where $p_>$ ($p_<$) is the greater (lesser) of $p$, $p'$ and $\alpha\equiv p_</p_>$. The $C_n{}^1\equiv[\sin(n+1)\chi]/\sin\chi$ obey the convenient orthogonality condition

$$\langle C_n{}^1(z)C_m{}^1(z)\rangle=\delta_{nm}. \tag{B3}$$

The integrals appearing in text involve structures proportional to both $1/q^2$ and $(1/q^2)^2$. One needs the following averages involving $1/q^2$:

$$\langle q^{-2}\rangle=p_>{}^{-2},\quad \langle p_\mu'/q^2\rangle=\tfrac{1}{2}\alpha^2p^{-2}p_\mu,$$
$$\langle p'^\mu p'^\nu/q^2\rangle=(p'/p_>)^2[\tfrac{1}{4}(1-\tfrac{1}{3}\alpha^2)\eta^{\mu\nu}+\tfrac{1}{3}\alpha^2p^\mu p^\nu p^{-2}]. \tag{B4}$$

To deduce, for example, the second identity, one writes $\langle p'^\mu/q^2\rangle=ap^\mu/p^2$ where $a=pp'\langle z/q^2\rangle$. One then makes use of Eq. (B2) along with the recursion relation

$$zC_n{}^1(z)=\tfrac{1}{2}[C_{n+1}{}^1(z)+C_{n-1}{}^1(z)],\quad C_{-1}{}^1\equiv0 \tag{B5}$$

to evaluate $a$.

The integrals needed involving $(1/q^2)^2$ are

$$\langle q^{-4}\rangle=p_>{}^{-4}(1-\alpha^2)^{-1},$$
$$\langle p_\mu'/q^4\rangle=p_\mu p^{-2}p_>{}^{-2}\alpha^2(1-\alpha^2)^{-1},$$
$$\langle p_\mu'p_\nu'/q^4\rangle=(\alpha^2/p^2)[\tfrac{1}{4}\eta_{\mu\nu}+p_\mu p_\nu p^{-2}\alpha^2(1-\alpha^2)^{-1}], \tag{B6}$$
$$\langle p_\mu'p_\nu'p_\alpha'/q^4\rangle=(p_>{}^2/p^4)\alpha^4[\tfrac{1}{6}(p^\mu\eta^{\alpha\nu}+p^\nu\eta^{\alpha\mu}+p^\alpha\eta^{\mu\nu})+\alpha^2(1-\alpha^2)^{-1}p_\mu p_\nu p_\alpha p^{-2}].$$

These may most easily be deduced by inserting expression (B2) for each factor of $1/q^2$ and again using Eq. (B5).

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# Further Evidence for Pignotti's $R$ Trajectory*

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The high-energy $K^\pm p$ and $K^\pm n$ total cross section and the $K^-+p\to\bar{K}^0+n$ charge-exchange data contain further evidence for the Regge trajectory $R$ proposed by Pignotti. The signature factor is important in fitting these data; thus there is also some support for the Regge-pole hypothesis itself.

## I. INTRODUCTION

RECENTLY, Pignotti suggested the existence of a new octet of even-signature boson Regge trajectories.[1] They are expected to lie near the $\rho$ trajectory and thus to give no $0^+$ bound states or resonances; however, they may give $2^+$, etc., resonances, and it has been suggested that the $A_2$ meson may lie on one of these trajectories.[2]

Some evidence for the $I=1$ member of this octet, called $R$, was found by Ahmadzadeh.[3] He showed that the differences between high-energy $pp$ and $np$ total cross sections, together with $n+p\to p+n$ charge-exchange data, are readily explained by using a combination of the $\rho$ and $R$ trajectories, whereas $\rho$ alone fails.[4]

The present note shows there is further evidence for $R$ in the differences of $K^\pm p$ and $K^\pm n$ total cross sections,[5] and in $K^-+p\to\bar{K}^0+n$ charge exchange.[6] Here again $\rho$ is inadequate, but the addition of $R$ explains the discrepancies in a natural way.

From a theoretical viewpoint these $KN$ and $\bar{K}N$ processes have many similarities to $NN$ and $\bar{N}N$ scattering; isospin considerations are the same and so are the Regge trajectories that one assumes to dominate forward scattering.[7] Our formalism is therefore related to that of Ahmadzadeh[3]; our arguments, however, are different. The data we consider have three new features: (a) The $KN$ and $\bar{K}N$ data are more precise[8] than the corresponding $NN$ and $\bar{N}N$ data. (b) The charge exchange, $K^-+p\to\bar{K}^0+n$, is the direct analog of $\bar{p}+p\to\bar{n}+n$ rather than the $n+p\to p+n$ case already studied

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[7] Not all the trajectories are common, of course; for example, those associated with $0^-$ or $1^+$ mesons do not affect $KN$ scattering.

[8] For example, total cross sections are more accurately known for $\bar{K}N$ than for $\bar{N}N$ scattering. See Ref. 5.

in Ref. 3. The $\rho$ and $R$ contributions therefore combine in a different way, and a different kind of test of the formalism is made. (c) $KN$ and $NN$ may be similar theoretically, but are quite independent experimentally.

## II. FORMALISM

We discuss only the scattering amplitude at zero angle. At this angle are found the most stringent constraints, which provide the clearest evidence of $R$.[9]

We assume that high-energy forward scattering in the range to be discussed is controlled by Regge poles in the crossed channel. Mandelstam has shown that there are probably branch points in the complex angular-momentum plane, which cannot be ignored asymptotically.[10] Nevertheless, there seems to be a good chance that over a wide range—perhaps up to 100 GeV or more—these branch points are not yet important and the Regge poles dominate.[11]

These Regge poles can have isospin $I=0$ or 1 and $G$ parity, $G=\pm1$. Let us denote the contribution to the $K^+p$ elastic amplitude from the Regge poles with common isospin and $G$ parity by the symbol $(I,G)$. Then the $KN$ and $\bar{K}N$ amplitudes of interest can be written as follows:

$$\begin{aligned}
A(K^++p\to K^++p)&=(0,1)+(0,-1)+(1,1)+(1,-1),\\
A(K^++n\to K^++n)&=(0,1)+(0,-1)-(1,1)-(1,-1),\\
A(K^-+p\to K^-+p)&=(0,1)-(0,-1)+(1,1)-(1,-1),\\
A(K^-+n\to K^-+n)&=(0,1)-(0,-1)-(1,1)+(1,-1),\\
A(K^++n\to K^0+p)&=2[(1,1)+(1,-1)],\\
A(K^-+p\to \bar{K}^0+n)&=2[-(1,1)+(1,-1)].
\end{aligned}\qquad(1)$$

The isospin dependence of forward scattering is thus due to Regge poles with $I=1$. The obvious candidate is $\rho$, which has $G=+1$; we shall also consider $R$, which has $G=-1$. We write their contributions to the $K^+p$ amplitude in the following form:

$$\begin{aligned}
A_\rho(0)&=B_\rho\left(\frac{s-m_K^2-m_N^2}{s_0}\right)^{\alpha_\rho}\times[1-\exp(-i\pi\alpha_\rho)]/\sin\pi\alpha_\rho,\\
A_R(0)&=B_R\left(\frac{s-m_K^2-m_N^2}{s_0}\right)^{\alpha_R}\times[1+\exp(-i\pi\alpha_R)]/\sin\pi\alpha_R.
\end{aligned}\qquad(2)$$

$\alpha_\rho$ and $\alpha_R$ are the $\rho$ and $R$ trajectories at squared momentum transfer $t=0$; $s$ is the invariant total energy squared; $s_0$ is a scaling constant which may be chosen arbitrarily; $m_K$ and $m_N$ are the kaon and nucleon rest masses. The coefficients $B_\rho$ and $B_R$ are related to the residues and are assumed real; the phase of each contribution thus comes entirely from the "signature factor" $[1\pm\exp(-i\pi\alpha)]$.

In terms of the four real parameters $\alpha_\rho$, $\alpha_R$, $B_\rho$, and $B_R$, the forward $K^-p$ charge-exchange cross section is

$$\frac{d\sigma}{dt}(K^-+p\to\bar{K}^0+n)_{t=0}=\frac{m_N^2}{4\pi k^2 s}|A_R(0)-A_\rho(0)|^2,\quad(3)$$

where $k$ is the c.m. momentum of $K$ and $N$. Using the optical theorem, we also find the total-cross-section differences.

$$\sigma_T(K^-p)-\sigma_T(K^-n)=\frac{2m_N}{k\sqrt{s}}\operatorname{Im}[A_R(0)-A_\rho(0)],\quad(4)$$

$$\sigma_T(K^+p)-\sigma_T(K^+n)=\frac{2m_N}{k\sqrt{s}}\operatorname{Im}[A_R(0)+A_\rho(0)].\quad(5)$$

## III. DISCUSSION

If $\rho$ alone accounts for these isospin-dependent effects, two predictions can be made at any energy (apart from predictions relating to energy dependence):

(i) $\sigma_T(K^-p)-\sigma_T(K^-n)=\sigma_T(K^+n)-\sigma_T(K^+p)$;

(ii) $$\frac{d\sigma}{dt}(K^-+p\to\bar{K}^0+n)_{t=0}=\frac{1}{16\pi}[\sigma_T(K^-p)-\sigma_T(K^-n)]^2\left[1+\tan^2\left(\frac{\pi\alpha_\rho}{2}\right)\right].$$

With $\alpha_\rho\approx0.5$, the value established by other experiments (e.g., Refs. 3 and 9), prediction (ii) says that the forward charge-exchange cross section is roughly twice the "optical" lower limit. Both these predictions conflict with data.

Figure 1 shows the Brookhaven data[5] for the cross-section differences; the solid curve is a least-squares fit to these 15 points with $\rho$ alone, taking $\alpha_\rho=0.5$ and optimizing $B_\rho$. $\chi^2$ for this curve is 70; half of this comes from one point at 8 GeV/$c$, but even without this point the fit to the data is bad. Allowing $\alpha_\rho$ to vary has little effect; $\chi^2$ drops to 69. So prediction (i) fails.

[9] We have also made Regge-pole fits to the data at other angles, in the range $0\leq|t|<1$ (GeV/$c$)$^2$, for the $\pi N, KN, \bar{K}N, NN$, and $\bar{N}N$ systems.

[10] S. Mandelstam, Nuovo Cimento **30**, 1127 and 1148 (1963).

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Figure 2 shows the CERN/Zurich $K^- + p \to \bar{K}^0 + n$ data at 9.5 GeV/$c$,[6] which indicate a forward cross section near 200 $\mu$b/(GeV/$c$)$^2$. At this incident momentum, the Brookhaven data[5] indicate $\sigma_T(K^-p) - \sigma_T(K^-n) = 2.0 \pm 0.4$ mb, implying an optical lower limit of $210 \pm 80$ $\mu$b/(GeV/$c$)$^2$. Prediction (ii) states that the forward cross section should be *twice* the optical limit; it is therefore unsatisfactory. The experimental uncertainty is rather large, admittedly, and this particular point is not conclusive by itself. However, it supports our previous conclusion that $\rho$ alone is inadequate.

The addition of $R$, however, simultaneously removes both contradictions. If we take $B_R \approx B_\rho$ and $\alpha_R \approx \alpha_\rho$, the $\rho$ and $R$ contributions to Eq. (4) tend to reinforce, while those to Eq. (5) tend to cancel, in agreement with experiment. These relations involve the imaginary part of the amplitude only. At the same time, because $\rho$ and $R$ have opposite signature factors, the real parts in Eq. (3) tend to cancel, while the imaginary parts add. Thus the forward charge-exchange cross section should be close to the optical limit. In fact, the $\rho$ and $R$ contributions do not have exactly the same energy dependence, but the argument above remains qualitatively true. For a quantitative argument, we make a fit to the data.

The dashed lines in Fig. 1 show the least-squares fits with $\rho$ and $R$ together, fixing $\alpha_\rho = 0.5$ and $\alpha_R = 0.3$ (the latter value suggested by Ahmadzadeh). $\chi^2$ is now only 18, a reasonable value. Allowing $\alpha_\rho$ and $\alpha_R$ to vary has little effect. The forward charge-exchange cross section corresponding to this fit, at 9.5 GeV/$c$, is 245 $\mu$b/(GeV/$c$)$^2$, with an uncertainty of some 10 to 20%. Note that this value is based on the total-cross-section differences alone. It is in reasonable agreement with the data (Fig. 2).

Our arguments rely on the high precision of the data, of course. If there were a systematic increase of both $\sigma_T(K^+n)$ and $\sigma_T(K^-n)$ by about 0.7 mb, the difficulties with predictions (i) and (ii) would vanish: But we know of no reason for such a correction. It would require a 50% increase in the Glauber "shadow" term to produce this effect. In applying the Glauber formula, the real parts of the forward-scattering amplitudes are ignored in Ref. 5. For $\pi N$ scattering they are known to be negligible; if, for $KN$ and $\bar{K}N$ scattering, they are not negligible, the effect will be to reduce rather than increase the shadow term.

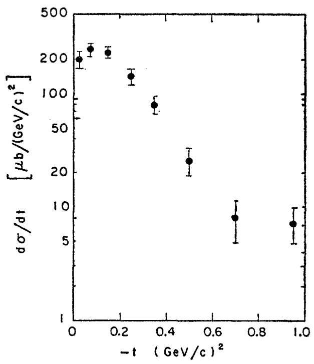


FIG. 2. The CERN/Zurich $K^+ + p \to K^0 + n$ differential cross section at 9.5 GeV/$c$.

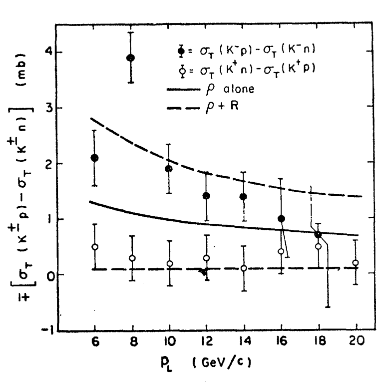


FIG. 1. Total cross-section differences of Ref. 5. The solid curve is the fit to both sets of data together, assuming $\rho$ alone. The dashed curves are the fits in the two sets of data separately, assuming $\rho$ plus $R$.

It is interesting to compare the roles of $\rho$ and $R$ in $K^- + p \to \bar{K}^0 + n$ and in the other charge-exchange process $K^+ + n \to K^0 + p$. In the former case the real parts tend to cancel, but in the latter their relative sign is reversed, and it is the imaginary parts that tend to cancel. So the $\rho + R$ model predicts that the forward $K^+ + n \to K^0 + p$ cross section greatly exceeds the optical limit at the energies we have been considering.

The cases of $NN$ and $\bar{N}N$ charge exchange are analogous, as we have already remarked. In a $\rho + R$ model, only the spin-independent amplitude enters at $t = 0$. The process $p + n \to n + p$ studied by Ahmadzadeh[3] is analogous to $K^+ + n \to K^0 + p$; the imaginary parts tend to cancel and the real parts dominate. For $\bar{p} + p \to \bar{n} + n$, the real parts tend to cancel, and the imaginary parts dominate.

Note that in the Regge-pole formalism, with its complex signature factors, an amplitude can change from being mostly real to mostly imaginary when one of the contributions changes sign. This kind of effect is certainly suggested by the $K^-p$ and $np$ charge-exchange data.[12] Other well-known mechanisms do not give such an effect in any simple way: elementary-particle exchange gives an essentially real amplitude, even with initial- and final-state absorption; direct absorption gives an essentially imaginary amplitude.

The importance of the signature factors in fitting the isospin dependence of $KN$ and $\bar{K}N$ data, and in reconciling the apparent differences between $K^-p$ and $np$ charge exchange, give some empirical support to the Regge-pole hypothesis.

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[12] It will be interesting to see whether reactions $K^+ + n \to K^0 + p$ and $\bar{p} + p \to \bar{n} + n$ confirm this effect.