Optical Analysis of the 900-MeV π^--p Resonance*

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A model for elastic scattering near the 900-MeV π^--p resonance is proposed, consisting of a Breit-Wigner term together with an optical background. The optical background is chosen to fit experimental data beyond the 900-MeV resonance; the spin of the Breit-Wigner term is then adjusted to give an elastic cross section in agreement with experiment. A distorted-wave Born-approximation calculation shows a spin-$\frac{5}{2}$ resonance term to be in agreement with the data, in accord with the usual spin assignment. Parity of the Breit-Wigner term cannot be decided from the elastic energy spectra and angular distributions alone; however, if the elastic resonance is identified with the observed Λ-K resonance near 900 MeV, one finds a suitable branching ratio only for the $D_{5/2}$ choice. The resulting Λ-K angular distributions, as well as the production cross section as a function of energy, agree generally with experiment. It is therefore proposed that the 900-MeV π^--p resonance has negative parity relative to the proton.

I. INTRODUCTION

A MODEL is proposed to account for some features of the total and integrated elastic cross sections as well as elastic angular distributions near the 900-MeV π^--p resonance. The model consists of an optical term and a single Breit-Wigner resonance.[1] The optical term accounts phenomenologically for inelastic background while the Breit-Wigner term accounts for the resonance structure of the elastic and total cross sections.

The optical parameters are determined from very high-energy data and the Breit-Wigner parameters are then chosen to fit the total and integrated cross sections at the resonant energy. It is found that a spin-$\frac{5}{2}$ resonance gives the best value for the elastic cross section in agreement with the usual spin assignment.[2] The angular distributions are in general agreement with experimental angular distributions,[3] being dominated at forward angles by the optical term and near 180° in the c.m. system by the resonance term.

The resonance in the elastic π-p channel couples to each open inelastic channel. If the resonance term is taken to be $D_{5/2}$ the size of the $\pi^-+p\rightarrow K+\Lambda$ resonance will have approximately the experimental value, whereas an $F_{5/2}$ assignment leads to an inelastic resonance too small by a factor of 10.

Assuming a $D_{5/2}$ resonance, the energy spectrum and angular distributions are computed for the Λ-K channel using the model proposed by Hoff[4] (K^* exchange together with a $P_{1/2}$ resonance) with three modifications: (a) The resonance is taken to be $D_{5/2}$ rather than $P_{1/2}$. (b) The resonance parameters are computed from the elastic partial width. (c) The calculations are carried out using the distorted-wave Born approximation to include optical effects systematically. The resulting inelastic energy spectrum and the Λ-K angular distributions are in fair agreement with experiment.

It is therefore suggested that the $D_{5/2}$ spin-parity assignment (although in disagreement with the usual $F_{5/2}$ Regge pole assignment[5]) does lead to a simple understanding of the Λ-K resonance, as well as an explanation of the elastic angular distributions near the resonant energy.

II. ELASTIC π^--p CROSS SECTION

The elastic π^--p differential cross section is determined in terms of the amplitudes f_1 and f_2 by[6]

$$d\sigma/d\Omega=|f_1|^2+|f_2|^2+2\cos\theta\,\mathrm{Re}(f_1f_2^*)\,, \qquad (1)$$

where f_1 and f_2 are given by the partial-wave expansions

$$\begin{aligned} f_1&=\sum_{l=1}^{\infty}(f_{l-1}{}^{+}-f_{l+1}{}^{-})P_l'(\cos\theta)\,,\\ f_2&=\sum_{l=1}^{\infty}(f_l{}^{-}-f_l{}^{+})P_l'(\cos\theta)\,; \end{aligned} \qquad (2)$$

$f_l{}^{\pm}$ being the partial wave amplitudes for $J=l\pm\frac{1}{2}$. Each partial-wave amplitude will be given as the sum of an optical term and a resonant term evaluated using the distorted-wave Born approximation:

$$\begin{aligned} f_{l0}{}^{\pm}&=(1/2ip)(e^{2i\delta_l\pm}-1)\,,\\ f_{lr}{}^{\pm}&=e^{2i\delta_l}\,\frac{\Gamma_1}{2p(W_r-W-i\Gamma/2)}\,,\quad l=l_r\,,\quad J=J_r\,,\\ &=0\,,\quad \text{otherwise}. \end{aligned} \qquad (3)$$

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[1] Examples of optically distorted Born approximation calculations are found in Robert Serber, Phys. Rev. Letters **10**, 357 (1963); L. Durand, III, and Y. T. Chiu, *ibid.* **12**, 399 (1964); L. Durand, III, and Y. T. Chiu, Phys. Rev. **137**, B1530 (1965); Kurt Gottfried and J. D. Jackson, CERN report (unpublished).

[2] M. Roos, Phys. Letters **8**, 1 (1964).

[3] J. A. Helland, C. D. Wood, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. **134**, B1079 (1964).

[4] G. T. Hoff, Phys. Rev. **131**, 1302 (1963).

[5] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8**, 41 (1962); P. Carruthers, *ibid.* **10**, 540 (1963).

[6] The amplitudes of f_1 and f_2 are introduced, for example, in G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

$e^{2i\delta_l^\pm}$ represent phase-shift factors due to optical background alone. W and p are the center-of-mass energy and momentum. The resonant energy $W_r = 1688$ MeV and the total width $\Gamma = 100$ MeV. The elastic partial width Γ_1 is determined by fitting the total cross section at $W = W_r$.

The optical parameters are chosen so that the differential cross section, far beyond the resonance, agrees with experiment. At the relatively high momentum of 4 BeV/c, the elastic cross section satisfies the empirical formula[7]

$$d\sigma/dt = (\sigma^2/16\pi)e^{t/2\nu^2}, \tag{4}$$

where $\sigma = 32.3$ mb and $\nu = 1.73\ m_\pi$. The quantity $t = -2p^2(1-\cos\theta)$ is the square of the center-of-mass momentum transfer. The scattering is consistent with the assumption that the amplitudes f_1 and f_2 are both imaginary. In fact, if we take[8]

$$f_{10} = i(p\sigma/4\pi)e^{t/4\nu^2}, \tag{5}$$

$$f_{20} = 0, \tag{6}$$

the optical theorem gives for the total cross section,

$$\sigma_T = (4\pi/p)\mathrm{Im}[f_{10}(0) + f_{20}(0)] = \sigma. \tag{7}$$

This is in agreement with experiment. If the optical amplitudes are substituted into Eq. (1) for the differential cross section, Eq. (4) is recovered.

The optical amplitudes given in Eqs. (5) and (6) are not the most general choice consistent with Eq. (4) for the differential cross section and Eq. (7) for the total cross section. They do provide, however, a specific choice in accord with the notion that optical effects can be described by a spin-independent optical potential.[9]

The partial-wave amplitudes $f_l^\pm$ are determined from f_1 and f_2 by

$$f_l^\pm = \tfrac{1}{2}\int_{-1}^{1} d\mu[f_1 P_l(\mu) + f_2 P_{l\pm1}(\mu)], \tag{8}$$

from which one finds

$$e^{2i\delta_l^+} = e^{2i\delta_l^-} = 1 - (\sigma\nu^2/2\pi)Z_l(p^2/2\nu^2), \tag{9}$$

where $Z_l(x) = 2xe^{-x}i_l(x)$ with $i_l(x)$ a modified spherical Bessel function.[10]

[7] Aachen-Birmingham-Bonn-Hamburg-London-München Collaboration, Nuovo Cimento **31**, 729 (1964).

[8] An essentially identical choice is made by Gottfried and Jackson, Ref. 1.

[9] The replacement $f_{10} \to f_{10}\cos^2(\alpha/2)$, $f_{20} \to f_{20}\sin^2(\alpha/2)$ gives a total cross section still in agreement with experiment but causes the differential cross section to be multiplied by

$$\cos^2(\theta/2) + \sin^2(\theta/2)\cos^2\alpha.$$

To the extent that the scattering is forward in the c.m. system this factor cannot be distinguished from 1 and the spin-dependent amplitudes explain the high-energy data as well as the spin-independent amplitudes of Eqs. (5) and (6).

[10] The dominant term in the asymptotic series forlarge positive x is $Z_l(x) \to e^{-l(l+1)/2x}$.

Substituting the resonant term from Eq. (3) into Eq. (2), one finds

$$\begin{aligned} f_{1r}^+ &= f_{lr}P_{l+1}'(\cos\theta), \\ f_{2r}^+ &= -f_{lr}P_l'(\cos\theta), \\ f_{1r}^- &= -f_{lr}P_{l-1}'(\cos\theta), \\ f_{2r}^- &= f_{lr}P_l'(\cos\theta). \end{aligned} \tag{10}$$

Here $f_{1r}^\pm$ and $f_{2r}^\pm$ represent contributions to f_1 and f_2 due to a resonance with angular momentum $J = l \pm \frac{1}{2}$. In Eqs. (10) use has been made of the fact that for the distorted wave Breit-Wigner term $f_{lr}^+ = f_{lr}^- = f_{lr}$.

Combining Eqs. (5) and (6), and (10) for the amplitudes f_1 and f_2 the total cross section is found to be

$$\sigma_T(W) = \sigma + \frac{\pi(2J+1)\Gamma_1\Gamma}{2p^2((W_r - W)^2 + \Gamma^2/4)}e^{2i\delta_l}, \tag{11}$$

from the optical theorem.

Setting $W = W_r$, Eq. (11) can be solved for Γ_1. Table I gives Γ_1/Γ for various choices of J and l. We use $\sigma_T(W_r) = 58.0$ mb and determine $e^{2i\delta_l}$ from Eq. (9).

The differential cross section may be decomposed into three terms: an optical term, a resonance term, and an interference term. Calling these contributions $I_0(\theta)$, $I_r(\theta)$, and $I_i(\theta)$, one determines from Eq. (1):

$$\begin{aligned} I_o(\theta) &= \frac{\sigma^2 p^2}{16\pi^2}e^{-(p^2/\nu^2)(1-\cos\theta)}, \\ I_r(\theta) &= \frac{\Gamma_1^2}{4p^2[(W_r - W)^2 + \Gamma^2/4]}e^{4i\delta_l}F_{J-1/2}(\cos\theta), \\ I_i(\theta) &= \frac{\sigma(2J+1)\Gamma\Gamma_1}{16\pi[(W_r - W)^2 + \Gamma^2/4]}e^{2i\delta_l}e^{-(p^2/2\nu^2)(1-\cos\theta)} \\ &\quad \times P_l(\cos\theta), \end{aligned} \tag{12}$$

where

$$F_l(\mu) = [P_{l+1}'(\mu)]^2 + [P_l'(\mu)]^2 - 2\mu P_{l+1}'(\mu)P_l'(\mu).$$

Integrating Eqs. (12) over angle one finds for the elastic cross section a decomposition $\sigma_E(W) = \sigma_0(W)$

TABLE I. Elastic branching ratio and elastic cross secton at resonance for various choices of J and l.

J	l	Γ_1/Γ	$\sigma_e(W_r)$ (mb)
$\frac{1}{2}$	0	7.23	69.9
	1	3.42	63.1
$\frac{3}{2}$	1	1.71	41.0
	2	1.12	34.3
$\frac{5}{2}$	2	0.748	26.9
	3	0.626	23.1
$\frac{7}{2}$	3	0.469	19.4
	4	0.440	17.9

$+\sigma_r(W)+\sigma_i(W)$ where

$$\sigma_0(W)=\frac{\sigma^2\nu^2}{8\pi}(1-e^{-2p^2/\nu^2}),$$

$$\sigma_r(W)=\frac{\pi(2J+1)\Gamma_1^2}{2p^2[(W_r-W)^2+\Gamma^2/4]}e^{4i\delta_l}, \qquad (13)$$

$$\sigma_i(W)=\frac{\sigma\nu^2(2J+1)\Gamma_1\Gamma}{4p^2[(W_r-W)^2+\Gamma^2/4]}e^{2i\delta_l}Z_l(p^2/2\nu^2).$$

The values of Γ_1/Γ, from the third column of Table I, are used in Eq. (13) to give the values of $\sigma_E(W_r)$ shown in the fourth column. These values seem to favor the $D_{5/2}$ assignment when compared with the experimental value 26.58±0.61 mb. This choice must be considered in the light of the particular choice of optical amplitudes made in Eqs. (5) and (6). If one makes the replacement $f_{10}\leftrightarrow f_{20}$ the resonance and optical contributions to Eqs. (12) and (13) remain unchanged but the interferences are changed. With this new choice of spin dependence in the optical amplitudes the rows of Table I corresponding to a given J are interchanged, and the $F_{5/2}$ choice gives the value of $\sigma(W_r)$ closer to experiment.

We give in Figs. 1 and 2 some elastic angular distributions for the $D_{5/2}$ and $F_{5/2}$ resonances compared with experiment. The curves for $D_{5/2}$ and $F_{5/2}$ cases will again be interchanged if the optical amplitudes f_{10} and f_{20} are interchanged. Both choices are in reasonable agreement with experiment; however, for neither choice is the downward swing seen in the experimental data near 180° predicted by the model.

III. Λ-K PRODUCTION

Let us now consider the decomposition given in Fig. 3 of the $\pi^-+p\rightarrow K+\Lambda$ production amplitude into a resonance of spin J and a K^* exchange. The production amplitudes f_1 and f_2 may again be decomposed into partial-wave sums according to Eq. (2). The production amplitudes associated with the resonance are given in the distorted wave Born approximation as

$$f_{lr}^{\pm}=\frac{(\Gamma_1\Gamma_2)^{1/2}}{2p(W_r-W-i\Gamma/2)}D_l, \qquad (14)$$

where Γ_1 and Γ_2 are partial widths for the initial and final states and $D_l=e^{i\delta_l^1+i\delta_l^2}$ is the product of optical phase factors for the initial and final states. These phase shifts will be given by Eq. (9) where it is assumed that the optical parameters σ and ν are identical for the Λ-K and the π^--p interactions. One has

$$D_l=\left[1-\frac{\sigma\nu^2}{2\pi}Z_l(p^2/2\nu^2)\right]^{1/2}\left[1-\frac{\sigma\nu^2}{2\pi}Z_l(p'^2/2\nu^2)\right]^{1/2}, \qquad (15)$$

where p and p' are the c.m. momenta for the incident and final particles, respectively. Equations (10) then describe the resonant contribution to the production amplitudes f_1 and f_2.

The K^* partial-wave amplitudes can be deduced from Fig. 3 provided a specific form of the interaction is

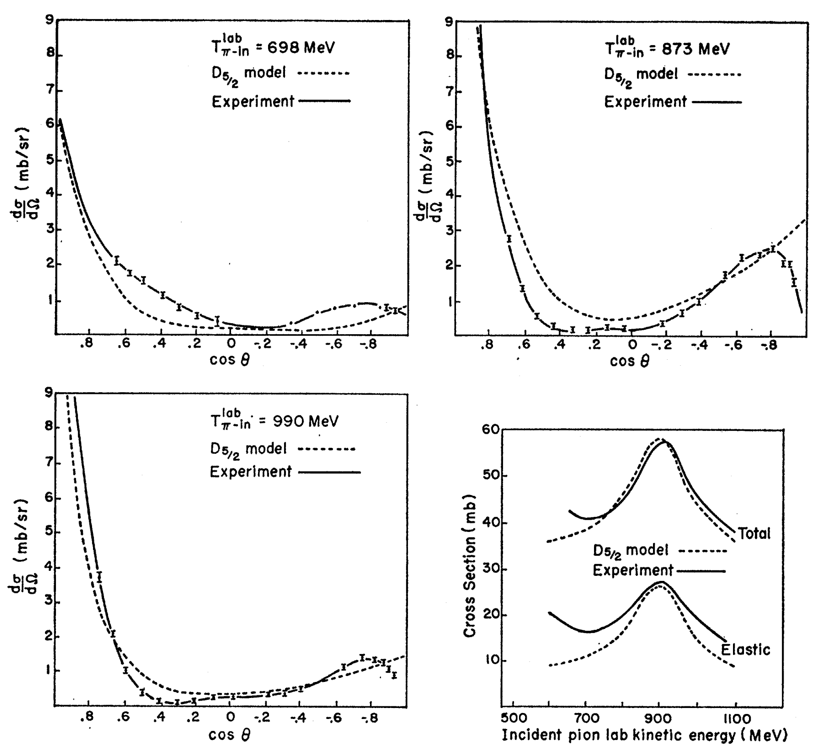


FIG. 1. Comparison of the $D_{5/2}$ model calculation with experiment.

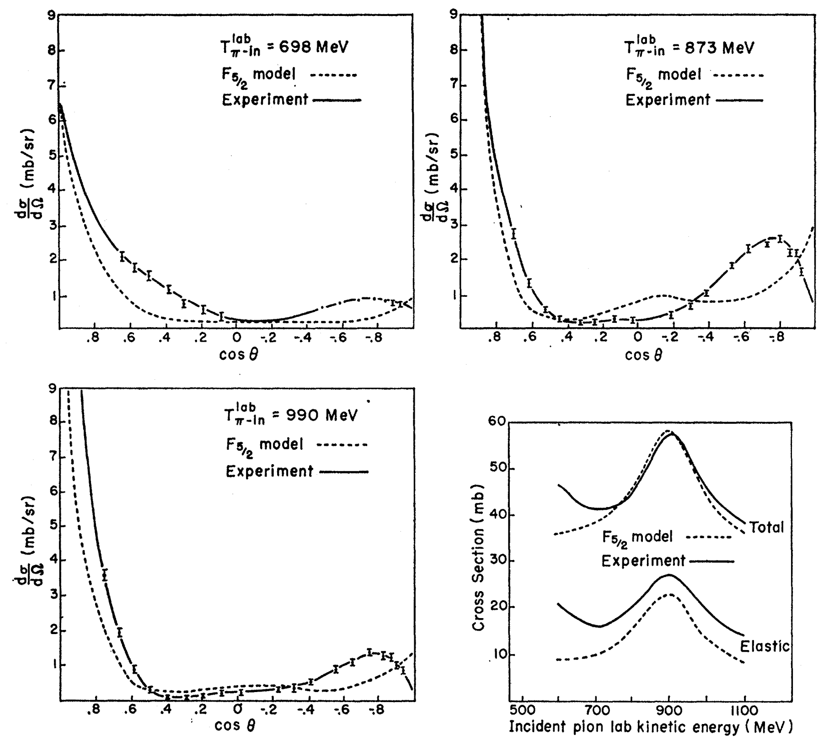


FIG. 2. Comparison of the $D_{5/2}$ model calculation with experiment.

given. We assume an interaction between the K^* and vector currents which are conserved in the limit of degenerate masses. We find

$$
\begin{aligned}
f_{lK^*}{}^{\pm} &= N[GQ_l(\beta)+HQ_{l\pm1}(\beta)]D_l\,,\\
N &= -g_{p\Lambda K^*}g_{\pi^-KK^*}/4\pi\,,\\
G &= \frac{1}{2Wp}\left(\frac{(E+m)(E'+m')}{2pp'}\right)^{1/2}\\
&\quad\times\left[2W-m-m'+\frac{(m'-m)(\mu'^2-\mu^2)}{M_2}\right],\\
H &= \frac{1}{2Wp}\left(\frac{(E-m)(E'-m')}{2pp'}\right)^{1/2}\\
&\quad\times\left[2W+m+m'-\frac{(m'-m)(\mu'^2-\mu^2)}{M_2}\right],\\
\beta &= (2EE'+M^2-m'^2-m^2)/2pp'\,.
\end{aligned}
\qquad (16)
$$

In Eqs. (16), E and m are the energy and mass of the proton, E' and m' are the energy and mass of the Λ; μ, μ', and M are the masses of π, K, and K^*, respectively. Q_l is a Legendre function of the second kind.

The partial-wave amplitudes in Eq. (16) are summed according to Eq. (2) to give K^* production amplitudes f_{1K^*} and f_{2K^*}. In the limit of no optical absorption, $D_l=1$, one has

$$
f_{1K^*}=NG/(\beta-\cos\theta)\,,\quad f_{2K^*}=NH/(\beta-\cos\theta)\,,\qquad (17)
$$

which agrees with the corresponding formulas of Hoff.[4]

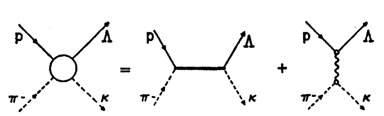
The differential cross section for production is given in terms of the amplitudes f_1 and f_2 by Eq. (1). This cross section may be decomposed into a resonance term, a K^* term, and an interference contribution as

$$
\begin{aligned}
I_r(\theta) &= \frac{\Gamma_1\Gamma_2}{4p^2[(W_r-W)^2+\Gamma^2/4]}D_l^2F_{J-1/2}(\cos\theta)\,,\\
I_{K^*}(\theta) &= |f_{1K^*}|^2+|f_{2K^*}|^2+2\cos\theta\,\mathrm{Re}(f_{1K^*}f_{2K^*}{}^*)\,,\\
I_i{}^{\pm}(\theta) &= \frac{(\Gamma_1\Gamma_2)^{1/2}(2J+1)(W_r-W)}{2p[(W_r-W)^2+\Gamma^2/4]}D_l[f_{1K^*}P_l(\cos\theta)\\
&\qquad +f_{2K^*}P_{l\pm1}(\cos\theta)]\,.
\end{aligned}
\qquad (18)
$$

The factor $F_{J-1/2}(\cos\theta)$ is defined in Eq. (12). The differential cross section is integrated to give $\sigma(W)=\sigma_r(W)+\sigma_{K^*}(W)+\sigma_i(W)$ where

$$
\begin{aligned}
\sigma_r(W) &= \frac{\pi\Gamma_1\Gamma_2}{2p^2[(W_r-W)^2+\Gamma^2/4]}D_l^2\,,\\
\sigma_{K^*}(W) &= \sum_{l=0}^{\infty}(2l+2)(|f_{lK^*}{}^+|^2+|f_{(l+1)K^*}{}^-|^2)\,,\\
\sigma_i{}^{\pm}(W) &= \frac{2\pi(\Gamma_1\Gamma_2)^{1/2}(2J+1)(W_r-W)}{p[(W_r-W)^2+\Gamma^2/4]}D_lf_{lK^*}{}^{\pm}\,.
\end{aligned}
\qquad (19)
$$

FIG. 3. Decomposition of the Λ-K production amplitude.



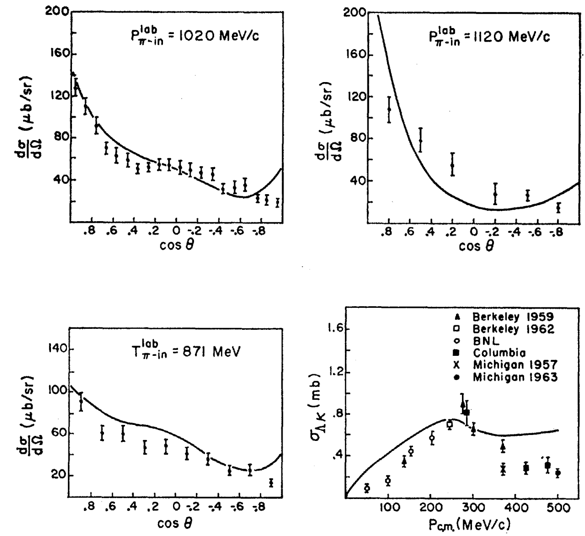


FIG. 4. Comparison of the $D_{5/2}$ model calculation with experiment.

In Eqs. (18) and (19) Γ_2 is a parameter which can be determined once, Γ_1 is given by the branching law

$$\frac{\Gamma_2}{\Gamma_1}=\frac{(E'\pm m')(p')^{2J}}{(E\pm m)(p)^{2J}} \quad \text{for} \quad J=l\pm\tfrac{1}{2}, \tag{20}$$

where it is assumed that the coupling for the resonance to the π^--p and K-Λ systems is identical, $g_{N^*\pi^-p}=g_{N^*K\Lambda}$. Using the values of Γ_1/Γ given in Table I we find

$$\begin{aligned}\Gamma_1\Gamma_2/\Gamma^2&=6.64\times10^{-3} \quad \text{for} \quad D_{5/2},\\ &=6.20\times10^{-4} \quad \text{for} \quad F_{5/2}.\end{aligned} \tag{21}$$

Interchanging the columns of Table I as discussed in Sec. II does not appreciably change the numbers in Eq. (21). The contribution of the resonance to the cross section at $W=W_r$ is 0.23 mb for $D_{5/2}$ and 0.025 mb for $F_{5/2}$. The experimental data indicate a rise above background of about 0.3 mb which favors the $D_{5/2}$ assignment independently of the choice of optical amplitudes.

Assuming a $D_{5/2}$ resonance, angular distributions at several energies and the integrated production cross section are computed from Eqs. (18) and (19). The results are given in Fig. 4, together with the corresponding experimental values.[11]

To obtain reasonable comparison with experiment it was necessary to increase the value of $(g_{\pi KK^*}{}^2/4\pi)\times(g_{P\Lambda K^*}{}^2/4\pi)$ to 1.2 from the value of 0.115 quoted by Hoff.[4] This increase can be understood if one notes that, at the energies considered, the parameter β in the Born amplitudes of Eq. (17) is larger than 2 and the K^* contribution to the amplitude is dominantly s-wave. The s-wave amplitude is strongly absorbed since the corresponding optical parameter $D_0 \ll 1$; consequently, the coupling must be increased.

As a result of the rather large K^* coupling constant, one tends to doubt polarization calculations based on interferences between the diagrams of Fig. 3 alone. For this reason, no attempt is made to explain the polarization data using this model.

IV. SUMMARY

A definite prescription has been given for subtracting back-ground from the π^--p total cross section in the vicinity of a resonance. This procedure is similar to the usual method of subtracting the constant high energy cross section from the experimental total cross section and interpreting the remainder in terms of a Breit-Wigner formula. The major difference is that a specific model is proposed to account for the constant high-energy cross section; the elastic cross section and the angular distributions computed from this model are found to agree, generally, with experiment. The spin of the resonance is found to be $\frac{5}{2}$ by comparing the model calculations with experimental elastic cross sections.

To determine the parity of the resonance a simple two-channel problem is considered. It is shown that a $D_{5/2}$ resonance couples to a resonance in the ΛK channel of approximately the correct strength to account for the observed ΛK cross section. The resonance model for ΛK production proposed by Hoff is therefore modified by replacing the $P_{1/2}$ resonance by a $D_{5/2}$ term and computing the distorted-wave approximation. The production cross section and angular distributions are again found to agree generally with experiment.

[11] Joseph Keren, Phys. Rev. **133**, B457 (1964); L. Bertanza, P. L. Connoly, B. B. Culwick, F. R. Eisler, T. Morris *et al.*, Phys. Rev. Letters **8**, 332 (1962).