from the percent intensities of the gamma rays de-exciting the levels fed by the respective beta-ray spectra. These values of $\log ft$ are summarized in Table III. The initial nucleus of each beta spectrum of Table III is taken to be that indicated by earlier investigations.[8,10]

V. DISCUSSION OF RESULTS

The shell-model orbitals of Te^{129m} and Te^{129} are $h_{11/2}$ and $d_{3/2}$, the 106-keV transition[4] being $M4$. The ground state of I^{129} has a measured spin[14] of $\frac{7}{2}$, shell-model orbital $g_{7/2}$. By lifetime measurements[15] and study[10] of the $(L+M)$-shells conversion coefficient, the 27-keV transition has been shown to be $M1$, the spin and parity of the 27-keV state in I^{129} being $\frac{5}{2}+$, shell-model orbital $d_{5/2}$. If indeed the allowed transitions of Table III do stem from Te^{129}, the possible spins and parities of the 482-, 797-, 1112-, and 1222-keV levels would be $\frac{3}{2}+$ or $\frac{5}{2}+$. The spin and parity of the 725-keV level, fed by a first-forbidden spectrum initiating at Te^{129m}, could be $\frac{7}{2}+$ or $\frac{9}{2}+$. The 1065-keV level does not de-excite with a transition to either the ground state or the 27-keV state. Its spin is therefore assumed to be $13/2+$, making the spin of the 725-keV state more probably $\frac{9}{2}+$. These various possible assignments of spin and parity would suggest that the bulk of the observed gamma-ray transitions in I^{129} are $M1$ or $E2$ or a mixture thereof. Others[10] have suggested possible spins and parities of $\frac{9}{2}+$, $11/2+$, or $13/2+$ for the 1385-keV level. All of these possible values are consistent with the gamma transitions from that level as shown in the decay scheme of Fig. 5.

Some theoretical efforts have been made to compute the energies of the excited states of I^{129}. Banerjee and Gupta[11] have based their theoretical calculation upon a model which assumes the nucleus to be an even-even core, with its spectrum of vibrational levels, and an odd nucleon giving rise to single-particle states. They find thirty-three excited states between the ground state and an excitation energy of 1075 keV. Somewhat better agreement has been found with the calculations of Kisslinger and Sorensen,[12] which are based upon the assumption of spherical nuclear shape with residual forces. They report the possibility of ten excited states on approximately the same energy interval. The spins predicted are such that virtually all the levels of either theoretical calculation should have been excited in the decay of Te^{129m}–Te^{129}. It is concluded that the proposed decay scheme produces best qualitative agreement with the works of Kisslinger and Sorensen.[12]

[14] Ralph Livingston, O. R. Gillian, and Walter Gordy, Phys. Rev. **76**, 149 (1949).
[15] D. W. Hafemeister, G. DePasquali, and H. deWaard, Phys. Rev. **135**, B1089 (1964).

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Nuclear Structure and Parity Impurities*

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Nuclear-structure calculations, relevant to the evaluation of irregular gamma-ray transition amplitudes (parity-forbidden multipoles), have been performed. General expressions have been derived for the case of one-particle states in odd-A nuclei with spherical or spheroidal shape. In particular, the pseudoscalar operator $\boldsymbol{\sigma}\cdot\mathbf{p}$ is discussed. The case of regular $M1$ plus irregular $E1$ transition is considered as an application. Results are derived for the transitions of 482 keV in Ta^{181}, 343 keV in Lu^{175}, and 14 keV in Fe^{57}. For Ta^{181}, it is possible to calculate the magnitude and the sign of the nuclear matrix-element ratio R. Comparing the theoretical result for the circular polarization, $P\propto FR$, with recent measurement, gives the following limits for the amplitude factor F: $9\lesssim 10^7\times F\lesssim 110$. This result agrees in sign and order of magnitude with estimates derived from the current-current theory of weak interactions.

I. INTRODUCTION

RECENTLY, experimental proof has been obtained for the existence of parity admixtures in nuclear states.[1,2] Such impurities are predicted by theories of weak interactions. The current-current hypothesis[3] implies a weak nucleon-nucleon force, with the same parity-violating properties as the interaction responsible for beta decay. From this theory a form of the weak

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[1] F. Boehm and E. Kankeleit (private communication); *Proceedings of the International Conference on Nuclear Physics, Paris, July 1964* (Publication par le Centre National de la Recherche Scientifique, Paris, 1964), Vol. II, p. 1181.
[2] Yu. G. Abov, P. A. Krupchitsky and Yu. A. Oratovsky, Phys. Letters **12**, 25 (1964).
[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); M. Gell-Mann, Rev. Mod. Phys. **31**, 834 (1959).

pseudoscalar term in the nuclear Hamiltonian can be derived and its magnitude estimated.[4,5]

For the observation of parity impurities, such nuclear processes are favorable where the regular process is hindered,[6] e.g., retarded electromagnetic transitions. A relevant measure of the degree of retardation is provided by the ratio R between the nuclear matrix elements of the irregular (parity-forbidden) and regular process.[6] The other quantity of interest is F, which characterizes the parity-admixture amplitudes in the nuclear wave functions[7] (in the present paper F is regarded as a weak-interaction parameter). The observable effect depends on the product FR; thus, in cases of interest, the inherently small value of F is compensated by a large value of R. General estimates of the quantities F and R are presented in Refs. 4 and 5. The present work deals with more detailed nuclear-structure considerations, relevant to the calculation of the ratio R for gamma-ray transitions. This is of interest, since the result makes possible a comparison between experiment and predictions based on weak-interaction theories. Alternatively, such considerations may provide a means for deriving the value of F from experiment.

In Ref. 1, the circular polarization P of a gamma-ray has been measured; P is proportional to the product FR (see e.g., Ref. 8). In the present paper general expressions for P in terms of transition amplitudes are first presented. Then the use of the independent-particle model is described with regard to evaluating the irregular transition amplitude for one-particle levels in odd-A nuclei. The interaction of the form $\boldsymbol{\sigma}\cdot\mathbf{p}$, suggested by previous authors,[4,5] is discussed with particular attention to the role of the ls term in the irregular-EL case. The case of dipole radiation is considered in greater detail. The theory is applied to the 482-keV transition in Ta^{181}, where the circular polarization has been measured, to the analogous 343-keV transition in Lu^{175}, and to the 14-keV transiton in Fe^{57}. The results and conclusions are briefly summarized. The basic nuclear-model expressions used in this work are briefly presented in the Appendix.

II. GENERAL EXPRESSIONS

In general, the probability for a transition $I_i \rightarrow I_f$ with the emission of an electromagnetic quantum with polarization τ and wave number $k=\omega/c$, is given by

$$t_\tau = C'k\{\sum_{\sigma,L} |A_{if}(\sigma,L)|^2 + 2\tau\,\mathrm{Re}[\sum_L A_{if}(0,L)A_{if}(1,L)^*]\}. \quad (1)$$

The amplitudes A_{if} and multipole operators $\Theta(\sigma,L)$ are defined as follows:

$$A_{if}(\sigma,L)=S_{if}\langle I_f\|\Theta(\sigma,L)\|I_i\rangle,$$
$$\Theta_\mu(\sigma,L)=(C''/c)i^{L+\sigma}\mathbf{j}\cdot\mathbf{A}_\mu(\sigma,L), \quad (2)$$

where $S_{if}\equiv[(2I_f+1)/(2I_i+1)]^{1/2}$, $\mathbf{j}$ is the nuclear current, and $C'C''^2=4\pi/\hbar$ holds. For the reduced matrix elements and the standard multipole-fields, $\mathbf{A}_\mu(\sigma,L)$, the conventions according to Rose[9,10] are used. The symbol σ has the value 0 for magnetic, and 1 for electric 2^L pole.[11] By definition, $\tau=+1$ when the polarization and propagation direction of the photon are parallel, $\tau=-1$ when they are antiparallel (conventionally called left and right polarization, respectively). In the following, phases of operators and wave functions are chosen so as to ensure a time-reversal invariant description, and thus real values of all matrix elements.

Let (σ,L) denote the lowest multipole order which is allowed according to the usual angular-momentum and parity selection rules. This regular transition may in practice contain a regular multipole admixture (σ',L'), where $L'=L+1$ and $\sigma'\neq\sigma$ hold. The irregular part of interest is (σ',L). For example the regular transition may be $M1+E2$, and the irregular transition $\tilde{E}1$ [irregular part is indicated by a tilde, if the general notation (σ',L) does not apply]. The expression for the circular polarization of the gamma ray is obtained from Eq. (1) and reads:

$$P\equiv\frac{t_+-t_-}{t_++t_-}=\frac{2A_{if}(\sigma,L)A_{if}(\sigma',L)}{A_{if}(\sigma,L)^2+A_{if}(\sigma',L')^2}$$
$$=\frac{2}{1+q^2}\frac{A_{if}(\sigma',L)}{A_{if}(\sigma,L)}; \quad (3a)$$

$$q=A_{if}(\sigma',L')/A_{if}(\sigma,L). \quad (3b)$$

It should be noted that the total transition probability is given by the expression

$$T_\gamma=t_++t_-=2C'k[A_{if}(\sigma,L)^2+A_{if}(\sigma',L')^2].$$

The nuclear Hamiltonian is generally written as the sum of a scalar operator H_0, which is the regular part, and a pseudoscalar operator H', which is a parity-violating perturbation:

$$H=H_0+H'. \quad (4)$$

Both parts are assumed to be Hermitian and invariant under time reversal. The parity-impure initial and final

[4] R. J. Blin-Stoyle, Phys. Rev. **118**, 1605 (1960); **120**, 181 (1960).
[5] F. C. Michel, Phys. Rev. **133**, B329 (1964) [gives a review and also contains references to earlier work].
[6] D. H. Wilkinson, Phys. Rev. **109**, 1603 (1958).
[7] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[8] L. Krüger, Z. Physik **157**, 369 (1959).
[9] M. E. Rose, *Multipole Fields* (John Wiley & Sons, Inc., New York, 1955).
[10] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).
[11] Note that the interference terms in Eq. (1) contain only products of amplitudes with the same rank L but with different σ.

states of the transition are then given by

$$|\tilde{I}_i\rangle = |\pi_i,I_i\rangle + \sum_\alpha \langle\pi_\alpha,I_\alpha|H'|\pi_i,I_i\rangle \times (E_i-E_\alpha)^{-1}|\pi_\alpha,I_\alpha\rangle, \quad (5a)$$

$$|\tilde{I}_f\rangle = |\pi_f,I_f\rangle + \sum_\beta \langle\pi_\beta,I_\beta|H'|\pi_f,I_f\rangle \times (E_f-E_\beta)^{-1}|\pi_\beta,I_\beta\rangle. \quad (5b)$$

Here, the indices α and β label the admixed states, and (π,I) labels generally an eigenstate of H_0, i.e., $H_0|\pi,I\rangle = E|\pi,I\rangle$. In Eq. (5) we have $I_\alpha = I_i$, $\pi_\alpha = -\pi_i$, and $I_\beta = I_f$, $\pi_\beta = -\pi_f$. The irregular transition-amplitude, $A_{if}(\sigma',L)$, is given by

$$A_{if}(\sigma',L) = \sum_\alpha A_\alpha''(\sigma',L)/(E_i-E_\alpha) + \sum_\beta B_\beta''(\sigma',L)/(E_f-E_\beta), \quad (6a)$$

$$A_\alpha''(\sigma',L) = S_{if}\langle\pi_f,I_f\|\Theta(\sigma',L)\|\pi_\alpha,I_\alpha\rangle \times \langle\pi_\alpha,I_\alpha|H'|\pi_i,I_i\rangle, \quad (6b)$$

$$B_\beta''(\sigma',L) = S_{if}\langle\pi_f,I_f|H'|\pi_\beta,I_\beta\rangle \times \langle\pi_\beta,I_\beta\|\Theta(\sigma',L)\|\pi_i,I_i\rangle. \quad (6c)$$

III. THE INDEPENDENT-PARTICLE MODEL AND ODD-*A* NUCLEI

To start with, the total Hamiltonian for nucleus, radiation and their interaction is represented in the following semiclassical way[9]:

$$H_{\rm tot} = \sum_\nu \{(2M_0)^{-1}[\mathbf{p}_\nu - (e_\nu/c)\mathbf{A}]^2 + V(\mathbf{r}_\nu) - \mu_\nu e\hbar(2M_0c)^{-1}\boldsymbol{\sigma}_\nu\cdot\mathbf{H}\} + H_{\rm res} + H' + H_{\rm rad}, \quad (7)$$

where $\mathbf{H} = \nabla\times\mathbf{A}$ and $\nabla\cdot\mathbf{A} = 0$. The first part is a sum of single-particle operators, labelled by the nucleon index ν. It does not include residual interactions, which are all represented by $H_{\rm res}$. The perturbation H' is the same as in Eq. (4), and $H_{\rm rad}$ stands for the free radiation field; e_ν and μ_ν are the values of the charge and magnetic moment of the νth particle; M_0 is the mass of the nucleon; $V(\mathbf{r})$ is the average single-particle potential of the nucleus.

In the limit of long wavelength ($kR_0 \ll 1$, R_0 = the nuclear radius) one finds, as a consequence of Eq. (7),

$$A_{if}(\sigma,L) \simeq (-1)^{\sigma+1}C''k^L[(L+1)/L]^{1/2} \times [(2L+1)!!]^{-1}\langle I_f\|\Omega(\sigma,L)\|I_i\rangle S_{if}, \quad (8)$$

where $\Omega(\sigma,L)$ is a sum, $\Omega_\mu(\sigma,L) = \sum_\nu \omega_\mu(\sigma,L)_\nu$, of single-particle operators having the following well-known form[9,12]:

$$\omega_\mu(1,L) \equiv (EL)_\mu = e_L i^L r^L Y_\mu^L(\hat{r}); \quad (9a)$$

$$\omega_\mu(0,L) \equiv (ML)_\mu = (e\hbar/2M_0c)i^{L-1}[2g_l\mathbf{l}/(L+1) + g_\sigma\boldsymbol{\sigma}] \cdot\nabla[r^L Y_\mu^L(\hat{r})]. \quad (9b)$$

Here, e_L is the effective charge for EL radiation. The nuclear gyromagnetic factor, g_l, equals the charge of the nucleon (0 or 1), and g_σ may have an effective value which is usually smaller in magnitude than the free-particle magnetic moment. For convenience the following notation is introduced:

$$U(\sigma,L) \equiv S_{if}\langle I_f\|\Omega(\sigma,L)\|I_i\rangle, \quad (10)$$

which is simply related to the reduced transition probability, $B(\sigma,L) = |U(\sigma,L)|^2$. The circular polarization is now given by

$$P = -2(1+q^2)^{-1}U(\sigma',L)/U(\sigma,L), \quad (11)$$

which is not strictly exact but can be considered a good approximation. Equation (6) can be taken as an expression for U instead of A_{if}, if the operator Θ is replaced by Ω.

In the nuclear Hamiltonian, Eq. (4), the independent-particle model is introduced as described in the Appendix, Eq. (A1). Then also H' is written in the form $H' = \sum_\nu h_\nu'$, where h' is as yet unspecified (it will be discussed in the next section).

One of the most interesting cases at hand for experimental investigation of parity impurities in nuclear states seems to be transitions between low-lying one-particle levels in nuclei with odd mass-number. The remaining part of the paper will be restricted to this case. Because of the presence of strong pairing correlations in nuclei, caused by the short-range forces of $H_{\rm res}$, the seniority (or quasiparticle) description is applicable for these states,[13] as outlined in the first part of the Appendix. The multipole and H' matrix elements therefore reduce according to Eq. (A6). For the high-lying admixed excitations, α and β of Eq. (6), the effects of the correlations are unimportant. Equation (A7) is then a good approximation, and one can set,

$$E_i - E_\alpha = \pm(\epsilon_i - \epsilon_\alpha), \quad E_f - E_\beta = \pm(\epsilon_f - \epsilon_\beta), \quad (12)$$

where plus holds for particle, minus for hole excitation. For the matrix elements, one of the situations (I)–(III) will apply (see the Appendix), and one can assume S to be negligible in the situation (III). Then for the terms contributing to Eq. (6), either situation (I) or (II) will apply, the labels being denoted by α_+, β_+ and α_-, β_-, respectively. Furthermore, the S factors are approximately independent of the excitation [see Eq. (A9)], so that, for the quantities A_α'', B_β'', Eq. (6), a common factor is obtained which is denoted by $S_\pm(\pm 1)^{\sigma'+1}$; with the sign rules stated in the Appendix, $S_\pm$ is positive [follows from Eq. (A8)]. Applying also Eq. (12), one

[12] Such higher terms have been omitted which are either relativistic corrections or, for the (ML) operator, minor corrections arising if $V(\mathbf{r})$ has spin dependence.

[13] For general information about the nuclear-structure and nuclear-model considerations applied in this work (in particular about rotating nuclei, collective motion, pairing correlations, seniority, quasi-particles, blocking, the BCS method, etc.), the reader is referred for example to the notes from A. Bohr and B. R. Mottelson, *Lectures on Nuclear Structure and Energy Spectra* (Copenhagen, 1962), and to the excellent monograph by G. E. Brown, *Unified Theory of Nuclear Models* (North-Holland Publishing Company, Amsterdam, 1964). See also A. K. Kerman, *Nuclear Reactions*, edited by P. M. Endt, and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, Chap. X.

finds:

$$U(\sigma',L)=S_{\pm}(\pm 1)^{\sigma'}\big[\textstyle\sum_{\alpha_{\pm}} A_{\alpha_{\pm}}'(\sigma',L)/(\epsilon_i-\epsilon_{\alpha_{\pm}}) + \sum_{\beta_{\pm}} B_{\beta_{\pm}}'(\sigma',L)/(\epsilon_f-\epsilon_{\beta_{\pm}})\big]; \quad (13)$$

$$A_{\alpha}'(\sigma',L)=\langle f[\![\omega(\sigma',L)]\!]\alpha\rangle\langle\alpha|h'|i\rangle; \quad B_{\beta}'(\sigma',L)=\langle f|h'|\beta\rangle\langle\beta[\![\omega(\sigma',L)]\!]i\rangle. \quad (14)$$

The quantities denoted by $\langle[\![\;]\!]\rangle$ are defined by Eq. (A4) in the case of a spherical nucleus, where the usual nuclear shell model is applicable, and by Eq. (A5) in the case of a nucleus with great deformation, where one applies a modification of the shell model, including a spheroidal equilibrium shape of the nuclear surface.[14,15]

If the effects of residual inteactions on the matrix elements are neglected, one may set $S_{\pm}\simeq 2^{-1/2}$ for a spherical nucleus (if $|f\rangle$ is the ground state), and $S_{\pm}=1$ for a deformed nucleus. Due to the correlations, $S_{\pm}$ might be slightly reduced [also small additional contributions may occur in Eq. (13)]. The effect of possible collective motion on one-particle transition amplitudes may be represented by effective charges or effective nuclear g factors. However, configuration mixing (or band mixing) in the initial and final states might sometimes significantly modify the result [the form of Eq. (13), as well as the value of the S factors]. These questions will be illustrated later in connection with the applications to irregular $\tilde{E}1$ transition.

IV. THE CASE OF $h'=G''\boldsymbol{\sigma}\cdot\boldsymbol{p}$ AND THE ROLE OF THE ls TERM

The single-nucleon pseudoscalar interaction, emerging[4,5] to lowest order from the current-current theory of weak interactions, has the form of the "helicity operator," $\boldsymbol{\sigma}\cdot\mathbf{p}$, of the nucleon. Other simple operators, which may be constructed, subject to the required symmetry-properties, have the character of relativistic corrections to this operator. The subsequent considerations are therefore restricted to

$$h'=G''\boldsymbol{\sigma}\cdot\mathbf{p}=(F\hbar/M_0R_0)\boldsymbol{\sigma}\cdot\mathbf{p}, \quad (15)$$

where G'' is a constant, characterizing the strength of the nuclear weak interaction (G'' depends on isospin and mass number[5]). The dimensionless parameter F represents the average magnitude of parity-impurity amplitudes in nuclear states. Estimates of F have been presented, e.g., by Michel.[5] The transition amplitude ratio of Eq. (11) may be factorized as follows:

$$U(\sigma',L)/U(\sigma,L)=FR(\sigma',L;\sigma,L), \quad (16)$$

which provides the desired decomposition in a weak-interaction parameter F, and a pure nuclear-structure ratio R.

[14] For further information about the nuclear shell model see, for example, A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963). The "Nilsson-Model" is introduced in Ref. 15.

[15] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

To facilitate the further discussion of the expression $U(\sigma',L)$ with h', given by Eq. (15), the following single-particle quantities are defined:

$$A_{\alpha}(\sigma',L)=\langle f[\![\omega(\sigma',L)]\!]\alpha\rangle\langle\alpha|(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)|i\rangle, \quad (17a)$$

$$B_{\beta}(\sigma',L)=\langle f|(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)|\beta\rangle\langle\beta[\![\omega(\sigma',L)]\!]i\rangle, \quad (17b)$$

$$u_A(\sigma',L)=\langle f[\![\omega(\sigma',L)(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)]\!]i\rangle, \quad (18a)$$

$$u_B(\sigma',L)=\langle f[\![(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)\omega(\sigma',L)]\!]i\rangle, \quad (18b)$$

$$u(\sigma',L)=(1/2)[u_A(\sigma',L)+u_B(\sigma',L)]. \quad (18')$$

The following sum rules hold:

$$u_A(\sigma',L)=\textstyle\sum_{\alpha} A_{\alpha}(\sigma',L); \quad u_B(\sigma',L)=\sum_{\beta} B_{\beta}(\sigma',L). \quad (19)$$

With the expressions, Eq. (9), for the operators (ML) and (EL), one finds $u_A(\tilde{E}L)=u_B(\tilde{E}L)=u(\tilde{E}L)$, whereas the operators $u_A(\tilde{M}L)$ and $u_B(\tilde{M}L)$ generally need not be equal. It is appropriate to compare the quantities $U(\sigma',L)$ and $u(\sigma',L)$; the ratio $Q=F^{-1}U(\sigma',L)/u(\sigma',L)$ should in general be of the order of 1. The ratio $u(\sigma',L)/U(\sigma,L)$, on the other hand, is expected to be large in the cases of interest, due to nuclear-structure effects [(σ,L) is hindered]. In the case of irregular $\tilde{E}L$ transitions, a reduction of the ratio $|Q|$ compared to 1 is also expected; special attention will be paid to this question.

If the nuclear Hamiltonian were *spin-independent* the following relations would hold for the independent-particle model:

$$[h_0,\boldsymbol{\sigma}\cdot\mathbf{p}]=0, \quad \langle\eta'[\![\boldsymbol{\sigma}\cdot\mathbf{p}]\!]\eta\rangle=(M_0/\hbar)(\epsilon_\eta-\epsilon_{\eta'})\langle\eta'[\![\boldsymbol{\sigma}\cdot\mathbf{r}/i]\!]\eta\rangle, \quad (20)$$

η, η' meaning any labels i, f, α or β. Then one would also find the following relation, from Eqs. (13) and (14), since the energy denominators are cancelled:

$$U(\sigma',L)=FS_{\pm}(\pm 1)^{\sigma'} \times\big[\textstyle\sum_{\alpha_{\pm}} A_{\alpha_{\pm}}(\sigma',L)-\sum_{\beta_{\pm}} B_{\beta_{\pm}}(\sigma',L)\big]. \quad (21)$$

If the sums were performed over all α and β values, this expression would be proportional to the quantity $\langle f[\![[\omega(\sigma',L),(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)]]\!]i\rangle$, which is the relation found by Michel.[5] This does not seem to be strictly valid though, even when Eq. (20) is fulfilled. Yet, $U(\sigma',L)$ should be roughly proportional to the difference $u_A(\sigma',L)-u_B(\sigma',L)$, and thus tend to vanish for the case $\tilde{E}L$ because of cancellation. However, when h_0 is spin-dependent, Eqs. (20) and (21) no longer hold. Thus, in principle there is not even approximate cancellation between the two sums of Eq. (13) for the case of $\tilde{E}L$. Consequently, $U(\tilde{E}L)$ is essentially determined by the *spin-dependent part of* h_0.

In the nuclear shell model the spin-dependence is represented by the well-known ls term.[14] The actual form of h_0 is described in the Appendix; see Eq. (A10). Considering spherical nuclei, it seems to be an empirical fact that the presence of the ls term affects the radial wave functions $R_{nlj}(r)$ only slightly. On the other hand, the ls term has a great effect on the single-particle energies. These facts are born out e.g., in calculations with a diffuse nuclear potential[16]; in particular the condition $R_{nlj}(r)\simeq R_{nl}(r)$ is found to be valid to good approximation. One may therefore make use of Eq. (A14) and insert in Eq. (13) the expressions

$$\begin{aligned} A_\alpha'(\sigma',L)&\simeq[\epsilon_i^0-\epsilon_\alpha^0]FA_\alpha(\sigma',L)\,,\\ B_\beta'(\sigma',L)&\simeq[\epsilon_\beta^0-\epsilon_f^0]FB_\beta(\sigma',L)\,, \end{aligned} \tag{22}$$

while the energy denominators are $\epsilon_i(\kappa)-\epsilon_\alpha(\kappa)$, and $\epsilon_f(\kappa)-\epsilon_\beta(\kappa)$, respectively. Here, A_α and B_β are given by Eq. (17). For $\kappa\neq 0$, the energy ratios differ from 1 (by a term proportional to κ, to lowest order), preventing the A' and B' sums of Eq. (13) from cancelling. The result differs greatly from Eq. (21). In the case of a nucleus with spheroidal deformation, the above considerations for a spherical nucleus are still essentially valid, including Eq. (22). (ϵ_η^0 refers generally to the eigenvalue ϵ_η with $\kappa=0$; see the Appendix.)

V. THE CASE OF $M1$ PLUS $\tilde{E}1$

The case where the irregular $E1$ transition interferes with the regular, hindered $M1$ transition is considered. The operators read, from Eq. (9),

$$(E1)=e_1 i(3/4\pi)^{1/2}\mathbf{r}\,,$$
$$(M1)=(e\hbar/2M_0c)(3/4\pi)^{1/2}(g_l\mathbf{l}+g_\sigma\boldsymbol{\sigma})\,.$$

For simplicity, the following dimensionless quantities are introduced:

$$a_\alpha=\langle f[\![(\mathbf{r}/R_0 i)]\!]\alpha\rangle\langle\alpha|(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)|i\rangle\,, \tag{23a}$$

$$b_\beta=\langle f|(\boldsymbol{\sigma}\cdot\mathbf{r}/R_0 i)|\beta\rangle\langle\beta[\![(\mathbf{r}/R_0 i)]\!]i\rangle\,, \tag{23b}$$

replacing A_α and B_β of Eq. (17);

$$a_\alpha'=-[e_1R_0(3/4\pi)^{1/2}(\epsilon_i^0-\epsilon_\alpha^0)F]^{-1}A_\alpha'(\tilde{E}1)\,, \tag{24a}$$

$$b_\beta'=-[e_1R_0(3/4\pi)^{1/2}(\epsilon_f^0-\epsilon_\beta^0)F]^{-1}B_\beta'(\tilde{E}1)\,, \tag{24b}$$

replacing A_α' and B_β' of Eq. (14);

$$C_\pm=\sum_{\alpha\pm}\frac{\epsilon_i^0-\epsilon_{\alpha_\pm}{}^0}{\epsilon_i(\kappa)-\epsilon_{\alpha_\pm}(\kappa)}a_{\alpha_\pm}'-\sum_{\beta\pm}\frac{\epsilon_f^0-\epsilon_{\beta_\pm}{}^0}{\epsilon_f(\kappa)-\epsilon_{\beta_\pm}(\kappa)}b_{\beta_\pm}'; \tag{25}$$

$$\begin{aligned} D&=-(e_1R_0)^{-1}(3/4\pi)^{-1/2}u(\tilde{E}1)\\ &=\langle f[\![(\mathbf{r}/iR_0)(\boldsymbol{\sigma}\cdot\mathbf{r}/iR_0)]\!]i\rangle\,, \end{aligned} \tag{26}$$

replacing u of Eq. (18′). Using

$$\begin{aligned} A_+&\equiv\textstyle\sum_{\alpha_+}a_{\alpha_+}\,, & A_-&\equiv\textstyle\sum_{\alpha_-}a_{\alpha_-}\,,\\ B_+&\equiv\textstyle\sum_{\beta_+}b_{\beta_+}\,, & B_-&\equiv\textstyle\sum_{\beta_-}b_{\beta_-}\,, \end{aligned} \tag{27}$$

we write the sum rule, Eq. (19), in the form

$$D=A_++A_-=B_++B_-\,. \tag{28}$$

The quantity defined by $X=\langle f[\![(g_l/g_\sigma)\mathbf{l}+\boldsymbol{\sigma}]\!]i\rangle$ represents the hindered $M1$ amplitude, $U(M1)=S_1(3/4\pi)^{1/2}\times(e\hbar/2M_0c)g_\sigma X$. Here, S_1 is the S-factor [Eq. (A6)] for $M1$.

The expression for R now reads as follows:

$$R_1\equiv R(\tilde{E}1;M1)=\pm(S_\pm/S_1)f_1(A,Z)C_\pm/X\,; \tag{29}$$

$$\begin{aligned} f_1(A,Z)&=(2/g_\sigma)(e_1/e)(R_0M_0c/\hbar)\\ &\simeq(11/g_\sigma)(e_1/e)A^{1/3}\,. \end{aligned} \tag{30}$$

If effects of correlations are disregarded, S_1 is the statistical factor for $M1$, and one can set $S_1\simeq S_+\simeq S_-\simeq 2^{-1/2}$ for spherical nucleus, $S_1=1$ for deformed nucleus. In Eq. (30) one inserts, for g_σ, either the free-particle magnetic moment (μ_p or μ_n), or an effective value which may be about 30 to 40% smaller. The factor e_1 is the effective $E1$ charge; for pure single-particle transition it would be equal to $(g_l-Z/A)e$. However, for admixtures of higher excitations, collective effects may contribute, via the giant dipole resonance, tending to increase the value of e_1. Thus $e_1=(g_l-Z/A)e+e_{\text{coll}}$ holds, where $0\leq e_{\text{coll}}\lesssim e$. Results will be presented for $e_{\text{coll}}=0$, as well as for $e_{\text{coll}}=e$; g_l is 0 for neutron, 1 for proton.

The expressions derived so far, will in Sec. VI be applied for calculating R_1 in a few cases. Of course, the important problem is to evaluate $C_\pm/X$ (or $C_\pm/D$ and D/X), including the determination of the sign. Since the spin-orbit splitting is fairly well established[14] (κ is known to be positive), one expects generally C_+ and C_- to be well-determined [they should have opposite signs, see Eq. (25)]. Also, the quantity D [see Eq. (26)] seems to be well defined, in general. For the magnitude and the sign of X (or D/X), one must usually resort to experimental information. Furthermore, in the case of neutron transitions, the value of the factor f_1, Eq. (30), may be ambiguous.

In the following calculations, the *harmonic-oscillator potential* (h.o.p.) will be used. This is briefly described in the Appendix; see Eq. (A16) *et seq.* In the spherical case, it should be noted that Eq. (A14) holds exactly for h.o.p.; thus for the quantities defined by Eqs. (23) and (24) one obtains[17]:

$$a_\alpha=a_\alpha'\,,\quad b_\beta=b_\beta'\,. \tag{31}$$

Another great simplification appears in this case, since $\mathbf{r}$ and $\mathbf{r}\cdot\boldsymbol{\sigma}$ connect only states with $|\Delta N|=1$: There exists only one term of each kind α_+, α_-, β_+ and β_-; consequently, there are only four quantities a and b to compute, being equal to A_+, A_-, B_+ and B_-, Eq. (27). For deformed nuclei ($\delta>0$), this simplification and Eq. (31) do not hold strictly. However, one can assume N to be a good quantum number (see the Appendix), and then Eq. (31) is still valid.[17]

[16] J. Blomqvist and S. Wahlborn, Arkiv Fysik **16**, 545 (1960).

[17] The validity of Eq. (31) also means that Eq. (22) holds exactly. Furthermore, the following fact (independent of h.o.p.) should be emphasized: The result of the calculation (i.e., the value of the ratio R_1) is essentially determined by the spin-orbit splitting of the single-particle energy levels; otherwise, the spacing of the levels enters roughly by an over-all factor and is of less importance.

VI. APPLICATIONS

Ta^{181} 482-keV Transition; $5/2^+ \rightarrow 7/2^+$

This transition is predominantly $E2(\simeq 97\%)$, and its half-life has been measured to be 1.0×10^{-8} sec. The $M1$ part is hindered by a factor of 10^5–10^6. From angular-correlation measurements,[18] the ratio q [Eq. (3b)] has been obtained; the magnitude is $|q|=6.4\pm 0.8$. The $M1$ internal conversion coefficient[18] is found to be 0.40 ± 0.15, nearly 10 times larger than the tabulated value. The penetration factor[19] has been evaluated from experiment and turns out to have the value[20]

$$p_{M1}\equiv U(M1;\text{pen.})/U(M1)=+210\pm 30. \quad (32)$$

The single-particle operator for $(M1;\text{pen.})$ is obtained (approximately) by replacing, in the expression for the operator $(M1)$, the vector $\boldsymbol{\mu}=g_l\mathbf{l}+g_\sigma\boldsymbol{\sigma}$ by the vector[19]

$$(r/R_0)^2[\boldsymbol{\mu}+g_\sigma\boldsymbol{\sigma}-g_\sigma\hat{r}(\boldsymbol{\sigma}\cdot\hat{r})]. \quad (33)$$

Since the $\mathbf{l}$- and $\boldsymbol{\sigma}$-matrix elements both are greatly reduced, only the last term of Eq. (33) contributes. In fact, the matrix element $U(M1;\text{pen.})$ is of allowed character for $2d_{5/2}\rightleftarrows 1g_{7/2}$ or for $5/2^+[402]\rightleftarrows 7/2^+[404]$ (see the Appendix regarding asymptotic quantum numbers[21]).

The fact that p_{M1} is known experimentally makes it possible to predict the sign of R_1. One may put $FR_1=p_{M1}U(\bar{E}1)/U(M1;\text{pen.})$. Furthermore the last term of Eq. (33) is just the same operator as occurring in $u(\bar{E}1)$, Eq. (26); in fact $U(M1;\text{pen.})=S_1g_\sigma(3/4\pi)^{1/2}\times(e\hbar/2M_0c)D$, where the factor S_1 is the same as the one appearing in Eq. (29). Thus R_1 can be written in the form

$$R_1=p_{M1}f_1(A,Z)(S_\pm/S_1)Y_\pm,\quad Y_\pm\equiv\mp C_\pm/D. \quad (34)$$

If the nucleus is assumed to be spherical, the transition is $2d_{5/2}\rightarrow 1g_{7/2}$, with the only possible parity admixtures being $1f_{7/2}$, $1f_{5/2}$, $2f_{7/2}$ and $2f_{5/2}$. One finds exactly (using h.o.p.) $A_+=A_-=B_+=B_-$, so that all nuclear matrix-elements cancel in the ratio between $C_\pm$ and D [see Eqs. (25), (27) and (28)]. As a result,[17] $Y_\pm$ is given simply in terms of the energy ratios of Eq. (25), and this expression clearly vanishes at $\kappa=0$. The energy-level order is roughly correct, in the spherical case, for $\kappa=0.075$ and $\lambda=0.45$ in Eq. (A18). It holds to second order in κ: $Y_\pm=3.85\kappa(1\pm 1.10\kappa)$. The ratio $Y_\pm$ as function of κ is drawn in the first diagram of Fig. 1, where various assumptions are compared.

In actuality, the Ta^{181} nucleus is deformed. The calculation for $\delta>0$ is more complicated, and has been

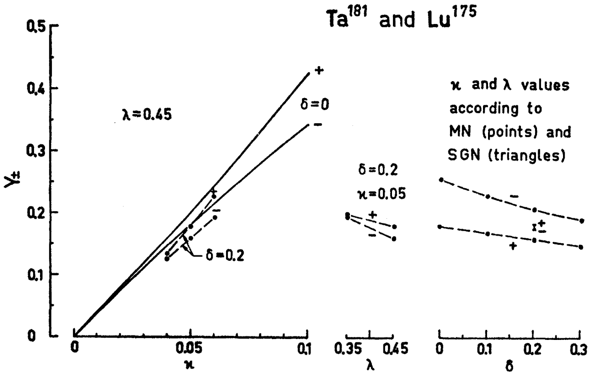

FIG. 1. The ratio $Y_\pm=\mp C_\pm/D$ [see definitions in the text, Eqs. (34), (25), and (26)] as a function of the nuclear potential parameters κ, λ and δ. MN means Ref. 21; SGN means Ref. 15.

performed by the aid of a computer (IBM 7094). It leads to a result which is roughly the same as for $\delta=0$. In fact, even the relative position of the two levels $5/2^+[402]\uparrow$ and $7/2^+[404]\downarrow$ is found not to be greatly affected by the value of δ; see e.g., Ref. 21. The result is shown in Fig. 1. We see that Y_+ and Y_- do not differ too widely; the sign is well determined.[17] In the present case, the $5/2^+$ level is a hole state, and so Y_- should be used. Experimentally, the deformation[21] is roughly $\delta=0.2$, and one may take $\kappa\simeq 0.06$, $\lambda\simeq 0.45$. The value found from Fig. 1 is then $Y_-=0.22\pm 0.04$. The details of the calculations are shown, for one particular case, in Tables I and II. Table I shows that there are still essentially only four admixing states, and Table II shows that A_+, A_-, B_+ and B_- are approximately equal, also for $\delta>0$. *The calculation is therefore insensitive to details of the nuclear wave functions.* From Table II it is also seen that the energy difference $(\epsilon_f-\epsilon_i)$ is approximately reproduced $(\hbar\omega_0\simeq 7.6$ MeV).

It is of interest to calculate the magnitude of p_{M1}, using the experimental value of the $(M1;\gamma)$ lifetime, which gives $|p_{M1}|=(S_1g_\sigma/2.79)(240\pm 40)(R_0/b_0)^2|D|$.

TABLE I. Calculated values of the quantities a_α, b_β [see Eq. (23)], and $\Delta\epsilon/\Delta N$ for $\delta=0.2$, $\kappa=0.06$ and $\lambda=0.45$. The ratio $\Delta\epsilon/\Delta N$ equals $(\epsilon_i-\epsilon_\alpha)/(N_i-N_\alpha)$ or $(\epsilon_f-\epsilon_\beta)/(N_f-N_\beta)$ (unit $\hbar\omega_0$). (The 482-keV transition in Ta^{181} or the 343-keV transition in Lu^{175}.)

Orbital $\alpha_\pm$ $\beta_\pm$	Assignment $[Nn_z\Lambda]\Sigma$	$(R_0^2/b_0^2)a_\alpha$; $(R_0^2/b_0^2)b_\beta$	$\Delta\epsilon/\Delta N$
α_-	303↓	−1.426	0.878
	312↑	−0.036	1.466
α_+	503↓	−1.420	1.245
	512↑	+0.002	0.622
	523↓	+0.008	0.472
	532↑	+0.000	−0.237
β_-	303↑	−1.447	1.322
β_+	503↑	−1.427	0.811
	514↓	+0.001	0.635
	523↑	+0.002	−0.142

[18] Z. Grabowski, B.-G. Pettersson, T. R. Gerholm, and J. E. Thun, Nucl. Phys. 24, 251 (1961).
[19] E. L. Church and J. Weneser, Ann. Rev. Nucl. Sci. 10, 193 (1960). See also A. S. Reiner, Nucl. Phys. 5, 544 (1958).
[20] T. R. Gerholm (private communication); T. R. Gerholm, B.-G. Pettersson, and Z. Grabowski, Nucl. Phys. (to be published).
[21] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter 1, No. 8 (1959).

TABLE II. Calculated quantities in the case $\delta=0.2$, $\kappa=0.06$ and $\lambda=0.45$. (The 482-keV transition in Ta^{181} or the 343-keV transition in Lu^{175}.)

Quantity		Value
$(\epsilon_f-\epsilon_i)/\hbar\omega_0$		0.044
$(R_0/b_0)^2\times$	A_+	−1.461
	A_-	−1.410
	B_+	−1.447
	B_-	−1.424
	C_+	0.653
	C_-	−0.553
	D	−2.871
	Y_+	0.228
	Y_-	0.193

(The uncertainty is derived from the experimental errors of q and the $M1$ conversion coefficient.) The calculations give a very stable value of D, $(R_0/b_0)^2D=-(2.88\pm0.03)$, and the result is $|p_{M1}|\simeq(S_1g_\sigma/2.79)(700\pm100)$. Thus, with $S_1=1$ and $g_\sigma=2.79$, a value much larger than the experimental one [Eq. (32)] is found. However, including the effects of pairing correlations, particularly with blocking,[13,22] certainly brings down the value of S_1; possible effects of band mixing give the same tendency. Furthermore, one expects $g_\sigma<2.79$. Together, therefore, these effects may substantially reduce the estimate of $|p_{M1}|$, and there need not be any discrepancy with experiment. The calculated value of D can therefore be assumed to be reasonably correct. Equating the result for $|p_{M1}|$ with the experimental value, Eq. (32), gives the estimate $(S_1g_\sigma/2.79)\simeq0.30\pm0.06$.

The results of the calculations are inserted in Eq. (34) (with $S_-=1$), giving $R_1=-(3.4\pm0.8)\times10^3(e_1/e)$. From Eqs. (11) and (16) one finally obtains

$$P=-(1.6\pm0.5)(e_1/e)\times10^2F=-(1.6_{-0.9}^{+1.8})\times10^2F, \quad (35)$$

where the limits refer to e_1 being $0.6e$ and $1.6e$. The experimental result by Boehm and Kankeleit[1] reads

$$P=-(5\pm2)\times10^{-4}. \quad (36)$$

Combining the extreme limits of Eqs. (35) and (36) gives the following range of possible F values, which are compatible with experiment and with the present nuclear-structure consideration:

$$9\lesssim10^7\times F\lesssim110. \quad (37)$$

Lu^{175} 343-keV Transition; $5/2^+\rightarrow7/2^+$

This $M1$ transition is also hindered (by about 600 times). Experiment gives the value $(4.7\pm0.4)\times10^{-10}$ sec for its γ lifetime,[23] and the estimate $|q|\simeq0.1$ for the $(E2/M1)$ ratio. The penetration factor p_{M1} of the $M1$ internal conversion is not known, however. The information available is sufficient for evaluating the magnitude, but not the sign, of R_1 [Eq. (29)]. The transition involves the same orbitals[21] as the 482-keV transition in Ta^{181} (also, in both cases, the final state is the ground state). One may evaluate $|R_1|$, using the theoretical result obtained for Ta^{181}; in particular, the information in Fig. 1 is applicable. For Lu^{175}, $5/2^+$ is a particle state. Thus, $|R_1|=e_1R_0(3/4\pi)^{1/2}|C_+/U(M1)|$, where $|C_+|=0.063\pm0.010$ (from $C_+=-DY_+$) and $U(M1)$ is obtained from the partial $(M1)_\gamma$ life-time. The expression for the polarization reads:

$$|P|=(0.7\pm0.1)(1+q^2)^{-1/2}(e_1/e)\times10^2|F|=(0.7_{-0.3}^{+0.6})\times10^2|F|. \quad (38)$$

Comparing this result with Eqs. (35) and (36) for Ta^{181}, the following reasonable prediction for Lu^{175} is found: $|P|\simeq(2\pm1)\times10^{-4}$. The sign of P is expected to be opposite to that of p_{M1}.

Fe^{57} 14-keV Transition; $3/2^-\rightarrow1/2^-$

The half-life of this well-known transition is 1×10^{-7} sec. One may assume that the amount of $E2$ is negligible, i.e., $q\simeq0$. The retardation factor for $M1$ is then about 140. The hindrance is due to the complicated structure of the states, which have strong configurational admixtures, adding to the seniority-1 components built on the orbitals $2p_{1/2}$ and $2p_{3/2}$. The strong configuration-interaction is enough to shift the level order, so that $1/2^-$ becomes the ground state. The state vectors (one quasiparticle[13]) are given by,

$$|\pi_i,I_i\rangle=C_{3/2}|2p_{3/2}\rangle+\text{configurational admixtures},$$
$$|\pi_f,I_f\rangle=C_{1/2}|2p_{1/2}\rangle+\text{configurational admixtures}.$$

The available configurations are such that their contributions to $M1$ are fairly small; therefore, one may set

$$S_1C_{1/2}C_{3/2}\simeq140^{-1/2}, \quad (39)$$

where S_1 is the $M1$ statistical factor. Theoretical analysis[24] of properties of Fe^{57} levels indicates that the main component of the $1/2^-$ state is $(p_{3/2}{}^2)_0p_{1/2}(42\%)$, and the $3/2^-$ state is predominantly $(p_{3/2}{}^3)_{3/2}(72\%)$. This result would require the factor S_1 to have a rather small value (there might also be destructive interference in the $M1$ amplitude from the admixtures).

In calculating the parity impurities, it is assumed that these to first-order enter only as parity admixtures to the $2p_{1/2}$ and $2p_{3/2}$ orbitals: $\alpha_+=2d_{3/2}$, $\alpha_-=1d_{3/2}$, $\beta_+=3s_{1/2}$, $\beta_-=2s_{1/2}$. Using the h.o.p., one finds:

$$A_+=-(7\sqrt{3}/6)(b_0/R_0)^2,\ B_+=-(2\sqrt{3}/3)(b_0/R_0)^2;$$
$$A_-=-(\sqrt{3}/3)(b_0/R_0)^2,\ B_-=-(5\sqrt{3}/6)(b_0/R_0)^2; \quad (40a)$$

$$D=-(3\sqrt{3}/2)(b_0/R_0)^2,\quad X=2\sqrt{3}/3; \quad (40b)$$

$$C_\pm/X=-\tfrac{1}{8}(b_0/R_0)^2[(9\pm5)/(1\pm2.20\kappa)-(9\mp1)(1\mp1.10\kappa)]. \quad (40c)$$

Equation (40c) has been obtained with $\lambda=0.45$. Clearly the quantity C_+ is negative and C_- positive ($\kappa<0.1$); they do not approach 0 when $\kappa\rightarrow0$ but the sum C_++C_-

[22] S. Wahlborn, Nucl. Phys. 37, 554 (1962).
[23] B. Deutch, Nucl. Phys. 30, 191 (1962).
[24] I. Hamamoto and A. Arima, Nucl. Phys. 37, 457 (1962).

does. Therefore, the ratio $\mp C_{\pm}/X$ is positive, and R_1 [Eq. (29)] has the same sign as $f_1(A,Z)$. Since g_σ is negative and $g_l=0$ for a neutron, R_1 turns out positive if $e_{\text{coll}}=0$, but negative if $e_{\text{coll}}=e$. Thus, one cannot with certainty predict the sign of R_1; within the present considerations, it is possible, however, to derive an upper limit for $|R_1|$.

For $\kappa=0.06$, Eq. (40c) gives: $-C_+/X=+0.081$, $+C_-/X=+0.104$. Since the states are actually quite complicated, pairing correlations will greatly modify the wave functions, making the result intermediate between these values. For the same reason, S_1 is reduced relative to S (S stands for $S_+\simeq S_-$). Assuming the product $C_{1/2}\,C_{3/2}$ to cancel in the ratio $U(\tilde{E}1)/U(M1)$, one finds

$$|R_1|=|(2.0\pm0.2)(-e_1/e)(-1.91/g_\sigma)(S/S_1)| \lesssim (1.0\pm0.1)\phi, \quad (41)$$

where ϕ is a factor for possible enhancement of $\tilde{E}1$ relative to $M1$. The expected reduction of S_1 and $|g_\sigma|$ may increase the value of ϕ by as much as a factor of $\simeq3$. However, there is also the possibility that $\tilde{E}1$ does not suffer from the same "hinderedness" as does $M1$; this could give rise to another factor of roughly 10 [see Eq. (39)]. One may therefore set the limits $1\leq\phi\lesssim30$.

The result reads, if the value $F=4.4\times10^{-7}$ according to Michel[5] is used:

$$|P|\lesssim60|F|=2.6\times10^{-5}. \quad (42)$$

This should be considered a fairly conservative upper limit (provided the estimate of F is correct). In fact, $|P|$ might be smaller than the value of Eq. (42) by an order of magnitude. Attempts have been made to measure P for Fe^{57}, the present experimental result[25] being $P=(2\pm6)\times10^{-5}$.

VII. SUMMARY AND CONCLUSIONS

The general expressions for the irregular transition-amplitude $U(\sigma',L)$, for the case of low-lying states in odd-A nuclei, are presented in Eqs. (13) and (14). These expressions have been derived within the independent-particle model and are restricted to seniority-1 states. Prescriptions for their use with the shell model are presented in the text for spherical as well as spheroidal nuclei (see also the Appendix). The particular form of the perturbation $h'=G''\boldsymbol{\sigma}\cdot\mathbf{p}$ leads to simplifications [as indicated by Eq. (22)]. For irregular $\tilde{E}L$ transition, the ls-term is of crucial importance, and special attention has been paid to this question.

In the case of $M1+\tilde{E}1$, the nuclear matrix-element ratio R_1 is given by Eq. (29). The use of harmonic-oscillator potential for calculating the amplitudes leads to simplifications, in particular Eq. (31), but does not crucially influence the result. In connection with the illustrating applications, effects of correlations on the calculated matrix elements have been discussed.

[25] E. Kankeleit (private communication); *Proceedings of the International Conference on Nuclear Physics, Paris, July 1964* (Publication par le Centre National de la Recherche Scientifique, Paris, 1964), Vol. II, p. 1206. See also L. Grodzins and F. Genovese, Phys. Rev. **121**, 228 (1961).

The 482-keV transition in Ta^{181} is a remarkably fortunate case, since the penetration factor[19] p_{M1} has been experimentally determined[20]; this makes it also possible to predict the sign of R_1 (and hence of $P\propto FR$). It is interesting to compare the tentative limits of F, which are obtained in this work, with Michel's estimate[5] $F=8.4\times10^{-7}$, based on the current-current theory of weak interactions. The agreement is good for the lower limit of F according to Eq. (37). In the analogous case of the 343-keV transition in Lu^{175}, the calculations indicate a value of $|P|$ [Eq. (38)] roughly half of that for Ta^{181}. The 14-keV transition in Fe^{57} is a more complicated case, because of strong configuration-mixing, and the result [Eq. (42)] is not very decisive; the predicted upper limit of $|P|$ is smaller than the experimental error.[25]

One may conclude that it is possible to calculate, under certain circumstances, fairly unambiguously the value of the nuclear matrix-element ratio R, using the appropriate version of the independent-particle model. Such calculation is important, since it makes it possible to compare the result of experimentally observed effects of parity admixtures (like the circular polarization P) with predictions of weak-interaction theories. In particular it has been found that the Ta^{181} experiment[1] does not contradict the current-current hypothesis.[3] [*Note added in proof.* The measurement of the circular polarization for Ta^{181} has recently been repeated [F. Boehm and E. Kankeleit (private communication)] giving the improved result $P=-(2.0\pm0.4)\times10^{-4}$, which should replace the value quoted in the text [Eq. (36)]. It has, furthermore, been brought to the author's attention that the collective contribution to the $E1$ effective charge should actually have a negative and rather small value, reducing for proton transition the ratio e_1/e, and hence the last members of Eqs. (35) and (38), by a factor of about 2, without changing the sign. It should also be mentioned that 3-quasiparticle excitations, which have not been explicitly considered in deriving Eq. (13), may generally give rise to a correction, although this is relatively unimportant unless the single-particle level distribution is very nonuniform at the Fermi surface; conservative estimates for Ta^{181} and Lu^{175} give a correction factor with the value 0.8 ± 0.3. Combining the above-mentioned facts, we find that the conclusion drawn in this article from the Ta^{181} measurement is not changed. The tentative result for the constant F seems somewhat more diffuse, however, and it is adequate to state the limits in the form $10^{-6}\lesssim F\lesssim10^{-5}$.]

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APPENDIX: NUCLEAR-MODEL EXPRESSIONS

The Independent-Particle Model and Seniority-1 States

The nuclear Hamiltonian is given by

$$H_0=\sum_\nu h_{0,\nu}+H_{\rm res}\,,\quad h_0=-\hbar^2(2M_0)^{-1}\nabla^2+V(\mathbf{r})\,,\quad \text{(A1)}$$

where h_0 defines the independent-particle model used, and $H_{\rm res}$ represents the residual forces. The eigenstates of H_0 and h_0 are written

$$H_0|\xi,\pi,I,M\rangle=E(\xi,\pi,I)|\xi,\pi,I,M\rangle\,,\quad h_0|\eta\rangle=\epsilon_\eta|\eta\rangle\,.\quad \text{(A2)}$$

The symbol ξ represents all quantum numbers needed in addition to the parity (π) and angular momentum (I,M); the index η labels the single-particle states. Consider any independent-particle operator of tensor rank L (component μ): $O_\mu(\theta,L)=\sum_\nu o_\mu(\theta,L)_\nu$. Here, $\theta=0$ for time-even, $\theta=1$ for time-odd operator; for example, $\theta=0$ for h_0, and $\theta=1-\sigma$ for the electromagnetic multipole operator $\omega(\sigma,L)$ (see the text). One can write

$$\langle\xi'\pi'I'\|O(\theta,L)\|\xi\pi I\rangle=\sum_{\eta,\eta'}S(\xi\pi I,\xi'\pi'I';\eta,\eta';\theta,L)\langle\eta'[\![o(\theta,L)]\!]\eta\rangle\,,\quad \text{(A3)}$$

where the S factors generally contain the effects of $H_{\rm res}$.

In defining the last quantity of Eq. (A3) it is necessary to distinguish between the cases of the nucleus having spherical or nonspherical equilibrium shape.[13,14] In the first case, $V(\mathbf{r})$ is symmetric with respect to all directions in the nucleus; then the angular momentum (j,m) of the single-particle orbital is a good quantum number, and the definition reads:

$$\langle j'[\![o(\theta,L)]\!]j\rangle=(2j'+1)^{1/2}(2j+1)^{-1/2}\langle j'\|o(\theta,L)\|j\rangle\,.\quad \text{(A4)}$$

If the nucleus has a static deformation, it will here be assumed that the shape is axially symmetric (spheroidal) and that the adiabatic description of particle and rotational motion can be used. The angular momentum is then a sum of an intrinsic and a collective part, $\mathbf{I}=\mathbf{J}+\mathbf{R}$; the component K of $\mathbf{J}$ along the symmetry axis of the nucleus is a good quantum number. The definiton reads

$$\begin{aligned}\langle I',K'[\![o(\theta,L)]\!]I,K\rangle &=(IKL,K'-K|I'K')\langle K'|o_{K'-K}(\theta,L)|K\rangle\\ &\quad+(-1)^{I'+K'}(IKL,-K'-K|I',-K')\\ &\quad\times\langle -K'|o_{-K'-K}(\theta,L)|K\rangle\,.\end{aligned}\quad \text{(A5)}$$

The degeneracy of an eigenstate $|\eta\rangle$ is $2j+1$ in the spherical case, and 2 in the spheroidal case. The Fermi level is denoted by ϵ_F. A certain orbital η is referred to as a particle orbital if $\epsilon_\eta\geq\epsilon_F$, and a hole orbital if $\epsilon_\eta<\epsilon_F$.

The considerations are here limited to low-lying intrinsic states of nuclei with odd mass-number A. These states are assumed to have seniority=1. The usefulness of the seniority quantum number is the main consequence of the strong pairing-correlations known to act in nuclei[13] (due to the short-range part of $H_{\rm res}$). Then Eq. (A3) contains only one term, the label η being uniquely determined by the quantum numbers ($\xi\pi I$). In the spherical case, η stands for $I=j$, and η' for $I'=j'$. For the intrinsic states of deformed nuclei, η stands for $I=K$, and η' for $I'=K'$. Thus, η and η' label the states uniquely; $\eta\neq\eta'$ is assumed to hold. One may write

$$\langle\xi'\pi'I'\|O(\theta,L)\|\xi\pi I\rangle=S(\eta,\eta';\theta,L)\langle\eta'[\![o(\theta,L)]\!]\eta\rangle\,.\quad \text{(A6)}$$

One of the following three situations is encountered:

(I) Both η and η' are particle orbitals (including ground state).

(II) Both η and η' are hole orbitals, or one of them is a hole and the other the ground state.

(III) One of the two orbitals η and η' is a hole, and the other a particle orbital *other than* the ground state.

In the situation (III), the S factor vanishes or is small compared to 1, in general. In the following, only the situation (I) or (II) will be considered.

If the further effects of residual interactions are neglected, the energy eigenvalues of H_0 are given by

$$E(\xi,\pi,I)_{\rm seniority=1}\equiv E(\eta)=|\epsilon_\eta-\epsilon_F|\,.\quad \text{(A7)}$$

The properties of the S-factors are fairly simple in this case. The following sign rule holds:

$$(\pm1)^{\theta+1}S(\eta,\eta';\theta,L)>0\,.\quad \text{(A8)}$$

Here, as well as in the main text, *the upper sign always refers to the situation (I), the lower sign to the situation (II).* For a spherical nucleus, the order of magnitude of $|S|$ is 1 (the actual value is determined by the shell-filling). For a deformed nucleus, $|S|$ is exactly 1.

The above results for energies and S-factors may be modified due to the effects of residual interactions, notably pairing correlations and blocking.[13,22] However, for high-energy one-particle excitations, these modifications are relatively unimportant; the magnitude of the S-factors is somewhat reduced relative to 1. A clue to these effects is generally provided by the BCS approximation[13,22,26]; this gives for the S factors in the seniority-1 case:

$$S(\eta,\eta';\theta,L)\simeq uu'+(-1)^{\theta+1}vv'\,.\quad \text{(A9)}$$

[26] S. Wahlborn, Nucl. Phys. 58, 209 (1964).

The Shell Model and the Use of Harmonic-Oscillator Potential

In the nuclear shell-model one takes, generally,[14]

$$h_0 \equiv h_0(\kappa,\delta) = -\hbar^2(2M_0)^{-1}\nabla^2 + V(r) + 2\kappa\phi(r)\mathbf{l}\cdot\mathbf{s} + \delta\psi(\mathbf{r}), \quad \text{(A10a)}$$

where, in particular, one may take

$$\phi(r) = -b^2 V'(r)/r, \quad \psi(\mathbf{r}) = -\hbar^2(M_0 b^4)^{-1}[z^2 - (r^2/3)]. \quad \text{(A10b)}$$

Here, b is a characteristic length, and κ and δ are empirical parameters.

The Spherical-Nucleus Case ($\delta=0$)

If $\kappa=0$ the eigensolutions read,

$$h_0|nlm_l\rangle = \epsilon_{nl}|nlm_l\rangle, \quad |nlm_l\rangle = |R_{nl}(r)i^l Y_{m_l}{}^l(\hat{r})\rangle. \quad \text{(A11)}$$

The following relation is of use in the present work:

$$\langle n'l'\|\mathbf{p}\|nl\rangle = M_0\hbar^{-1}(\epsilon_{nl} - \epsilon_{n'l'})\langle n'l'\|\mathbf{r}/i\|nl\rangle. \quad \text{(A12)}$$

The radial quantum-number n is usually taken to be a nonzero integer. For $\kappa>0$, the solution is given by

$$h_0|nljm\rangle = \epsilon_{nlj}(\kappa)|nljm\rangle; \quad |nljm\rangle = |R_{nlj}(r)i^l[Y^l(\hat{r})\chi^{1/2}]_m{}^j\rangle, \quad \text{(A13)}$$

where the bracket $[\ ]_m{}^j$ means vector coupling; $j=l\pm 1/2$. The following statement is valid, as a *corollarium* of Eq. (A12): If $R_{nlj}(r)=R_{nl}(r)$ is independent of j, then

$$\langle n'l'j'|\boldsymbol{\sigma}\cdot\mathbf{p}|nlj\rangle = M_0\hbar^{-1}[\epsilon_{nl}{}^0 - \epsilon_{n'l'}{}^0] \times\langle n'l'j'|\boldsymbol{\sigma}\cdot\mathbf{r}/i|nlj\rangle. \quad \text{(A14)}$$

Here $\epsilon_{nl}{}^0 = \epsilon_{nlj}(0)$, by definition.

The Spheroidal-Nucleus Case ($\delta>0$)

The solution for $\kappa>0$, $\delta>0$ is expressed in terms of the eigenstates for $\kappa=0$, $\delta=0$:

$$h_0|\gamma\pi K\rangle = \epsilon_{\gamma\pi K}(\kappa)|\gamma\pi K\rangle; \quad |\gamma\pi K\rangle = \sum_{nl\Lambda} c_{nl\Lambda}(\gamma\pi K)|nl\Lambda\Sigma\rangle. \quad \text{(A15)}$$

Here $|nl\Lambda\Sigma\rangle = |R_{nl}(r)i^l Y_\Lambda{}^l(\hat{r})\chi_\Sigma{}^{1/2}\rangle$, $K=\Lambda+\Sigma$; K has been defined earlier, π=parity, and γ represents all other quantum numbers needed. The matrix element $\langle\gamma'\pi'K|\boldsymbol{\sigma}\cdot\mathbf{p}|\gamma\pi K\rangle$ can readily be evaluated, since Eq. (A12) is applicable for each component.

The Harmonic-Oscillator Potential (h.o.p.)

The Hamiltonian, used in this work, is given by[15]

$$h_0 = [-\hbar^2(2M_0)^{-1}\nabla^2 + (\hbar\omega_0/2)(r/b_0)^2] - \kappa\hbar\omega_0(2\mathbf{l}\cdot\mathbf{s} + \lambda\mathbf{l}^2) + \delta\hbar\omega_0(r/b_0)^2[(z/r)^2 - (1/3)], \quad \text{(A16)}$$

where $b_0 \equiv (\hbar/M_0\omega_0)^{1/2}$. The following values are used:

$$\hbar\omega_0 \simeq 43A^{-1/3}\ \text{MeV}, \quad R_0 \simeq 1.2A^{1/3}\ \text{F}, \quad b_0/R_0 \simeq 0.82A^{-1/6}. \quad \text{(A17)}$$

In Eq. (A16), λ is another empirical parameter.[15]

For $\delta=0$, the number of oscillators, $N=2(n-1)+l$, is a good quantum-number. The energy eigenvalues are given by

$$\epsilon_{Nlj}(\kappa) = \{(N+3/2) - \kappa[f(l,j) + \lambda l(l+1)]\}\hbar\omega_0, \quad \text{(A18)}$$

where $f(l, l\pm 1/2) = \pm(l+1/2\mp 1/2)$. One finds that $R_{Nlj}(r)=R_{Nl}(r)$ does not depend on j, so that Eq. (A14) is valid.

For $\delta>0$, N is not strictly a good quantum number. However, one has found that a good approximation[15] results if it is assumed that N has a fixed value for the solution, Eq. (A15). Then the eigenvalues of h_0 are written $\epsilon_{\gamma NK}(\kappa)$, and the eigenvectors $|\gamma NK\rangle = \sum_{l\Lambda} a_{l\Lambda}(\gamma NK)|Nl\Lambda\Sigma\rangle$ [note that $\pi=(-1)^K$]. It is conventional to label the eigenstates by the so-called asymptotic quantum numbers,[13,21] $[Nn_z\Lambda_a]\Sigma_a$, $K=\Lambda_a+\Sigma_a$.

The evaluation of matrix elements with h.o.p. is a relatively simple matter. For the matrix elements of $\boldsymbol{\sigma}\cdot\mathbf{r}$, Eq. (A14), general expressions are presented in Ref. 26, including the radial matrix-elements. In the deformed case, where Eq. (A5) is applicable, we use

$$\langle\gamma'N'K'|\boldsymbol{\sigma}\cdot\mathbf{p}|\gamma NK\rangle = (N-N')M_0\omega_0\delta_{KK'}\sum_{l\Lambda}\sum_{l'\Lambda'} a_{l\Lambda}(\gamma NK)a_{l'\Lambda'}(\gamma'N'K) \times\langle N'l'\Lambda'\Sigma'|\boldsymbol{\sigma}\cdot\mathbf{r}/i|Nl\Lambda\Sigma\rangle, \quad \text{(A19)}$$

where $i^{-1}\langle N'l'\Lambda'\Sigma'|\boldsymbol{\sigma}\cdot\mathbf{r}|Nl\Lambda\Sigma\rangle = (4\pi/3)^{1/2}(-1)^{\Sigma'-\Sigma}i^{l-l'-1} \times(N'l'|r|Nl)\langle\Sigma'|\sigma_{\Sigma'-\Sigma}|\Sigma\rangle\int Y_{\Lambda'}{}^{l'*}Y_{\Lambda'-\Lambda}{}^1 Y_\Lambda{}^l d\Omega$, etc. (see, e.g., Ref. 10). The matrix elements of the electromagnetic multipole operators $\omega(\sigma,L)$ (see the main text) are evaluated according to standard methods[9,10,15] (see also Ref. 27 for $M1$ and $E2$ in the deformed case). For generating the Nilsson wave functions, a previously developed computer program has been used; see Ref. 27.

General Note

Finally it should be noted that it is, of course, possible to generalize the treatment to other kinds of states than those considered here, and to nuclei with even mass-number. Concerning odd-mass deformed nuclei, the present work is restricted to the case $I_f=K_f$, $I_i=K_i$. Furthermore, in the actual applications ($M1+\bar{E}1$ in Ta^{181} and Lu^{175}), the relations $L=I_f-I_i$ and $K_i+K_f>L$ are valid, making the two Clebsch-Gordan coefficients of Eq. (A5) equal to 1 and 0, respectively, which is a somewhat special situation. If the condition $K_i+K_f\leq L$ holds, the second term of Eq. (A5) may contribute. By the use of Eq. (A5) one can easily include rotational states in the treatment; however, the triangular conditions limit the number of possible contributions (in particular, the condition $|K'-K|\leq L$ is restrictive).

[27] R. T. Brockmeier, S. Wahlborn, E. J. Seppi, and F. Boehm, Nucl. Phys. **63**, 102 (1965).