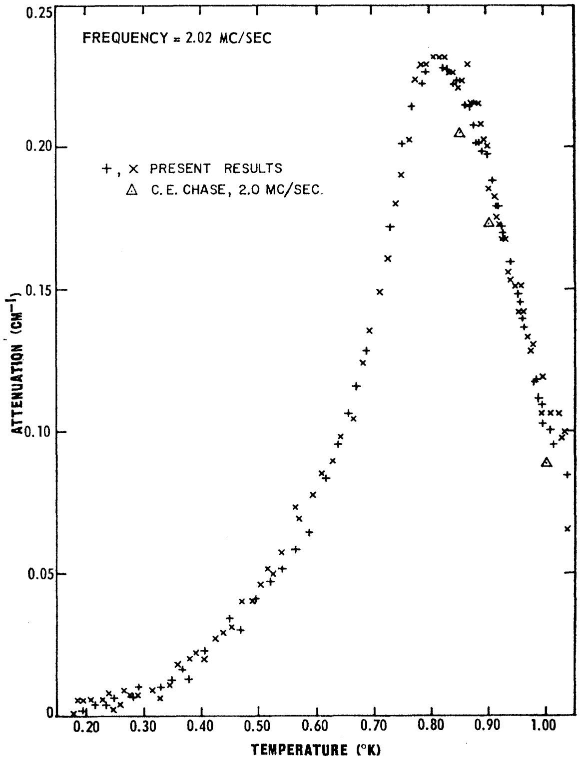

FIG. 1. Attenuation versus temperature at 2.02 Mc/sec. Attenuation values have been assigned by extrapolating the signal-amplitude measurements to $T=0$°K, assuming that $\alpha\sim T^3$. Signal amplitudes for the points indicated by × were approximately half those for the points indicated by +. The results of Chase are taken from Ref. 26.

The measurements coincide if we reduce our temperature values by 2 to 3%; however, the proper correction is unknown, and consequently the magnitude of any additional adjustment that must be made in our attenuation values to bring them into coincidence with those of Chase cannot be determined.

The numerical values plotted in Fig. 1 have been obtained by extrapolating our measurements to $T=0$°K, rather than by fitting the results above the attenuation peak to Chase's values. The method is based upon two assumptions: (1) The amplitudes of the sound pulses are sufficiently small, so that none of the absorption at low temperatures arises from finite amplitude effects; (2) below some upper limiting temperature $T_{\max}$, the true attenuation follows a simple power law,

$$\alpha=AT^n, \tag{2}$$

where the coefficient A depends only upon frequency. The exponent n is a constant, not necessarily an integer. The signal amplitude will obey the equation

$$\ln S(T)=\ln S_0-AdT^n. \tag{3}$$

For a given exponent n, the constants A and S_0 are calculated by the method of least squares, using signal amplitudes $S_i(T_i)$ from a given run for which $T_i\leq T_{\max}=0.5$°K. It is then determined for what value of n the total squared deviation or residual R_n will be minimum, where

$$R_n=\sum_i\left[A_nT_i^n-d^{-1}\ln(S_{0n}/S_i)\right]^2. \tag{4}$$

(The subscript n is used to indicate that A and S_0 are not varied independently.) For the two runs in question, $n=3.1$ and 3.2; however, R_n is a slowly varying function of n, and we do not consider the difference between these numbers and $n=3$ to be significant. In adjusting the data plotted in Fig. 1, we have therefore used the value S_{03}, corresponding to $n=3.0$. The coefficient A_3 has an average value 0.34 for the two runs.

Figure 2 is a log-log plot of attenuation versus temperature for both demagnetizations at 2.02 Mc/sec, the dashed line representing the empirical equation $\alpha=0.34\ T^3$. Note that the region over which the equation and the experimental points are in agreement is larger than that for which the least-squares fit was carried out, extending up to approximately 0.65°K.

The extrapolation procedure just described has been applied to more than thirty demagnetizations at six different frequencies. In twenty of these, the exponent n for which R_n is minimum lies in the range 2.6 to 3.4. For this reason we have chosen as the reference amplitude S_0 for each of our runs the value derived from a least-squares fit of the measured signal amplitudes to Eq. (3), with $n=3.0$. This method of converting signal

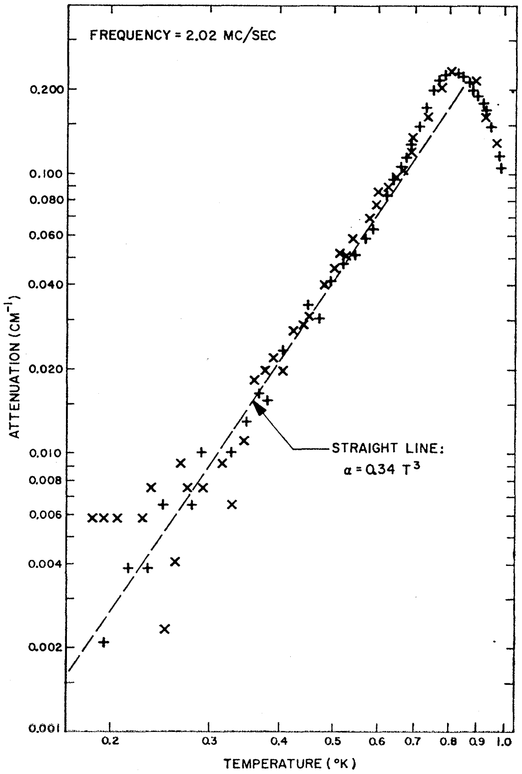

FIG. 2. Logarithmic plot of the points shown in Fig. 1.

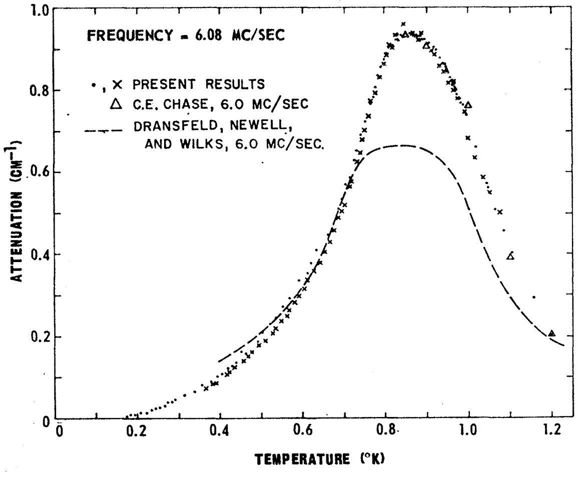

FIG. 3. Attenuation versus temperature at 6.08 Mc/sec. Attenuation values have been assigned on the basis of a T^3 extrapolation. The signal amplitudes for the points indicated by × were approximately one-fourth those for the points indicated by •. The data of Chase are taken from Ref. 26. The dashed line representing the results of Dransfeld, Newell, and Wilks is derived from Fig. 4 of Ref. 5.

amplitudes to attenuation values in no way changes the general shape of an attenuation-versus-temperature curve such as that presented in Fig. 1, but it necessarily affects conclusions about the temperature dependence of the attenuation at temperatures well below the peak. This point will be discussed at greater length in the next section.

Results of two demagnetizations at 6.08 Mc/sec, carried out on the same day as the 2-Mc/sec runs just described, are shown in Fig. 3. The agreement between our values and those given by Chase[26] for 6.0 Mc/sec is excellent. The dashed line is derived from the measurements of DNW at 6.0 Mc/sec. Their results agree with those reported here in the temperature region 0.5 to 0.7°K, and with those of Chase above 1.2°K (which are not shown in Fig. 3), but in the vicinity of the attenuation peak they lie considerably lower than either Chase's values or ours. We have encountered similar flattening of the attenuation peak in our own experiments when there was excessive interference between the received pulse and either the tail of the transmitted pulse, lengthened by prolonged ringing of the input crystal, or a cw signal leaking through from the pulsed oscillator as a result of inadequate gating.

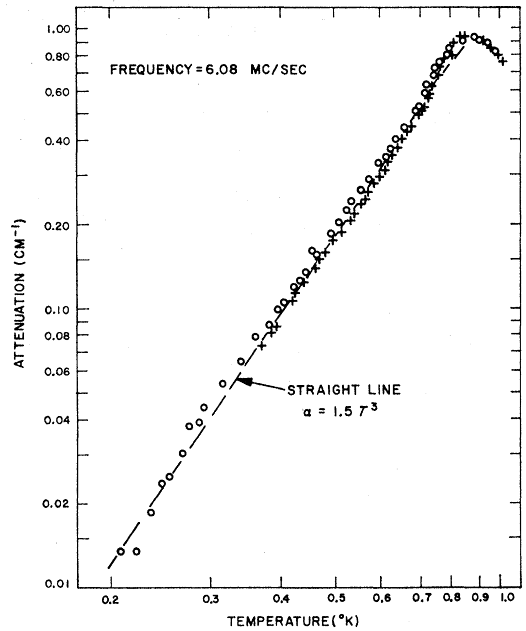

FIG. 4. Logarithmic plot of the points shown in Fig. 3. (The symbols ○ and + correspond, respectively to the symbols • and × of Fig. 3).

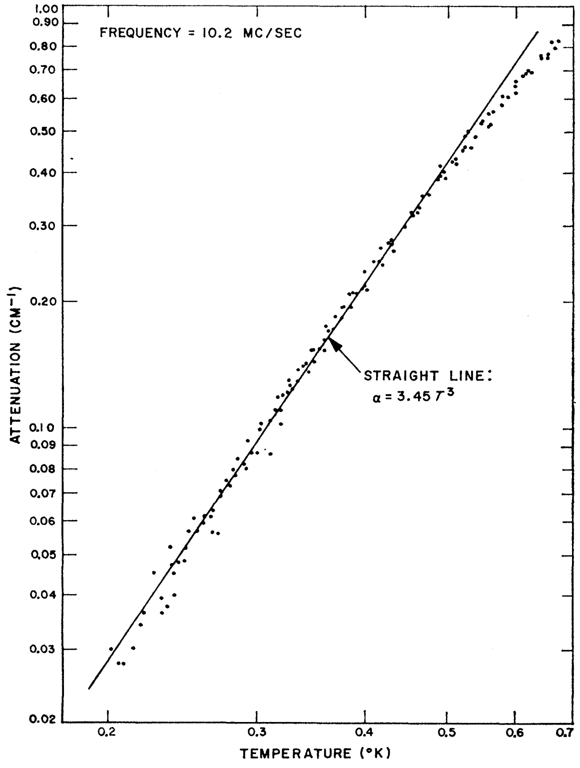

FIG. 5. Attenuation versus temperature at 10.2 Mc/sec.

In Fig. 4, the results at 6.08 Mc/sec are compared with the empirical equation $\alpha = 1.5\ T^3$. The coefficient is an average derived from five demagnetizations, the two described above and three others carried out on another day.

Demagnetizations at 10.2 Mc/sec were performed on two separate days. Between 0.7 and 1.1°K the amplitude of the received signal was so small that we could not measure it, so we have no results in the immediate

---

[26] C. E. Chase, Ph.D. thesis, Cambridge University, 1954 (unpublished).

vicinity of the absorption peak. Attenuation values derived from four demagnetizations are shown in Fig. 5.

From 12 such runs, an average value 3.45 is obtained for the coefficient A_3. Above 0.5°K the points lie below the straight line which represents the T^3 law. This departure is attributable to the fact that, for each of the four runs plotted, the best fit to Eq. (3) occurs for n somewhat less than 3. If the values shown are increased by a few percent, they fall more nearly on a single straight line with a smaller slope.

Signal-amplitude measurements were made at the frequencies 3.91 and 11.7 Mc/sec by exciting a second pair of crystals at their fundamental and third-harmonic frequencies. The crystal spacing was 5.05 cm. Attenuation values derived from two demagnetizations at 11.7 Mc/sec are shown in Fig. 6, together with smoothed values of the results of Chase and Herlin[10] ($f=12.1$ Mc/sec, $d=1.96$ cm), and Whitney[11] ($f=11.8$ Mc/sec, $d=1.48$ cm). Although there appear to be systematic differences between individual runs, taken together the points are consistent with the straight line representing the empirical equation $\alpha=4.5\ T^3$.

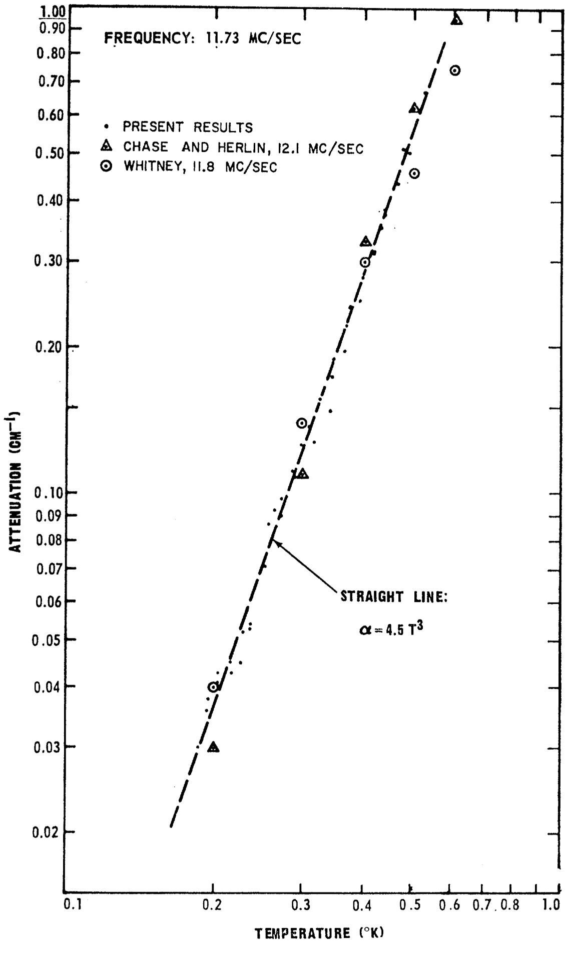

FIG. 6. Attenuation versus temperature at 11.73 Mc/sec. Results of earlier experiments at approximately 12 Mc/sec are shown for comparison.

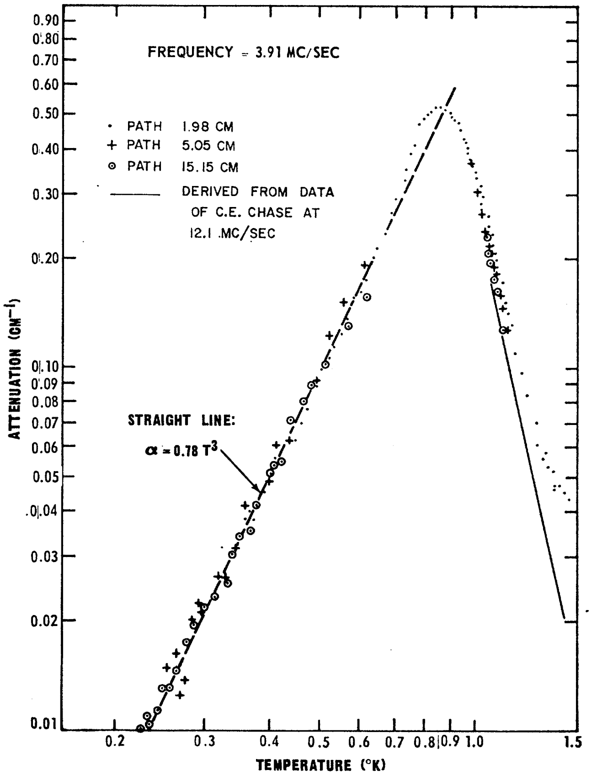

FIG. 7. Attenuation versus temperature at 3.91 Mc/sec. The solid line is derived from the measurements of Chase at 12.1 Mc/sec (Ref. 26) by assuming that $\alpha\propto\omega^2$ above 1.1°K.

The points shown in Fig. 7 for the frequency 3.91 Mc/sec were obtained under a variety of experimental conditions. The pluses are derived from signal-amplitude measurements made using the first transmitted pulse, as in the runs described above. The enclosed dots represent measurements of the amplitude of the first echo at the receiving crystal, a pulse that has been reflected twice, once by each crystal, for which the effective path length is therefore 15.15 cm. The warmup times for these and five similar runs were quite short, averaging seven minutes from the lowest temperatures reached to 0.5°K. (At the conclusion of this experiment the cones were relapped.) Nevertheless these results agree very well with the third set of points plotted in Fig. 7, shown as dots, for which the warmup time was 14 min to 0.6°K (18 min to 0.9°K). The crystal spacing for this demagnetization was 1.98 cm. For all of the runs at 3.91 Mc/sec, the average value of A_3 is 0.78.

The data at 3.91 Mc/sec and 1.98 cm presented above, as well as those to be discussed next, were taken during the course of measurements of the velocity of sound near 1°K and below.[27] In Fig. 8, the points shown as

---

[27] W. M. Whitney and C. E. Chase, Phys. Rev. Letters **9**, 243 (1962).

pluses represent the results of one of several runs carried out at 1.00 Mc/sec with a crystal spacing of 5.05 cm and warmup times of approximately $\frac{1}{2}$h to 0.9°K. For a second series of measurements, results from which are shown as solid circles, the crystal separation was lengthened to 9.96 cm to increase the accuracy of the attenuation values. The results of the two experiments are in reasonable accord. In the figure, points below 0.4°K have been suppressed because of their scatter, and the extrapolation calculation was applied only to those points in the temperature range 0.4 to 0.6°K. The dashed line in Fig. 8 represents the equation $\alpha=0.13\,T^3$. The points shown as triangles are Chase's values at 12.1 Mc/sec,[26] corrected to 1.00 Mc/sec by making use of the fact that $\alpha\propto f^2$ above approximately 1.1°K.

IV. ANALYSIS AND DISCUSSION

1. Low-Temperature Region

Temperature Dependence

An analysis of our results to determine the temperature dependence of the attenuation on the low-temperature side of the peak is complicated by the fact that we have made assumptions about this behavior in order to convert signal-amplitude measurements to attenuation values. The hypothesis that the attenuation obeys a simple power law at sufficiently low temperatures is compatible with the theoretical results referred to in the introduction, according to which the attenuation ought to approach zero as T^4 (or, according to Khalatnikov,[18] as T^6). The course followed by our data at all frequencies is, in fact, consistent with a power law up to temperatures just short of the peak. More complex behavior could well be masked by the scatter of data, particularly at the lower frequencies. Because of such scatter, the precision with which it can be established that the best value of the exponent n is, say, 3.0 instead of 4.0, is limited. To provide an illustration, we have recalculated attenuation values for the 2.02 Mc/sec data with $n=4$ in Eq. (3), and have replotted them in Fig. 9. Agreement between the points and the line $\alpha=0.68T^4$ is reasonably good, although it would clearly be better if the line had a slope smaller than 4.

Our most satisfactory runs for an assessment of the temperature dependence are those at 6, 10, and 12 Mc/sec, for which the resolution was good and the scatter, for a given run, correspondingly small. The minima exhibited by the residuals R_n are quite sharp, and the results are consistent from one set of data to the next. Out of 16 individual demagnetizations at these three frequencies, the value of n for which R_n is minimum lies in the range 2.8 to 3.2 for 11, and for no run is $n>3.2$. We conclude that, at these frequencies, the power law followed by the absorption coefficient is more likely T^3 than T^4.

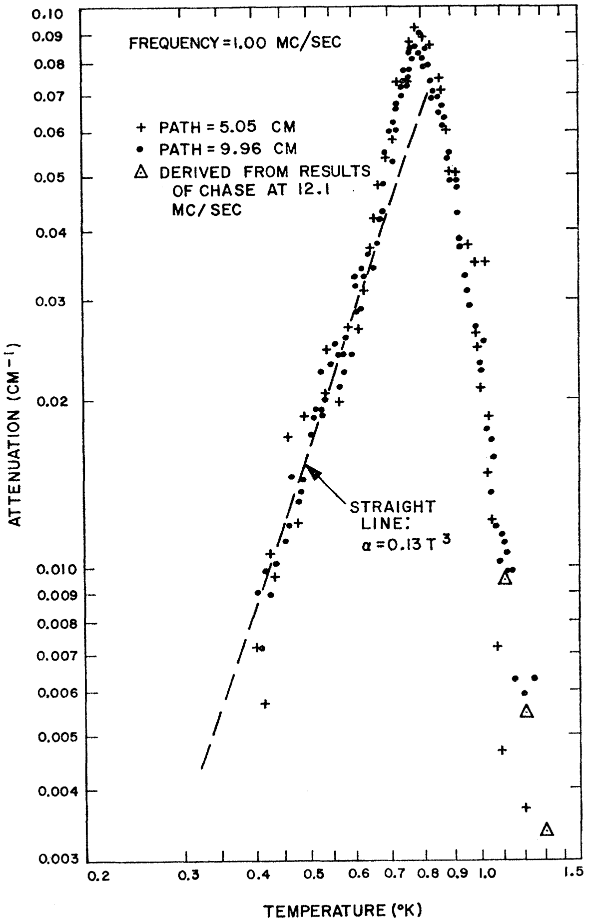

FIG. 8. Attenuation versus temperature at 1.00 Mc/sec. Points below 0.4°K have been suppressed because of their scatter. The points shown as triangles represent results of Chase at 12.1 Mc/sec (Ref. 26) reduced to 1.0 Mc/sec by assuming that $\alpha\propto\omega^2$.

Data taken at the three lower frequencies 1, 2, and 4 Mc/sec exhibit considerable scatter within the temperature region over which the least-squares fit is carried out. The residual R_n is a slowly varying function of n. Its minimum is poorly defined, and any estimate of the best value of n is subject to large uncertainty. For the data at these frequencies, the exponent n is more likely to lie inside the range 3–4 than outside. Measurements with greater resolution than those presented here would be required to back up a stronger statement; however, in part 2 of this section it will be shown that there is additional evidence that supports a T^3 power-law behavior.

Uncertainty in the determination of the temperature dependence will also accompany attempts to calculate attenuation values by matching signal-amplitude measurements with Chase's values above the peak. The nature of the ambiguity is illustrated in Fig. 10. A log-log plot of the quantity $\alpha_0=0.34\,T^3$, representing the general course of the 2-Mc/sec results, yields a straight line. If the values of α_0 are displaced upward and downward by the small amount 0.003 cm^{-1}, equal

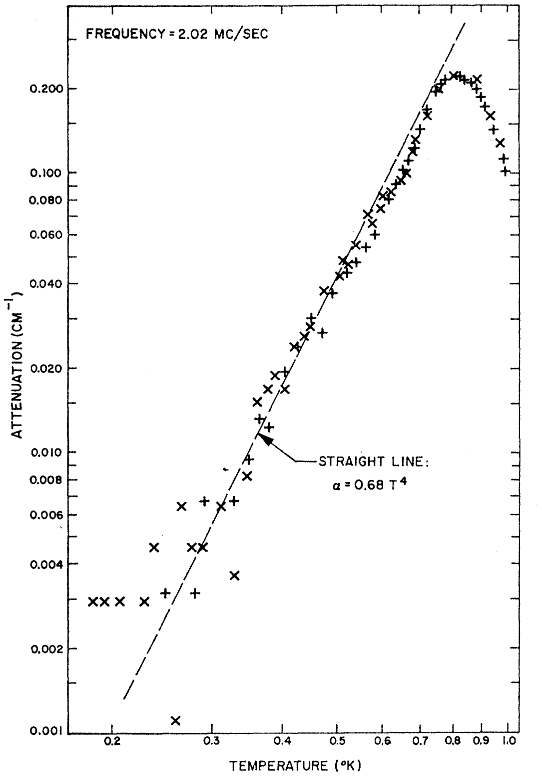

FIG. 9. Attenuation versus temperature at 2.02 Mc/sec. The signal-amplitude values are the same as those from which the points of Fig. 1 and 2 were derived, but attenuation values have been assigned on the basis of a T^4 extrapolation rather than T^3.

roughly to our resolution in measuring changes in attenuation, the results yield curves that deviate from the straight line at lower temperatures. Points scattered randomly about one of these curves could well appear to obey one power law at higher temperatures, another one below. It was an effect similar to that exhibited by the lowest of the three curves in Fig. 10 that was noted in earlier experiments,[10,11] in which the matching technique was used. It now seems likely that the apparent change in power law seen below 0.4°K was spurious, since it can be eliminated by adjusting the reported values of attenuation by amounts that are smaller than the probable errors introduced by fitting the results to Chase's values. In view of the fact that our extrapolation procedure eliminates such a change by hypothesis, we cannot cite our results as evidence that it is not a real effect. We can state, however, that if there is a change in the power-law behavior at temperatures much below 0.4°K, where the total attenuation is comparable with the estimated experimental errors, its presence cannot be established by either of the two earlier experiments referred to, or by the present one.

Frequency Dependence

At temperatures sufficiently far below the peak, the attenuation displays the same, or nearly the same, temperature dependence at each of the six frequencies for which we have results. The frequency dependence in this temperature region can thus be derived from the coefficient A_3, values of which (indicated by plusses) are plotted logarithmically against frequency in Fig. 11. Apparently a simple power law describes the frequency behavior over the range 1 to 12 Mc/sec; the dotted line drawn through the points has a slope of $\frac{3}{2}$. The open circles represent values of A_4, derived from our signal-amplitude measurements by assuming that the attenuation follows a T^4 power law instead of T^3. The upper dotted line passing through these points also has slope $\frac{3}{2}$. The fact that both A_3 and A_4 exhibit the same frequency dependence is not surprising, because a change in exponent in Eq. (3) from $n=3.0$ to $n=4.0$ leads to differences between the resulting values of attenuation that are quite small, amounting at most to a few percent of the peak values. The conclusion that $\alpha \propto \omega^{3/2}$ on the low-temperature side of the peak is thus largely independent of the decision that the temperature dependence is more nearly T^3 than T^4. From the characteristics of the lower line in Fig. 11, we obtain $\alpha = 0.11\, f^{3/2}T^3$, where α is in cm^{-1} and f is the frequency in megacycles per second.

Smoothed values of the attenuation at 0.4°K derived from our measurements (using A_3) are plotted against frequency in Fig. 12 in order to compare them with the results of previous experiments at vapor pressure. The straight line that best fits these points has slope 1.5 ± 0.1.

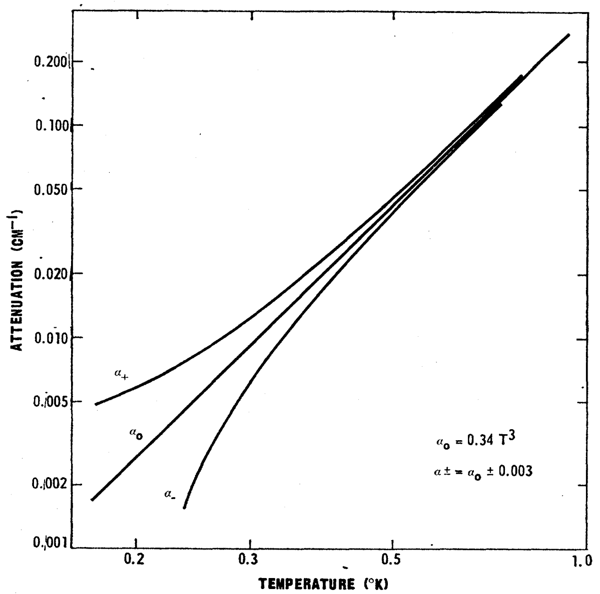

FIG. 10. A logarithmic plot of attenuation against temperature, assuming a relationship $\alpha_0 = 0.34\,T^3$, yields the straight line. When the values of α_0 are shifted upward or downward by small amounts, the two curved lines are obtained. We suggest that the apparent changes in the power-law behavior of the attenuation at low temperatures noted in previous work (Refs. 10 and 11) arose from small systematic errors made in assigning attenuation values.

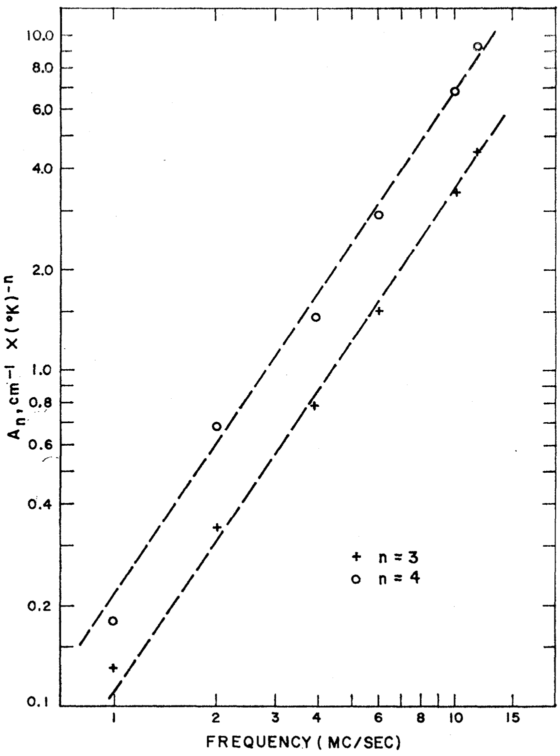

FIG. 11. Logarithmic plots of the coefficient A_n against frequency for $n=3.0$ (pluses) and $n=4.0$ (open circles). The two dashed lines have slope $\frac{3}{2}$.

Discussion

A theory developed by Khalatnikov[3] successfully accounts for the absorption on the high-temperature side of the peak. The underlying point of view is similar in certain respects to that which leads to the Stokes and Kirchhoff equations for the absorption of sound in ordinary fluids,[28] and, in fact, Khalatnikov's expression for the attenuation coefficient [see Eq. (10) below] has the same form as the classical ones in the low-frequency limit. It is assumed that the passage of a sound wave produces small departures from equilibrium in the local pressure and temperature. Collisions among the elementary excitations of the liquid, the phonons and rotons, enable the system to relax toward its new equilibrium state. Since the macroscopic changes that follow from these collisions are irreversible, some of the mechanical energy carried by the sound wave is converted locally into heat, and the wave is attenuated.

The validity of the above description rests upon the condition that the number of elementary collisions during one cycle of the sound wave be large, so that a macroscopic state approximating an equilibrium state can be established; i.e., the average time interval τ between collisions must be small in comparison with the period of the sound wave, $\omega\tau \ll 1$. Calculations performed by Landau and Khalatnikov[29] and by Khalatnikov,[3,20] together with experimental values of the phonon mean-free-path derived from thermal conductivity measurements,[30] indicate that the collision times τ are comparable, in the vicinity of the attenuation peak, with the periods of the ultrasonic waves that have been used in previous experiments, and that they increase rapidly with decreasing temperature. Consequently, sound propagation on the low-temperature side of the peak will be characterized by the condition $\omega\tau \gg 1$, and a classical description, i.e., one concerned with the dynamical behavior of macroscopic states of the system, may be inappropriate.

Several authors have developed explanations of the absorption on the low-temperature side of the peak in which emphasis is placed not upon the changes induced by the sound wave in the thermodynamic state of the system, but rather upon the direct interaction between the wave and the elementary excitations. Their work reflects an approach originated by Landau and Rumer[31] in a calculation of the absorption of sound in solids. According to them, the condition $\omega\tau \gg 1$ makes it possible to treat the sound wave as a beam of low-energy

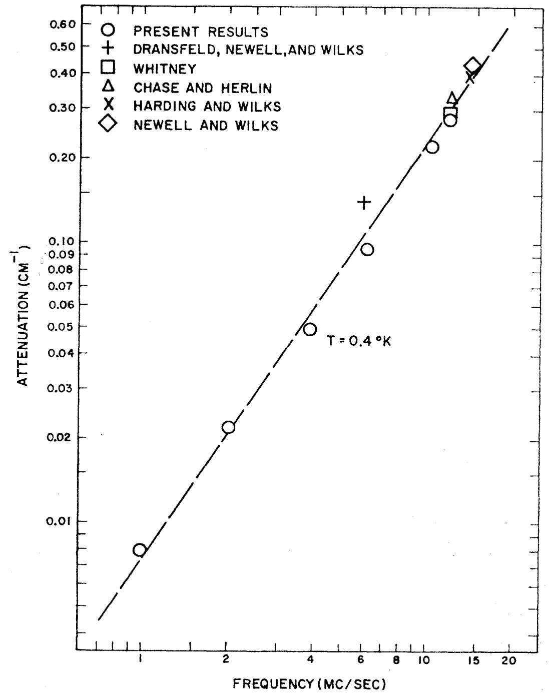

FIG. 12. Logarithmic plot of attenuation against frequency at 0.4°K. Results of previous experiments are shown for comparison. The slope of the dashed line, determined from a least-squares fit to all the points shown, is 1.5±0.1.

---

[28] J. J. Markham, R. T. Beyer, and R. B. Lindsay, Rev. Mod. Phys. 23, 353 (1951).

[29] L. D. Landau and I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. 19, 637, 709 (1949).

[30] R. W. Whitworth, Proc. Roy. Soc. (London) A246, 390 (1958).

[31] L. Landau and G. Rumer, Z. Phys. Sowjetunion 11, 18 (1937).

acoustic quanta, and to interpret absorption as a reduction in the flux of these quanta. Their calculations are based upon the so-called three-phonon scattering process, in which a sound quantum $\mathbf{p}$ interacts with a thermal phonon $\mathbf{p}_1$ and is absorbed, yielding a single thermal phonon $\mathbf{p}_2$. In liquid helium below 0.6°K, the dominant thermal excitations are phonons, and attempts have been made to explain sound absorption in this temperature region in terms of the three-phonon interaction. In a recent paper by Khalatnikov,[18] the four-phonon process is considered. The principal results of these investigations will be summarized below, and shown to be incompatible with our measurements.

(a) *Three-phonon process.* The absorption arising from the three-phonon interaction in liquid helium has been considered by Dransfeld,[15] Kawasaki,[14] Woodruff,[16] and Kawasaki and Mori.[17] A study of certain characteristics of the three-phonon process with applications to liquid helium has been made by Simons.[32] Although the starting points in these investigations appear to be quite different, the final results are very nearly the same. The central idea in Dransfeld's calculation is that certain thermal phonons, namely, those traveling along with the wave, will have a long time to interact with it and absorb energy from it because of the condition $\omega\tau \gg 1$. Kawasaki evaluates the matrix element associated with the transition $\mathbf{p}+\mathbf{p}_1 \rightarrow \mathbf{p}_2$, using wave functions of the form introduced by Feynman[33] for the low-lying states of liquid helium. In a later paper, Kawasaki and Mori calculate the absorption using a fluctuation-dissipation theory, the fluctuations being those induced by the three-phonon interaction. Woodruff's theory represents a modification of a calculation of the absorption of sound in insulators by Woodruff and Ehrenreich,[34] which is stated to be valid, strictly speaking, only under the condition $\omega\tau < 1$. It is considered that the small density changes accompanying a sound wave modulate the energies of the thermal phonons. The total energy imparted to the assembly of phonons is redistributed by three-phonon collisions. Woodruff determines the modified phonon-distribution function by solving a Boltzmann equation, and from this calculates the absorption of sound.

It can be argued that a three-phonon interaction is not allowed in a liquid. According to the usual description of such an event by collision theory, the energy and momentum of the interacting quanta must be conserved in the process. Landau and Rumer show that the necessary conditions cannot be met if the velocities of the three participating phonons are the same. The implication of this requirement for most substances is that at least one of the phonons must be transversely polarized. It is, in fact, the absorption of transverse sound waves that they determine in their calculation. Three-phonon scattering in liquid helium would involve longitudinally polarized quanta only. For such an interaction to take place with simultaneous fulfillment of the conservation laws, it is necessary that the velocity of a thermal phonon exceed that of a sound quantum.[35] According to Landau and Khalatnikov,[29] the dispersion law for thermal phonons in liquid helium has the form

$$\epsilon(p) = pc(1-\gamma_p p^2), \tag{5}$$

where $\epsilon(p)$ is the energy of a phonon with momentum p; c is the low-frequency sound velocity; and $\gamma_p \sim 2.8\times 10^{37}$ (sec/g cm)^2. The phonon velocity thus decreases with increasing wave number, and consequently a three-phonon interaction cannot take place with strict conservation of both energy and momentum. It was for this reason that such processes were ruled out by Landau and Khalatnikov[29] in their calculation of the shear viscosity, and by Khalatnikov[3] in his treatment of the second viscosity.

According to Kawasaki, energy need not be strictly conserved in a three-phonon scattering event; it is necessary only that the amount $\delta\epsilon$ by which energy is not conserved be less than the average uncertainty in the energy of the thermal phonons, i.e., $\delta\epsilon < \hbar/\tau$, where τ is the phonon lifetime. This idea and its ramifications have also been explored by Simons.[32] From the dispersion relation, Eq. (5), and the assumption $p_{\text{av}} = kT/c$, Kawasaki obtains $\delta\epsilon = 3\gamma_p \hbar\omega (kT/c)^2$. Following Dransfeld[36] we write the inequality in the form

$$\omega\tau < (\theta/T)^2, \quad \theta = (c/k)(3\gamma)^{-1/2}. \tag{6}$$

For the value of γ_p given following Eq. (5) and for $c = 2.38\times 10^4$ cm/sec,[27] we find that $\theta \sim 19$°K.

Absorption by the three-phonon process in liquid helium can also be justified from Dransfeld's point of view.[37] According to the original presentation,[15] in which dispersion was ignored, thermal phonons traveling parallel with the wave will be able to keep up with it. If there is dispersion, they will fall behind. The results of the analysis will, however, remain unchanged provided that the distance by which the phonons trail the wave, after a time τ has elapsed, is small in comparison with the wavelength: $(c-v_g)\tau < c/\omega$, where $v_g = \partial\epsilon/\partial p$ is the phonon group velocity. This inequality leads directly to Eq. (6) when v_g is evaluated from the dispersion relation (5). Thus Dransfeld's criterion for the occurrence of the three-phonon interaction and that of Kawasaki are equivalent.

The ideas outlined above receive support from recent experiments by Ciccarello and Dransfeld,[38] who

---

[32] S. Simons, Proc. Phys. Soc. (London) **82**, 401 (1963).

[33] R. P. Feynman, Phys. Rev. **94**, 262 (1952).

[34] T. O. Woodruff and H. Ehrenreich, Phys. Rev. **123**, 1553 (1961).

[35] I. Pomeranchuk, Phys. Rev. **60**, 820 (1941).

[36] K. Dransfeld, Z. Physik **179**, 525 (1964).

[37] R. Nava, R. Azrt, I. Ciccarello, and K. Dransfeld, Phys. Rev. **134**, A581 (1964).

[38] I. S. Ciccarello and K. Dransfeld, Phys. Rev. **134**, A1517 (1964).

find that the absorption of longitudinal waves in MgO and ruby at 0.5 and 3 Gc/sec can be ascribed to the collinear three-phonon process. It can be demonstrated using measured values of the phonon mean free path[30] that the inequality (6) is satisfied in liquid helium at all temperatures above 0.25°K if the frequency is less than 15 Mc/sec. Nevertheless, the qualitative predictions concerning the temperature and frequency dependence of the attenuation arising from the three-phonon process are not in agreement with our experimental results.

The various three-phonon theories lead to the following expression for the absorption coefficient:

$$\alpha=(\pi/2)A^2\omega U/\rho c^3\,, \tag{7}$$

where ρ is the density of the liquid and U is the phonon energy density,

$$U=(4\pi^5/15)kT(kT/hc)^3\,. \tag{8}$$

The dimensionless constant A in Eq. (7) expresses the strength of the interaction between the sound wave and the thermal phonons; its exact value reflects assumptions made in the development of the theories. According to Dransfeld, $A=\gamma_g=(\rho/c)\partial c/\partial\rho$. Kawasaki obtains $A=\gamma_g+1$. The theory of Kawasaki and Mori yields $A_{\mathrm{ad}}=2\gamma_g+7/3$ and $A_{\mathrm{is}}=\gamma_g+2$; which of the two values applies at temperatures below 0.4°K depends upon whether the sound propagation in this temperature region has an adiabatic or an isothermal character. Woodruff finds that $A^2=(\gamma_g+1)^2+(4/3\pi)F_2$, where $F_2=\mathrm{Im}(v_n/v_s)$, v_n and v_s being the velocities of the normal and superfluid components of liquid helium. The value of F_2, which is not provided by the treatment, is adjusted to give best agreement with the experimental results. With $F_2=0$, the results of Kawasaki and Woodruff are the same.

Combination of Eqs. (7) and (8) gives $\alpha=C\omega T^4$, as compared with the observed behavior $\alpha\propto\omega^{3/2}T^3$. According to the theory developed by Kawasaki and Mori, the temperature dependence of the attenuation gradually departs from T^4 behavior as the temperature increases above 0.4°K, and the frequency dependence changes from ω to ω^2. Above 0.7°K their theory is incomplete because the phonon-roton interaction has been ignored. Up to approximately 0.7°K, attenuation values calculated from their equations are in good agreement with earlier experimental results at 12 Mc/sec[10,11]; however, since a linear frequency dependence is predicted for much of this region, the correspondence will be less satisfactory at other frequencies. Furthermore, the frequency and temperature dependences are coupled in such a way that the temperature range over which the T^4 variation persists will increase as the frequency increases. In contrast, our results at the frequencies 6, 10, and 12 Mc/sec give much clearer indication of T^3 behavior than do the results at the three lower frequencies. Moreover, the results of prior measurements of the absorption coefficient cited in the introduction, most of which were carried out at frequencies above 10 Mc/sec, provide further evidence that the temperature power law at higher frequencies is more likely to be T^3 than T^4.

With the exception of F_2, there are no adjustable constants in the theoretical expressions for the absorption coefficient. Despite the fact that they do not correctly predict the temperature and frequency dependence, it is nevertheless true that these equations yield values of the absorption that are of the same order of magnitude as those measured over the range of frequencies covered in our experiment. We evaluate Eq. (7), substituting $\gamma_g=2.65$,[39] $\rho=0.145$ g/cm^3,[40] $c=2.38\times10^4$ cm/sec,[27] and appropriate values for the constants in (8). With Kawasaki's value $A=\gamma_g+1$, we obtain $C=8\times10^{-8}$ (sec/cm) (°K)$^{-4}$. At 1 and 12 Mc/sec, the product ωC will be, respectively, 0.5 and 6 cm^{-1}(°K)$^{-4}$. These numbers can be compared with values of A_4 derived from our data by forcing a fit to a T^4 power law: From Fig. 11 we obtain $A_4=0.2$ and 9 cm^{-1} (°K)$^{-4}$ at the same two frequencies. For the range of frequencies 1 to 12 Mc/sec, the calculated and observed absorption curves have a point of intersection somewhere in the temperature region 0.2 to 0.8°K.

At temperatures low enough or frequencies high enough that Eq. (6) is not valid, the absorption attributable to the three-phonon process in solids is expected to become independent of frequency and to vary with temperature as T^n, with $n\geqslant4$.[32,38] Behavior of this type is exhibited by the absorption of longitudinal waves in quartz in the frequency range 0.5–9 Gc/sec,[38] but we find no evidence for it in liquid helium over the temperature range covered in our experiment.

Dransfeld[36] has suggested that, in sufficiently small containers of liquid helium, collisions of the thermal phonons with the walls will prevent their lifetimes from increasing indefinitely, thus enabling three-phonon scattering to proceed at all temperatures below a certain upper limit. One might then expect that, at temperatures low enough so that Eq. (6) would otherwise be violated, the attenuation would show some dependence on the crystal spacing d. This effect would not be observed in the present experiment because, as mentioned before, the inequality (6) is in fact satisfied for the temperature and frequency ranges covered in the measurements. Only at 4 Mc/sec do we have data extending reliably below 0.4°K that represent more than a single value of d, and they display no systematic differences from run to run. The fact that attenuation values derived from all our measurements at different frequencies and crystal spacings exhibit the same qualitative behavior is evidence that the size of the sound chamber had no important bearing on our results.

(*b*) *Four-phonon process*. Khalatnikov[18] has calcu-

---

[39] K. R. Atkins and R. A. Stasior, Can. J. Phys. **31**, 1156 (1953).

[40] E. C. Kerr and R. D. Taylor, Ann. Phys. **26**, 292 (1964).

lated the absorption arising from the interaction $\mathbf{p}+\mathbf{p}_1\rightarrow\mathbf{p}_2+\mathbf{p}_3$ at low temperatures. An expression is obtained for the lifetime τ of the acoustic quanta, from which the attenuation coefficient is derived: $\alpha=1/\tau c\propto\omega T^6/\gamma_p$. By choosing $\gamma_p=1.5\times10^{35}$ (sec/g cm)^2, Khalatnikov obtains approximate numerical agreement with the results of Chase and Herlin at 0.3°K. However, from an analysis of the dispersion curve obtained by Henshaw and Woods[41] from inelastic neutron-scattering experiments, it can be shown[36] that the energy-versus-momentum relation for the phonon part of the spectrum is consistent with Eq. (5), with $\gamma_p\sim2.5\times10^{37}$ (sec/g cm)^2. This number agrees well with the original estimate of Landau and Khalatnikov, $\gamma_p=2.8\times10^{37}$ (sec/g cm)^2. If γ_p were in fact this large, as seems likely, the absorption arising from the four-phonon process would be two orders of magnitude too small to account for the measured attenuation values, apart from the fact that the predicted temperature and frequency dependences are different from what we observe.

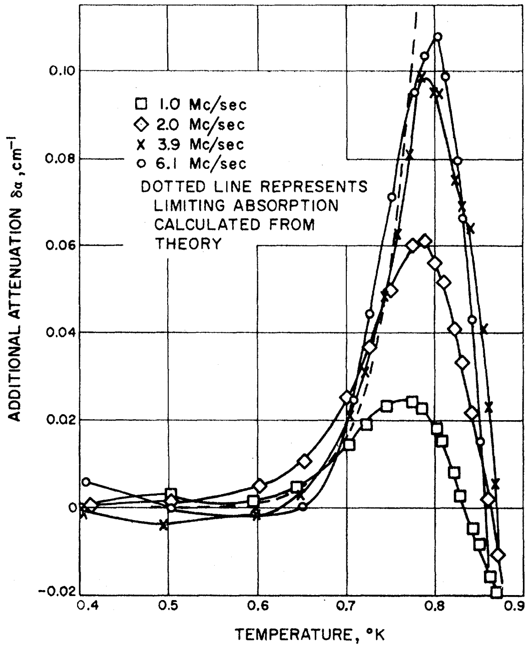

FIG. 13. The additional attenuation $\delta\alpha$ plotted against temperature in the vicinity of 0.8°K. The points shown have been obtained by subtracting the quantity $A_3(f)T^3$ from the individual attenuation values at a given frequency f, and then averaging the results over fixed temperature intervals. The dashed line represents the predicted limiting attenuation at infinite frequency arising from thermal conductivity, calculated from Eq. (9) and the expression for κ given by Khalatnikov.

2. Absorption Near 0.8°K

The experimental results at four frequencies, 1, 2, 4, and 6 Mc/sec, extend over the temperature region that includes the attenuation peak. When $\ln\alpha$ is plotted against $\ln T$, as in Figs. 2, 4, 7, and 8, the data just below the peak exhibit a somewhat stronger temperature dependence than T^3. This behavior indicates a change in the dominant attenuation mechanism, or perhaps the appearance of another mechanism that contributes additively to the total absorption. We will show that our data are consistent with the second possibility. The additional absorption is quite satisfactorily described by an equation derived according to the classical or hydrodynamical approach, and can be linked directly with the parameter κ introduced by Khalatnikov[19] to characterize the thermal conductivity of the normal fluid.

We will assume that most of the absorption near the peak arises from the mechanism that is effective below 0.5°K, and that the power-law behavior $\alpha\propto T^3$ observed in that region persists up to the peak. The characteristics of the additional absorption are ascertained by subtracting the amount A_3T^3 from the measured attenuation values: The remainders $\delta\alpha$ are plotted against temperature in Fig. 13, the points of which represent smoothed values obtained by averaging the data over temperature intervals varying from 0.1°K below 0.6 to 0.02°K near 0.8.

The qualitative behavior of the four curves is characteristic of a relaxation process. The absorption arising from a single relaxation mechanism is described by the equation[28]

$$\alpha=S\omega^2\tau/(1+\omega^2\tau^2)\,, \tag{9}$$

where the relaxation time τ and the coefficient S are functions of temperature and pressure but not frequency. According to Eq. (9), the peak attenuation, which occurs at a temperature for which $\omega\tau=1$, would be proportional to frequency. Its position would shift to lower temperatures at reduced frequency if, as one might expect, the relaxation time τ increases with decreasing temperature. The attenuation would be independent of frequency in regions where $\omega\tau\gg1$, and proportional to ω^2 in regions where $\omega\tau\ll1$. Most of these features are evident in Fig. 13, although the steep descent of $\delta\alpha$ on the high-temperature side of the peak must be attributed principally to changes in the dominant absorption mechanism, and no meaningful qualitative comparison with Eq. (9) can be made in this region.

Viewed individually, the four sets of data shown in Fig. 13 display systematic variations that preclude a detailed quantitative comparison with Eq. (9) over an extended temperature region. We can nevertheless derive useful information from the frequency dependence at temperatures near the maxima, where attenuation values are least affected by errors in temperature. In Fig. 14 we plot $f^2/\delta\alpha$ against f^2 (f=frequency) for $T=0.8$°K, using values of $\delta\alpha$ taken from Fig. 13. The fact that these points lie nearly on a straight line indicates compatibility with the form of Eq. (9). From

---

[41] D. G. Henshaw and A. D. B. Woods, Phys. Rev. **121**, 1266 (1961).

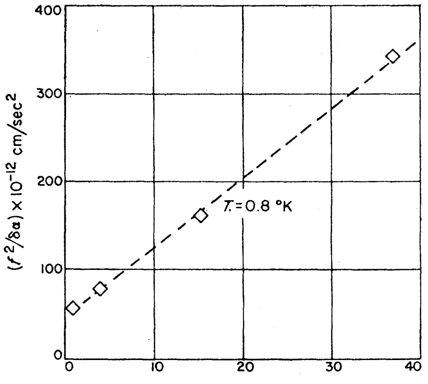

FIG. 14. The ratio $f^2/\delta\alpha$ plotted against f^2 for $T=0.8$°K. The fact that the four points lie nearly on a straight line indicates compatibility with the form of Eq. (9).

the slope and intercept of the line, the parameters S and τ can be determined. The numbers obtained are presented below.

Hydrodynamic equations for liquid helium II, in which terms that take account of dissipative processes have been incorporated, have been derived by Khalatnikov.[19] When these equations are solved for sound propagation, the following expression is obtained for the attenuation coefficient:

$$\alpha=(\omega^2/2\rho c^3)\{\tfrac{4}{3}\eta+\zeta_2+(\gamma_c-1)\kappa/C_p\}\,, \tag{10}$$

where C_p is the heat capacity per unit mass at constant pressure; γ_c is the ratio of specific heats C_p/C_v; η is the shear viscosity; ζ_2 is one of three independent coefficients of second viscosity associated with dilatation of the fluid; and κ is a coefficient of thermal conductivity.

The conduction of heat in liquid helium II is accomplished principally by convective counterflow of the normal-fluid and superfluid components. According to Khalatnikov,[19] the diffusion of thermal excitations in the normal fluid provides an additional mechanism for heat transfer, and it is this process, which is linear in the temperature gradient, that is characterized by the coefficient κ. The behavior of this parameter can be traced directly to specific interactions among the thermal excitations,[20] phonon-roton scattering being the most important process, and it is found that κ increases rapidly with decreasing temperature, approaching a variation as $1/T^6$ at very low temperatures.

Khalatnikov has shown that the attenuation of second sound arises primarily from the thermal conductivity, a prediction that subsequent experiments have fully substantiated.[42] Above 1°K, the contribution of this mechanism to the absorption of ordinary sound is quite small in comparison with that of second viscosity, which is responsible for approximately 90% of the observed attenuation, and that of shear viscosity, which accounts for most of the remainder. However, on the low-temperature side of the peak, where ζ_2 is vanishing while κ is rapidly increasing, the relative contribution of thermal conductivity to the total absorption ought to be considerably greater.[5]

That part of Eq. (10) which contains κ as a factor can be written in the form

$$\alpha_\kappa=\{(\gamma_c-1)/2c\}\omega^2\tau_\kappa\,,\quad \tau_\kappa=\kappa/\rho c^2C_p\,. \tag{11}$$

[The time constants τ in Eq. (9) and τ_κ in Eq. (11) characterize macroscopic behavior of the fluid, and are not to be confused with the microscopic collision times that are related to the mean free paths of the thermal excitations.] Because of the manner in which it was derived from the hydrodynamic equations, Eq. (11) is valid only at temperatures for which $\omega\tau_\kappa\ll 1$. For ordinary fluids, the generalized Kirchhoff expression for the attenuation arising from the thermal conductivity κ is identical in form with Eq. (9),[28] with $S_\kappa=(\gamma_c-1)/2c$, and τ_κ defined as in (11). To derive a corresponding relationship for liquid helium from the two-fluid hydrodynamic equations, it is expedient to ignore altogether the second-order terms associated with the coefficients of first and second viscosity. Making use of the approximation $(\gamma_c-1)\ll 1$, we obtain

$$\alpha_\kappa=S_\kappa\omega^2\tau_\kappa\{(1-c_2^2/c^2)^2+\omega^2\tau_\kappa^2\}^{-1}\,, \tag{12}$$

where c_2 is the velocity of second sound. The difference in form between Eqs. (9) and (12) turns out to be of slight numerical significance. At temperatures for which $\omega^2\tau_\kappa^2\lesssim 1$, $(c_2/c)^2\ll 1$, and at temperatures below approximately 0.7°K, where c_2 and c are comparable in magnitude, $\omega^2\tau_\kappa^2\gg 1$. For the numerical work below we may therefore set $c_2/c=0$.

Experimental values of S and τ derived from Fig. 14 can now be compared directly with calculated values of S_κ and τ_κ. The expressions given by Khalatnikov yield $\kappa=9.2\times 10^5$ ergs/cm sec °K at 0.8°K. The specific heat ratio γ_c is evaluated from the thermodynamic identity $\gamma_c-1=T\beta^2c^2/C_p$, where β is the coefficient of thermal expansion. It is estimated that $\beta=4.4\times 10^{-4}$ (°K)$^{-1}$ at 0.8°K. This value is deduced by extrapolating the experimental results of Atkins and Edwards,[43] using the expression given by them for the coefficient of expansion of an ideal gas of phonons and rotons. These numbers, together with values taken from the literature for the remaining parameters, yield $S_\kappa=8\times 10^{-9}$ sec/cm, $\tau_\kappa=5\times 10^{-8}$ sec, as compared with $S=9\times 10^{-9}$ sec/cm, $\tau=7\times 10^{-8}$ sec, derived from our data. At 0.775°K, the calculated values are $S_\kappa=9\times 10^{-9}$ sec/cm, $\tau_\kappa=9\times 10^{-8}$ sec, as compared with $S=10\times 10^{-9}$ sec/cm, $\tau=9\times 10^{-8}$ sec.

We have also computed the ratio $\alpha_{\kappa\infty}=S_\kappa/\tau_\kappa$ for

---

[42] W. B. Hanson and J. R. Pellam, Phys. Rev. **95**, 321 (1954); K. R. Atkins and K. H. Hart, Can. J. Phys. **32**, 381 (1954); K. N. Zinov'eva, Zh. Eksperim. i Teor. Fiz. **31**, 31 (1956) [English transl.: Soviet Phys.—JETP **4**, 36 (1957)].

[43] K. R. Atkins and M. H. Edwards, Phys. Rev. **97**, 1429 (1955).

temperatures below the maximum values of $\delta\alpha$. This quantity represents the maximum attenuation in the limit $\omega\tau_\kappa \gg 1$, corresponding to high frequencies or long relaxation times for which α_κ becomes independent of frequency. A plot of $\alpha_{\kappa\infty}$ against temperature thus represents a limiting curve toward which the experimental curves $\delta\alpha$-versus-T should converge at sufficiently low temperatures. In Fig. 13, $\alpha_{\kappa\infty}$ is represented as a dotted line. It is apparent that this curve is quite consistent with the average behavior of the four sets of experimental points below approximately 0.75°K.

The success with which the properties exhibited by $\delta\alpha$ can be correlated with those expected of α_κ leads us to identify thermal conduction as the mechanism responsible for the change in the characteristics of the total absorption at temperatures just below those of the maxima. The good agreement between theory and experiment strengthens our earlier conclusion that the temperature dependence of the attenuation at temperatures below the peak is more nearly T^3 than T^4. If our experimental results had been fitted to the empirical equation $\alpha = A_4 T^4$, the values of the differences $\alpha - A_4 T^4$ near the peak would be much smaller than the values of $\delta\alpha$ presented in Fig. 13, or even negative (see Fig. 9), and a correlation like that demonstrated above could not be established.

The possibility that thermal conductivity contributes significantly to the attenuation of ordinary sound at low temperatures was suggested by DNW, who attempted to attribute all of the absorption below the peak to this mechanism. It was realized that, at sufficiently low temperatures, the law obeyed by the absorption coefficient would depart from that predicted by Eq. (11), but it was assumed that the departure would arise from a relaxation or change in temperature dependence of the thermal conductivity itself. As we have shown, it is the form of Eq. (11) that requires modification; the temperature variation of κ predicted by Khalatnikov is consistent with our results.

From measurements of the thermal conductivity of liquid helium II in narrow tubes, Whitworth[30] concludes that the phonon mean free path λ_{mfp} lies in the range 0.01 to 0.035 cm from 0.6 to 0.7°K. The sound wavelengths for the frequencies 1 and 6 Mc/sec are 0.024 and 0.004 cm, respectively. Thus the absorption associated with thermal conductivity in liquid helium II below 1°K is correctly accounted for on the basis of a hydrodynamic description of sound propagation, even at temperatures for which $\lambda_{\mathrm{mfp}}/\lambda \gtrsim 1$.

V. CONCLUSION

From the evidence presented herein, we conclude that the principal contribution to the absorption of sound in liquid helium below approximately 0.8°K and over the frequency range 1–12 Mc/sec obeys the law $\alpha \propto \omega^{3/2} T^3$. Small departures from this behavior over the temperature region 0.6–0.8°K are attributed to additional absorption arising from thermal conduction.

Although the measured values of the absorption coefficient have roughly the magnitude predicted by theories based upon the three-phonon interaction, the observed dependence upon frequency and temperature differs from that expected, $\alpha \propto \omega T^4$. This fact requires an explanation, since the conditions under which it is believed that the three-phonon process may influence the absorption of longitudinal sound waves appear to be satisfied over the intervals of temperature and frequency involved in the measurements.

The conclusion that the attenuation is proportional to $\omega^{3/2}$ is very little affected by the validity of the extrapolation procedure followed in determining the temperature variation. We know of no absorption mechanism for which such a frequency dependence is predicted. Experiments are being carried out at the Jet Propulsion Laboratory to look further into this unusual behavior.

ACKNOWLEDGMENTS

We express our appreciation to J. Keefe for helpful suggestions concerning the construction of the apparatus, to F. Bunick, W. Mosby, and A. MacDonald for their careful fabrication and assembly of the conical valve, to T. Oversluizen for constant technical advice and assistance, and to R. R. Oder, A. A. Thiele, and other members of the laboratory for their assistance in recording data and for supplying extra hands whenever needed. The results at 1 Mc/sec and some of those at 4 Mc/sec were taken in the course of another experiment conducted with C. E. Chase. We are grateful to him for his collaboration in this part of the experiment, and for many useful and stimulating discussions during the preparation of this paper. We are also indebted to J. Wilks for some very helpful comments concerning the manuscript.