where $\delta=\delta_{12}$ is taken to be the $\pi\pi$ $T=0$ scattering phase shift at relative momentum $k=k_{12}$.

We then take as hypothesis $k\cot\delta=c_1+c_2k^2+c_3(k^2)^2+\cdots$. We have attempted to fit the π^0 spectrum with various numbers of terms (up to four) in the expansion. It is easy to fit the spectrum; none of the choices made fit the branching ratio (see Fig. 4). The best fit with the first 3 terms in the expansion gives R(predicted)$=1.40$. The best fit taking c_1 and c_3 as the parameters gives R(predicted)$=1.35$.

Since the magnitude of f is essentially smooth, as shown by the spectrum, it seems that a rapid variation of the phase of f is required to bring down the overlap integral in (2) and fit the branching ratio. Accordingly we have tried an expansion of $f=\rho e^{i\phi}$ of the form

$$\rho=1+\epsilon y+\cdots.$$

$$\phi=\gamma y+\cdots\ (\epsilon,\gamma\text{real}).$$

$\Gamma_\eta(+-0)\sim|1+\epsilon y|^2$ and thus the spectrum is fitted with $\epsilon=-0.41$ (as in linear matrix-element theory), independent of γ. The value $\gamma=1.60\pm0.40$ then fits the experimental branching ratio. Such a value of γ implies a shift in phase of f by $183°\pm46°$ in crossing the physical region.

We do not know what the meaning of such a form for f would be; in any case there is strong evidence for an interesting structure in the 3π final state in η decay.

Note added in proof. L. Brown and H. Faier[9] have recently obtained a prediction for the branching ratio R that agrees more closely with the experimental result.

[9] L. Brown and H. Faier, paper presented at the Conference on Symmetry Principles at High Energy, Coral Gables, Florida, 1965 (unpublished).

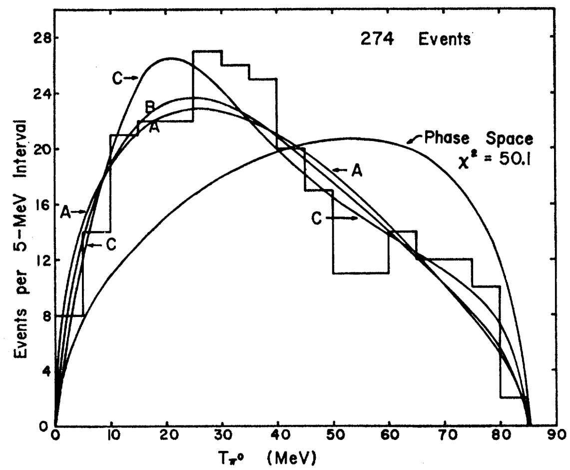

FIG. 4. Kinetic-energy distribution of π^0 in η decay, $\eta\to\pi^+\pi^-\pi^0$. Curve A: Linear matrix element; $a=-0.4$, $\chi^2=9.9$, R(predicted) $=1.63$. Curve B: Brown and Singer; $m=425$ MeV, $\Gamma=\gamma k=118$ MeV, $\chi^2=8.4$, R (predicted)$=1.47$. Curve C: $k\cot\delta=c_1+c_3(k^2)^2$, $c_1=1.26f^{-1}$, $c_3=-4.6f^3$, $\chi^2=7.7$, R (predicted)$=1.35$.

Assuming a modified Breit–Wigner form for the proposed $T=0$ dipion resonance and using a compilation of 708 events, including the 274 reported on here, they are able to predict a value as low as R(predicted)$=1.19$. Thus, if the proposed resonance is found to exist, it may be the explanation for the rapid variation in phase of f.

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Possible Electromagnetically Induced Muonium-Antimuonium Conversion*

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The possibility of the existence of an electromagnetically induced transition between muonium and antimuonium is examined. An experiment is suggested involving the formation of muonium by the injection of positive muons into a helium-filled resonant cavity, which is excited at a frequency corresponding to the difference in interaction energy between muonium and helium, on the one hand, and that between antimuonium and helium, on the other. The sign of antimuonium formation is the observation of the fast electrons from μ^- decay. The dependence of the number of these on which of the various cavity modes is excited gives information on the relative intrinsic parity of muonium and antimuonium. If this turns out to be odd, then this measurement, when combined with the usual relation for the product of the intrinsic parities of a Dirac particle and its antiparticle, would determine the relative intrinsic parity of the muon and electron to be imaginary. The conservation of parity in electromagnetic phenomena and the absence of electromagnetic μ-e transitions would then both find their natural explanation in the single assumption that the observation of electromagnetic phenomena must be compatible with invariance under space inversion.

I. INTRODUCTION

THE motivation for this paper is twofold. First, we hope to present a fairly detailed discussion of a type of experiment which could be used to investigate the possible existence of an electromagnetically induced transition between muonium and antimuonium. An interaction that results in such transitions would also lead to the process

$$e^-+e^-\to\mu^-+\mu^-. \quad (1)$$

* Work done under the auspices of the U. S. Atomic Energy Commission.

The experimental investigation of this reaction, however, requires high-energy electrons, as, for example, in the colliding-beams experiment being planned at Stanford,[1] and hence is much more expensive than the type of experiment we have in mind. On the other hand, it is possible that reaction (1) will occur but will be mediated by some interaction that does not entail the induced conversion of muonium ($M \equiv \mu^+e^-$) into antimuonium ($\bar{M} \equiv \mu^-e^+$) by an external electromagnetic field. Thus we believe that the experiment we are suggesting is in part a simple alternative and in part a valuable supplement to the production experiment (1).

The second purpose of the experiment we are proposing is to determine the relative intrinsic parity of the muonium and antimuonium systems. There is, at present, no experimental information regarding the relative M-$\bar{M}$ parity, because no transitions between the two systems have been observed and because the relative μ-e parity, which, if known, would determine the relative M-$\bar{M}$ parity, can not be determined from the observed μ-e transitions since these violate parity. The importance of the possibility of determining the relative M-$\bar{M}$ parity for the understanding of parity conservation in electromagnetic phenomena and of the absence of the processes

$$\mu \to e+\gamma\,, \tag{2}$$

$$\mu \to 3e\,, \tag{3}$$

has been discussed by one of us.[2] The considerations of Ref. 2 can be briefly summarized as follows.

If the M-$\bar{M}$ parity turns out to be odd, $\eta(M)=-\eta(\bar{M})$, then this information coupled with the relation $\eta(l^+)\times\eta(l^-)=-1$ for the product of the intrinsic parities of a Dirac particle l and its antiparticle, would determine the relative parity of the muon and electron of the same charge to be imaginary, $\eta^2(\mu^\pm)=-\eta^2(e^\pm)$. This result has important implications for processes (2) and (3). There are, *a priori*, three possible alternatives concerning the existence and characteristics of these reactions: (a) They are absent. (b) They occur but do not exhibit the asymmetries that indicate parity violation in the usual sense. (c) They occur and do exhibit these asymmetries.

If now the relative μ-e parity is determined to be imaginary, or more precisely, if the single-particle states of the μ and the e of the same charge are determined to belong to eigenvalues of the operator corresponding to double space inversion that differ by (-1), then the measurement of *any* operator connecting the states in processes (2) and (3) is incompatible with space-reflection symmetry. That is, the very occurrence of these processes would violate invariance under space reflection, since no combination of spatial parities could then compensate for the changes in intrinsic parities. The assumption made in Ref. 2, that it is a fundamental property of electromagnetic phenomena that their *observation*—i.e., their occurrence itself—must be compatible with space-reflection symmetry, thus excludes both alternative (b) and alternative (c). This assumption can thus account both for parity conservation in the usual sense in electromagnetic phenomena and, if the relative μ-e parity is imaginary, for the absence of the processes (2) and (3).

The condition that the measurement of a subset of observables be compatible with invariance under space reflection is to be distinguished from the condition that parity be conserved in the usual sense, namely, that the relevant Hamiltonian not contain terms with opposite space-reflection properties. The latter condition cannot account for the absence of processes (2) and (3), for, in this case, these decays could occur without exhibiting left-right asymmetries. That is, alternative (b) could *not* be excluded on the basis of this condition alone.

II. CONVERSION OF MUONIUM

Conversion of muonium into antimuonium has been discussed by Pontecorvo[3] and by Feinberg and Weinberg.[4] The basis of the considerations of these authors is the observation that in the presence of an interaction that causes M-$\bar{M}$ interconversion the eigenstates of the total Hamiltonian are linear combinations M_1, M_2 of M and $\bar{M}$. Consequently, a state which is initially pure muonium will in time develop a component and antimuonium, and, in fact, the fractions of the system that decay as μ^+ and μ^- will be (damped) oscillatory functions of time. The authors of Refs. 3 and 4 conclude that the conversion process would be very difficult to observe, in effect, because in a medium suitable for formation of muonium that M_1-M_2 energy difference is so large as to quench very strongly the $M \to \bar{M}$ conversion.

We would like to consider the possibility that this transition can be induced by an external electromagnetic field. It is useful to examine the differences in the physical mechanism for the M-$\bar{M}$ conversion process considered in Refs. 3 and 4 and that considered by us, and the implication for the observability of this process entailed by these differences.

In the absence of the interaction that converts muonium and antimuonium into each other and in the absence of inhomogeneous electromagnetic fields these two states are degenerate. The conversion mechanism discussed in Refs. 3 and 4 depends essentially on interference between these two quantum states. It is thus a

[1] G. K. O'Neill, *Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959* (CERN, Geneva, 1959), p. 125; W. K. H. Panofsky, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 769; B. Richter, *Proceedings of the International Conference on Theoretical Aspects of Very High-Energy Phenomena, CERN, 1961* (CERN, Geneva, 1961), p. 57.

[2] R. Spitzer, Nucl. Phys. **51**, 553 (1964).

[3] B. Pontecorvo, Zh. Eksperim. i Teor. Fiz. **33**, 549 (1957) [English transl.: Soviet Phys.—JETP **6**, 429 (1958)].

[4] G. Feinberg and S. Weinberg, Phys. Rev. **123**, 1439 (1961).

manifestation of the superposition principle of quantum mechanics and as such an essentially quantum effect.

In order for these two states to interfere, their energy difference must be comparable to or *smaller* than the muon decay width. Thus, if the difference in energy between two states is much greater than the width of either state, the two states are by definition resolved, which eliminates the possibility of interference between them. From another point of view, in order that any interference effect between two states separated by an energy difference Δ not average out to zero, the time τ of the measurement of the relative phase cannot be large compared to the period Δ^{-1} with which the phase rotates, $\Delta \lesssim \tau^{-1}$. For a decay, however, τ^{-1} can be at most of the order of the decay width λ, $\tau^{-1} \gtrsim \lambda$. Hence, the condition on the observatility of the relative phase of two such states is that their energy be at most of the order of the natural width, $\Delta \lesssim \lambda$. The result of Ref. 4, that the probability $\bar{P}(\infty)$ of the muon decaying as μ^- rather than as μ^+ is quenched strongly as the energy difference Δ' between M_1 and M_2 becomes appreciably larger than the muon decay rate, is therefore to be expected.

In our case, the mechanism for M-$\bar{M}$ conversion differs in principle from that discussed in Refs. 3 and 4 in that the phenomenon of induced emission (or absorption) is already present in classical theory. More to the point, the experiment involves not an interference effect but rather the determination of an energy difference Δ between the states M and $\bar{M}$, which requires a time not less than Δ^{-1}. Thus the condition for observability of the effect we consider is that Δ should be *larger* than the muon decay width.

It is therefore not surprising that experimental conditions unfavorable for the observation of one effect because they result in too large an energy splitting should turn out to be favorable for the observation of the other. Indeed we find that in our case $\bar{P}(\infty)$ is independent of Δ, provided the resonance condition is satisfied.

To be sure, the authors of Ref. 4 consider the effect of a time-dependent external field, but only for the purpose of decreasing the energy difference Δ'. They find that such a field does not eliminate the quenching.

We assume that the off-diagonal matrix element of the part of the Hamiltonian that leads to M-$\bar{M}$ conversion is of the form

$$H_{ab}' = \tfrac{1}{2} R^* e^{-i\omega t}, \tag{4}$$

where R depends on the intensity of the rf field whose frequency is ω. This can be considered to be our basic dynamical assumption. We also assume that H' conserves parity in the usual sense (which assumption again differs from that made in Ref. 4) and conserves muons modulo 2. We do not inquire further into the structure of the part of H' that connects muonium and antimuonium, except to mention that it cannot be a minimal electromagnetic interaction, which would lead to the unobserved decays (2) and (3). It is to be understood, however, that the matrix element (4) for one-photon emission is to include the effect of conventional electromagnetic coupling to as many orders as may be necessary. Also the diagonal matrix elements H_{aa}' and H_{bb}' are to include the effect of any external medium.

To the best of our knowledge, there is no experimental evidence against the existence of such an interaction. The apparent absence of the transition $M \to \bar{M} + \gamma$ in the absence of an rf field of appropriate frequency is not surprising, since spontaneous emission is generally a negligible phenomenon at microwave and lower frequencies. We expressly assume that by increasing sufficiently the number of photons in the resonant cavity in which the M-$\bar{M}$ conversion is to be effected, the transition rate will increase to an observable level.

The equations of motion for the system and their solution can be obtained by standard methods. Details are given in Appendix A, and in this section we only outline the main steps of the calculation. We treat the decay of the muon phenomenologically in terms of a decay constant γ, which is assumed to be the same for muonium and for antimuonium. This is justified by the expectation that the decay characteristics of the μe system should be very similar to those of free muons.

With these assumptions, the equations of time-dependent perturbation theory become

$$\begin{aligned} i\dot{a} &= H_{aa}'a + H_{ab}'b - \tfrac{1}{2} i\gamma a, \\ i\dot{b} &= H_{ba}'a + H_{bb}'b - \tfrac{1}{2} i\gamma b, \end{aligned} \tag{5}$$

where a, b are the amplitudes of the states $|M\rangle$, $|\bar{M}\rangle$, respectively. At resonance, the fraction of the original muonium which remains at time t as muonium is

$$|a|_0^2 = \tfrac{1}{2} e^{-\gamma t}(1 + \cos|R|t), \tag{6}$$

while the fraction that has turned into antimuonium is

$$|b|_0^2 = \tfrac{1}{2} e^{-\gamma t}(1 - \cos|R|t). \tag{7}$$

The total probability that the muon decays as antimuonium (still at resonance) is

$$P(\bar{M}) = \tfrac{1}{2}|R|^2/(\gamma^2 + |R|^2). \tag{8}$$

In the next section, we consider the physical origin of the energy difference between muonium and antimuonium in a gas-filled resonant cavity and discuss the behavior of the system in the cavity.

III. DETAILS OF THE EXPERIMENT

The experiment we propose consists of two stages: (1) the formation of muonium in a helium-filled cavity excited at a frequency corresponding to the energy difference between muonium and antimuonium; and (2) the subsequent detection of the high-energy electrons which are a sign of antimuonium formation.

TABLE I. The interaction energy and auxiliary quantities for helium-muonium and helium-antimuonium interaction. $\Delta(R)$ is the difference between the interaction energy of helium and muonium and that of helium and antimuonium. I_{13}' is an overlap integral defined by Eq. (B16) of the text. R is the separation distance between helium and the μe system. All quantities are in atomic units.

R	$I_{13}'(R)$	$\Delta(R)$	$R^2\Delta(R)$
0.05	0.09621	76.18	0.1904
0.10	0.4148	34.55	0.3455
0.20	0.6476	13.65	0.5460
0.30	0.6451	7.395	0.6651
0.40	0.6181	4.557	0.7296
0.60	0.5479	2.091	0.7528
0.80	0.4680	1.103	0.7059
1.488	0.2210	0.2061	0.4563
2.233	0.07757	0.05308	0.2648
2.977	0.02360	0.01546	0.1365
3.721	0.006578	0.00442	0.0612
4.465	0.001731	0.00121	0.0241
5.209	0.0004383	0.000318	0.00863
5.953	0.0001081	0.0000809	0.00287
6.698	0.00002613	0.0000201	0.00090

The energy difference between muonium and antimuonium arises (in part) from the difference of interaction energies of the two systems with the helium in the cavity. This is calculated in Appendix B. The main reason for this difference is the fact that in the muonium-helium system all three electrons are indistinguishable, while in the antimuonium system only two have the same charge. We neglect completely, in calculating the M-$\bar{M}$ energy difference, the possible shift in resonance frequency due to the rf field and, in Appendix A, the annihilation of positrons as a mechanism for antimuonium depletion. The justification for this is that we expect the theoretical uncertainties introduced by these approximations to be smaller than those due to the use of approximate wave functions in calculating the energy splitting. Indeed, we do not claim the calculations to be very accurate, so that a certain amount of scanning of a limited frequency range around the calculated frequency might be necessary in the actual experiment.

In estimating the value of the energy difference Δ relevant to the resonance experiment, we have calculated an average, as defined by Eq. (B24), over the whole medium of the energy due to interaction with individual helium atoms. This is the relevant quantity if the characteristic periods of the system are such that a time average of the interaction energy is justified. If, however, the collision frequency in the gas is much smaller than the frequency corresponding to the most probable energy difference, it is the most probable rather than the average energy that will be appropriate. These two quantities could be significantly different in magnitude since, as is evident from Table I, the integral (B24) is weighted very heavily for values of R that, at standard pressure and temperature, are much smaller than the most probable nearest neighbor. The appropriate Δ to be calculated thus depends to a certain extent on the conditions under which the experiment is to be carried out, and further discussion of the magnitude of this quantity would not be very meaningful without more detailed knowledge of the practical limitations on these conditions.

We used helium in our considerations because the calculation of the interaction energy between helium and the μe system can be carried out almost entirely analytically. Although we know of no reason why muonium formation in helium should not be possible in principle, the only experiments in which muonium formation in a gas has, to our knowledge, been observed have used argon.[5] If this difference turns out to be a crucial one in the practical realization of the experiment we suggest, the calculation of the interaction energy would have to be repeated using argon wave functions.

The M-$\bar{M}$ conversion can be detected by observing the fast electrons resulting from the decay of negative muons. If only low-energy incident muons are used, all electrons generated by scattering processes will be easily distinguishable from those resulting from μ^- decay by their substantially lower energies. A possible alternative source of high-energy electrons is the annihilation of positrons from μ^+ decay and subsequent pair-production by the photon. However, even this mechanism yields an electron with only about one-fourth of the energy of that coming from μ^- decay. The observation of a sufficiently fast electron would thus confirm the existence of the conversion process.

The characteristics of the two resonant modes of interest to use are given in Appendix C. We consider a rectangular cavity excited in the TM_{110} mode, with the muons incident along the z axis of the cavity. At the frequencies of interest, only $n=1$ to $n=1$ transitions need be taken into account. Since 0-0 transitions are forbidden for one-photon processes, which are all that we consider, the M-$\bar{M}$ transition must be one between either the states $F=0$ and $F=1$ or the states $F=1$ and $F=1$. The former can occur only by dipole radiation, the latter by either dipole or quadrupole radiation. Strictly speaking, the energy difference Δ should be modified in the case of 0-1 transitions to include the hyperfine energy of the μe system, but for sufficiently high frequencies this can be neglected. For reasons which will be made clear below, it is sufficient for our purposes to consider only the dipole transitions and hereinafter we confine our attention to these.

In the expansion of the TM_{100} mode in multipole fields about an origin anywhere on the z axis (as defined in Appendix C), only the electric, and not the magnetic, dipole field has a nonvanishing amplitude. Hence only an electric dipole M-$\bar{M}$ transition can be induced for muonium formed on the z axis of the cavity, so that the conversion could occur, in this case, only if the relative

[5] W. E. Cleland, J. M. Bailey, M. Eckhause, V. W. Hughes, R. M. Mobley, R. Prepost, and J. E. Rothberg, Phys. Rev. Letters **13**, 202 (1964).

M-$\bar{M}$ parity is odd. The reason it is not necessary for our purposes to consider the quadrupole transitions is that it is the magnetic rather than the electric quadrupole field that has a nonvanishing component for an expansion about the z axis, and the parity selection rule is the same as for the electric-dipole transition.

Actually, the muon beam has a finite width, so that the multipole expansion about points slightly off the z axis has to be considered. The amplitudes of the dipole fields are then given by Eqs. (C19) and (C20). We see that both the electric and magnetic dipole fields now have nonvanishing amplitudes. Although the former are very much larger than the latter, it is not possible to establish from this fact alone whether the M-$\bar{M}$ transition, assuming it takes place, was induced by the electric or magnetic dipole field, because the absolute strength of the interaction is an unknown parameter. The occurrence itself of the M-$\bar{M}$ conversion induced by the TM_{110} mode, under the conditions described above, would therefore not be sufficient to determine the relative M-$\bar{M}$ parity.

In principle, this parity could be determined by measuring the polarization of the antimuonium. In practice, it may be simpler to follow an alternative procedure.

Consider the experimental arrangement described above with the TM_{110} mode now replaced by the TE_{101} mode. For this mode, the amplitudes of the dipole fields centered about an origin near the z axis are given by Eqs. (C27) and (C28). The average intensities of the electric- and magnetic-dipole fields are, in this case, of the same order of magnitude. Assuming the conversion process is detected with the TM_{110} mode excited, the relative M-$\bar{M}$ parity could then be determined by repeating the experiment with the TE_{101} mode excited with the same intensity. If in the second experiment the number of fast electrons is comparable to that in the first one, the conversion will have been induced by the electric dipole field, and the relative M-$\bar{M}$ parity is odd. If changing the mode results in an increase in the number of fast electrons by a factor of order of the square of the ratio of the dimension of the cavity to that of the muon beam, the conversion will have been induced by the magnetic-dipole field, and the relative M-$\bar{M}$ parity is even. An intermediate result would imply that the process does not conserve parity. The last two possibilities, though of intrinsic interest themselves, would not lead to the previously proposed[2] explanation for the absence of the decays (2) and (3).

IV. SUMMARY

We have suggested an experiment to detect the possible existence of an electromagnetically induced conversion of muonium into antimuonium. The experiment consists of the formation of muonium in a gas-filled resonant cavity excited at a frequency corresponding to the M-$\bar{M}$ energy difference, and the subsequent detection of high-energy electrons resulting from μ^- decay. The ratio of the number of electrons obtained when a given cavity mode is excited to that when a different mode is excited provides information on the relative intrinsic M-$\bar{M}$ parity. If this relative parity is odd, then the relative μ-e parity is imaginary. Parity conservation in electromagnetic phenomena and the absence of purely electromagnetic transitions between single-particle states of the muon and electron could then be understood on the basis of a single previously suggested[2] assumption.

ACKNOWLEDGMENTS

We want to thank B. DeFacio and R. Arndt for assistance in the numerical calculations and A. Karo for supplying us with some of the values of the integral I_{13}' used in Appendix B. We are grateful to T. Bowen for correspondence helpful in clarifying several points related to a possible experiment.

APPENDIX A

The equations of time-dependent perturbation theory can be put in the form

$$\begin{aligned} i\dot{a}&=H_{aa}'a+H_{ab}'b-\tfrac{1}{2}i\gamma a\,,\\ i\dot{b}&=H_{ba}'a+H_{bb}'b-\tfrac{1}{2}i\gamma b\,, \end{aligned} \tag{A1}$$

where a, b are the amplitudes of the states of muonium and antimuonium, respectively. The decay is treated phenomenologically by introducing a decay constant γ.

We now define

$$H_{aa}'\equiv\Delta+\delta,\quad H_{bb}'\equiv\delta\,, \tag{A2}$$

and redefine the amplitudes a, b so as to absorb into each an over-all phase involving the part of the mass shift δ that is common to muonium and antimuonium. With the assumption (4) for H_{ab}', Eqs. (A1) then become

$$\begin{aligned} i\dot{a}&=\Delta a+\tfrac{1}{2}R^*e^{-i\omega t}b-\tfrac{1}{2}i\gamma a\,,\\ i\dot{b}&=\tfrac{1}{2}\,\mathrm{Re}^{i\omega t}a-\tfrac{1}{2}i\gamma b\,. \end{aligned} \tag{A3}$$

In writing the matrix element H_{ab}' in the form (4), we have kept only the part of the oscillatory term that can lead to resonance ($\omega=\Delta$), so that Eqs. (A3) are valid for either positive or negative ω and Δ, provided a negative frequency is interpreted as energy absorbed from the rf field.

The general solutions of Eqs. (A3) are

$$\begin{aligned} a&=A_1e^{\mu_1 t}+A_2e^{\mu_2 t}\,,\\ b&=(2i/R^*)\big[(\mu_1+\tfrac{1}{2}\gamma+i\Delta)A_1e^{(\mu_1 t+i\omega)t}\\ &\qquad+(\mu_2+\tfrac{1}{2}\gamma+i\Delta)A_2e^{(\mu_2+i\omega)t}\big]\,, \end{aligned} \tag{A4}$$

where

$$\begin{aligned} \mu_1&=-\tfrac{1}{2}(\gamma+i\omega+i\Delta)+\tfrac{1}{2}i\alpha\,,\\ \mu_2&=-\tfrac{1}{2}(\gamma+i\omega+i\Delta)-\tfrac{1}{2}i\alpha\,,\\ \alpha&=((\Delta-\omega)^2+|R|^2)^{1/2}. \end{aligned} \tag{A5}$$

For the initial conditions

$$a(0)=1, \quad b(0)=0, \tag{A6}$$

corresponding to pure muonium, we get for the probability that at time t we still have muonium

$$|a|^2=\frac{(\Delta-\omega-\alpha)^2}{4\alpha^2}e^{-\gamma t}\left[1+\left(\frac{\alpha+\Delta-\omega}{\alpha-\Delta+\omega}\right)^2 +2\left(\frac{\alpha+\Delta-\omega}{\alpha-\Delta+\omega}\right)\cos\alpha t\right], \tag{A7}$$

whereas the probability that the system has turned into antimuonium is

$$|b|^2=(|R|^2/2\alpha^2)e^{-\gamma t}(1-\cos\alpha t). \tag{A8}$$

At resonance, i.e., for $\Delta=\omega$, Eqs. (A7) and (A8) reduce to

$$|a|_0^2=\tfrac{1}{2}e^{-\gamma t}(1+\cos|R|t) \tag{A7$'$}$$

and

$$|b|_0^2=\tfrac{1}{2}e^{-\gamma t}(1-\cos|R|t), \tag{A8$'$}$$

which are Eqs. (6) and (7) of the text. The ratio of muons that decay as μ^+ to those that decay as μ^- is therefore an oscillatory function of time.

The total probability, up to time T after formation of muonium, that the muon decays as μ^- (still at resonance) is

$$\begin{aligned} P(\bar{M},T)&\equiv\gamma\int_0^T|b|_0^2dt \\ &=\tfrac{1}{2}(1-e^{-\gamma T})-\tfrac{1}{2}[\gamma/(\gamma^2+|R|^2)] \\ &\quad\times\{\gamma-e^{-\gamma T}[\gamma\cos|R|T-|R|\sin|R|T]\}. \end{aligned} \tag{A9}$$

For $T=\infty$, this gives

$$P(\bar{M})\equiv P(\bar{M},\infty)=\tfrac{1}{2}|R|^2/(\gamma^2+|R|^2), \tag{A10}$$

which is Eq. (8) of the text. We see that for $|R|\gg\gamma$, the probability $P(\bar{M})$ approaches one-half. This is what one would expect, since if the period of oscillation between muonium and antimuonium is much shorter than the muon lifetime, the time averages of the intensities of muonium and antimuonium states should be equal.

Denoting by $P(M)$ the total probability that (at resonance) the muon decays as μ^+, and using the relation

$$P(M)+P(\bar{M})=1,$$

we find

$$|R|^2/\gamma^2=\tfrac{1}{2}P(\bar{M})/[P(M)-P(\bar{M})]. \tag{A11}$$

APPENDIX B

The notation for the coordinates appearing in the Hamiltonian of the He-(bound μe) system is shown in Fig. 1. Here A denotes the helium nucleus, B the muon, and 1, 2, 3 the three electrons. The distance R between the two "nuclei" is considered to be a fixed parameter.

It is convenient to decompose the Hamiltonian for the He-M system as follows:

$$H_M=H_a+H_b+U_M, \tag{B1}$$

$$H_a=-\frac{1}{2m}(\Delta_1+\Delta_2)-z'e^2\left(\frac{1}{r_{a1}}+\frac{1}{r_{a2}}\right), \tag{B2}$$

$$H_b=-\frac{1}{2m}\Delta_3-\frac{e^2}{r_{b3}}, \tag{B3}$$

$$U_M=(z'-2)e^2\left(\frac{1}{r_{a1}}+\frac{1}{r_{a2}}\right)-\frac{2e^2}{r_{a3}}-e^2\left(\frac{1}{r_{b1}}+\frac{1}{r_{b2}}\right) +e^2\left(\frac{1}{r_{12}}+\frac{1}{r_{13}}+\frac{1}{r_{23}}+\frac{2}{R}\right), \tag{B4}$$

where Δ_i is the Laplacian operator acting on the coordinates of electron i. The Hamiltonian is separated in this fashion because we construct the approximate wave functions used in our treatment from the normalized $1s$ hydrogen-like wave functions

$$\begin{aligned} a(i)&=(z'^3/\pi a_0^3)^{1/2}\exp[-z'r_{ai}/a_0], \\ b(i)&=(1/\pi a_0^3)^{1/2}\exp[-r_{bi}/a_0], \end{aligned} \tag{B5}$$

where a_0 is the Bohr radius. We then have

$$H_aa(1)a(2)=E_aa(1)a(2), \quad H_bb(3)=E_0b(3), \tag{B6}$$

with

$$E_a=2z'^2E_0 \tag{B7}$$

and where

$$E_0=-e^2/2a_0 \tag{B8}$$

is the energy of the hydrogen atom in the ground state (-13.6 eV$=-\frac{1}{2}$ a.u.).

We choose $z'=27/16$, which minimizes the energy of the free helium atom when the helium wave function is written as the product of two $1s$ hydrogenic wave functions, and the nuclear charge is considered as a variable parameter.[6] In some sense, then, the three parts (B2)–(B4) of the Hamiltonian represent the helium atom, muonium, and the interaction.

For our approximate wave function, we choose

$$\begin{aligned} \psi_M=C[&a(1)a(2)b(3)\chi(1,2)\alpha(3) \\ &-a(1)b(2)a(3)\chi(1,3)\alpha(2) \\ &+b(1)a(2)a(3)\chi(2,3)\alpha(1)], \end{aligned} \tag{B9}$$

where

$$\chi(i,j)=2^{-1/2}[\alpha(i)\beta(j)-\beta(i)\alpha(j)] \tag{B10}$$

is the singlet spin function and $\alpha(i)$, $\beta(j)$ are the usual orthonormal spin functions. The wave function (B9) is normalized to unity, which gives for the normalization constant

$$C=[3(1-S^2)]^{-1/2}, \tag{B11}$$

[6] L. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1935).

$$S=\frac{8z'^{3/2}}{R(z'^2-1)^2}\left\{\left[R+\frac{4'z}{z'^2-1}\right]e^{-z'R}+z'\left[R-\frac{4}{z'^2-1}\right]e^{-R}\right\}. \quad \text{(B12)}$$

This wave function is properly antisymmetrized to represent a three-electron system.

The energy of the system is given in this approximation by

$$E(R)=\int \psi_M{}^* H_M \psi_M d\tau_1 d\tau_2 d\tau_3. \quad \text{(B13)}$$

Substituting the Hamiltonian (B1) and the wave function (B9) into Eq. (B13), we get

$$\begin{aligned}E(R)=&E_a+E_0\\&+(1-S^2)^{-1}\left\{\int d\tau_1 d\tau_2 d\tau_3 a^2(1)a^2(2)b^2(3)U_M\right.\\&\left.-\int d\tau_1 d\tau_2 d\tau_3 a^2(1)a(2)b(2)a(3)b(3)U_M\right\}. \quad \text{(B14)}\end{aligned}$$

Using the specific form of U_M, as given by Eq. (B4), and Eqs. (B7) and (B8), we get (in atomic units)

$$\begin{aligned}E(R)=&(2/R)-\tfrac{1}{2}-Z'^2+(1-S^2)^{-1}\{-z'(2-z')(2-S^2)\\&+\tfrac{5}{8}z'-(2-S^2)I_{1b}+2(I_{12}-I_{1a})+I_{1b}'\\&+(2-z')I_{1a}'-2(I_{12}'-I_{1a}')-I_{13}'\}. \quad \text{(B15)}\end{aligned}$$

This result agrees with Eqs. (A2), (A3) of Ref. 7, for the values $\lambda=0$, $z=1$ in these equations. The various terms are defined as follows:

$$\begin{aligned}I_{1a}&\equiv\int d\tau_1 b^2(1)\frac{1}{r_{a1}},\\ I_{1b}&\equiv\int d\tau_1 a^2(1)\frac{1}{r_{b1}},\\ I_{12}&\equiv\int d\tau_1 d\tau_2 a^2(1)b^2(2)\frac{1}{r_{12}},\\ I_{1a}'&\equiv S\int d\tau_1 a(1)b(1)\frac{1}{r_{a1}}, \quad \text{(B16)}\\ I_{1b}'&\equiv S\int d\tau_1 a(1)b(1)\frac{1}{r_{b1}},\\ I_{12}'&\equiv S\int d\tau_1 d\tau_2 a^2(1)a(2)b(2)\frac{1}{r_{12}},\\ I_{13}'&\equiv\int d\tau_1 d\tau_2 a(1)b(1)a(2)b(2)\frac{1}{r_{12}}.\end{aligned}$$

[7] E. A. Mason, J. Ross, and P. N. Schatz, J. Chem. Phys. **25**, 626 (1956).

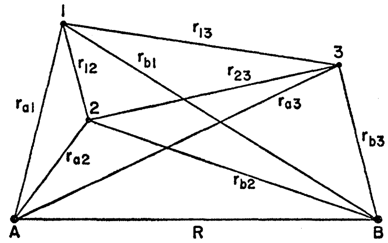

FIG. 1. Coordinates appearing in the Hamiltonian of the He-(bound μe) system.

In arriving at Eq. (B15), we have also used the relations

$$\int d\tau_1 a^2(1)\frac{1}{r_{a1}}=z' \quad \text{(B17)}$$

and

$$\int d\tau_1 d\tau_2 a^2(1)a^2(2)\frac{1}{r_{12}}=\frac{5z'}{8}, \quad \text{(B18)}$$

which are given in Pauling and Wilson's book.[6]

All the integrals in Eq. (B16), except the last one, can be found in the literature in closed form.[8] The integral I_{13}' is given as an infinite series.[9]

We next consider the interaction of antimuonium with helium. The treatment is very similar to that of the muonium-helium interaction. In particular, the first two parts of the Hamiltonian, H_a and H_b, are still given by Eqs. (B2) and (B3), but the interaction Hamiltonian $U_{\bar{M}}$ will have some different signs:

$$\begin{aligned}U_{\bar{M}}=&(z'-2)e^2\left(\frac{1}{r_{a1}}+\frac{1}{r_{a2}}\right)+\frac{2e^2}{r_{a3}}+e^2\left(\frac{1}{r_{b1}}+\frac{1}{r_{b2}}\right)\\&+e^2\left(\frac{1}{r_{12}}-\frac{1}{r_{13}}-\frac{1}{r_{23}}-\frac{2}{R}\right). \quad \text{(B19)}\end{aligned}$$

The most important difference, however, between the muonium-helium and the antimuonium-helium interaction is the fact that in the latter the three electrons do not all have the same charge, and hence the wave function is not to be antisymmetrized in the coordinates of all three electrons. In particular, we choose, in this case,

$$\psi_{\bar{M}}=a(r_1)a(r_2)b(r_3)\chi(1,2), \quad \text{(B20)}$$

and hence we get, again in atomic units,

$$\begin{aligned}\bar{E}(R)=&-z'^2-\tfrac{1}{2}-2(2-z')z'+2I_{1a}+2I_{1b}\\&-\frac{2}{R}+\frac{5z'}{8}-2I_{12}. \quad \text{(B21)}\end{aligned}$$

We consider the difference between the two interaction energies given by Eqs. (B15) and (B21), which

[8] M. P. Barnett and C. A. Coulson, Trans. Roy. Soc. (London) **A243**, 221 (1951); C. C. J. Roothaan, J. Chem. Phys. **19**, 1445 (1951).

[9] K. Rudenberg, J. Chem. Phys. **19**, 1459 (1951).

turns out to be

$$\Delta(R)\equiv E(R)-\bar{E}(R)=\frac{4}{R}+\frac{z'S^2}{1-S^2}[\tfrac{5}{8}-(2-z')]+\frac{2(2-S^2)}{1-S^2}(I_{12}-I_{1a})-\frac{4-3S^2}{1-S^2}I_{1b}+(1-S^2)^{-1}$$
$$\times[I_{1b}'+(2-z')I_{1a}'-2(I_{12}'-I_{1a}')-I_{13}']. \quad \text{(B22)}$$

Substituting the closed expressions for all the integrals of Eq. (B16) except the last one, we get

$$\Delta(R)=\frac{S^2}{1-S^2}\left(\frac{5z'}{8}-(2-z')z'-\frac{1}{R}\right)+\frac{4}{1-S^2}\left\{\left[\frac{3z'^2-1}{R(z'^2-1)^3}-\frac{2z'^2-1}{(z'^2-1)^2}\right]e^{-2R}+\left[\frac{z'^3(z'^2-2)}{(z'^2-1)^2}-\frac{z'^4(3-z'^2)}{R(z'^2-1)^3}\right]e^{-2z'R}\right\}$$
$$-\frac{S^2}{1-S^2}\left\{2\left[\frac{3z'^2-1}{R(z'^2-1)^3}-\frac{2z'^2-1}{(z'^2-1)^2}\right]e^{-2R}+\left[\frac{1}{R}\left(3-\frac{2(3z'^2-1)}{(z'^2-1)^3}\right)+3z'-\frac{2z'}{(z'^2-1)^2}\right]e^{-2z'R}\right\}$$
$$+\frac{S}{1-S^2}\left\{\left[\frac{5z'^{3/2}}{4(z'^2-1)}+\frac{8z'^{3/2}(15z'^2-1)}{(9z'^2-1)^2}\right]+\frac{1}{R}\left[\frac{4z'^{3/2}(2z'-\frac{5}{8})}{(z'^2-1)^2}-\frac{16z'^{3/2}(21z'^2-1)}{(9z'^2-1)^3}\right]\right\}e^{-R}$$
$$-\frac{S}{1-S^2}\left\{\left[\frac{4z'^{3/2}}{z'^2-1}+\frac{1}{R}\frac{4z'^{3/2}(2z'-\frac{5}{8})}{(z'^2-1)^2}\right]e^{-z'R}-\left[\frac{16z'^{5/2}}{(9z'^2-1)^2}+\frac{1}{R}\frac{16z'^{3/2}(21z'^2-1)}{(9z'^2-1)^3}\right]e^{-3z'R}\right\}-\frac{I_{13}'}{(1-S^2)}. \quad \text{(B23)}$$

The value of the last integral, I_{13}', is given in Table I.

Ultimately, we are interested in the total difference in the energy of muonium and antimuonium due to the interaction of each with the whole medium. We therefore want to average the difference in interaction energies over all the helium atoms with which the μe system interacts. The number of helium atoms at distance R from the μe system is $4\pi R^2\rho dR$, where ρ is the density of the helium gas, so we calculate the quantity

$$\Delta_{\text{av}}\equiv\int_0^\infty \Delta(R)4\pi R^2\rho dR. \quad \text{(B24)}$$

We assume the ideal gas law, which for helium holds to within 1% for temperatures up to 200°C and pressures up to 250 atm. The integral in Eq. (B24) was evaluated numerically. The result is

$$\Delta_{\text{av}}=5.16\times10^5P(300/T)\ \text{Mc-sec}^{-1}, \quad \text{(B25)}$$

where P is the pressure of the helium gas in atmospheres and T is the temperature in °K.

As can be seen from Table I, much of the contribution to Δ_{av} comes from very small values of R, for which our approximate wave functions are not expected to be very accurate because of the mutual distortion of the atoms. The present calculation should therefore be considered only as indicative of the range of frequencies to be scanned.

APPENDIX C

The general multipole expansion of the electromagnetic fields in a resonant cavity can be written in the form[10]

$$\mathbf{E}(\mathbf{x})=\sum_{lm}\left[a_{lm}j_l(kr)\mathbf{X}_{lm}+\frac{i}{k}b_{lm}\nabla\times j_l(kr)\mathbf{X}_{lm}\right],$$
$$\mathbf{B}(\mathbf{x})=\sum_{lm}\left[-\frac{i}{k}a_{lm}\nabla\times j_l(kr)\mathbf{X}_{lm}+b_{lm}j_l(kr)\mathbf{X}_{lm}\right], \quad \text{(C1)}$$

where

$$\mathbf{X}_{lm}(\theta\phi)=[l(l+1)]^{-1/2}\mathbf{L}Y_l^m(\theta\phi) \quad \text{(C2)}$$

are the normalized vector spherical harmonics, and we have included only the regular radial function j_l because the fields inside a cavity are finite everywhere.

The coefficients a_{lm} and b_{lm} are the amplitudes of, respectively, magnetic and electric (lm) multipole fields. They can be calculated for given electric and magnetic fields from the expressions[10]

$$a_{lm}j_l(kr)=\int\mathbf{X}_{lm}^*\cdot\mathbf{E}(\mathbf{x})d\Omega,$$
$$b_{lm}j_l(kr)=\int\mathbf{X}_{lm}^*\cdot\mathbf{B}(\mathbf{x})d\Omega. \quad \text{(C3)}$$

Consider now a rectangular cavity with dimensions a, b, d in the x, y, z directions, which is excited in the TM_{110} mode. The z direction is taken to coincide with that of the incident μ beam, and we choose the origin of the coordinate system at an arbitrary point on the axis of the cavity defined by the intersection of the two planes that bisect the cavity in the x and y directions. In other words, the origin is in the center of the

[10] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, Inc., New York, 1962), Chap. 16.

cavity with respect to the x and y directions but at an arbitrary point of the cavity in the z direction. We shall refer to the axis in the z direction with coordinates $x=y=0$ as the z axis.

The electric and magnetic fields for this mode are then[11] (omitting a constant amplitude)

$$\begin{aligned} &E_z=\cos(\pi x/a)\cos(\pi y/b)\,,\\ &E_x=E_y=0\,,\\ &B_z=0\,,\\ &B_x=-\zeta(\pi/b)\cos(\pi x/a)\sin(\pi y/b)\,,\\ &B_y=\zeta(\pi/a)\sin(\pi x/a)\cos(\pi y/b)\,,\end{aligned}\tag{C4}$$

where (we use Gaussian units)

$$\zeta\equiv[ik/(k_c{}^2)_{11}](\epsilon\mu)^{1/2}\tag{C5}$$

is defined in terms of the dielectric constant ϵ, the permeability μ and the two parameters

$$(k_c{}^2)_{mn}=(m\pi/a)^2+(n\pi/b)^2\,,\tag{C6}$$

$$k^2=k_c{}^2+(p\pi/\alpha)^2\,.\tag{C7}$$

The TM_{110} mode is characterized by $m=n=1$, $p=0$, so for this mode we have $k^2=k_c{}^2$.

For this mode, the expressions (C3) for the multipole coefficients become

$$\begin{aligned} a_{lm}&=4\pi mi^l[l(l+1)]^{-1/2}\,\mathrm{Re}Y_l{}^m(\Omega_k), &&\text{for } l \text{ even}\\ &=0, &&\text{for } l \text{ odd}\end{aligned}\tag{C8}$$

and

$$\begin{aligned} b_{lm}&=0, \qquad\qquad \text{for } l \text{ even}\\ &=2\pi^2 i^l\zeta[l(l+1)]^{-1/2}\{[(l-m)(l+m+1)]^{1/2}\\ &\quad\times[b^{-1}\,\mathrm{Im}Y^{m+1}(\Omega_k)+a^{-1}\,\mathrm{Re}Y_l{}^{m+1}(\Omega_k)]\\ &\quad+[(l+m)(l-m+1)]^{1/2}[b^{-1}\,\mathrm{Im}Y_l{}^{m-1}(\Omega_k)\\ &\qquad-a^{-1}\,\mathrm{Re}Y_l{}^{m-1}(\Omega_k)],\quad \text{for } l \text{ odd}.\end{aligned}\tag{C9}$$

The angles in the argument of $Y_l{}^m$ are defined by

$$\begin{aligned} &\cos\theta_k=0, &&\sin\theta_k=1\\ &\cos\phi_k=(\pi/ak)\,, &&\sin\phi_k=(\pi/bk)\,.\end{aligned}\tag{C10}$$

In particular, for $l=1$ we have

$$a_{1m}=0;\tag{C11}$$

that is, in the expansion of the TM_{110} mode in multipole fields about an origin anywhere on the z axis, the amplitude of the magnetic dipole field is zero. Furthermore,

$$\begin{aligned} &b_{11}=b_{1-1}=0\,,\\ &b_{10}=(6\pi\epsilon\mu)^{1/2}\,,\end{aligned}\tag{C12}$$

so that the electric dipole fields with respect to an origin anywhere on the z axis are

$$\begin{aligned} &\mathbf{E}^{ed}(\mathbf{x})=(i/k)(6\pi\epsilon\mu)^{1/2}\nabla\times j_1(kr)\mathbf{X}_{10}\,,\\ &\mathbf{B}^{ed}(\mathbf{x})=(6\pi\epsilon\mu)^{1/2}j_1(kr)\mathbf{X}_{10}\,.\end{aligned}\tag{C13}$$

In order to make use of the selection rules for angular momentum in multipole transitions, we want the origin of the spherical-wave expansion at the center of mass of the bound μe system. Since the muon beam has a finite width, we are therefore also interested in the multipole fields as seen from an origin slightly off the z axis of the cavity. We therefore transform to a coordinate system centered a small distance off the z axis,

$$\begin{aligned} &x=x'+\alpha, \quad |\pi\alpha/a|\ll 1\,,\\ &y=y'+\beta, \quad |\pi\beta/b|\ll 1\,,\\ &z=z'\end{aligned}\tag{C14}$$

and define the quantities

$$\begin{aligned} &\gamma_\alpha\equiv\cos(\pi\alpha/a)\simeq 1, \quad \lambda_\alpha\equiv\sin(\pi\alpha/a)\simeq\pi\alpha/a\,,\\ &\gamma_\beta\equiv\cos(\pi\beta/b)\simeq 1, \quad \lambda_\beta\equiv\sin(\pi\beta/b)\simeq\pi\beta/b\,.\end{aligned}\tag{C15}$$

We shall refer to the axis in the z direction with coordinates $x'=y'=0$ as the z' axis.

In terms of the new coordinate system, the nonvanishing components of the fields (C4) are, to first order in the small quantities λ_α, λ_β,

$$\begin{aligned} E_{z'}&\simeq\gamma_\alpha\gamma_\beta\cos\frac{\pi x'}{a}\cos\frac{\pi y'}{b}-\gamma_\alpha\lambda_\beta\cos\frac{\pi x'}{a}\sin\frac{\pi y'}{b}-\gamma_\beta\lambda_\alpha\sin\frac{\pi x'}{a}\cos\frac{\pi y'}{b}\,,\\ B_{x'}&\simeq-\zeta\frac{\pi}{b}\left[\gamma_\alpha\gamma_\beta\cos\frac{\pi x'}{a}\sin\frac{\pi y'}{b}+\gamma_\alpha\lambda_\beta\cos\frac{\pi x'}{a}\cos\frac{\pi y'}{b}-\gamma_\beta\lambda_\alpha\sin\frac{\pi x'}{a}\sin\frac{\pi y'}{b}\right],\\ B_{y'}&\simeq\zeta\frac{\pi}{a}\left[\gamma_\alpha\gamma_\beta\sin\frac{\pi x'}{a}\cos\frac{\pi y'}{b}-\gamma_\alpha\lambda_\beta\sin\frac{\pi x'}{a}\sin\frac{\pi y'}{b}+\gamma_\beta\lambda_\alpha\cos\frac{\pi x'}{a}\cos\frac{\pi y'}{b}\right].\end{aligned}\tag{C16}$$

Using these expressions and the same procedure as before, we get for the coefficients of the multipole fields about an origin at an arbitrary point on the z' axis

$$\begin{aligned} a_{lm}&\simeq 4\pi mi^l[l(l+1)]^{-1/2}\gamma_\alpha\gamma_\beta\,\mathrm{Re}Y_l{}^m(\Omega_k), &&\text{for } l \text{ even}\\ &\simeq 4\pi mi^{l+1}[l(l+1)]^{-1/2}\{\gamma_\beta\lambda_\alpha\,\mathrm{Re}Y_l{}^m(\Omega_k)-i\gamma_\alpha\lambda_\beta\,\mathrm{Im}Y_l{}^m(\Omega_k)\}\,, &&\text{for } l \text{ odd}\end{aligned}\tag{C17}$$

[11] *Americal Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1957), Sec. 5, p. 65.

and

$$b_{lm}\simeq\frac{2\pi^2 i^l\zeta}{[l(l+1)]^{1/2}}\bigg\{[(l-m)(l+m+1)]^{1/2}\bigg[\bigg(\frac{i\gamma_\beta\lambda_\alpha}{b}+\frac{\gamma_\alpha\lambda_\beta}{a}\bigg)\operatorname{Im}Y_l^{m+1}(\Omega_k)-\bigg(\frac{\gamma_\alpha\lambda_\beta}{b}-\frac{i\gamma_\beta\lambda_\alpha}{a}\bigg)\operatorname{Re}Y_l^{m+1}(\Omega_k)\bigg]$$
$$+[(l+m)(l-m+1)]^{1/2}\bigg[\bigg(\frac{i\gamma_\beta\lambda_\alpha}{b}-\frac{\gamma_\alpha\lambda_\beta}{a}\bigg)\operatorname{Im}Y_l^{m-1}(\Omega_k)-\bigg(\frac{\gamma_\alpha\lambda_\beta}{b}+\frac{i\gamma_\beta\lambda_\alpha}{a}\bigg)\operatorname{Re}Y_l^{m-1}(\Omega_k)\bigg]\bigg\},\quad\text{for } l \text{ even}$$
$$\simeq\frac{2\pi^2 i^l\zeta\gamma_\alpha\gamma_\beta}{[l(l+1)]^{1/2}}\bigg\{[(l-m)(l+m+1)]^{1/2}\bigg[\frac{1}{b}\operatorname{Im}Y_l^{m+1}(\Omega_k)+\frac{1}{a}\operatorname{Re}Y_l^{m+1}(\Omega_k)\bigg]$$
$$+[(l+m)(l-m+1)]^{1/2}\bigg[\frac{1}{b}\operatorname{Im}Y_l^{m-1}(\Omega_k)-\frac{1}{a}\operatorname{Re}Y_l^{m-1}(\Omega_k)\bigg]\bigg\},\quad\text{for } l \text{ odd.}\qquad\text{(C18)}$$

In particular, the coefficients of the dipole fields are (to lowest order in α,β)

$$a_{1,\pm1}\simeq-[\pi^2(3\pi)^{1/2}/k][(\alpha/a^2)\pm i(\beta/b^2)],\quad a_{10}\simeq0,\qquad\text{(C19)}$$

and

$$b_{1,\pm1}\simeq0,\quad b_{10}\simeq(6\pi\epsilon\mu)^{1/2}.\qquad\text{(C20)}$$

We see that the amplitudes of the electric dipole field in the multipole expansion about the μe system are, for this mode, larger than those for the magnetic dipole field by at least a factor of the order of the ratio of the dimension of the cavity to the beam width.

We now repeat the calculation for the TE_{101} mode, again taking the z direction to coincide with that of the muon beam.

The fields in this mode, in terms of a coordinate system with origin at a *corner* of the cavity, are[11]

$$\begin{aligned}B_z&=\mu\cos(\pi x/a)\sin(\pi x/d),\\ B_x&=-(\mu a/d)\sin(\pi x/a)\cos(\pi z/d),\\ B_y&=0,\\ E_z&=E_x=0,\\ E_y&=-\zeta'\sin(\pi x/a)\sin(\pi z/d),\end{aligned}\qquad\text{(C21)}$$

where

$$\zeta'\equiv(ika/\pi)(\mu/\epsilon)^{1/2},\qquad\text{(C22)}$$

and for this mode $(k_c^2)_{10}=(a/\pi)^2$ is not equal to k^2.

As in the case of the TM_{110} mode, we again obtain the coefficients of the multipole expansion about a point on the z' axis and a distance z_0 in from the face of the cavity. In terms of the parameters

$$\eta\equiv\cos(\pi z_0/d),\quad\tau\equiv\sin(\pi z_0/d),\qquad\text{(C23)}$$

the results are

$$\begin{aligned}a_{lm}\simeq&-\pi i^{l+1}\zeta'[l(l+1)]^{-1/2}\\ &\times\{[(l-m)(l+m+1)]^{1/2}Y_l^{m+1}(\Omega_k)\\ &\times[(l+m)(l-m+1)]^{1/2}Y_l^{m-1}(\Omega_k)\}\\ &\times\{\gamma_\alpha\tau[1-(-1)^m]+\lambda_\alpha\eta[1+(-1)^m]\},\quad\text{for } l\text{ even}\\ \simeq&\,\pi i^l\zeta'[l(l+1)]^{-1/2}\\ &\times\{[(l-m)(l+m+1)]^{1/2}Y_l^{m+1}(\Omega_k)\\ &-[(l+m)(l-m+1)]^{1/2}Y_l^{m-1}(\Omega_k)\}\\ &\times\{-\gamma_\alpha\eta[1-(-1)^m]+\lambda_\alpha\tau[1+(-1)^m]\},\quad\text{for } l\text{ odd}\end{aligned}\qquad\text{(C24)}$$

and

$$\begin{aligned}b_{lm}\simeq&-2\pi i^l\mu[l(l+1)]^{-1/2}\\ &\times\{(a/2d)[((l-m)(l+m+1))^{1/2}Y_l^{m+1}(\Omega_k)\\ &+((l+m)(l-m+1))^{1/2}Y_l^{m-1}(\Omega_k)]\\ &\times[\gamma_\alpha\eta(1-(-1)^m)-\lambda_\alpha\tau(1+(-1)^m)]\\ &+mY_l^m(\Omega_k)[\lambda_\alpha\tau(1+(-1)^m)-\gamma_\alpha\eta(1-(-1)^m)]\},\quad\text{for } l\text{ even}\\ \simeq&-2\pi i^{l+1}\mu[l(l+1)]^{-1/2}\\ &\times\{(a/2d)[((l-m)(l+m+1))^{1/2}Y_l^{m+1}(\Omega_k)\\ &+((l+m)(l-m+1))^{1/2}Y_l^{m-1}(\Omega_K)]\\ &\times[\gamma_\alpha\tau(1-(-1)^m)+\lambda_\alpha\eta(1+(-1)^m)]\\ &-mY_l^m(\Omega_k)[\lambda_\alpha\eta(1+(-1)^m)+\gamma_\alpha\tau(1-(-1)^m)]\},\quad\text{for } l\text{ odd.}\end{aligned}\qquad\text{(C25)}$$

For the TE_{101} mode, the angles in the argument of Y_l^m are defined by

$$\cos\theta_k=(\pi/dk),\quad\sin\theta_k=(\pi/ak),\quad\cos\phi_k=1,\quad\sin\phi_k=0.\qquad\text{(C26)}$$

For the special case of a dipole field, Eqs. (C24) and (C25) reduce to

$$\begin{aligned}a_{11}&=-(a\eta/d)(3\pi\mu/\epsilon)^{1/2},\\ a_{1-1}&=(a\eta/d)(3\pi\mu/\epsilon)^{1/2},\\ a_{10}&=(\pi\alpha\tau/a)(6\pi\mu/\epsilon)^{1/2},\end{aligned}\qquad\text{(C27)}$$

and

$$b_{1\pm1}=[\pi\mu a\tau(3\pi)^{1/2}/k][(1/d^2)+(1/a^2)],\quad b_{10}=0.\qquad\text{(C28)}$$

In this case, the amplitudes of some of the components of the magnetic dipole fields are of the same order as those of the electric dipole fields. On the z axis we have $\alpha=0$ and $a_{10}=0$. The parameters η and τ, and therefore the nonvanishing amplitudes, are oscillatory functions of position along the z or z' axis. The average intensity of the electric and magnetic dipole fields centered at a point on the z or z' axis will therefore be comparable.