Spontaneous Symmetry Breakdown and the μ-e-γ Interaction

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Spontaneous breakdowns of symmetries have been examined for a system of two charged fields of zero bare mass (the "muon" and "electron" fields) interacting minimally with the electromagnetic field. Upon arranging the two fields into an "isotopic" doublet, the Lagrangian is seen to possess $SU(2)$ symmetry. Three possibilities are available: (a) no spontaneous breakdown of the $SU(2)$ symmetry is allowed and the muon-electron system remains a degenerate doublet; (b) a partial breakdown occurs in which a mass splitting develops but the heavier muon remains stable; (c) a complete breakdown occurs in which the muon decays into an electron plus a photon. Using the high-energy scheme of Baker, Johnson, and Willey, approximate solutions for the one-fermion Green's function and vertex function are examined. (The approximation scheme has the advantage that no *ad hoc* cutoffs need be invoked.) The solutions obtained permit case (b) to occur but not case (c), provided improper Lorentz invariance is imposed. It is shown, at least for the one-fermion Green's function, that no solutions breaking P, C, or T invariance can arise.

I. INTRODUCTION

ONE of the more remarkable features of particle and resonance phenomena is the large number of conservation laws that appear to govern the interactions. Aside from the conserved quantities arising from the space-time invariances (i.e., energy, momentum, angular momentum), perhaps only charge conservation has at present a reasonably fundamental theoretical basis. Further, with the exception of heavy particle number, the remaining quantities are not exactly conserved, breakdowns occurring in varying degrees. The "classical" way of accounting for a breakdown in conservation laws is, of course, to assume that in addition to the part invariant under the corresponding symmetry group, the Hamiltonian has a (presumably small) noninvariant perturbing term. Indeed, there exists experimental evidence that such an approach is at least phenomenologically correct in a number of cases. Thus, conservation of total isotopic spin and of strangeness is violated by the order of electromagnetic and weak interactions, respectively. On the other hand, some symmetries appear to be broken without any obvious dynamical agency being present. For example, consider interchange of muon and electron fields (with corresponding interchange of their neutrinos). As far as is known, the entire Lagrangian is invariant under this transformation except for the mass terms. However, the mass splitting is a hundred times larger than any known dynamical interaction in which leptons participate. It has also recently been suggested[1] that another example of "nondynamical" symmetry breakdown might be the loss of $SU(3)$ symmetry.

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[1] S. L. Glashow, Phys. Rev. **130**, 2132 (1963).

Some time ago, Nambu and Goldstone[2] pointed out that a given symmetry might indeed break down spontaneously (i.e., without introducing a dynamical perturbation) if solutions to the field theory could be found with a nonsymmetric vacuum state. A number of field theories, exhibiting this phenomenon in approximate solutions, have been examined. More recently, Baker and Glashow have suggested that the muon-electron mass splitting might arise in this fashion.[3]

The purpose of this paper is to investigate the possibility of such spontaneous breakdowns in the electromagnetic interactions of the muon-electron system.[4] If the muon-electron mass degeneracy is lifted, the decay

$$\mu^{\pm} \to e^{\pm} + \gamma \tag{1.1}$$

is, of course, feasible energetically. As is well known, this decay cannot proceed via weak interactions owing to the existence of two neutrinos, i.e., weak interactions conserve μ number and e number separately. Experimentally, the branching ratio for this mode is now less than about[5] 2×10^{-8}. The purely electromagnetic interaction between the electron, muon, and photon also gives rise to separately conserved μ and e currents. Thus, the only way reaction (1.1) could arise electrodynamically would be if a spontaneous breakdown of the muon-electron symmetry could occur. In Sec. II, possible types

[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961); J. Goldstone, Nuovo Cimento **19**, 154 (1961).
[3] M. Baker and S. L. Glashow, Phys. Rev. **128**, 2462 (1962). The mass splitting problem is discussed by a perturbation analysis.
[4] The possibility that massless-boson modes may be associated with such spontaneous breakdowns is not discussed here. It would seem to be an open question at present, with some evidence that in gauge theories such modes do not develop: See P. W. Higgs, Phys. Letters **12**, 132 (1964); and M. Baker, K. Johnson, and B. W. Lee, Phys. Rev. **133**, B209 (1964).
[5] S. Parker, H. L. Anderson, and C. Rey, Phys. Rev. **133**, B1768 (1964).

of electromagnetic breakdowns are analyzed. It is seen that three *a priori* possibilities can occur: (a) the muon-electron system is completely degenerate; (b) a mass splitting can exist but decay (1.1) is forbidden; (c) both a mass splitting and decay (1.1) occur. These possibilities correspond to increasingly asymmetric vacuum states. Clearly case (b) is what exists in nature.

In Sec. III, an approximate solution of the one-fermion Green's function is discussed. The approximation used is the high-energy scheme of Baker, Johnson, and Willey.[6] This scheme has the advantage of leading to finite results and thus not requiring *ad hoc* cutoffs. If one limits the analysis to those solutions possessing improper Lorentz covariance, it is seen that case (c) is forbidden, while case (b) can indeed occur. Actually, as is shown in Appendix A, all solutions automatically preserve[6a] P, C, and T. In Sec. IV, a higher approximation involving a spontaneous symmetry breakdown occurring first in the vertex function is examined. Again improper Lorentz covariance appears to preclude possibility (c) while still allowing possibility (b). To within the validity of the approximation scheme used, then, the spontaneous breakdown idea seems capable of accounting for the existing facts in the muon-electron system.

II. INVARIANCE CONDITIONS ON GREEN'S FUNCTIONS

In this section we investigate the conditions imposed upon the muon-electron-photon Green's functions by the different invariances of the Lagrangian. Denoting the "muon" and "electron" fields by $\psi_1(x)$ and $\psi_2(x)$, respectively, the Lagrangian with minimal electromagnetic coupling reads[7]

$$\mathcal{L}(x)=-\bar{\psi}_1(x)\left(\gamma^\mu\frac{1}{i}\partial_\mu+m_0\right)\psi_1(x)+e_0\bar{\psi}_1(x)\gamma^\mu\psi_1(x)A_\mu(x)-\bar{\psi}_2(x)\left(\gamma^\mu\frac{1}{i}\partial_\mu+m_0\right)\times\psi_2(x)+e_0\bar{\psi}_2(x)\gamma^\mu\psi_2(x)A_\mu(x)+L_M, \quad (2.1)$$

where $A_\mu(x)$ is the electromagnetic potential and L_M is the free Maxwell Lagrangian. We have taken m_0 and e_0, the bare mass and charge, to be the same for the electron and muon, so that the Lagrangian possesse the maximum symmetry.

Owing to the fact that the electron and muon do not interact directly but only via the photon field,[8] there exist separate phase invariances for the electron and muon fields:

$$\begin{aligned}\psi_1&\to e^{i\epsilon_1}\psi_1, & \psi_2&\to\psi_2,\\ \psi_1&\to\psi_1, & \psi_2&\to e^{i\epsilon_2}\psi_2.\end{aligned} \quad (2.2)$$

Consequently, the electron and muon currents, $J_1(x)$ and $J_2(x)$ [$J_i(x)\equiv\bar{\psi}_i\gamma^\mu\psi_i$], are *separately* conserved and a decay such as (1.1) can only occur if there is a spontaneous breakdown of these phase symmetries. To facilitate the discussion, let us introduce the "isotopic" notation $\psi(x)\equiv(\psi_1(x),\psi_2(x))$; then $\mathcal{L}$ takes the form

$$\mathcal{L}=-\bar{\psi}(x)\left(\gamma^\mu\frac{1}{i}\partial_\mu+m_0\right)\psi(x)+e_0\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)+L_M \quad (2.3)$$

showing that $\mathcal{L}$ is invariant under the symmetry group $U(2)$ of arbitrary unitary rotations in the "isotopic" space:

$$\begin{aligned}\psi(x)&\to e^{i\epsilon_0}\exp(i\boldsymbol{\epsilon}\cdot\boldsymbol{\tau})\psi(x);\\ \bar{\psi}(x)&\to\bar{\psi}(x)\exp(-i\boldsymbol{\epsilon}\cdot\boldsymbol{\tau})e^{-i\epsilon_0}.\end{aligned} \quad (2.4)$$

Here ϵ_0 and ϵ_m ($m=1, 2, 3$) are arbitrary real constants and the τ_m are the usual 2×2 Pauli matrices. The full group $U(2)$ contains the factor $U(1)$, the one-dimensional group of phase transformations $\exp(i\epsilon_0)$. Invariance under $U(1)$ gives rise to conservation of *total* electromagnetic charge (i.e., the sum of μ and e number). The relative properties of the muon and electron are thus governed by $SU(2)$, and we will restrict our discussion to this group (as we do not wish to consider breakdowns of charge conservation).

We begin by considering the one-fermion propagator, $G_{ij}(x-x')$, defined by

$$G_{ij}(x-x')\equiv i\langle0|T[\psi_i(x)\bar{\psi}_j(x')]|0\rangle, \quad (2.5)$$

where $|0\rangle$ is the physical vacuum state and T represents the usual fermion time-ordering operation.

It is convenient to view the G_{ij} of Eq. (2.5) as a 2×2 isotopic matrix. It can then be written in terms of Pauli matrices as

$$G(x-x')=G_0(x-x')\mathbf{1}+\mathbf{G}(x-x')\cdot\boldsymbol{\tau}, \quad (2.6)$$

where G_0 and G_m are, respectively, the isotopic scalar and isotopic vector form factors. Spontaneous symmetry breakdown occurs when the vacuum state does not share in the operator invariances of the Lagrangian

[6] M. Baker, K. Johnson, and R. Willey, Phys. Rev. Letters **11**, 518 (1963); Phys. Rev. **136**, B209 (1964). See also lectures by K. Johnson, Brandeis Summer Institute, 1964 (Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1965). This work will be referred to as BJW in text.

[6a] A previous statement by the authors that parity-breaking terms could also arise [Phys. Letters **13**, 256 (1964)] is incorrect, as such terms can actually be rotated away by means of Eq. (A.10). We are grateful to Dr. Th. A. J. Maris, Dr. V. E. Herscovitz, and Dr. G. Jacob for bringing this to our attention.

[7] Units such that $\hbar=1=c$ are used. Greek indices run from 0 to 3, $x^0=ct$, Latin indices over 1, 2, 3. The Dirac matrices are defined by the anticommutation relations $\{\gamma^\mu,\gamma^\nu\}=-2\eta^{\mu\nu}$, where $\eta^{\mu\nu}$ is the Lorentz metric with signature $(-1, +1, +1, +1)$. The symbol $\bar{\psi}(x)$ means $\psi^\dagger(x)\gamma^0$ where "†" means Hermitian conjugate and $\gamma_5\equiv\gamma^0\gamma^1\gamma^2\gamma^3(\gamma_5{}^\dagger=-\gamma_5)$.

[8] As shown by G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Letters **3**, 524 (1959), and N. Cabbibo and R. Gatto, Phys. Rev. **116**, 1334 (1959), any gauge-invariant off-diagonal terms that might be added to $\mathcal{L}$ (e.g., $\bar{\psi}_1(\gamma^\mu\partial_\mu-ieA_\mu)\psi_2$) can always be eliminated by taking new linear combinations of the fermion fields which reduce the Lagrangian again to diagonal form. Thus (2.1) represents the most general minimally coupled Lagrangian.

so that vacuum expectation values of operators will not necessarily be invariant under (2.4). Let U_m be the unitary transformations generating the group transformations of $SU(2)$:

$$U_m\psi(x)U_m^{-1}=e^{i\epsilon_m\tau_m}\psi(x)\,,\qquad U_m\bar{\psi}(x)U_m^{-1}=\bar{\psi}(x)e^{-i\epsilon_m\tau_m}\,. \tag{2.7}$$

Inserting factors of $U_m^{-1}U_m$ between all the operators of Eq. (2.2) leads to the identity

$$G(x-x')=e^{i\epsilon_m\tau_m}i\langle 0|U_m^{-1}T(\psi(x)\bar{\psi}(x'))\times U_m|0\rangle e^{-i\epsilon_m\tau_m},\quad m=1,2,3. \tag{2.8}$$

The degree of invariance of G depends upon the effect of U_m on the vacuum. Three possibilities are available: (a) The vacuum is symmetric under the entire group $SU(2)$. (b) The vacuum is symmetric only under a subgroup of $SU(2)$. For $SU(2)$, the only subgroups are the one-dimensional rotation groups. Without loss of generality, we may call the preferred symmetry axis $m=3$ and consider only the subgroup of rotations around this axis. (c) The vacuum is not symmetric under the *entire* $SU(2)$ group. We begin with possibility (a) which implies (with the conventional choice of phase factor)

$$U_m|0\rangle=|0\rangle,\quad m=1,2,3\,. \tag{2.9}$$

Equation (2.8) then reduces to

$$[\tau_m,G]=0\,,\quad m=1,2,3 \tag{2.10}$$

for infinitesimal ϵ_m. In terms of the notation of Eq. (2.6), this implies that the entire isotopic vector form factor vanishes, $G_m\equiv 0$, and hence

$$G_{ij}(x)=G_0(x)\delta_{ij}\,. \tag{2.11}$$

Thus the electron and muon Green's functions are identical and a completely degenerate doublet remains. This is the familiar situation when no symmetry breakdown occurs. Case (b) corresponds to Eq. (2.9) [and consequently Eq. (2.10)] holding only for $m=3$. One finds now that only the $m=1,2$ isotopic vector form factors vanish. Thus,

$$G_{ij}(x)=G_0(x)\delta_{ij}+G_3(x)(\tau_3)_{ij}\,. \tag{2.12}$$

Choosing the conventional diagonal representation for τ_3, one has

$$G_{11}=G_0+G_3\,,\quad G_{22}=G_0-G_3\,,\quad G_{12}=0=G_{21}\,. \tag{2.13}$$

The electron and muon Green's functions are now distinct, as would be the case if a mass splitting were to occur. However, since a representation can be found where $G_{ij}(x)$ is diagonal for *all* x, one would expect that no mixing could occur between the two particles, and hence decay process (1.1) is still forbidden. This will be borne out below.

For the final case (c), Eq. (2.9) no longer holds at all, and consequently the group symmetry of the Lagrangian produces no *a priori* conditions on G_{ij}. In general, then $G_{11}\neq G_{22}$ and G_{12}, $G_{21}\neq 0$ here. We will see later that this situation allows both the mass splitting as well as the decay to occur. However, when G_{ij} has off-diagonal matrix elements, it is no longer proper to interpret G_{11} as the electron propagator and G_{22} as the muon propagator. Rather, the electron and the muon are the two poles of the matrix G in momentum space (the electron, by definition, being the lighter particle). The free particle muon and electron spinors are thus determined by solving the eigenvalue equation $G^{-1}(p)u(p)=0$, where $G^{-1}(p)$ is the matrix inverse of $G_{ij}(p)$ and $G(p)$ is the Fourier transform of $G(x)$:

$$G(p)\equiv\int d^4x\, e^{-ipx}G(x)\,. \tag{2.14}$$

For simplicity, consider the case where parity invariance is maintained (the more general discussion is given in Appendix A). One may then write $G^{-1}(p)\equiv\gamma p h(p^2)+k(p^2)$, where h and k are isotopic matrices and space-time scalars. The spinor $u(p)$ may be factored into a product of a Dirac spinor $u_D(p)$ [obeying $(\gamma p+m)\times u_D(p)=0$] and an isotopic spinor v. Then v obeys the relation

$$[-mh(-m^2)+k(-m^2)]v=0\,. \tag{2.15}$$

Writing $h^{-1}k=a+\mathbf{b}\cdot\boldsymbol{\tau}$, one expects two solutions $v_{(\pm)}$ of Eq. (2.15) to exist,[9] defined by the solutions of

$$[b(-m_\pm^2)\mp\mathbf{b}(-m_\pm^2)\cdot\boldsymbol{\tau}]v_{(\pm)}=0\,. \tag{2.16}$$

The mass eigenvalues $m_\pm$ are given by

$$m_\pm=a(-m_\pm^2)\pm b(-m_\pm^2),\quad b\equiv(\mathbf{b}^2)^{1/2}\,. \tag{2.17}$$

Note that in the limit when G_{ij} is diagonal [cases (a) and (b) above], i.e., when for *all* momenta one can choose $b_2=0=b_1$, $v_{(+)}$ and $v_{(-)}$, respectively, reduce to the orthogonal isotopic spin-up and spin-down functions appropriate for muon and electron. [$v_{(+)}$ is the muon spinor since $m_+>m_-$.] For the general situation, however, $v_{(+)}$ and $v_{(-)}$ are not expected to be orthogonal since they represent two null eigenvectors of $G^{-1}(p)$ at two different momenta (i.e., at $p^2=-m_+^2$ and $p^2=-m_-^2$).

The above analysis can be extended to the higher Green's functions. We consider briefly here the muon-electron-photon vertex function $\Gamma^\mu{}_{ij}(x,x';\xi)$ which may be defined by

$$-e_0\int G_{ik}(x-x'')\Gamma^\nu{}_{kl}(x'',x''';\xi')G_{lj}(x'''-x')\times D_\nu{}^\mu(\xi'-\xi)=\langle 0|T[\psi_i(x)\bar{\psi}_j(x')A^\mu(\xi)]|0\rangle\,. \tag{2.18}$$

In Eq. (2.18) the function $D_\nu{}^\mu(\xi-\xi')$ is the one-photon

[9] We are assuming that the transcendental equation (2.17) has only one physically acceptable solution for m_+ and one for m_-.

propagator. In zeroth approximation (neglecting closed loops) it is given by

$$D_\mu{}^{\nu(0)}(k)=[\delta_\mu{}^\nu-\lambda_\mu(k)k^\nu-\lambda_\nu(k)k^\mu][k^2-i\epsilon]^{-1}, \quad (2.19)$$

where

$$D_\mu{}^\nu(k)=\int d^4\xi\, e^{-ik\xi}D_\mu{}^\nu(\xi) \quad (2.20)$$

and $\lambda_\mu(k)$ is an arbitrary gauge function. The isotopic structure of $\Gamma^\mu{}_{ij}$ also depends on the three possible degrees of symmetry of the vacuum state. Thus, for the totally symmetric vacuum of case (a) above, the three-point function $\langle 0|T(\psi_i\bar{\psi}_jA^\mu)|0\rangle$ must be proportional to δ_{ij} and hence $\Gamma^\mu{}_{ij}=\Gamma^\mu{}_{(0)}(xx'\xi)\delta_{ij}$. For case (b), again choosing the invariance axis to be $m=3$, one has $\Gamma^\mu{}_{ij}=\Gamma^\mu{}_{(0)}\delta_{ij}+\Gamma^\mu{}_{(3)}(\tau_3)_{ij}$. Finally, for case (c), where the vacuum possesses no symmetry, $\Gamma^\mu{}_{ij}$ can take on the general off-diagonal form $\Gamma^\mu{}_{ij}=\Gamma^\mu{}_{(0)}\delta_{ij}+\mathbf{\Gamma}^\mu\cdot(\boldsymbol{\tau})_{ij}$. It is convenient to Fourier analyze the vertex function according to

$$\Gamma^\mu{}_{ij}(x,x',\xi)$$

$$=(2\pi)^{-8}\int d^4p d^4p' e^{ip(x-\xi)}e^{-ip'(x'-\xi)}\Gamma^\mu{}_{ij}(p,p'). \quad (2.21)$$

The μ-e-γ decay amplitude is then proportional to

$$\bar{u}_{(-)}(p)\Gamma^\mu(p,p')u_{(+)}(p')e_\mu(p-p'), \quad (2.22)$$

where $e_\mu(k)$ is the polarization vector of the emitted photon. Expression (2.22) vanishes for cases (a) or (b) since then $v_{(-)}{}^\dagger v_{(+)}=0=v_{(-)}{}^\dagger\tau_3 v_{(+)}$. A nonzero decay probability can thus occur only for the completely asymmetric vacuum [case (c)] and provided the vertex function does not have a zero for p' on the muon mass shell or p on the electron mass shell.[10]

Aside from the $U(2)$ internal symmetry discussed above, the Lagrangian of Eq. (2.1) possesses other symmetries. In the approximation scheme to be used in the next section, it is essential that the bare mass m_0 be set to zero. As discussed by Baker, Johnson, and Willey (BJW),[6] the Lagrangian then possesses invariance under the γ_5 transformation:

$$\psi(x)\to e^{+\epsilon\gamma_5}\psi(x), \quad \bar{\psi}(x)\to\bar{\psi}(x)e^{+\epsilon\gamma_5} \quad (2.23)$$

where $\gamma_5\equiv\gamma^0\gamma^1\gamma^2\gamma^3$ and $\gamma_5{}^\dagger=-\gamma_5$. Writing $G_{ij}(p)$ in terms of spatial vector and scalar form factors,

$$G_{ij}(p)=\gamma p g_{ij}(p^2)+f_{ij}(p^2), \quad (2.24)$$

one sees that the condition that the vacuum be γ_5 invariant reads[6]

$$\{\gamma_5,G_{ij}(p)\}=0, \quad (2.25)$$

and so f_{ij} must vanish. This result would imply that the muon and electron have zero clothed mass. As pointed out by BJW, it is the possibility of a spontaneous breakdown of this invariance that allows the fermions to grow a mass in the first place, though as seen above, a further breakdown in the $SU(2)$ symmetry is needed for a mass difference to develop.

We conclude this section by summarizing for future reference the properties of the fermion propagator obtained from the spectral representation. We write

$$G_{ij}(x-x')=i[W_{ij}(x-x')\theta(x-x') \\ -\bar{W}_{ji}(x'-x)\theta(x'-x)], \quad (2.26)$$

where $\theta(x)$ is the step function [$\theta(x)=1$ for $x^0>0$ and zero otherwise] and W and $\bar{W}$ are the Wightman functions

$$W_{ij}(x-x')\equiv\langle 0|\psi_i(x)\bar{\psi}_j(x')|0\rangle, \\ \bar{W}_{ji}(x'-x)\equiv\langle 0|\bar{\psi}_j(x')\psi_i(x)|0\rangle. \quad (2.27)$$

Following the standard arguments,[11] the assumptions of positiveness of the energy spectrum and proper Lorentz covariance leads to the spectral representation for W_{ij}:

$$W_{ij}(x-x')$$

$$\equiv-(2\pi)^{-3}\int d^4p\, e^{ip(x-x')}\theta(-p^2)\theta(p^0)\rho_{ij}(p^2), \quad (2.28)$$

where ρ_{ij} is given by

$$\rho_{i\alpha,j\beta}(p_n)=-(2\pi)^3\sum\nolimits_{n,m,p_n}\langle 0|\psi_{i\alpha}(0)|p_n,n\rangle \\ \times g^{nm}\langle p_{n,m}|\bar{\psi}_{j\beta}(0)|0\rangle, \quad p_n{}^2<0. \quad (2.29)$$

Here α and β are the Dirac spinor indices. We assume that the Hilbert space metric, g^{nm}, is Hermitian ($g^{nm*}=g^{mn}$) and diagonal in the energy-momentum variables $p_n{}^\mu$. Proper Lorentz covariance then implies that

$$\rho_{ij}(p)=[f_{ij}(-p^2)+\bar{f}_{ij}(-p^2)\gamma_5] \\ +\gamma p[g_{ij}(-p^2)+i\gamma_5\bar{g}_{ij}(-p^2)], \quad (2.30)$$

where the form factors f, $\bar{f}$, g, and $\bar{g}$ are scalar functions of p^2. Equation (2.29) requires that ρ obeys the reality condition

$$\rho_{ij}{}^*=(\gamma^0\rho_{ji}\gamma^0)^\sim, \quad (2.31)$$

where the tilde means transpose in the Dirac spinor space. From this one sees that the form factors are all *Hermitian* isotopic matrices, i.e.,

$$f_{ij}{}^*=f_{ji}, \quad \bar{f}_{ij}{}^*=\bar{f}_{ji}, \text{ etc.} \quad (2.32)$$

The further assumption of local commutativity allows one, as usual,[12] to relate $\bar{W}_{ji}$ to W_{ij} and so to obtain

[10] Accidental cancellations might also cause the vanishing of expression (2.22). As mentioned above, $v_{(+)}$ and $v_{(-)}$ need no longer be orthogonal for case (c). Thus the isotopic diagonal part of $\Gamma^\mu{}_{ij}$ contributes to Eq. (2.22) and could conceivably cancel the off-diagonal parts.

[11] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, and Company, New York, 1961), p. 672.

[12] See Ref. 11, p. 734.

the Lehmann representation for the Green's function G_{ij}:

$$G_{ij}(x-x')=(2\pi)^{-4}\int_0^\infty d\kappa^2\{[f_{ij}(\kappa^2)+\gamma_5\bar{f}_{ij}(\kappa^2)] +\gamma p[g_{ij}(\kappa^2)+i\gamma_5\bar{g}_{ij}(\kappa^2)]\} \times e^{ip(x-x')}[p^2+\kappa^2-i\epsilon]^{-1}. \quad (2.33)$$

Invariance under the discrete operations of charge conjugation, and space and time reflection also imposes conditions on the spectral functions. Thus, if the vacuum is invariant under spatial reflections, one finds that G_{ij} obeys

$$G_{ij}(\mathbf{p},p^0)=\gamma^0 G_{ij}(-\mathbf{p},\,p^0)\gamma^0, \quad (2.34)$$

and hence the form factors that are coefficients of γ_5, $\bar{f}_{ij}$, and $\bar{g}_{ij}$ vanish. Invariance under time reflection implies

$$G_{ij}(\mathbf{p},p^0)=[C\gamma_5\gamma^0 G_{ji}(-\mathbf{p},\,p^0)\gamma_5\gamma^0 C^{-1}]^\sim, \quad (2.35)$$

where C is the charge conjugation matrix $[C\gamma^\mu C^{-1}=-\tilde{\gamma}^\mu,\ C^+=C^{-1},\ \tilde{C}=-C]$. Condition (2.35) states that f_{ij}, g_{ij} and $\bar{g}_{ij}$ are symmetric isotopic matrices (and hence real), while $\bar{f}_{ij}$ is antisymmetric (and hence pure imaginary). Finally, invariance under charge conjugation yields

$$G_{ij}(p)=[CG_{ji}(-p)C^{-1}]^\sim \quad (2.36)$$

which implies that f_{ij}, $\bar{f}_{ij}$, and g_{ij} are symmetric, while $\bar{g}_{ij}$ is antisymmetric.[13] Finally we note that under charge conjugation invariance, the vertex function obeys the equation

$$\Gamma^\mu{}_{ij}(p,p')=-[C\Gamma^\mu{}_{ji}(-p',\,-p)C^{-1}]^\sim. \quad (2.37)$$

III. APPROXIMATE SOLUTIONS FOR G_{ij}

In the previous section, a discussion was given of the symmetries that might possibly undergo spontaneous breakdown for the muon-electron system. Whether or not a given breakdown actually occurs depends, of course, on the dynamical equations of the theory, e.g., the coupled Green's function equations. In this section we given an approximate solution of these equations for the one-fermion Green's function G_{ij}. The approximation we will use is the high-energy scheme of Baker, Johnson, and Willey.[6] This method has the advantage of giving finite answers at each stage of approximation for all quantities, at least when closed loop vacuum polarization effects are neglected.[14] We shall deal here with the first (BJW) approximation for G_{ij}. (The corresponding approximation for the vertex function will be examined in the next section.) Assuming the convergence of the successive approximations, the first approximation should give the rigorous *form* for G_{ij} in momentum space as $p^2\to\infty$. It is as good as second-order perturbation theory for $G^{-1}{}_{ij}(p)$ in the vicinity of poles of G_{ij}. The accuracy of the approximation is, however, unclear in the vicinity of the origin in momentum space. Unfortunately, it is necessary to impose a boundary condition on $G_{ij}(p)$ at $p^2=0$ and so the structure of the solution there does enter into our considerations. We will see, though, that the boundary condition itself, at least, is actually a property of the rigorous G_{ij} and not just an accident of the approximation used.

[13] One sees that G_{ij} is automatically invariant under the product TCP (even if not under T, C, and P separately) since this invariance is already implied in the Lehmann representation [Eq. (2.9)] for the Green's function with Feynman boundary conditions. The proof of invariance uses, of course, only proper Lorentz invariance in its derivation.

[14] It is possible that the vacuum polarization effects are also finite when four-sided closed loops are included in the calculations. See K. Johnson, Ref. 6.

We begin with a brief summary of the (BJW) scheme and its extension to our system. The Schwinger-Dyson equation for $G_{ij}(p)$ reads

$$G^{-1}{}_{ij}(p)=(\gamma p+m_0)\delta_{ij} +\frac{ie_0^2}{(2\pi)^4}\int d^4p'\gamma^\mu G_{im}(p')\Gamma^\nu{}_{mj}(p',p)D_{\nu\mu}(p-p'), \quad (3.1)$$

where $G_{ij}^{-1}(p)$ is the Dirac and isotopic matrix inverse of $G_{ij}(p)$. If, indeed, the integral on the right-hand side of Eq. (3.1) were convergent, one might hope that it would vanish in the limit $p^2\to\infty$. Then $G(p)$ would have the following asymptotic form:

$$G^{-1}{}_{ij}(p)\sim(\gamma p+m_0)\delta_{ij}, \quad (3.2)$$

a result which also is expected from the Lehmann representation (2.33) and the canonical commutation rules. In general, $D_{\mu\nu}(k)$ has the form

$$D_{\mu\nu}(k)=[\eta_{\mu\nu}-\lambda_\mu(k)k_\nu-\lambda_\nu(k)k_\mu]D(k^2), \quad (3.3)$$

where $\lambda_\mu(k)$ is an arbitrary gauge function. If again, vacuum polarization effects were finite, the gauge-invariant function $D(k^2)$ should behave, for large k^2, as

$$D(k^2)\sim 1/k^2. \quad (3.4)$$

The convergence of the integral in Eq. (3.1) is governed by these asymptotic forms for $G_{ij}(p')$ and $D_{\mu\nu}(p'-p)$ and also the one for $\Gamma^\mu{}_{ij}(p',p)$. Integral equations for the vertex function can be obtained conveniently by the device of introducing an external current $\mathbf{J}^\mu(x)$, the Green's functions then becoming functionals of $\mathbf{J}^\mu(x)$. The left-hand side of Eq. (2.18) is then just

$$-[\delta G_{ij}(x,x')/\delta\mathbf{J}_\mu(\xi)]_{\mathbf{J}^\mu\to 0}. \quad (3.5)$$

Carrying out the indicated differentiation of $G_{ij}(xx')$, one can obtain a series expression for $\Gamma^\mu{}_{ij}(p',p)$:

$$\Gamma^\mu(p',p)=\gamma^\mu-\frac{ie_0^2}{(2\pi)^4}\int d^4p''\gamma^\alpha \times G(p''-p+p')\Gamma^\mu(p''-p+p',\,p'') \times G(p'')\Gamma^\beta(p'',p)D_{\alpha\beta}(p-p'')+\cdots. \quad (3.6)$$

A successive iteration of this equation (i.e., first replace Γ^μ in the integrals by γ^μ, etc.) gives rise to an alternate series for $\Gamma^\mu{}_{ij}$:

$$\Gamma^\mu(p',p)=\gamma^\mu-\frac{ie_0^2}{(2\pi)^4}\int d^4p''\gamma^\alpha \times G(p''-p+p')\gamma^\mu G(p'')\gamma^\beta D_{\alpha\beta}(p-p'')+\cdots. \quad (3.7)$$

The usefulness of Eq. (3.7) resides in the fact that it expresses Γ^μ as a functional of the rigorous one-particle Green's functions G_{ij} and $D_{\alpha\beta}$. If this form were inserted into Eq. (3.1) and closed loop effects neglected, one would obtain a nonlinear integral equation for G_{ij}. The approximation scheme of BJW corresponds to successively inserting the first term only, the first two terms only, etc. into Eq. (3.1). Thus, to first approximation one has

$$G^{-1}(p)=\gamma p+m_0+\frac{ie_0^2}{(2\pi)^4}\int d^4p'\gamma^\mu G(p')\gamma^\nu D_{\nu\mu}(p-p') \quad (3.8)$$

and so forth. Two possible infinities exist in the integral of Eq. (3.1): The wave function renormalization $Z_2(=Z_1)$ and the fermion self-mass δm. By an appropriate choice of gauge, BJW show that the former infinity can be removed. In the approximation of Eq. (3.8), this preferred gauge is the Landau gauge,[15] i.e., $\lambda_\mu=\frac{1}{2}k_\mu/k^2$ in Eq. (3.3). (Modifications in the gauge are needed for the higher approximations.) The self-mass is gauge invariant, and cannot be eliminated by such a device. One finds, however, that if one makes the choice $m_0=0$ (so that *all* the mass is dynamical in origin), then δm is finite. This can be easily seen in the approximation of Eq. (3.8). The convergence of the integral is govern by the asymptotic form of G_{ij} and $D_{\mu\nu}$, i.e., Eqs. (3.2) and (3.4). If one inserts these into Eq. (3.8), the integral is easily seen to converge in the Landau gauge provided m_0 is set to zero. BJW show that the same result holds in the higher approximations. In general, to any order of approximation, one finds that the form factors of $G_{ij}(p)$ behave asymptotically as $(1/p^2)^{\lambda(e_0)}$. The first approximation (3.8) determines the constant λ correct to order e_0^2, the second approximation to order e_0^4, etc. Thus, Eq. (3.8) gives rigorously the asymptotic *form* of $G_{ij}(p)$ with a presumably good approximation to the exponent λ (assuming the series for λ converges).[16]

It is highly convenient, at each stage of the approximation scheme, to maintain Ward's identity rigorously, so that no apparent violations of charge conservation appear. This can be achieved most easily by using neither Eq. (3.6) nor Eq. (3.7) to develop an approximation scheme for $\Gamma^\mu{}_{ij}(p',p)$, but rather a third form. Thus, if G_{ij} is to be calculated to a given approximation, then the $\Gamma^\mu{}_{ij}$ calculated by inserting that G_{ij} into Eq. (3.5) will formally obey Ward's identity,

$$(p'-p)_\mu\Gamma^\mu{}_{ij}(p',p)=G^{-1}{}_{ij}(p')-G^{-1}{}_{ij}(p). \quad (3.9)$$

Thus in the approximation of Eq. (3.8), the corresponding vertex function turns out to be the solution of the following equation:

$$\Gamma^\mu(p',p)=C\gamma^\mu-\frac{ie_0^2}{(2\pi)^4}\int d^4p''\gamma^\alpha G(p''-p+p') \times\Gamma^\mu(p''-p+p',\,p'')G(p'')\gamma^\beta D_{\alpha\beta}(p''-p), \quad (3.10)$$

where $G(p)$ is the solution of Eq. (3.8) and[17]

$$C=1-(3e_0^2/32\pi^2).$$

[Note that Eq. (3.10) is neither the first two terms of Eq. (3.6) nor of Eq. (3.7).] To maintain Ward's identity, then, a linear integral equation must be solved for $\Gamma^\mu{}_{ij}$. Solutions to Eq. (3.10) will be examined in the next section.

We turn now to the solution of Eq. (3.8) (with $m_0=0$) in the approximation when $D_{\mu\nu}$ is replaced by its high-energy form[18] $D_{\mu\nu}{}^{(0)}$. We restrict ourselves to parity-conserving solutions. (The effects of parity violation are discussed in Appendix A.) One may therefore write

$$G_{ij}(p)=\gamma p g_{ij}(p^2)+f_{ij}(p^2) \quad (3.11)$$

and similarly

$$G^{-1}{}_{ij}(p)=\gamma p h_{ij}(p^2)+k_{ij}(p^2). \quad (3.12)$$

Equations (2.8) and (2.9) show that the form factors

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[15] More precisely, one can use any gauge that asymptotically approaches the Landau gauge sufficiently rapidly.

[16] Difficulties, of course, would arise if the bare charge were large (as might be the case in vector-meson models for strong interactions).

[17] One would have expected the numerical factor C to be unity since the γ^μ term arises from the usual replacement of γp by $\gamma(p-e_0A)$ in Eq. (3.8) when an external field A_μ is present [i.e., $\Gamma^\mu=-\delta G^{-1}/\delta(e_0A_\mu)$]. As discussed by BJW, the result $C\neq1$ arises as follows: While the integral in Eq. (3.8) *is* convergent in the Landau gauge, it is not *absolutely* convergent for the parts proportional to γp. Consequently, translations of integration variables, e.g., $p^{\mu\prime}=k^\mu+p^\mu$, change the value of these terms (by a finite amount). Further, just such translations are needed in verifying Eq. (3.9), and if one chose $C=1$, Eq. (3.9) would be valid only upon using k^μ as the integration variable in the mass operator. As we shall see below, however, the choice of $p^{\mu\prime}$ as the integration variable is uniquely determined by the requirement that the Lehmann asymptotic form be satisfied for $G^{-1}(p)$ (i.e., asymptotically, the coefficient of γp in G^{-1} is unity). Then the choice of $C\neq1$ is needed to compensate for the translation $k\rightarrow p'$ so that Ward's identity actually be satisfied. As pointed out by BJW, the coefficient of γ^μ in Eq. (3.10) is somewhat ambiguous, anyway, since it arises from taking the variational derivative of the singular function $G_{ij}(x,x')$ [Eq. (3.5)]. Some condition, such as Ward's identity, is thus needed to determine its value. Actually, none of the results of this paper are effected by the value of C.

[18] For the one-field case, the solution of Eq. (3.8) has been examined in the asymptotic domain by BJW. An approximate solution for the one-field case (with $D_{\mu\nu}$ replaced by $D_{\mu\nu}{}^{(0)}$), valid both in the vicinity of the pole and in the asymptotic domain, has also been given by Th. Maris, V. Herscovitz, and G. Jacob, Phys. Rev. Letters **12**, 313 (1964).

g_{ij}, f_{ij}, h_{ij}, k_{ij} are Hermitian isotopic matrices for space-like p^2, i.e., $p^2>0$. Inserting Eqs. (3.11) and (3.12) into Eq. (3.8), one finds, in the Landau gauge, that

$$\gamma p h(p^2)+k(p^2)=\gamma p+\frac{ie_0^2}{(2\pi)^4}\int d^4p' \times[g(p'^2)\{\gamma^\mu\gamma p'\gamma_\mu-\gamma q\gamma p'\gamma q q^{-2}\}q^{-2}] -3\frac{ie_0^2}{(2\pi)^4}\int d^4p' f(p'^2)q^{-2}, \quad (3.13)$$

where $q^\mu\equiv p'^\mu-p^\mu$. The analysis is simplified by the following device[6]: When all momenta are space-like, one is free to make the analytic continuation to the Euclidean metric by replacing p^0 by ip^0 and p'^0 by ip'^0. The angular integrations are then easily performed in the four-dimensional Euclidean momentum space. (A list of relevant formulas is given in Appendix B.) One is left with only a radial momentum integration. The first integral in Eq. (3.13) vanishes and so

$$h(x)=1\,, \quad (3.14)$$

while the remainder of Eq. (3.13) reduces to

$$k(x)=3\lambda\left[x^{-1}\int_0^x dx'x'f(x')+\int_x^\infty dx'f(x')\right]. \quad (3.15)$$

Here $x\equiv p^2>0$, and $\lambda=\alpha_0/4\pi=e_0^2/16\pi^2$. The functions k and f are, of course, related by the fact that $G^{-1}(p)$ is the matrix inverse of $G(p)$. Thus one finds [using Eq. (3.14)] that $g=-[x+k^2]^{-1}$ and

$$f(x)=k(x)[x+h^2(x)]^{-1}. \quad (3.16)$$

In order that the integral in Eq. (3.15) converge at infinity, $f(x)$ must vanish asymptotically no slower than $1/x^{1+\epsilon}$, $\epsilon>0$. This implies that $k(x)$ also vanishes at infinity and thus the solutions of the integral equation automatically satisfy the Lehmann condition (3.2) (with $m_0=0$). Equation (3.15) also enforces boundary conditions at the origin. Thus, for the first integral in Eq. (3.15) to converge, $f(x)$ can be no more singular than $1/x^{2-\epsilon}$, $\epsilon>0$, at the origin and hence $k(x)$ no more singular than $1/x^{1-\epsilon}$. However, on inserting this limit on $k(x)$ into Eq. (3.16) one sees that $f(x)$ must actually approach a finite constant at the origin and then by Eq. (3.15), so must $k(x)$. The fact that $k(x)$ must be regular at the origin is not an accident of the approximation scheme. One would expect the same condition on the solution of the rigorous Eq. (3.1). Thus since $k(p^2)=\frac{1}{4}\operatorname{tr}G^{-1}(p)$, one finds from Eq. (3.1) in the limit $p\to 0$

$$k(0)=\frac{ie_0^2}{(2\pi)^4}\times\frac{1}{4}\operatorname{tr}\int d^4p'\gamma^\mu G(p')\Gamma^\nu(p',0)D_{\nu\mu}(p'). \quad (3.17)$$

Thus $k(0)$ is finite provided the right-hand integral exists. One need only worry about an infrared divergence at $p'=0$ since if the integral did not converge elsewhere, $k(p^2)$ would not exist for $p\neq 0$. Since $G(p')$ has singularities only at the fermion poles, it must be regular at $p'=0$. The photon pole requires that $D(p')\sim 1/p'^2$ at the origin, while $d^4p'\sim p'^3dp'$. The integral will thus converge provided $\Gamma^\mu(p',0)$ is not too singular at $p'=0$. In fact, at least the series form (3.7) says that $\Gamma(p',0)$ tends to a finite value[19] at $p'=0$.

As in the one-field problem,[6] Eq. (3.15) may be converted into a second-order differential equation:

$$(xk)''+3\lambda k(x+k^2)^{-1}=0\,. \quad (3.18)$$

Here prime means derivative with respect to $x\equiv p^2$. Any solution of Eq. (3.15) is a solution of Eq. (3.18). The converse is, of course, not true. However, it is easy to see that any solution of Eq. (3.18) which obeys the previously stated boundary conditions (both at infinity and at the origin) is indeed a solution of Eq. (3.15). This is verified by inserting $-[xk(x)]''$ for $3\lambda f(x)$ into the integrals of Eq. (3.15) and showing that the right-hand side correctly reduces to $k(x)$ when the boundary conditions are satisfied.

Asymptotically, one easily finds the general solution to be

$$k(x)\sim A_1x^{-3\lambda}+A_2x^{-(1-3\lambda)}\,, \quad (3.19)$$

where A_1 and A_2 are two matrix constants of integration. Under our general assumption that α_0 is sufficiently small, both solutions satisfy the boundary conditions at infinity. A general solution near the origin may also be found easily. It takes the form

$$k(x)=a_{-1}x^{-1}+a_0+a_1x^2+\cdots\,, \quad (3.20)$$

where all the a_m, $m\geqslant 1$ can be determined by recursion relations in terms of the two independent matrices a_{-1}, a_0. Regularity at the origin forces one to choose $a_{-1}=0$. The a_m, $m\geqslant 1$ are then real algebraic functions of only one matrix of integration a_0:

$$k=k(x;\,a_0)\,. \quad (3.21)$$

Since k_{ij} is an Hermitian isotopic matrix for $x>0$, and for that domain is a real function of a_0, the matrix a_0 must also be chosen Hermitian. Now, in deriving the Green's function equations, only the field equations and canonical commutation relations need be used. The fermion fields used in the definition (2.2) of G_{ij} still leave the freedom of making *constant* unitary transformations $\psi(x)\to U\psi(x)$. Such transformations would change $k(x)$ according to $k(x)\to Uk(x;\,a_0)U^{-1}=k(x;\,Ua_0U^{-1})$. By an appropriate choice of U (which corresponds to fixing the choice of fermion fields ψ_i) one may diagonalize a_0 and

[19] These results stress the fact that infrared divergences do not arise in the *unrenormalized* Green's functions. This is true even for the singular case in which the clothed mass of the fermion vanishes (i.e., a "charged neutrino"). From Eq. (3.7), $\Gamma^\mu(p',0)$ could at worst be logarithmically divergent at the origin and hence the mass-operator integral of Eq. (3.17) still converges.

hence also $k_{ij}(x)$ for *all* momenta. We may, therefore, write

$$k_{ij}(x)=\alpha(x)\delta_{ij}+\beta(x)(\tau_3)_{ij}. \quad (3.22)$$

Thus the solution for $G_{ij}(p)$ that we have found corresponds to possibility (b) of Sec. II. Full $SU(2)$ symmetry is *not* broken and there still remains invariance under rotations about the third isotopic axis. This is the type of situation that allows for a mass splitting [Eq. (2.17)] but not necessarily a decay (1.1). To see the latter fact, one can easily generate iteration solutions of Eq. (3.10) [starting with the zeroth approximation $(\Gamma^\mu{}_{ij})^{(0)}=C\gamma^\mu\delta_{ij}$] after inserting the above solution for G_{ij}. Since G_{ij} is diagonal, the resultant $\Gamma^\mu{}_{ij}$ will also be diagonal, forbidding decay (1.1) by the discussion following Eq. (2.22).

The diagonal nature of G_{ij} follows directly from the boundary condition at the origin. This forces k_{ij} to depend only on one matrix of integration. [Thus the two matrices A_1 and A_2 in the solution (3.19) at infinity must actually be related to each other, if the boundary condition at the origin is to be maintained.] If k_{ij} had depended upon *two* constant matrices, both could not in general be simultaneously diagonalized. Then k_{ij} would have an off-diagonal part which would permit decay (1.1). The boundary condition at the origin is a rigorous one. However, the fact that just one solution satisfies it has only been established within the framework of our approximation.

As a final point, we might note that when a_0 is put into diagonal form, $G_{ij}(p)$ depends upon two arbitrary constants (the diagonal elements of a_0). These may be determined by requiring the two poles of $G_{ij}(p)$ to occur at the experimental masses of the muon and electron.

IV. VERTEX INTEGRAL EQUATION

In the previous section it was seen that Eq. (3.10) possessed a solution for $\Gamma^\mu{}_{ij}(p',p)$ diagonal in the isotopic indices (the ordinary iteration solution). It is natural to inquire whether Eq. (3.10) possesses any off-diagonal solutions (even though G_{ij} is diagonal), such solutions allowing decay (1.1) to occur. From the general analysis of Sec. II, it would appear doubtful that such a possibility could arise. It is conceivable, though, that a symmetry breakdown of the vacuum state might "accidentally" produce no effects on the two-point function and be observable only beginning in the vertex function. The calculations of this section give no indication, however, that this anomalous possibility occurs.

While a full solution of Eq. (3.10) is technically possible, the analysis necessary to produce it would be quite lengthy. A vector vertex function has twelve 2×2 isotopic matrix form factors (assuming parity conservation) and so there are 48 independent form factors in all. While Ward's identity allows one to determine 16 of these, the problem remains fairly formidable. We will, therefore, restrict ourselves to a calculation of $\Gamma^\mu(p',p)$ in the limit as $p'\to p$. More precisely, we consider an expansion of the following type:

$$\Gamma^\mu(p+\tfrac{1}{2}k,\, p-\tfrac{1}{2}k)=\Gamma^\mu(p,p)+k^\alpha\Gamma^\mu{}_\alpha(p)+\cdots, \quad (4.1)$$

the first nontrivial coefficient is $\Gamma^\mu{}_\alpha(p)$ and we will examine the solution of Eq. (3.10) for it. (There are still 16 independent form factors for this problem.) Ward's identity,

$$k^\mu\Gamma_\mu(p_+,p_-)=G^{-1}(p_+)-G^{-1}(p_-) \quad (4.2)$$

(where $p_\pm{}^\mu\equiv p^\mu\pm\frac{1}{2}k^\mu$), uniquely determines $\Gamma^\mu(p,p)$, while it requires that $k^\mu k^\alpha\Gamma_{\mu\alpha}(p)$ should vanish. Hence $\Gamma_{\mu\alpha}(p)$ must be antisymmetric in the tensor indices. Assuming parity invariance, there are then four independent structures for $\Gamma_{\mu\alpha}(p)$:

$$\Gamma_{\mu\alpha}(p)=\gamma_{[\mu}p_{\alpha]}\varphi^{(1)}(p^2)+\sigma_{[\mu\beta}p_\beta p_{\alpha]}\varphi^{(2)}(p^2) +\sigma_{\mu\alpha}\varphi^{(3)}(p^2)+\epsilon_{\mu\alpha\rho\sigma}\gamma_5\gamma^\rho p^\sigma\varphi^{(4)}(p^2), \quad (4.3)$$

where $\sigma_{\alpha\beta}\equiv[\gamma_\alpha,\gamma_\beta]$, $\gamma_5\equiv\gamma^0\gamma^1\gamma^2\gamma^3$, the Levi-Civita symbol has sign convention $\epsilon_{0123}=+1$ and the notation $A_{[\mu\alpha]}$ means $A_{\mu\alpha}-A_{\alpha\mu}$. The four form factors $\varphi^{(a)}(p^2)$ are space-time scalars and isotopic matrices. In an obvious notation, one may write $\Gamma_{\mu\alpha}(p)={}^{(a)}\Gamma_{\mu\alpha}\varphi^{(a)}(p^2)$. The condition of charge conjugation invariance, Eq. (2.37), implies

$$\Gamma^{\mu\alpha}{}_{ij}(p)=-[C\Gamma^{\mu\alpha}{}_{ji}(-p)C^{-1}]^\sim. \quad (4.4)$$

This imposes the following isotopic symmetry conditions on the form factors[20]:

$$\varphi^{(1)}{}_{ij}=-\varphi^{(1)}{}_{ji},\quad \varphi^{(a)}{}_{ij}=+\varphi^{(a)}{}_{ji},\quad a=2,3,4. \quad (4.5)$$

Equation (3.10) may conveniently be rewritten as

$$\Gamma^\mu(p_+,p_-)=C\gamma^\mu-\frac{ie_0{}^2}{(2\pi)^4}\int d^4p'\gamma^\alpha G(p_+') \times\Gamma^\mu(p_+',p_-')G(p_-')\gamma^\beta D_{\beta\alpha}(p-p'). \quad (4.6)$$

Differentiating with respect to k^ν and setting k^ν to zero, one obtains

$$\Gamma_{\mu\nu}(p)=\frac{ie_0{}^2}{(2\pi)^4}\int d^4p'\gamma^\alpha\frac{1}{2}\left[\frac{\partial G(p')}{\partial p'^\nu}G^{-1}(p') \times\frac{\partial G(p')}{\partial p'^\mu}-\mu\leftrightarrow\nu\right]\gamma^\beta D_{\beta\alpha}(p'-p)-\frac{ie_0{}^2}{(2\pi)^4}\int d^4p'\gamma^\alpha \times G(p')\Gamma_{\mu\nu}(p')G(p')\gamma^\beta D_{\beta\alpha}(p'-p). \quad (4.7)$$

The first integral in Eq. (4.7) is diagonal in the isotopic indices since G_{ij} is. As we are interested here only in seeing if there exist any off-diagonal components in $\Gamma^\mu{}_{ij}$ which would give rise to the decay (1.1), one may ignore this integral. The form of the second integral of

[20] The amplitudes that violate charge-conjugation invariance of course have symmetries opposite to those of Eq. (4.5). Since Eq. (3.10) is consistent with charge-conjugation invariance, the equations for the C-violating amplitudes decouple from the C-conserving ones.

Eq. (4.7) suggests that a convenient quantity to define is

$$M_{\mu\nu}(p)\equiv G(p)\Gamma_{\mu\nu}(p)G(p). \tag{4.8}$$

$M_{\mu\nu}(p)$ may also be expanded in terms of form factors in a fashion similar to $\Gamma_{\mu\nu}$, i.e., $M_{\mu\nu}(p)={}^{(a)}\Gamma_{\mu\nu}(p)\chi^{(a)}(p^2)$, the $\chi^{(a)}{}_{ij}$ obeying the same symmetry conditions (4.5) as the $\varphi^{(a)}{}_{ij}$. Since $G^{-1}(p)=\gamma p+k(p^2)$, Eq. (4.7) becomes

$$(\gamma p+k)\,{}^{(a)}\Gamma_{\mu\nu}(p)\chi^{(a)}(p^2)(\gamma p+k) =\frac{e_0^2}{(2\pi)^4}\int d^4p'q^{-2}[\gamma^\alpha\,{}^{(a)}\Gamma_{\mu\nu}(p')\gamma_\alpha -i\gamma q\,{}^{(a)}\Gamma_{\mu\nu}(p')\gamma q q^{-2}]\chi^{(a)}(p'^2), \tag{4.9}$$

where $q^\mu\equiv p'^\mu-p^\mu$. It is understood that Eq. (4.9) holds only for the antisymmetric isotopic parts since the first integral has already been neglected. (The transition to the Euclidean metric has also been made.) The angular integrations may now be performed explicitly with the aid of the results of Appendix B. Upon introducing the following notation:

$$\begin{aligned}\chi^{(1)}(p^2)&=\psi_1(p^2)\tau_2,\\ \chi^{(a)}(p^2)&=\rho_a(p^2)1+\sigma_a(p^2)\tau_3+\psi_a\tau_1,\\ k(p^2)&=\alpha(p^2)+\beta(p^2)\tau_3\quad(a=2,3,4),\end{aligned} \tag{4.10}$$

one finds, after a lengthy but straightforward calculation, that the off-diagonal parts of Eq. (4.9) reduce to four equations to determine the four $\psi_a(p^2)$. These read

$$\begin{aligned}
\varphi_1&\equiv[x+\alpha^2-\beta^2]\psi_1-4i\beta(x\psi_2+\psi_3)=\lambda\left[x^{-2}\int_0^x dx'x'^2\psi_1(x')+\int_x^\infty dx'\psi_1(x')\right],\\
\varphi_2&\equiv[x+\alpha^2-\beta^2]\psi_2+2\psi_3-i\beta\psi_1-\alpha\psi_4=-\lambda\int_0^x dx'\left\{\left(\frac{x'}{x}\right)^2\left(1-\frac{x'}{x}\right)\psi_2(x')+2\left(\frac{x'}{x}\right)^2\left(\frac{1}{x'}-\frac{1}{x}\right)\psi_3(x')\right\},\\
\varphi_3&\equiv[-x+\alpha^2-\beta^2]\psi_3+x\alpha\psi_4=\tfrac{1}{2}\lambda\left[\int_0^x dx'\left\{\left(\frac{x'}{x}\right)^2(x-x')\psi_2(x')+2\frac{x'}{x}\left(1-\frac{x'}{x}\right)\psi_3(x')\right\}+\int_x^\infty dx'(x-x')\psi_2(x')\right],\\
\varphi_4&\equiv[-x+\alpha^2-\beta^2]\psi_4-4\alpha\psi_3=-\lambda\left[\int_0^x dx'\left(\frac{x'}{x}\right)^2\psi_4(x')+\int_x^\infty dx'\psi_4(x')\right],
\end{aligned} \tag{4.11}$$

where $\lambda\equiv\alpha_0/4\pi$ and $x\equiv p^2>0$. From Eq. (4.9) it is clear that the left-hand sides of Eqs. (4.11) are related to the form factors $\varphi^{(a)}$ of $\Gamma_{\mu\nu}=G^{-1}M_{\mu\nu}G^{-1}$. Writing

$$\begin{aligned}\varphi^{(1)}(p^2)&=\varphi_1(p^2)\tau_2;\\ \varphi^{(a)}(p^2)&=\bar\rho_a(p^2)1+\bar\sigma_a(p^2)\tau_3+\varphi_a\tau_1\quad a=2,3,4,\end{aligned} \tag{4.12}$$

then the left-hand sides of Eq. (4.11) are just precisely the four form factors of the off-diagonal parts of $\Gamma_{\mu\nu}$, φ_a, expressed in terms of the ψ_a. One may easily reduce Eqs. (4.11) to a set of four second-order differential equations for the $\psi_a(x)$:

$$\begin{aligned}[x^{-1}(x^2\varphi_1)']'&=-2\lambda\psi_1,\\ [x^{-1}(x^2\varphi_4)']'&=2\lambda\psi_4,\\ [x\varphi_2+2\varphi_3]''&=-\lambda\psi_2,\\ [x^3\varphi_2]''&=-\lambda x(x\psi_2+2\psi_3).\end{aligned} \tag{4.12$'$}$$

We investigate first the boundary conditions at the origin. From Eq. (4.7) it is clear that $\Gamma_{\mu\nu}(p)$ is regular at $p=0$. [This follows from the fact that $G(p')$ is regular at $p'=0$ and so the kernel of Eq. (4.7) behaves as $\sim 1/(p-p')^2$ near $p'=0$ for small p.] From the definition of the form factors, Eq. (4.3), one has then the following limiting behaviors for $\varphi_a(x)$: $\varphi_a(x)\sim(1/x)^{\lambda_a}$ where $\lambda_1\leq\frac{1}{2}$, $\lambda_2\leq 1$, $\lambda_3\leq 0$, $\lambda_4\leq\frac{1}{2}$. Since $M_{\mu\nu}(p)=G(p)\times\Gamma_{\mu\nu}(p)G(p)$ is also regular at the origin, the $\psi_a(x)$ have an identical set of bounds at $x=0$. With these limits, one easily verifies that the right-hand sides of Eq. (4.11) actually approach a finite limit as $x\to 0$. Examination of the left-hand sides then allows one to conclude that all the $\psi_a(x)$ [and hence all the $\varphi_a(x)$] are actually regular at the origin. The integral equation thus requires one to look for solutions of Eq. (4.12′) which have the form

$$\psi_a(x)=\sum_0^\infty C_a(n)x^n. \tag{4.13}$$

Upon inserting Eq. (4.13) into Eqs. (4.12′), one obtains recursion relations among the Taylor coefficients. One finds that all the $C_a(n)$ can be determined in terms of four arbitrary ones: $C_1(0)$, $C_3(0)$, $C_3(1)$, $C_4(0)$. [The general solution of Eq. (4.12′) has eight constants of integration which implies that four of these solutions are singular at the origin.] Thus Eq. (4.12′) gives rise to a solution which depends linearly and homogeneously on these four constants.

We next investigate the boundary conditions at infinity. In order that the integrals on the right-hand side of Eq. (4.11) converge, $\psi_1(x)$ and $\psi_4(x)$ must vanish no slower than $1/x^a$, $a>1$, and $\psi_2(x)$ no slower than $1/x^b$, $b>2$. In a fashion analogous to the discussion following Eq. (3.18) one can then show that any solution of Eqs. (4.12′) obeying the boundary conditions both at the origin and at infinity is a solution of the integral equations (4.11). Aside from the above boundary condition at infinity, we also impose a second condition of regularity, i.e., that the integrals in Eqs. (4.7) or (4.9) actually converge absolutely at infinity. This requires that $M_{\mu\nu}(p)$ vanish faster than $1/x^{\alpha}$, $\alpha>2$, since $D(p')\sim 1/p'^2$ (and hence that $\Gamma_{\mu\nu}$ approaches zero at infinity). From Eq. (4.3), one sees that the above conditions on $\psi_1(x)$ and $\psi_4(x)$ must be strengthened to read that they vanish faster than $1/x^{a}$, $a>\frac{3}{2}$, and also that $\psi_3(x)$ vanishes at infinity.

Returning now to Eq. (4.12′), one can show that the general solution for large x begins as

$$\begin{aligned}\psi_{1,4}&\sim A_{1,4}x^{-1-\lambda}+B_{1,4}x^{-3+\lambda}+\cdots,\\ \psi_3&\sim -\tfrac{1}{2}x\psi_2+A_3x^{-2-\lambda}+B_3x^{-3+\lambda}+\cdots,\\ \psi_2&\sim A_2x^{-1-\lambda}+B_2x^{-2+\lambda}+\cdots,\end{aligned}\tag{4.14}$$

where A_a and B_a are the eight constants of integration. The boundary conditions at infinity then require that four of these, A_2, B_2, A_1, and A_4 vanish,[21] which represents four conditions on the general solution. In terms of the solution (4.13) regular at the origin, there are then four constraints on the constants $C_1(0)$, $C_3(0)$, $C_3(1)$, $C_4(0)$, in the form of four linear homogeneous algebraic equations. In general the four C's must then vanish and hence so do all the $\psi_a(x)$. Thus, to the approximation considered, the off-diagonal parts of $\Gamma_{\mu\nu}(p)$ are zero and the vertex function equation does not allow a broken symmetry solution permitting decay (1.1) to occur.[22]

V. DISCUSSION

In the previous sections, the spontaneous breakdown of internal symmetries has been examined for a system of two charged fields of zero bare mass (the "muon" and "electron" fields) interacting minimally with the electromagnetic field. As was seen, a mass splitting between electron and muon could develop. The masses appeared as two arbitrary constants of integration, and so the value of the mass splitting (i.e., the amount of breakdown) could not be predicted (a characteristic feature of spontaneous breakdowns). Since the one-field analysis of BJW shows that a single charged fermion can develop an arbitrary clothed mass,[23] it is perhaps not surprising that a difference of masses can develop in the two-fermion problem. It is gratifying, however, that the stability of the heavier particle is still maintained.[24] It is also interesting that spontaneous breakdown of the discrete operations of the Lorentz group is forbidden (as discussed in Appendix A)."

The electrodynamic system studied is a convenient one for examining symmetry breakdowns for several reasons. First the symmetry group involved, $SU(2)$, is particularly simple. Second, since the analysis involves the gauge-invariant electromagnetic couplings, none of the usual field-theoretical infinities arise, at least in the approximations examined. Thus, no cutoffs need be introduced before physical interpretations can be made. On the other hand, the technique that gives rise to a finite electrodynamics automatically requires that at least one broken symmetry (γ_5 invariance) occurs (so that fermions with zero bare mass can have nonzero clothed mass). Within such a framework, then, it is natural to consider seriously other possible spontaneous breakdowns. One of the original theoretical motivations for introducing two neutrinos was to prevent the photon decay of the muon. As we have seen, however, if spontaneous breakdowns are to be allowed, two neutrinos would not automatically stabilize the muon as a purely electromagnetic decay would still be feasible. The fact that this possibility does not seem to materialize is a property of the detailed dynamics of the electromagnetic interactions, and not merely of the symmetry group imposed on the Lagrangian.

APPENDIX A

In this appendix, the analysis of Sec. III is extended to consider the possibility that P, C, or T is spontaneously broken.[25] It will be seen that such breakdowns cannot, in fact occur.

The general form of $G_{ij}(p)$ now is

$$G_{ij}(p)=\gamma p[g_{ij}(p^2)+i\gamma_5\bar{g}_{ij}(p^2)]+[f_{ij}(p^2)+\gamma_5\bar{f}_{ij}(p^2)],\tag{A1}$$

whose inverse will be written as

$$G^{-1}{}_{ij}(p)=\gamma p[h_{ij}(p^2)+i\gamma_5\bar{h}_{ij}(p^2)]+[k_{ij}(q^2)+\gamma_5\bar{k}_{ij}(p^2)].\tag{A2}$$

[21] The requirement of absolute convergence sets A_1 and A_4 to zero. If it were not imposed, Eq. (4.9) would actually have divergent radial integrals in terms whose angular integrals averaged to zero.

[22] It should be noted that the off-diagonal terms in $\Gamma_{ij}{}^{\mu}$ really had to vanish to forbid decay (1.1) as no further freedom of isotopic rotation is left to diagonalize $\Gamma^{\mu}{}_{ij}$. Once a basis which diagonalizes G_{ij} is chosen, the μ and e field operators are completely fixed (except for the trivial isotopic rotations around the $m=3$ axis).

[23] The single arbitrary constant of integration in the one-field case may be used there to fix the mass, corresponding to our procedure for the two-field problem where two arbitrary constants remain.

[24] After completion of this paper, there has appeared an article by Th. Maris, V. Herscovitz, and G. Jacob, Nuovo Cimento **34**, 946 (1964) which reaches identical conclusions regarding the one-fermion Green's function.

[25] The usual argument that gauge invariance, together with CP conservation, implies that P must be separately conserved does not apply here for two reasons. First, it does not apply to spontaneous breakdown, being an argument made on the Lagrangian. Second, if, as is the case here, the bare mass vanishes, one may even have a "minimal," P-violating coupling term in the Lagrangian of the form $e_0[\bar{\psi}\gamma_\mu\psi+i\alpha\bar{\psi}\gamma_\mu\gamma_5\psi]A_\mu$, together with invariance with respect to the gauge transformation $A_\mu(x)\to A_\mu(x)+\partial_\mu\Lambda(x)$, $\psi\to\exp\{i[1+i\alpha\gamma_5]\Lambda(x)\}\psi(x)$.

Equation (3.8) now reads

$$G^{-1}(p)=\gamma p+\frac{ie_0^2}{(2\pi)^4}\int d^4p'q^{-2}[\gamma^\alpha\gamma p\gamma_\alpha$$
$$-i\gamma q\gamma p\gamma q q^{-2}][g(p'^2)-i\gamma_5\bar{g}(p'^2)]-\frac{3ie_0^2}{(2\pi)^4}\int d^4p'$$
$$\times q^{-2}[f(p'^2)-\gamma_5\bar{f}(p'^2)], \quad \text{(A3)}$$

where $q^\mu\equiv p'^\mu-p^\mu$. Again the angular part of the first integral vanishes and so

$$h_{ij}(p^2)=\delta_{ij}, \quad \bar{h}_{ij}=0 \quad \text{(A4)}$$

while the remaining terms reduce to

$$k(x)=3\lambda\left[\int_0^x dx'\left(\frac{x'}{x}\right)f(x')+\int_x^\infty dx' f(x')\right], \quad \text{(A5a)}$$

$$\bar{k}(x)=-3\lambda\left[\int_0^x dx'\left(\frac{x'}{x}\right)\bar{f}(x')+\int_x^\infty dx'\bar{f}(x')\right], \quad \text{(A5b)}$$

where $\lambda\equiv\alpha_0/4\pi$. Converting Eqs. (A5) to differential form yields

$$(xk)''=-3\lambda f(x), \quad \text{(A6a)}$$
$$(x\bar{k})''=3\lambda\bar{f}(x). \quad \text{(A6b)}$$

The relation betwben the form factors of $G(p)$ and $G^{-1}(p)$ is slightly more complicated now. Using Eq. (A4), one finds

$$\begin{aligned}&g(x)=-[1-Q^2]^{-1}[x+k^2+\bar{k}^2]^{-1},\\&\bar{g}(x)=-Qg(x),\\&f(x)=-[k-i\bar{k}Q]g(x), \quad \bar{f}(x)=(\bar{k}+ikQ)g(x),\end{aligned} \quad \text{(A7)}$$

where Q is the abbreviation

$$Q(x)\equiv-i[x+k^2+\bar{k}^2][\bar{k}(x),k(x)]. \quad \text{(A8)}$$

(The commutator is nonzero due to the isotopic dependence.) Asymptotically, then, $f\sim k/x$ and $\bar{f}\sim-(\bar{k}/x)$, and so both $k(x)$ and $\bar{k}(x)$ have the form for large x given in Eq. (3.19). Near the origin, however, Eqs. (A6) are highly coupled. Again only regular solutions are allowed and so one may expand $k(x)$ and $\bar{k}(x)$ near the origin:

$$k(x)=\sum a_m x^n, \quad \bar{k}(x)=\sum b_m x^m \quad \text{(A9)}$$

where all the a_m and b_m are Hermitian matrices. The recursion relations obtained from Eqs. (A6) show that all the Taylor coefficients can be determined in terms of the two arbitrary matrices a_0 and b_0. Thus we have for the form of the solution

$$k=k[x;a_0,b_0], \quad \bar{k}=\bar{k}[x;a_0,b_0]. \quad \text{(A10)}$$

One may easily check that $\bar{k}$ vanishes in the limit $b_0\rightarrow 0$ and then k correctly limits to the solution (3.21).

In analogy with the methods given following Eq. (3.21) to diagonalize the parity-conserving case, one may seek a constant unitary rotation eliminating $\bar{k}(x)$. This can be accomplished if a rotation eliminating $\bar{k}(0)$ exists, since then the whole series (A9) for $\bar{k}(x)$ vanishes. There is available, for this purpose, the counterpart of Eq. (2.4),

$$\begin{aligned}&\psi\rightarrow\exp[\alpha_0\gamma_5+\boldsymbol{\alpha}\cdot\boldsymbol{\tau}\gamma_5]\psi,\\&\bar{\psi}\rightarrow\bar{\psi}\exp[\alpha_0\gamma_5+\boldsymbol{\alpha}\cdot\boldsymbol{\tau}\gamma_5],\end{aligned} \quad \text{(A10)}$$

which transforms $K\equiv k+\gamma_5\bar{k}$ by

$$K(x)\rightarrow\exp[\alpha_0\gamma_5+\boldsymbol{\alpha}\cdot\boldsymbol{\tau}\gamma_5]\times K(x)\exp[\alpha_0\gamma_5+\boldsymbol{\alpha}\cdot\boldsymbol{\tau}\gamma_5]. \quad \text{(A11)}$$

We write $K(0)=A_0+\mathbf{A}\cdot\boldsymbol{\tau}+\gamma_5(B_0+\mathbf{B}\cdot\boldsymbol{\tau})$ and proceed as follows. First we eliminate the $\gamma_5\mathbf{B}\cdot\boldsymbol{\tau}$ structure using a transformation with $\alpha_0=0$ and $\boldsymbol{\alpha}=\lambda\mathbf{B}$ where

$$\tan 2\lambda\beta=-B/A_0; \quad B\equiv(\mathbf{B}^2)^{1/2}. \quad \text{(A12)}$$

$K(0)$ now has the following form: $K(0)=A_0'+\mathbf{A}'\cdot\boldsymbol{\tau}+\gamma_5 B_0'$. The elimination of all γ_5 terms is then completed by the following transformation, which eliminates the $\gamma_5 B_0'$ term without reinstating any $\gamma_5\mathbf{B}\cdot\boldsymbol{\tau}$ structure:

$$\begin{aligned}&\boldsymbol{\alpha}\equiv\rho\mathbf{A}', \quad C^2\equiv A_0'^2+B_0'^2-A'^2,\\&\tan 2\rho A'=-[C^2\pm(C^4+4A'^2B_0'^2)^{1/2}](2A'B_0')^{-1},\\&\tan 2\alpha_0=-[-(A_0'^2-B_0'^2-A'^2)\\&\qquad\pm(C^4+4A'^2B_0'^2)^{1/2}](2A_0'B_0')^{-1}.\end{aligned} \quad \text{(A13)}$$

Once the $\bar{k}(x)$ terms have been removed, the analysis proceeds as in Sec. 3, i.e., $k(x)$ may be diagonalized using transformations (2.4). This automatically implies that C and T invariance are also preserved [by Eqs. (2.35) and (2.36)].

APPENDIX B

In this section we list some of the formulas needed to perform the angular integrals of Eqs. (3.13) and (4.9). In the Euclidean metric, one may introduce four-dimensional spherical coordinates $d^4p'=p'^3dp'd^3\Omega'$, where $\int d^3\Omega' d^3\Omega'=\frac{1}{2}\pi(4\pi)=2\pi^2$. We therefore define the angular average of a function by

$$\langle f(p^\alpha,p'^\beta)\rangle\equiv(2\pi)^{-2}\int d^3\Omega' f(p^\alpha,p'^\beta). \quad \text{(B1)}$$

In general, it is convenient to choose the polar axis in the direction of p^μ so that $p^\alpha p_\alpha'=pp'\cos\chi$ where χ is the polar angle. One can then expand the "potential function" $1/q^2\equiv1/(p'-p)^2$ in terms of Gegenbauer polynomials[26]:

$$q^{-2}=p_>^{-2}\sum_0\alpha^n C_n^1(z), \quad z\equiv\cos\chi, \quad \text{(B2)}$$

[26] C. F. W. Magnus and F. Oberhettinger, *Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1949).

where $p_>$ ($p_<$) is the greater (lesser) of p, p' and $\alpha \equiv p_</p_>$. The $C_n{}^1 \equiv [\sin(n+1)\chi]/\sin\chi$ obey the convenient orthogonality condition

$$\langle C_n{}^1(z) C_m{}^1(z)\rangle = \delta_{nm}. \tag{B3}$$

The integrals appearing in text involve structures proportional to both $1/q^2$ and $(1/q^2)^2$. One needs the following averages involving $1/q^2$:

$$\begin{aligned}&\langle q^{-2}\rangle = p_>{}^{-2}, \quad \langle p_\mu'/q^2\rangle = \tfrac{1}{2}\alpha^2 p^{-2} p_\mu, \\ &\langle p'^\mu p'^\nu/q^2\rangle = (p'/p_>)^2[\tfrac{1}{4}(1-\tfrac{1}{3}\alpha^2)\eta^{\mu\nu} + \tfrac{1}{3}\alpha^2 p^\mu p^\nu p^{-2}].\end{aligned} \tag{B4}$$

To deduce, for example, the second identity, one writes $\langle p'^\mu/q^2\rangle = a p^\mu/p^2$ where $a = pp'\langle z/q^2\rangle$. One then makes use of Eq. (B2) along with the recursion relation

$$zC_n{}^1(z) = \tfrac{1}{2}[C_{n+1}{}^1(z) + C_{n-1}{}^1(z)], \quad C_{-1}{}^1 \equiv 0 \tag{B5}$$

to evaluate a.

The integrals needed involving $(1/q^2)^2$ are

$$\begin{aligned}\langle q^{-4}\rangle &= p_>{}^{-4}(1-\alpha^2)^{-1}, \\ \langle p_\mu'/q^4\rangle &= p_\mu p^{-2} p_>{}^{-2}\alpha^2(1-\alpha^2)^{-1}, \\ \langle p_\mu' p_\nu'/q^4\rangle &= (\alpha^2/p^2)[\tfrac{1}{4}\eta_{\mu\nu} + p_\mu p_\nu p^{-2}\alpha^2(1-\alpha^2)^{-1}], \\ \langle p_\mu' p_\nu' p_\alpha'/q^4\rangle &= (p_>{}^2/p^4)\alpha^4[\tfrac{1}{6}(p^\mu\eta^{\alpha\nu} + p^\nu\eta^{\alpha\mu} + p^\alpha\eta^{\mu\nu}) \\ &\quad + \alpha^2(1-\alpha^2)^{-1} p_\mu p_\nu p_\alpha p^{-2}].\end{aligned} \tag{B6}$$

These may most easily be deduced by inserting expression (B2) for each factor of $1/q^2$ and again using Eq. (B5).

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Further Evidence for Pignotti's R Trajectory*

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The high-energy $K^\pm p$ and $K^\pm n$ total cross section and the $K^- + p \to \bar{K}^0 + n$ charge-exchange data contain further evidence for the Regge trajectory R proposed by Pignotti. The signature factor is important in fitting these data; thus there is also some support for the Regge-pole hypothesis itself.

I. INTRODUCTION

RECENTLY, Pignotti suggested the existence of a new octet of even-signature boson Regge trajectories.[1] They are expected to lie near the ρ trajectory and thus to give no 0^+ bound states or resonances; however, they may give 2^+, etc., resonances, and it has been suggested that the A_2 meson may lie on one of these trajectories.[2]

Some evidence for the $I=1$ member of this octet, called R, was found by Ahmadzadeh.[3] He showed that the differences between high-energy pp and np total cross sections, together with $n+p \to p+n$ charge-exchange data, are readily explained by using a combination of the ρ and R trajectories, whereas ρ alone fails.[4]

The present note shows there is further evidence for R in the differences of $K^\pm p$ and $K^\pm n$ total cross sections,[5] and in $K^- + p \to \bar{K}^0 + n$ charge exchange.[6] Here again ρ is inadequate, but the addition of R explains the discrepancies in a natural way.

From a theoretical viewpoint these KN and $\bar{K}N$ processes have many similarities to NN and $\bar{N}N$ scattering; isospin considerations are the same and so are the Regge trajectories that one assumes to dominate forward scattering.[7] Our formalism is therefore related to that of Ahmadzadeh[3]; our arguments, however, are different. The data we consider have three new features: (a) The KN and $\bar{K}N$ data are more precise[8] than the corresponding NN and $\bar{N}N$ data. (b) The charge exchange, $K^- + p \to \bar{K}^0 + n$, is the direct analog of $\bar{p}+p \to \bar{n}+n$ rather than the $n+p \to p+n$ case already studied

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[7] Not all the trajectories are common, of course; for example, those associated with 0^- or 1^+ mesons do not affect KN scattering.

[8] For example, total cross sections are more accurately known for $\bar{K}N$ than for $\bar{N}N$ scattering. See Ref. 5.