where the definition of $\beta_{(l,s)\mu M}{}^{i}(x,\omega)$ is quite obvious and analogous to Eq. (10), and where the matrix kernel

$$\Gamma_{j(l,s)\mu}{}^{i(\lambda,\sigma)\nu}(x,y)$$

is defined by

$$\Gamma_{j(l,s)}{}^{i(\lambda,\sigma)\nu}(x,y) = (1-\delta_{ij})\sum_{M''}\int \phi_i{}^{l,s}(\omega'')\Delta_{M''\mu}{}^{J*}Y_{l\mu}(\gamma_i'',0) \times\delta(x-\omega_i'')A_j{}^{\lambda,\sigma}(\omega'',z) \times Y_{\lambda\nu}{}^{*}(\gamma_j'',0)\Delta_{M''\nu}{}^{J}\delta(y-\omega_j'')d\omega''. \quad (23)$$

Equation (22) is closely analogous to (12), except that the dimensionality of the matrix is greater. In practical cases, one must say that these equations are much simpler than what they seem to be here, for the number of pole terms in each two-body channel will not be very large.

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New Measurement of the $\bar{K}^0$-K^- Mass Difference*

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The $\bar{K}^0$-K^- mass difference has been measured to be 3.90±0.25 MeV. The method employed involves the observation of nine examples of the reaction $K^-+p\rightarrow\bar{K}^0+n$, $\bar{K}^0\rightarrow\pi^++\pi^-$ in association with the proton recoil from the scattered neutron. The kinematic fitting of this event in the bubble chamber is very sensitive to the $\bar{K}^0$-K^- mass difference. Sources of systematic errors are also discussed.

I. INTRODUCTION AND EXPERIMENTAL METHOD

We have measured the $\bar{K}^0$-K^- mass difference using $\bar{K}^0$ produced by low-energy K^- interactions in the Saclay 81-cm hydrogen bubble chamber[1] at CERN.[2]

The reactions which we observed were

$$K^-+p\rightarrow\bar{K}^0+n \quad \searrow \quad \pi^++\pi^-, \quad (1)$$

$$n+p\rightarrow n+p. \quad (2)$$

We rescanned a sample of about 100 events which had been measured and fit the hypothesis of reaction (1). We searched for those events where there was, in addition, an example of reaction (2), an (n,p) scatter with a proton recoil, in the same photograph. We only considered events as candidates if the proton recoil was less than 7 cm away and if the proton recoil appeared to conserve momentum in reaction (1). Thirteen candidates were found. These events were remeasured to include the neutron direction and the recoil proton. The kinematic fitting program,[3] using the $\bar{K}^0$ momentum[4] as deduced from its decay and the neutron momentum as deduced from its recoil, tested the hypothesis of reaction (1). In this attempted kinematic fit all the vector momenta are known and therefore the results are very sensitive to the $\bar{K}^0$-K^- mass difference, provided, of course, that the recoil is truly associated. Our kinematic fitting program did not allow the $\bar{K}^0$ mass to be varied as an undetermined parameter so the following procedure was adopted. For each event, the kinematic fits were attempted in steps of 0.1 MeV over the region of $\bar{K}^0$ mass 491.0 to 504.0 MeV. The best value of the $\bar{K}^0$-K^- mass difference for each event was taken as that value of the mass difference for which the χ^2 of the fit was a minimum.[5] Accidental recoils and poor measurements were rejected by requiring that the goodness of this fit at the minimum correspond to a confidence level of greater than 1%. An error was assigned to each determination of the mass difference. This error was obtained by noting that the mass was being used as an independent degree of freedom and $(\chi^2)_{\min}+1$ corresponds to a change of a standard deviation in this one degree of freedom.

II. RESULTS

Table I lists data of the nine events which satisfy our criteria. The data listed are the average values obtained

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[1] P. Baillon, thesis, University of Paris, 1963 (unpublished).

[2] A description of the beam is given by B. Aubert, H. Courant, H. Filthuth, A. Segar, and W. Willis, in *Proceedings of the International Conference on Instrumentation for High Energy Physics at CERN* (North-Holland Publishing Company, Amsterdam, 1963).

[3] For a description of the program, KICK, see Reference Manual for Kick IBM Program, edited by A. H. Rosenfeld, University of California Radiation Laboratory Report No. UCRL 9099 (unpublished); and A. H. Rosenfeld and J. N. Snyder, Rev. Sci. Instr. **33**, 181 (1962).

[4] To the accuracy needed we can ignore the variation of the fitted $\bar{K}^0$ momentum with the assumed $K_1{}^0$ mass.

[5] The kinematic fitting program, KICK, uses linear constraints. We assume that this approximation does not systematically shift the position of $(\chi^2)_{\min}$.

from four measurements of each event. We rejected four events. Three of these did not fit the hypothesis of reactions (1) and (2) anywhere in the region of $\bar{K}^0$ masses from 491 to 504 MeV. This corresponds to a range of $\bar{K}^0$-K^- mass differences of $\sim\pm6.0$ MeV. All the nine accepted events fall in a region ±1.5 MeV about the central value. For this reason we conclude that our background of accidental recoils is negligible. One additional event was rejected because the confidence level of all four measurements was considerably less than 1%. The weighted average of the nine accepted events gives a $\bar{K}^0$-K^- mass difference of 3.90 ± 0.25 MeV. This value of the $\bar{K}^0$-K^- mass difference was calculated by weighting the measurements inversely as the square of their errors. The error in the mass difference was estimated by assuming that the theoretical internal errors listed in Table I are proportional to the true errors. Using the weighted sum of the squares of the individual deviations, one can estimate this proportionality factor. The error derived in this way is 1.4 times the value calculated from the theoretical internal errors. The $\bar{K}^0$-K^- mass difference is not sensitive to the absolute value of the K^- mass within a few MeV.[6,7] The effect of the uncertainty in the masses of the other particles, the neutron, the proton, and the pion is also negligible.

TABLE I. Summary of data on $\bar{K}^0$-K^- mass difference.

Frame number	$\bar{K}^0$-K^- mass difference (MeV)
1341 158	4.43 ± 0.6
1353 645	2.53 ± 1.1
1353 780	2.80 ± 0.5
1359 210	3.65 ± 0.9
1359 483	4.38 ± 0.4
1359 597	2.31 ± 0.7
1360 016	4.27 ± 0.6
1366 076	4.53 ± 0.4
1381 761	3.87 ± 0.3
Weighted average $\bar{K}^0$-K^- mass difference	$=3.90\pm0.25$ MeV

III. DISCUSSION

We have studied the sensitivity of this result to the uncertainty in the density of hydrogen in the bubble chamber and the uncertainty in the magnetic field. The details of the methods used to determine these constants are given in Ref. 8.

The error in our knowledge of the hydrogen density is $\pm0.5\%$. For such a change in the density of hydrogen, we have redetermined the value of the $\bar{K}^0$-K^- mass difference and find that the result changes by ±0.03 MeV. In a similar way, the effect of an 0.5% change in the magnetic field was investigated and found to result in a ±0.03-MeV change in the $\bar{K}^0$-K^- mass difference. We have investigated the sensitivity of the results to a somewhat different program which used a different geometrical reconstruction method (utilizing corresponding points) and different error calculations.[9] We have found that processing the same measurements through the different program gave a final result for the $\bar{K}^0$-K^- mass difference which differed by 0.05 MeV. In addition, we investigated the effect of the uncertainty in the bubble chamber camera positions by changing these optical constants and then reprocessing the events. We found that this resulted in about a ±0.05-MeV change in the $\bar{K}^0$-K^- mass difference. We therefore conclude that our result is not sensitive to many of the usual sources of systematic error.

Our results can be compared with a similar measurement using recoils by Rosenfeld *et al.*[10] of 3.3 ± 0.9 MeV and the currently accepted value of 4.2 ± 0.5 MeV.[7] Combining our measurement of the $\bar{K}^0$-K^- mass difference with the accepted value of the K^- mass 493.8 ± 0.2 MeV,[7] we get the result $M_{\bar{K}^0}=497.7\pm0.32$ MeV. This agrees with the recent result of Christenson *et al.*[11] of $M_{\bar{K}^0}=498.1\pm0.4$ MeV obtained from direct measurements on K^0 decays.

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[6] We have assumed $M_{K^-}=493.8$ MeV as given in Ref. 7.

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