Double-Charge-Exchange Scattering of Pions from Nuclei*

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Pions offer a unique possibility as probes of nuclear structure since they can exchange two units of electrical charge unaccompanied by other quantum numbers. We have calculated the double-charge-exchange cross section for the reaction $\pi^- + \mathrm{He}^3 \to \pi^+ + 3n$ in the impulse approximation using the Chew-Low model for the pion-nucleon interaction. Only the dominant 3-3 channel is retained. For incident pions in the energy region of several hundred MeV, values of the differential cross section of $d^2\sigma/d\Omega dE \approx 1$-$10$ μb/MeV are obtained for forward angles. Triple-scattering terms are also calculated and found to introduce corrections of $<10\%$ in $d^2\sigma/d\Omega dE$. Similar results are obtained when the work is extended to the reaction $\pi^+ + \mathrm{O}^{18} \to \pi^- + \mathrm{Ne}^{18}$ using shell-model wave functions.

DISCUSSION

It has been pointed out[1] that pions offer a unique opportunity to probe nuclear structure, since they can exchange two units of electrical charge unaccompanied by other quantum numbers in the reactions

$$\pi^{\pm} + A(Z) \to \pi^{\mp} + A(Z\pm 2). \qquad (1)$$

Such processes are of interest in nuclear-structure analyses for information they provide on the correlation between two identically charged nucleons in nuclei. What is more, since both the incident and emerging pions are electrically charged and can be detected with very high-energy resolution, excitations of individual nuclear levels in the final nucleus can be studied and the overlap of two nuclear states differing only by particular shell-model level assignments can be measured.

Approximate calculations of differential cross sections for reactions (1) are presented in this paper. The incident pion must scatter twice within each individual nucleus in order to transfer two units of charge, one each to two nucleons. Therefore, it is not enough to relate the scattering amplitude to experimental parameters for single pion-nucleon scattering using the impulse approximation. What is needed is an extrapolation of the scattering amplitude "off its mass shell" in order to be able to describe the virtual pion propagating between the two scatterings—viz.,

$$\pi^+ + A(Z) \to \{\pi^0 + A(Z+1)\} \to \pi^- + A(Z+2).$$

The Chew-Low theory[2] provides the necessary extrapolation for this calculation. The calculation is carried out in the energy range in which the pion-nucleon phase shifts are dominated by the 3-3 resonance, and all other channels are ignored for simplicity. The double-scattering formalism is then used in a straightforward manner to calculate (1) both for

$$\pi^- + \mathrm{He}^3 \to \pi^+ + 3n,$$

and for

$$\pi^+ + \mathrm{O}^{18} \to \pi^- + \mathrm{Ne}^{18}.$$

Although triple-scattering corrections to the double-scattering amplitude are computed, other higher order multiple scatterings as well as distortion of the incident and emerging pion waves are ignored. These corrections are expected largely to cancel out in ratios of cross sections to different levels of a final nucleus which are primarily sensitive to the wave functions of the states.

The calculational results presented here are not to be ascribed a quantitative significance. Rather they are intended to encourage interest on the part of our experimental colleagues in making accurate measurements by showing that:

(i) differential cross sections are in the range of

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[1] The idea grew in a discussion between S. D. Drell, H. J. Lipkin, and A. de-Shalit at the Weizmann Institute, Rehovoth, in December 1961, and was reported in seminars thereafter at CERN and Saclay, 1962. See also, discussion by T. Ericson at 1963 International Conference on High Energy Physics and Nuclear Structure CERN, 1963 CERN Report No. 63-28, July 1963, p. 68 (unpublished). An independent discussion and study of this reaction via a different process has been given by A. K. Kerman (at the CERN conference); see also A. K. Kerman and R. Logan, in Proceedings of the Symposium on Nuclear Spectroscopy with Direct Reactions, 1964 (to be published). Experimental observation of this process has recently been reported by L. Gilly, M. Jean, R. Meunier, M. Spighel, J. P. Stroot, P. Duteil, and A. Rode, Phys. Letters **11**, 244 (1964).

[2] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1571 (1956).

magnitudes

$$d^2\sigma/d\Omega dE \sim 1-10\ \mu\text{b/sr MeV}$$

and thus can be experimentally measured by existing synchrocyclotrons; and

(ii) interesting nuclear-wave-function correlations are probed; in particular, in the energy region of several hundred MeV the reduced wavelength of the virtual intermediate pion is

$$\lambda\!\!\!^{-}_r = 1/k_r \sim 1/300\ \text{MeV} \sim 0.7\times10^{-13}\ \text{cm},$$

which is short enough to probe nuclear correlations within the "healing distance" but not so short as to run up against the repulsive core.

CALCULATION

He^3 Target

We calculate, first of all, the cross section for the double-charge-exchange reaction $\pi^- + He^3 \to \pi^+ + 3n$ in the forward direction using the Chew-Low model[2] for the individual pion-nucleon interaction.

The target nucleus is described by the dominant fully space-symmetric S-state wave function whose space part has a Gaussian dependence on the nucleon separations.[3] The mixed symmetry states are an admixture of only about 4% and are neglected in this calculation. All final-state interactions are neglected and plane waves in a Slater determinant are used for the final-state wave functions. It is assumed that the momentum transferred to the nucleons is small, so that the final state has two neutrons in a relative s state and one neutron in a p state relative to the other two. We define our units such that $\hbar = c = m_\pi = 1$.

In the impulse approximation, the scattering matrix $T(\mathbf{k},\mathbf{k}_0)$ for a pion with initial wave number $\mathbf{k}_0$ and final wave number $\mathbf{k}$ is given by[4]

$$T(\mathbf{k},\mathbf{k}_0) = 2\int \frac{d^3\mathbf{q}\ \omega_q}{(2\pi)^3(q^2-k_0^2-i\epsilon)} \times \sum_{ij,\ i\neq j}^{3} [\tilde{t}_i(\mathbf{k},\mathbf{q})\tilde{t}_j(\mathbf{q},\mathbf{k}_0)],$$

where $\omega_q = (q^2+1)^{1/2}$ and where $\tilde{t}_i$ is the scattering matrix for the interaction of a pion and the ith nucleon. Since both the initial and final wave functions are antisymmetric under the interchange of any two nucleons, the term in square brackets above may be replaced by

$$3[\tilde{t}_3(\mathbf{k},\mathbf{q})\tilde{t}_2(\mathbf{q},\mathbf{k}_0) + \tilde{t}_2(\mathbf{k},\mathbf{q})\tilde{t}_3(\mathbf{q},\mathbf{k}_0)].$$

The operator $\tilde{t}(\mathbf{p},\mathbf{q})$ corresponds to a displacement operator times the usual $t(\mathbf{p},\mathbf{q})$ of Chew and Low, i.e.,

$$\tilde{t}_2(\mathbf{k},\mathbf{q})\tilde{t}_3(\mathbf{q},\mathbf{k}_0) = \exp[i(\mathbf{k}_0-\mathbf{k})\cdot(\mathbf{R}-\tfrac{1}{3}\boldsymbol{\rho})] \times\exp[i(\mathbf{k}_0+\mathbf{k})\cdot(\tfrac{1}{2}\mathbf{r})]\exp[-i\mathbf{q}\cdot\mathbf{r})t_2(\mathbf{k},\mathbf{q})t_3(\mathbf{q},\mathbf{k}_0),$$

where $\mathbf{r} = \mathbf{r}_3 - \mathbf{r}_2$, $\boldsymbol{\rho} = \mathbf{r}_1 - \frac{1}{2}(\mathbf{r}_2+\mathbf{r}_3)$, and $\mathbf{R} = \frac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)$, and $\mathbf{r}_i$ is the coordinate of the ith nucleon.

The scattering matrix in the 3-3 channel is given by

$$t(\mathbf{p},\mathbf{q}) = -2\pi(\omega_q\omega_p)^{-1/2}P_{33}(\mathbf{p},\mathbf{q})p^{-3}e^{i\delta(p)}\sin\delta(p),$$

where $P_{33}(\mathbf{p},\mathbf{q})$ is the spin-isospin projection operator for the 3-3 channel. For the phase shifts we use a parametrization given by McKinley[5] for the 3-3 channel:

$$q^{-3}e^{i\delta(q)}\sin\delta(q) = q^{-3}[\cot\delta(q)-i]^{-1} = (a+bq^2+cq^4-iq^3)^{-1},$$

where $a = 4.108$, $b = 0.7987$, and $c = 0.8337$.

The angular part of the integral over the intermediate momentum was evaluated by expanding the projection operator and displacement operators in spherical harmonics. This leads to an integral over the intermediate momentum of the form

$$\int dq \frac{2q^4 j_n(qr)}{(q^2-k_0^2-i\epsilon)(a+bq^2+cq^4-iq^3)},$$

with $n = 0$ or 2. To approximate this integral, we note that $(a+bq^2+cq^4-iq^3)^{-1}$ is sharply peaked at $q = q_r = 1.658$. Therefore we replaced $j_n(qr)$ by $j_n(q_r r)$. The remainder of the integral was evaluated for $|k-q_r| \gg \Gamma$ (where Γ is the width of the resonance) and for $|k-q_r| \ll \Gamma$. For intermediate values of q, a graphical interpolation was made.

After integrating over $\mathbf{R}$, $\mathbf{r}$, and $\boldsymbol{\rho}$, summing over spin and isospin variables, and integrating over the final phase space (many of the integrals over r and ρ were done numerically on an IBM 7090) we obtained a table of the differential cross section for several values of momentum transfer. These are given in Table I.

TABLE I. Values of the differential cross section for the double-charge-exchange reaction of π^- on helium-3 for two values of the momentum transfer. (Cross section in μb/sr MeV.)

k_0 (incident pion momentum in units of m_π)	$\left.\frac{d^2\sigma}{d\Omega dE}\right\vert_{(k_0-k=0.2)}$	$\left.\frac{d^2\sigma}{d\Omega dE}\right\vert_{(k_0-k=0.3)}$
1.0	0.0082	0.035
1.1	0.025	0.11
1.2	0.078	0.34
1.3	0.25	1.8
1.4	0.71	3.1
1.5	1.4	6.3
1.6	1.9	9.2
1.7	2.0	11.0
1.8	1.4	10.0
1.9	0.91	6.9
2.0	0.35	3.0

[3] L. I. Schiff, Phys. Rev. **133**, B802 (1964).
[4] G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952); S. D. Drell and L. Verlet, *ibid.* **99**, 849 (1955).
[5] J. M. McKinley, Rev. Mod. Phys. **35**, 788 (1963).

A calculation was made to determine the triple-scattering corrections to the previous result. The calculation was very similar to, although longer than, the double-scattering case. The result was expressed as a correction factor to the double-scattering result, i.e.,

$$d^2\sigma/d\Omega dE|_{(\text{double}+\text{triple})}=d^2\sigma/d\Omega dE|_{(\text{double})}(1+\alpha)\,.$$

Values of α for a momentum transfer of 0.3 $m_\pi c$ are given in Table II for different incident momenta.

TABLE II. Values of the correction factor α for triple scattering on helium-3.

k_0 (incident pion momentum in units of m_π)	α
1.1	0.011
1.2	0.034
1.3	0.046
1.4	0.079
1.5	0.084
1.6	0.076
1.7	0.063
1.8	0.048
1.9	0.031
2.0	0.015

O^{18} Target

This method of calculation was also extended to a heavier nucleus using a simple version of the shell model with the neglect of spin-orbit coupling effects. The transition O^{18}-Ne^{18} was chosen since these two nuclei in the ground state differ only in that the former has two $1d$ neutrons outside of a closed shell, while the latter has two $1d$ protons outside of a closed shell. Also, both nuclei have zero total angular momentum in the ground state.

Since the double-scattering T matrix depends on the separation between the two scattering centers, we must use a shell-model potential whose nuclear wave function is separable into relative coordinates. Denoting the nuclear wave function by $|n_1l_1,n_2l_2,\lambda\mu\rangle$, where nucleon 1 has radial quantum number n_1 and orbital-angular-momentum quantum number l_1, nucleon 2 has quantum numbers n_2 and l_2, and the total-angular-momentum and magnetic quantum number are λ and μ, respectively, we may separate into the relative coordinates of the two particles if an harmonic oscillator potential is used.[6] We wish to write

$$|n_1l_1,n_2l_2,\lambda\mu\rangle=\sum_{nlNL}|nl,NL,\lambda\mu\rangle\langle nl,NL,\lambda|n_1l_1,n_2l_2,\lambda\rangle\,,$$

where n and l are the quantum numbers associated with the relative coordinate $(2)^{-1/2}(\mathbf{r}_1-\mathbf{r}_2)=\mathbf{r}=(r,\theta,\varphi)$ and N and L are the quantum numbers associated with the coordinate

$$(2)^{-1/2}(\mathbf{r}_1+\mathbf{r}_2)=\mathbf{R}=(R,\Theta,\Phi)\,.$$

$\langle nl,NL,\lambda|n_1l_1,n_2l_2,\lambda\rangle$ are called transformation brackets and are tabulated.[7] The nuclear wave functions are expressible in terms of single-particle harmonic-oscillator wave functions as follows:

$$|n_1l_1,n_2l_2,\lambda\mu\rangle=\sum_{m_1m_2}\langle l_1l_2,m_1m_2|\lambda\mu\rangle \times Y_{l_1m_1}(\theta_1,\varphi_1)Y_{l_2m_2}(\theta_2,\varphi_2)R_{n_1l_1}(r_1)R_{n_2l_2}(r_2)$$

and

$$|nl,NL,\lambda\mu\rangle=\sum_{mM}\langle lL,mM|\lambda\mu\rangle \times Y_{LM}(\Theta,\Phi)Y_{lm}(\theta,\varphi)R_{NL}(R)R_{nl}(r)\,.$$

The symbols $\langle lL,mM|\lambda\mu\rangle$ are Clebsch-Gordon coefficients and $R_{nl}(r)$ are normalized harmonic-oscillator wave functions given by

$$R_{nl}(\rho)=\left(\frac{2n}{\Gamma(n+l+\frac{3}{2})}\right)^{1/2}\rho^l e^{-1/2\rho^2}L_n^{l+1/2}(\rho^2)\,,$$

where ρ is in units of $(\hbar/M\omega)^{1/2}$.

The parameter for the oscillator potential for O^{18} is $(\hbar/M\omega)^{1/2}=1.694$ f.[8] The parameter for Ne^{18} was assumed to be equal to that of O^{18}.

This calculation is identical to that for He^3 except for the different initial and final nuclear wave functions. Values of $d\sigma/d\Omega$ for forward scattering are given in Table III.[9]

TABLE III. Values of the double-charge-exchange differential cross section for O^{18}-Ne^{18} (ground state) in μb/sr.

k_0 (incident pion momentum in units of m_π)	$d\sigma/d\Omega$
0.5	0.0092
0.6	0.026
0.7	0.064
0.8	0.16
0.9	0.38
1.0	0.83
1.1	1.79
1.2	4.37
1.3	11.5
1.4	27.0
1.5	42.0
1.6	42.0
1.7	31.0
1.8	18.0
1.9	9.1
2.0	3.4

[6] M. Moshinsky, Nucl. Phys. **13**, 104 (1959).

[7] T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (University of Mexico, Mexico City, 1960).

[8] B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954).

[9] In this calculation, we have set $\exp[i(\mathbf{k}_0-\mathbf{k})\cdot\boldsymbol{\rho}]=1$. For angles away from the forward direction, this term will become a nuclear form factor and reduce the value of $(d\sigma/d\Omega)$ for $|\mathbf{k}_0-\mathbf{k}|\gtrsim m_\pi/A^{1/3}$.

TABLE IV. Ratios of the forward differential cross section for O^{18}-Ne^{18} (excited state) to that for O^{18}-Ne^{18} (ground state).

k_0 (incident pion momentum in units of m_π)	σ^*/σ
0.5	0.930
0.6	1.05
0.7	1.13
0.8	1.18
0.9	1.21
1.0	1.24
1.1	1.26
1.2	1.27
1.3	1.30
1.4	1.33
1.5	1.40
1.6	1.48
1.7	1.56
1.8	1.61
1.9	1.63
2.0	1.63

TABLE V. Values of the differential cross section for O^{18}-Ne^{18} (ground state) in the closure approximation.

k_0 (incident pion momentum in units of m_π)	$d\sigma/d\Omega$ (μb/sr)
0.5	0.060
0.6	0.17
0.7	0.40
0.8	0.90
0.9	1.9
1.0	3.8
1.1	7.4
1.2	17.0
1.3	45.0
1.4	110.0
1.5	210.0
1.6	280.0
1.7	305.0
1.8	260.0
1.9	170.0
2.0	77.0

The calculation was repeated assuming that Ne^{18} was left in an excited state with both protons in the $2s$ shell. The ratio of this cross section to that for Ne^{18} in its ground state is tabulated in Table IV. Ratios of this type are of particular interest since many of the higher order multiple-scattering and nuclear-potential effects[10] are expected to cancel out. Comparison of calculation with observed ratios is then sensitive to details of the desired nuclear correlations.

We have also evaluated the cross section of O^{18}-Ne^{18} (ground state) by the closure approximation, i.e.,

$$\frac{d\sigma}{d\Omega}=\int\sum_f |T|^2\frac{\omega_{k_0}}{k_0}2\pi\delta(\text{energy})\frac{k\omega_k d\omega_k}{(2\pi)^3}\,.$$

The results of this calculation are tabulated in Table V, and are primarily of interest for comparing with observation to measure the effect of absorption of the incident and emerging pion waves into other inelastic channels.

[10] We have not computed these for O^{18}. Their correction to the results in Tables III and IV are presumably more important than the analogous corrections of Table II for the He^3 nucleus since there is more nuclear matter off which the intermediate pion can scatter.