Beta-Gamma Angular-Correlation Measurements on Rb⁸⁶ and Rb⁸⁴ †

P. C. SIMMS,* A. NAMENSON,[‡] T. H. WEI, AND C. S. WU Department of Physics, Columbia University, New York, New York (Received 4 January 1965)

The energy dependence of the beta-gamma directional correlation of Rb⁸⁴ and Rb⁸⁶ were measured. The results are $\alpha = 0.084 \pm 0.015$ and $\beta = 0.020 \pm 0.012$ for Rb⁸⁶ and $\alpha = 0.038 \pm 0.012$ and $\beta = -0.009 \pm 0.009$ for Rb⁸⁴ when the directional-correlation function is written in the form $N(W,\theta) = 1 + (p^2/W)(\alpha + W\beta)P_2(\theta)$. The angular dependence of the circular polarization of Rb⁸⁶ was also measured. The results show that there is a large $P_{\mathbf{s}}(\theta)$ contribution to the circular-polarization correlation.

I. INTRODUCTION

`HE experimental work reported in this paper is part of a systematic study of the nuclearmatrix elements of first-forbidden beta transitions. The energy dependence of the beta-gamma directional correlation was measured for both Rb⁸⁶ and Rb⁸⁴. The angular dependence of the beta-gamma circular polarization was measured for Rb⁸⁶. The results obtained in these measurements are in substantial agreement with a number of other experiments^{1,2} which have been reported since this work was begun.

Several precautions were taken in the directionalcorrelation experiments to minimize the possibility of systematic errors. The directional correlation was measured over an angular range of 150° rather than the usual measurement of the anisotropy between 180 and 90°. A detailed statistical analysis of the data was performed on the basis of the fit of the experimental points to the theoretical distribution function. A Gerholm³ magnetic spectrometer was used for the beta particles so that the corrections that occur with a scintillation detector could be avoided. This was particularly important for the positron decay of Rb⁸⁴.

One important change was made in the usual technique of detecting circular polarization by forward Compton scattering from magnetized iron. A zero-field measurement was included in the automatic cycle of field "left" and field "right." When the asymmetry in the coincidence counting rate is due to the circular polarization of the gamma rays, it must be symmetric around the zero-field position.

For a first-forbidden beta transition followed by a gamma ray, the angular-correlation function has the following form:

$$N(W,\theta,s) = A_0(W) + sA_1(W)P_1(\theta) + A_2(W)P_2(\theta) + sA_3(W)P_3(\theta).$$
(1)

W is the electron energy, $P_n(\theta)$ are Legendre poly-

nomials, and θ is the angle between the direction of emission of the beta and gamma rays. The helicity factor s is +1 and -1 for right- and left-hand circular polarization, respectively. If the circular polarization is not observed (s=0), the general expression reduces to the formula for the directional correlation,

$$\mathbf{V}(\theta, W) = \mathbf{1} + \boldsymbol{\epsilon}(W) P_2(\theta) , \qquad (2)$$

where $\epsilon(W) = A_2(W)/A_0(W)$. The primary energy dependence of $\epsilon(W)$ is seen when it is written in the following form :

$$\epsilon(W) = (p^2/W)(\alpha + \beta W), \qquad (3)$$

where p is the electron momentum. Since β is usually much smaller than α , it is convenient to define the parameter $\epsilon'(W)$ which indicates the magnitude of the directional correlation with the major energy dependence removed.

$$\epsilon'(W) = \epsilon(W)/(p^2/W) = \alpha + \beta W.$$
(4)

In the notation of Kotani⁴ which has been widely used, α and β are

$$\alpha = R_3 / C'(W) , \qquad (5)$$

$$\beta = e/C'(W), \qquad (6)$$

$$C'(W) = 1 + aW + (b/W) + cW^2, \tag{7}$$

where R_3 , e, a, b, and c are parameters which depend on the nuclear matrix elements of the transition. One common mistake has been made in dealing with these equations When the experimental shape-correction factor C'(W) shows no appreciable energy dependence, C'(W) has been set equal to 1, and it is assumed that $\alpha = R_3$ and $\beta = e$. This is generally not a valid assumption. Frequently, even though there is no pronounced energy dependence, C'(W) is considerably smaller than 1 because the energy-dependent terms tend to combine and give a relatively constant negative number. Results for the parameters α and β will be given, but it must be understood that they do not represent any simple combination of matrix elements. They are, how-

[†] Supported in part by the U. S. Atomic Energy Commission. Present address: Department of Physics, Purdue University, Lafayette, Indiana.

[‡] Present address: Physics Division, Argonne National Laboratory, Argonne, Illinois.

¹ Directional-correlation measurements on Rb⁸⁶, H. Fischbeck and R. Wilkinson, Phys. Rev. **120**, 1762 (1960); J. P. Deutsch, L. Grenacs, J. Lehman, and P. Lipnik. J. Phys. Radium **22**, 659 (1961); J. H. Hamilton, B. G. Peterson, and J. M. Hollander,

Arkiv Fysik 19, 249 (1961); J. Alberghini and R. M. Steffen, Phys. Letters 7, 85 (1963).

 ³ F. Boehm and J. D. Rogers, Nucl. Phys. 45, 392 (1963).
 ³ T. R. Gerholm, Rev. Sci. Instr. 26, 1069 (1955).
 ⁴ T. Kotani, Phys. Rev. 140, 795 (1959).

ever, useful parameters to indicate the general magnitude and energy dependence of the directional correlation.

The beta-gamma circular polarization is usually defined in terms of $N(W,\theta,s)$ as

$$P_{\gamma}(W,\theta) = \frac{N(W,\theta,+1) - N(W,\theta,-1)}{N(W,\theta,+1) + N(W,\theta,-1)}.$$
 (8)

Then,

$$P_{\gamma}(W,\theta) = \frac{A_1(W)P_1(\theta) + A_3(W)P_3(\theta)}{A_0(W) + A_2(W)P_2(\theta)}.$$
 (9)

Kotani has rewritten the formula for the circular polarization in the following manner:

$$P_{\gamma}(W,\theta) = \omega(W,\theta)(p/W) \cos\theta, \qquad (10)$$

where

$$\omega(W,\theta) = \frac{R_4 + gW + hW^2 + lp^2 P_3(\theta)/P_1(\theta)}{C'(W)(1 + \epsilon(W)P_2(\theta))}.$$
 (11)

The parameters R_4 , g, h, and l are functions of the nuclear-matrix elements. This notation is convenient because frequently R_4 is larger than the other parameters, and $P_{\gamma}(W,\theta)$ has the simple energy and angular dependence seen in Eq. (10).

The arrangement of the formulas given by Kotani provides valuable insight into the general characteristics of first-forbidden transitions. When the transition is dominated by the combinations of nuclear-matrix elements which are enhanced by the effect of the Coulomb field on the emitted electrons, R_4 is the only matrix-element parameter which is of order one. All of the other matrix-element parameters are more than an order of magnitude smaller. Then the first-forbidden transition is said to agree with the ξ approximation, and it has characteristics similar to allowed transitions except that its ft value is larger. The shape correction factor is constant, ω is a constant, and ϵ' is a small constant. When these conditions are met, it is extremely difficult to determine the nuclear matrix elements.

There are two effects which can reduce the relative size of the matrix-element combinations which cause this ambiguity. Either the individual matrix elements in the combination can be inhibited by selection rules, or cancellations within the combinations can occur. When either of these effects are present, it is possible that the experimental observables will be much more significant. The shape-correction factor may not be constant, there may be a significant anisotropy in the directional correlation; the $P_3(\theta)$ term may be present in P_{γ} ; or there may be some unusual energy dependence in P_{γ} . Any of these observations are hopeful signs that the nuclear matrix elements can be determined for the transition.

These general guides for determining the nature of the transition from the primary characteristics of the experimental observables are certainly valuable, but

they must be used with caution. Rb⁸⁶ provides a clear example that they are useful. This isotope has an unusual directional correlation and circular polarization, and the relative magnitudes of the nuclearmatrix elements do not agree with the ξ approximation. On the other hand, Rb⁸⁴ has experimental observables which are characteristic of nuclei which obey the ξ approximation, and yet the matrix elements are very similar to the matrix elements of Rb⁸⁶. The analysis used to determine the matrix elements of Rb⁸⁶ and Rb⁸⁴ is considered in the following paper. Several other isotopes which seem to follow the ξ approximation will be considered in two subsequent papers.⁵ For these isotopes, it will be shown that the experimental parameters can be understood either with or without selection rule and cancellation effects. Thus it should be stressed that the possibility of selection-rule effects cannot be excluded just because a first-forbidden transition can be interpreted satisfactorily on the basis of the ξ approximation.

II. BETA-GAMMA DIRECTIONAL-CORRELATION MEASUREMENTS

Instrument

The primary components of the instrument are the Gerholm magnetic spectrometer and an automatic rotating mechanism for the NaI gamma detector (Fig. 1). The spectrometer was modified slightly so that the source could be moved to a position 1 in. from the face of the spectrometer. (The normal source position was 1 cm from the face.) This was one of several precautions taken to minimize the probability of gamma rays being scattered from the spectrometer into the gamma detector. The last coil on the beta-detector end of the spectrometer was disconnected and the beta detector was moved away from the source by $\frac{3}{4}$ in. from its original position. The baffle location was not changed. After the modification, the transmission was 2% when the resolution was 6%. The



FIG. 1. Beta-gamma directional-correlation apparatus.

⁵ P. C. Simms (to be published).

current for the spectrometer was provided by a solidstate power supply (Model 12C, Dynamic Controls Company, Cambridge, Massachusetts). The current regulation was better than 0.1% in 24 h.

The source chamber was made of brass sheets $\frac{1}{16}$ in. thick. The chamber was evacuated by an oil-diffusion pump backed by a mechanical pump. An air lock was provided so that sources could be changed easily. The automatic control mechanism positioned the gamma detector to within a half degree at five points around a 180° semicircle. The rotation mechanism was driven by a synchronous motor (Slo-syn, Superior Electric Company, Bristol, Connecticut). The motor came equipped with planetary gears to give a low shaft speed.

The gamma detector was a 2-in.×2-in. NaI scintillator mounted on a 14-stage photomultiplier tube which was protected by several layers of magnetic shielding. The detector was covered by a lead shield to further reduce the probability of gamma rays getting to the scintillator unless they came directly from the source. The energy resolution of the detector was 8% for the Cs137 gamma ray. The beta scintillator was changed from the original design of Gerholm. The volume of the scintillator was reduced by a factor of 3 with no loss in detection efficiency. This was important for reducing the gamma-ray background particularly with a positron source in the spectrometer. A light pipe 2 in. long was placed between the scintillator and a 14-stage photomultiplier tube. All optical connections were made with a bonding agent (Araldite 502, Ciba Products Company, Fairlawn, New Jersey). The scintillator was covered with MgO₂, and the light pipe was wrapped with aluminum foil to reflect any light which passed through the polished surfaces. The detector had 11% energy resolution for the 0.98-MeV conversion electron of Bi²⁰⁷.

The usual fast-slow coincidence arrangement was used for the electronic system. The fast and slow coincidence circuits have been described previously.6 Double-delay-line clipped linear amplifiers and singlechannel analyzers (Model 348, Franklin Electronics Company, Bridgeport, Pennsylvania) were provided for both detectors The gamma-channel gain was stabilized by a unit (Spectrostat Model 1001A, Cosmic Radiation Labs., Bellport, New York) which uses the counting rate of a single-channel analyzer to control the photomultiplier high voltage. The current capacity (1 mA) of this high-voltage supply was insufficient to provide the current (5 mA) required by the 14-stage photomultiplier bleeder string, so a cathode follower with a separate plate supply was inserted between the control unit and the photomultiplier. The double coincidence, triple coincidence, and single counting rates were recorded on scalers and printed along with the gamma-detector position by an automatic printing system. (Model 7060 Electronic Counters, Model 3130 Multicounter Programmer, Model 1452 Digital Recorder, Berkeley Division, Beckman Instrument Company.)

Calibration and Test

There were several standard procedures which were used in the preliminary tests and the measurement on rubidium. The beta single-channel analyzer was set on the electron peak in the scintillation spectrum. Since the detector had good energy resolution, a narrow window could be used. The narrow window and the small volume of the beta detector were important factors in minimizing the gamma background in the beta detector. The gamma single-channel analyzer was set to just include the photopeak of the gamma ray being observed. Although this reduced the counting efficiency and required that the gamma channel be quite stable, it was a further step to minimize gammaray scattering.

The counting rates were recorded at five angles—255, 220, 180, 140, and 105°. The counting period was 15 min. Frequently directional-correlation measurements are reported where only the anisotropy between 180 and 90° is measured. The complete distribution function was measured here so that a check for systematic errors could be made by comparison with the expected $P_2(\theta)$ distribution.

A measurement of the beta-gamma directional correlation of the allowed beta decay of Co⁶⁰ was performed to test for any intrinsic anisotropy in the instrument. Although the gamma-gamma coincidence background was small ($\sim 1.5\%$), it was still measured carefully. First, the beta scintillator was covered with a thin sheet of Lucite to stop the beta particles without attenuating the gamma rays. Then the gamma-gamma background was measured as a function of the position of the gamma detector with all adjustments the same as for the beta-gamma measurement. A simpler method was also used for comparison. The instrument was operated without the beta shield, but the magnetic field was set to zero. The gamma-gamma background measured by these two methods agreed to within the statistical error required for the correction. The results obtained after gamma-gamma and chance-coincidence corrections are

$\epsilon = 0.0003 \pm 0.001$.

The instrument was further tested by measuring the directional correlation between the 0.98-MeV conversion electron and the 0.57-MeV gamma ray of Bi²⁰⁷ (Fig. 2). The resolution of the spectrometer was adjusted to 2%. The source was a very thin film electroplated onto a $\frac{1}{4}$ -mil copper backing (prepared by Nuclear Science and Engineering Corporation, Pittsburgh, Pennsylvania). Considering all the precautions which have been discussed previously, the probability of detecting the 0.57-MeV gamma ray after scattering was

⁶ P. C. Simms, Rev. Sci. Instr. 32, 894 (1961).



FIG. 2. Simplified decay scheme of Bi²⁰⁷.

very small. However, the 1.06-MeV gamma ray could be scattered and recorded in the energy range of the gamma window. This would not affect the coincidence counting rate, but it would cause the gamma single counting rate to be a systematic function of angle. The results could be affected through the normalization factor which will be discussed later. The effect was studied by comparing the gamma single counting rates when the single-channel analyzer was set on the 0.57- and 1.06-MeV photopeaks. The ratio of the two counting rates was constant to within 0.1% for all angles.

Without applying solid-angle corrections, the result obtained by a least-squares fit was $A_2/A_0=0.153 \pm 0.002$. The solid-angle correction factor Q_2 was estimated from the geometry of the system to be $Q_2=0.625$. Thus the corrected value of A_2/A_0 is

$$A_2/A_0 = 0.245 \pm 0.004$$
.

This is in good agreement with the previously measured value of 7 0.24. Since the solid angle of the gamma detector was only 2%, most of the correction is due to the entrance angle of the spectrometer ($\bar{\theta}=19\%$).

Rb⁸⁶ Measurement

The Rb⁸⁶ was obtained as RbCl in dilute HCl from Oak Ridge National Laboratory. The specific activity was usually greater than 10 mCi/mg. The sources used for measurements above 250 keV were prepared by evaporation from solution under a low-intensity heat lamp. The backing was a $\frac{1}{4}$ -mil Mylar film which had been coated with carbon to provide electrical conductivity. Sources for use below 250 keV were prepared by vacuum evaporation onto aluminum-coated $\frac{1}{4}$ -mil Mylar. These sources were uniform and had a surface density of approximately 100 μ g/cm². All of the sources were covered with collodion films which had surface density of approximately 20 μ g/cm².

The decay scheme of Rb⁸⁶ is shown in Fig. 3. The directional correlation between β_2^- and the gamma ray was measured as a function of the electron energy. A comparison was made at 150 keV between the thin and regular sources, and there was no appreciable

difference in the anisotropy obtained. Thus the regular sources, which were easier to prepare, were satisfactory for use above 250 keV. The source strength used over most of the spectrum was approximately 500 μ Ci. With a resolving time of 5 nsec, the true-to-chance ratio was larger than 10 to 1. At the upper end of the β_2 spectrum the source strength was reduced to 100 μ Ci because of the high intensity of the beta transition to the ground state. The true-to-chance ratio was never smaller than 5 to 1.

Rb⁸⁴ Measurements

The Rb⁸⁴ was obtained as RbCl in dilute HCl from Nuclear Science and Engineering Corporation. The specific activity of the source was approximately 10 mCi/mg. The sources were prepared in the same manner as for Rb⁸⁶ except that no vacuum-evaporated sources were made. The experience with Rb⁸⁶ indicated that the sources prepared by evaporation from solution were adequate even at low energies.

The decay scheme of Rb⁸⁴ is shown in Fig. 4. The energy dependence of the directional correlation between β_1^+ and γ_1 was measured. The primary limitation on source strength was the high gamma counting rate due to the gamma rays following electron capture and those due to positron annihilation. The true-to-chance ratio was not a major limitation because the β^+ transition to the ground state is relatively weak. Of course the noncoincidence gamma background did contribute to the chance coincidences, but this factor does not increase with beta energy as in the case of Rb⁸⁶. The usual source strength was approximately 200 μ Ci which gave a gamma counting rate of approximately 5×10^4 counts per second and a true-to-chance ratio of greater than 10 to 1.

Since the beta scintillator was small and a singlechannel analyzer was set on the electron peak in the scintillation spectrum, the gamma background in the beta detector was small. Even when the single-channel analyzer was set for low-energy electrons, the coincidence counting rate at zero current was only 1% of the normal coincidence rate. Nevertheless, the directional correlation due to this background was checked both at zero field and with a shield over the beta detector. The uncertainty in correcting for the gamma background was negligible compared to the statistical error.



⁷ F. K. McGowan, Phys. Rev. 92, 524 (1953).

where



Processing of Experimental Data

The data was processed on an IBM-7094 Computer. The raw data was first corrected for source decay, chance coincidences, and gamma background coincidences. The corrected-coincidence counting rate was normalized by dividing by the gamma single-counting rate. Then ϵ was determined by a least square fit of the directional-correlation function $N(\theta)$ to all of the experimental points. The data was not averaged at each angle before the least-squares fit. All points which deviated from the best-fit function by more than three standard deviations were discarded, and the least-squares fit was repeated. (Runs were also discarded by beta and gamma test which are discussed below.) Then a χ^2 test was performed to check the validity of the data.

A modified form of the procedure given by Rose⁸ was used to include systematic effects in the experimental error Δ of ϵ .

 $\Delta^2 = R^2 \sigma^2$,

where

(12)

$$R^{2} = \left[\sum_{i=1}^{n} \omega_{i}^{2} (N_{ei} - N_{ci})^{2}\right] / \left[\sum_{i=1}^{n} \omega_{i}\right].$$
(13)

The statistical error σ was computed by the standard method for propagation of errors.⁸ In the formula for the residual R, the sum is over all experimental points. N_{ei} and N_{ei} are the least-squares fit and the experimental values of the correlation function, respectively. The weight factor ω_i is defined in terms of the statistical error on the individual points

$$\omega_i = \sigma_i^{-2}$$
.

Tests were also performed on the stability of the beta and gamma single-counting rates. As long as the variation in these counting rates are slow compared to the counting period at each angle, the data is satisfactory. A method is needed to reject points which are erratic after allowance has been made for the slow drift. The gamma counts for each run were first corrected for source decay, and then they were corrected for variations in the solid angle of the gamma counter as a function of position. The corrected results were fitted to a linear function F(t) which could have a different slope in the first- and second-half of a 24-h run.

$$F(t) = a_0 + a_1 f_1(t) + a_2 f_2(t)$$

$$f_1(t) = t, f_2(t) = 0$$
 for $t \le t'$
 $f_1(t) = t', f_2(t) = t - t'$ for $t > t'$

and t' is the midpoint of the 24-h run. The parameters a_i were determined by a least square fit to the complete set of data points. A typical value of drift was 0.01% per 15-min run. In order to reject the erratic runs, the average deviation from F(t) was computed. All runs which deviated from F(t) by more than three times the average deviation were discarded. The typical value of the average deviation was 0.2%. The same procedure was applied to the single-beta counting rates. Typical values for the drift and average deviation were 0.02 and 0.3%, respectively.

III. BETA-GAMMA CIRCULAR-POLARIZATION MEASUREMENTS

Instrument

The most important features of the instrument will be described here, and more details are given in Ref. 9. The standard technique of forward Compton scattering from polarized electron in magnetized iron was employed. However, there was one important change. Usually measurements are made when the electrons are polarized first in one direction and then in the opposite direction. The polarization-sensitive term in the Compton-scattering cross section reverses sign when the electron-polarization direction is reversed, and it is zero for zero polarization. Thus when there are three positions in the automatic cycle-field left, zero, and right-the asymmetry in the beta-gamma coincidence counting rate should be symmetric around the zerofield position. The zero field was obtained by running automatically through progressively smaller hysteresis loops.

The gamma photomultiplier tube was removed from the field of the analyzing magnet by inserting a light pipe 19 in. long between the NaI crystal and the tube. The phototube was further protected by several layers of magnetic shielding. The gain of the gamma detector was stabilized with a Spectrostat in the same manner described in Sec. II. Since there is no prominent peak in the scattered gamma spectrum, a source of Ce¹³⁹ was attached to the NaI scintillator to provide a photopeak at 165 keV. The beta detector also used a light pipe (8 in. long) and a number of magnetic shields to protect the photomultiplier from the field of the analyzing magnet.

The source chamber was evacuated by a mechanical pump to a pressure of approximately 50μ . The inside of the chamber was lined with Lucite to reduce beta

⁹ T. H. Wei, P. C. Simms, and C. S. Wu (to be published).

⁸ M. E. Rose, Phys. Rev. 91, 610 (1953).



FIG. 5. Rb⁸⁶ directional-correlation parameters $\epsilon = A_2/A_0$ and $\epsilon' = \epsilon W/p^2$.

scattering and magnetic shielding to reduce the field on beta photomultiplier and the electrons emitted from the source. The gamma rays passed through a thin window (1.5-mil Dural) in the chamber wall to the analyzer. The average entrance angle into the analyzer magnet was 22.5°.

In this instrument, the beta scintillation counter was directly exposed to the source, so the beta counting rate was very high. A transistorized double-delay line-clipped linear amplifier¹⁰ and single-channel analyzer¹¹ were built to handle this high counting rate. The amplifier had a fast rise time (14 nsec) so that the clipping lines could be short (100 nsec). The single-channel analyzer was designed to process these short pulses at a high rate and provide an output pulse which was timed by the zero crossing of the linear amplifier signal. The output timing was necessary so that the triple-coincidence resolving time could be short (50 nsec). This reduced the higher-order accidental coincidences which would occur with high counting rates if conventional circuits



 $\epsilon = A_2/A_0 \text{ and } \epsilon' = \epsilon W/p^2.$

¹⁰ J. Hahn and V. Guiragossian, IEEE Trans. Nucl. Sci. 10, 44 (1963).
¹¹ J. Hahn and T. Becker (to be published).

TABLE I. Rb⁸⁶ directional-correlation parameters.

W	e	$\Delta \epsilon$	ϵ'	$\Delta \epsilon'$
1.30	0.060	0.010	0.109	0.019
1.41	0.078	0.004	0.112	0.006
1.51	0.088	0.009	0.103	0.010
1.74	0.147	0.006	0.127	0.005
1.91	0.175	0.010	0.119	0.007
2.10	0.194	0.023	0.120	0.014
2.25	0.230	0.020	0.130	0.011

were used. The fast coincidence circuit was very similar to that used in the directional-correlation instrument.⁶

Measurements

The sources were prepared in the manner discussed in Sec. II. The instrument was tested and calibrated with Co⁶⁰. After all appropriate corrections had been made, the results obtained with Co⁶⁰ were used to determine the absolute magnitude of the circular polarization of Rb⁸⁶. The circular polarization was measured at three angles, $\theta = 180^{\circ}$, $\pm 120^{\circ}$, where θ is the angle between the axis of the beta counter and the axis of the analyzing magnet. The lower level of the singlechannel analyzer window was set at 100 keV and the upper level of 600 keV. The average value of v/c was 0.79. The beta counting rate was frequently higher than 2×10^5 counts per second. The magnetic field was switched at 5-min intervals and the time for each run was measured on a scaler driven by a precision rate pulser. The single counting rates at the three magnetic field positions differed by less than 0.1% when the data was averaged for a 24-h period. When the results were averaged for the entire measurement, the difference was less than 0.02%. This indicates that there was no systematic effect of the magnetic field on the detector.

Processing of Experimental Data

Even though the variation in single counting rate as a function of magnetic field was very small, the coincidence counting rate for each magnetic field setting (0,1,2) was normalized by dividing by the product of the single counting rates. The time factor was also included in the normalization. The relative counting difference from which the circular polarization was calculated is defined in the following manner:

$$R = 2 \frac{N_1 - N_2}{(N_1 + N_2) - 2C} \,. \tag{14}$$

TABLE II. Rb⁸⁴ directional-correlation parameters.

W	é	Δε	ε'	$\Delta \epsilon'$
1.409	0.0151	0.0047	0.0216	0.0067
1.621	0.0281	0.0045	0.0281	0.0045
1.853	0.0274	0.0044	0.0209	0.0034
2.097	0.0301	0.0069	0.0185	0.0043
2.223	0.0392	0.0119	0.0221	0.0067

	R_{10}	R_{20}	R
Co ⁶⁰ (180°) Rb ⁸⁶ (180°) (+120°) (-120°)	$(-0.460\pm0.056)\%$ $(-0.134\pm0.176)\%$ $(0.143\pm0.203)\%$ $(0.176\pm0.143)\%$	$\begin{array}{c}(0.438 \pm 0.056)\%\\(0.103 \pm 0.176)\%\\(-0.108 \pm 0.203)\%\\(-0.087 \pm 0.143)\%\end{array}$	$(0.898 \pm 0.056)\% \\ (0.237 \pm 0.176)\% \\ (-0.251 \pm 0.203)\% \\ (-0.263 \pm 0.143)\%$

TABLE III. Asymmetry parameters used to calculate $P_{\gamma}(\theta)$.

 N_{i} is the normalized coincidence counting rate, and C is the normalized chance coincidence rate. The factor used to check for systematic errors is

$$R_{j0} = 2 \frac{N_j - N_0}{N_j + N_0 - 2C}; \tag{15}$$

 R_{10} should equal $-R_{20}$ within the limits of error. The absolute magnitude of the circular polarization was obtained by comparison to the Co⁶⁰ calibration in the following way:

$$P_{\gamma} = -\frac{1}{3} \frac{v}{c} \frac{R_{\rm Rb}}{R_{\rm Co}} \frac{G_{\rm Co}}{G_{\rm Rb}} \frac{E_{\rm Co}}{E_{\rm Rb}}.$$
 (16)

The factors G and E are included to correct for the geometric attenuation and the variation in analyzer efficiency with gamma-ray energy.

IV. RESULTS

Directional Correlation

The value of $\epsilon(W)$ and $\epsilon'(W)$ for Rb⁸⁶ are given in Table I and plotted in Fig. 5. The curves in the figure represent the least-squares fit to the experimental data assuming that $\epsilon(W)$ has the form given in Eq. (3). The results for α and β are

 $\alpha = 0.084 \pm 0.015, \beta = 0.020 \pm 0.012.$

The results for Rb⁸⁴ are shown in Table II and Fig. 6.

 $\alpha = 0.038 \pm 0.012$, $\beta = -0.009 \pm 0.009$.

Circular Polarization of Rb⁸⁶

The asymmetry factors calculated by Eqs. (14) and (15) are given in Table III. The data for Co⁶⁰ was taken



FIG. 7. Rb⁸⁶ beta-gamma circular polarization as a function of angle measured at an average v/c of 0.79.

from Ref. 9. By comparing R_{10} and R_{20} , it is seen that in all cases the effect is symmetric around the zero field position. The results for the circular polarization are shown in Fig. 7 where the measurements at +120 and -120° have been averaged together. The previous measurement reported by Boehm and Rogers² is also included for comparison.

V. DISCUSSION

All of the experimental results^{1,2} show that Rb⁸⁶ has an unusually large value for the directional-correlation parameter ϵ . However, there is some disagreement about the exact energy dependence of $\epsilon(W)$. The measurements of Alberghini and Steffen¹ and the measurements of Fischbeck and Wilkinson¹ agree with the energy dependence reported here. The measurements of Deutsch et al.1 and Hamilton et al.1 indicate that the parameter β in Eq. (3) is approximately zero. The fact that ϵ is very large is important, but the slight disagreement about the energy dependence is not critical for a determination of nuclear matrix elements. Also, the results of Boehm and Rogers² and the results reported here for the circular polarization show that the angular dependence of $P_{\gamma}(\theta)$ does not follow $\cos\theta$. A_3 must be of the same order as A_1 and must have the opposite sign. Thus, it is clear that an unusual combination of matrix elements is responsible for the beta decay of Rb⁸⁶. An analysis of the data to determine the matrix elements is given in the following paper.

Rb⁸⁴ has very different characteristics. The directional-correlation parameter ϵ is small; and although the circular polarization measurement of Boehm and Rogers² indicates that there is a $P_3(\theta)$ term in $P_{\gamma}(\theta)$, the evidence is not nearly as clear as in the case of Rb⁸⁶. On the basis of these superficial considerations, one would expect that Rb⁸⁴ could be interpreted on the basis of the ξ approximation. However, the analysis in the following paper shows that the matrix elements do not agree with the ξ approximation.

Thus it is clear that it is difficult to draw any conclusions from a superficial examination of the data. The detailed calculations discussed in the following paper must be performed before the implications of the data are clear.

ACKNOWLEDGMENT

The author appreciates the assistance given by Mrs. Eunice Chang-Norton with the computer calculations.