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$F^{19}(d,\alpha)O^{17}$ Reaction at 9.2 MeV*†

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The differential cross sections for the $F^{19}(d,\alpha)O^{17}$ reaction which leave O^{17} in its ground and lowest four excited states have been measured using silicon surface-barrier detectors. Thin Teflon targets (370 to 720 μg/cm^2)were bombarded with 9.2-MeV deuterons and alpha spectra were obtained at 46 laboratory angles between 10 and 172.5°. The five angular distributions exhibited forward and backward peaking, an over-all oscillatory structure, and minima whose magnitudes differ significantly from zero. The angular distributions are analyzed in terms of an expression which is the sum of an isotropic term and one which arises from the simultaneous action of two-nucleon pickup and heavy-particle stripping (HPS) mechanisms. Good fits are obtained for the α_0, α_2, and α_4 distributions, but only the gross features of the α_1 and α_3 distributions could be fitted. A discussion regarding the identification of either the pickup or the knockout mechanism as the dominant forward-angle direct-interaction process for (d,α) reactions in light nuclei is presented. It is concluded that the experimental angular distributions are fitted equally well irrespective of which one is assumed to act along with HPS. An interpretation of the differential cross sections is made which is based on the speculation that direct-interaction and statistical compound-nucleus (SCN) processes contribute incoherently. A method for the decomposition of the cross section is proposed in which the SCN contribution is assumed to be isotropic. The integrated SCN component of the cross sections is shown to be closely proportional to $2I+1$.

I. INTRODUCTION

The successes of simple plane-wave Born-approximation (PWBA) theories which are based on single-nucleon stripping and pickup models in describing particular (d,p), (d,n), and (He^3,α) forward-angle cross sections have stimulated interest in the investigation of the applicability of these ideas to processes in which two or more nucleons are transferred. In this connection the (d,α) reaction has been the subject of rather extensive investigation. There is general agreement among those investigators who used deuterons with energies greater than 7 MeV that the dominant reaction mechanism is of the direct-interaction (DI) type as opposed to a statistical-compound-nucleus (SCN) process. Furthermore, there appears to be a general tendency for the angular distributions to rise at backward angles.[1–7] Although supporting evidence for this assertion includes only a few cases in which the experimental measurements are of good statistical quality and extend to backward angles,[3–7] the remaining pertinent experimental data does weakly suggest such a generalization. The presence of this back-angle peaking is important since the existing plane-wave pickup and knockout models can not account for it; hence, some theoretical modifications are required and/or another process is involved. In this connection, the studies of the $F^{19}(d,\alpha)O^{17}$ reaction at 14.7,[8] 11.4,[9] and 10.2 MeV[10,11] are of special interest. The higher energy investigation yielded angular distriubtions of rather poor statistical quality and somewhat limited angular range which seem to suggest a slight tendency for the cross sections to rise at backward angles. The 11.4- and 10.2-MeV angular distributions definitely show pronounced backward peaking. The present study of the $F^{19}(d,\alpha)O^{17}$ reaction consisted of two parts: The measurement of the angular distributions at a lower deuteron energy, 9.2 MeV, to see if the general over-all characteristics persisted, and an investigation of the adequacy in fitting these data of a theoretical differential cross section which incorporates heavy-particle-stripping (HPS) and pickup or knockout mechanisms.

II. EXPERIMENTAL

A description of the Purdue University 37-in. cyclotron experimental area; beam focusing, steering, and analyzing system; and the 30-in. scattering chamber in which these measurements were performed has been presented elsewhere.[12] The energy spectra of the emitted alpha particles were measured using silicon surface-barrier detectors and a conventional electronics configu-

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‡ National Defense Education Act Fellow, September 1960 to September 1963.

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ration which included a 256-channel pulse-height analyzer. The incident deuteron energy was 9.200 MeV with an rms energy spread of 30 keV. The beam cross section at the target was circular with a 5/64-in. diameter. The target-detector geometry was such that the azimuthal acceptance angle of the detector with respect to the center of the target was 2.3°, and the nominal solid angle subtended by the detector was 0.001 sr. Alpha-particle spectra were obtained at 46 laboratory angles from 10 to 172.5°.

The F^{19} targets were prepared by stretching commercial $\frac{1}{4}$-mil Teflon films.[13] All but one of the many targets used in this investigation had measured thicknesses which ranged from 0.109 mil (610 $\mu g/cm^2$) to 0.128 mil (720 $\mu g/cm^2$). A special target which had a measured thickness of 0.066 mil (370 $\mu g/cm^2$) was used in a reflection geometry at 90°. The thicknesses of the targets were determined by measuring the energy lost by alpha particles from a Bi^{212}-Po^{212}-Po^{214} source in passing through the target and using computed dE/dx curves for alpha particles in Teflon.[14] During exploratory runs it was found that the thickness of the Teflon targets decreased during deuteron bombardment. To monitor the thickness change for each target, a second silicon surface-barrier detector was used. The pulses from this detector were amplified in a charge-sensitive preamplifier and routed to a linear amplifier. Pulses which corresponded to an energy deposition in the detector of 5 MeV or more were then scaled. Periodic determinations of the number of monitor counts accumulated per unit charge were made during the course of each run. The average target thickness for a given run was taken to be proportional to the product of the initial target thickness times the scaled monitor counts per unit charge averaged over the period of the run. The nominal decrease in thickness during a run was 6 to 9%. No target was allowed to decrease in thickness by more than 30%. The above circumstances necessitated the use of 32 targets during the course of this investigation. To check reproducibility, points were frequently repeated with different targets. In addition, no two adjacent experimental points were obtained using the same target, nor were any adjacent points obtained in consecutive runs. These procedures insured the detection of any systematic decrease in cross section due to the above-mentioned target difficulties, and provided a periodic check on the performance of the equipment.

The estimated probable systematic error in the absolute cross sections due to uncertainties in target thickness, beam integration, and experimental geometry is ±15%. The experimental differential cross sections have been corrected only to first order for finite geometry.

III. RESULTS, ANALYSIS, AND DISCUSSION

A. General

There have been measurements of the $F^{19}(d,\alpha)O^{17}$ differential cross sections at five energies: 10.2,[10,11] 11.4,[9] 13.0,[15] 14.7,[8] and 27.5 MeV.[16] In each investigation the experimental results were analyzed in terms of one or more PWBA theories: knockout[17] and deuteron pickup[17] (10.2-, 11.4-, 13.0-, and 27.5-MeV data); and two-nucleon pickup[18–20] (11.4- and 27.5-MeV data). The summarizing conclusion implied by these investigators is that a pickup mechanism offers the best explanation of the (d,α) reaction on F^{19}. This conclusion was in part due to the assumed shell-model structure of the F^{19} ground state, three nucleons outside an O^{16} core,[21] for which the pickup of two loosely bound nucleons seemed more likely than the knockout of an alpha particle from the core. It should be noted that Pellegrini[22] has shown that within the Butler formalism[23] the plane-wave theoretical descriptions of the knockout and deuteron pickup process are almost identical. Furthermore, if the finite size of the incident and exit particles are neglected, then both the knockout[24] and two-nucleon pickup models[18] predict cross sections which are proportional to a sum of spherical Bessel functions. The arguments of the Bessel functions are slightly different in the two cases, but can be made the same by using a slightly larger interaction radius for the knockout case. Mead and Cohen[25] have performed a low-resolution survey of (d,α) reactions with 28 nuclei from $Z=28$ to $Z=83$. They asserted that the Z dependence of the shape of the total alpha-particle spectrum could not be explained by a knockout process, but could be satisfactorily interpreted in terms of a two-nucleon pickup process. However, there is no *a priori* reason to believe that the dominant mechanism for the (d,α) reaction in light nuclei is the same as that for heavier nuclei.

If only the ground and first excited states of O^{17} are considered, then in accord with previous investigators the pickup model seems the most appropriate. However, as will be discussed later, when the low-lying negative parity states of O^{17} are considered, existing pickup models are inapplicable. The knockout description is,

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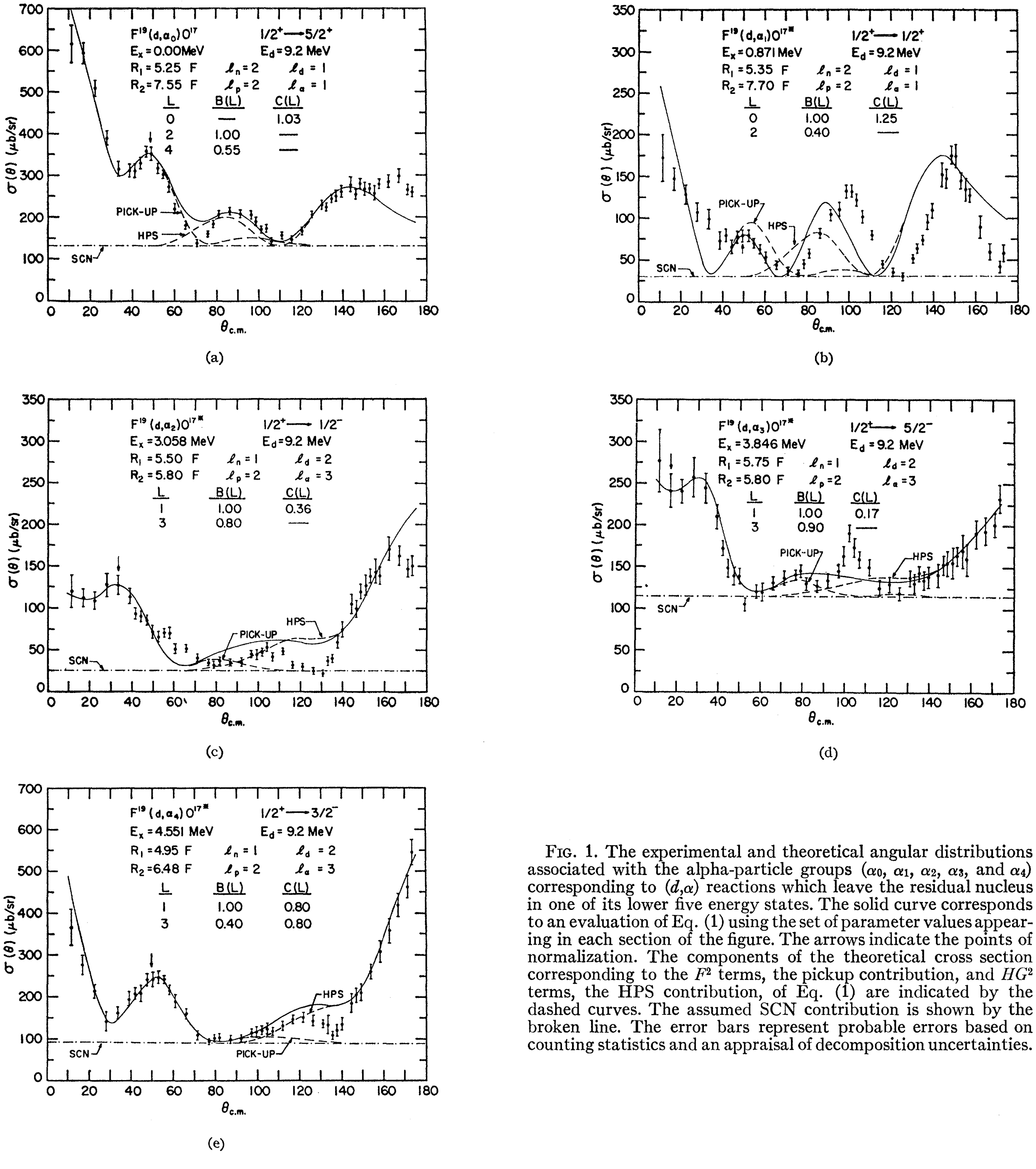

(a)

(b)

(c)

(d)

(e)

FIG. 1. The experimental and theoretical angular distributions associated with the alpha-particle groups (α_0, α_1, α_2, α_3, and α_4) corresponding to (d,α) reactions which leave the residual nucleus in one of its lower five energy states. The solid curve corresponds to an evaluation of Eq. (1) using the set of parameter values appearing in each section of the figure. The arrows indicate the points of normalization. The components of the theoretical cross section corresponding to the F^2 terms, the pickup contribution, and HG^2 terms, the HPS contribution, of Eq. (1) are indicated by the dashed curves. The assumed SCN contribution is shown by the broken line. The error bars represent probable errors based on counting statistics and an appraisal of decomposition uncertainties.

on the other hand, applicable to all low-lying states of O^{17}, but, as is the case with the heavy-particle stripping formalism, requires a less attractive model for the ground state of F^{19}.

It appears that there is at present no definitive experimental evidence or compelling physical argument to dictate which mechanism, pickup or knockout, is operative in the reaction $F^{19}(d,\alpha)O^{17}$. Furthermore, the near equivalence of the theoretical angular distributions predicted by these models renders a particular choice as a basis for data analysis arbitrary. Consequently, a two-nucleon pickup mechanism has been assumed in fitting the present experimental data.

The angular distributions shown in Fig. 1 display the following general characteristics: forward and backward peaking; an over-all oscillatory structure; and minima whose magnitudes in all cases differ significantly from zero. Existing plane-wave knockout and pickup theories can not account for the backward peaking. An interaction which is known to yield back-angle peaking is heavy-particle stripping (HPS). In an attempt to improve agreement between theory and experiment HPS has been included with two-nucleon pickup in the analysis of the present data. Furthermore, plane-wave DI theories predict angular distributions in which the minima have values close to zero in contrast to these experimental results. This could imply that there is a contribution to the cross section from a mechanism other than a DI process. In (d,α) reactions in which DI transitions are forbidden by isotopic spin and/or angular-momentum selection rules the cross sections are small, though not zero.[1,26–28] In these cases the nonzero cross sections are interpreted as due to the SCN mechanism. If there is a SCN contribution to the cross section when the DI process is inhibited, then there is no good reason to expect a total absence of the SCN contribution when the DI mechanism is not strongly inhibited. This could explain why (d,α) experimental differential cross sections seldom if ever are observed to approach zero. The SCN theory might be expected to be applicable in the present experimental situation because an excitation energy of 25.7 MeV is reached in the compound nucleus where the level density is certain to be very high. The rms spread in incident beam energy (30 keV) plus the energy spread due to finite target thickness (45 keV) would allow the excitation of many states in the compound nucleus. If it is assumed that the experimental cross section is a superposition of two incoherent components, a SCN and a DI contribution, and if use is made of the fact that DI theories predict angular distributions whose minima approach zero, then a decomposition of the experimental angular distribution may be possible. To obtain some idea of the general character of the SCN contribution, angular distributions from (d,α) reactions in which the DI mechanism is inhibited may be examined. In these cases the angular distributions show very little structure,[1,26–28] and closely approximate an isotropic distribution. By assuming an isotropic SCN contribution to the cross section, neglecting any interference between DI and SCN processes, and fixing the magnitude of the SCN cross section at the lowest point of the experimental angular distributions, the decomposition of the experimental cross section into two parts may be effected. The results of this procedure are presented in Table I. It should be noted that Jänecke[28] calculated the SCN contribution for the reaction $\mathrm{Ca}^{40}(d,\alpha)\mathrm{K}^{38}$, and for each state the calculated magnitude was very close to the lowest value in the experimental angular distribution.

TABLE I. Spins, parities, and energies of some states in O^{17} and the decomposition of their production cross sections in the (d,α) reaction on F^{19} at 9.2 MeV.

State O^{17}	Energy (MeV)	I^π	σ_I[a] (mb)	$d\sigma_{\mathrm{CN}}$[b] (μb/sr)	$\sigma_{\mathrm{CN}}/\sigma_I$	$\sigma_{\mathrm{CN}}/\sigma_{\mathrm{DI}}$[c]
0	0	$\frac{5}{2}+$	2.93	130	0.55	1.20
1	0.871	$\frac{1}{2}+$	1.00	30	0.37	0.59
2	3.058	$\frac{1}{2}-$	0.77	25	0.42	0.71
3	3.846	$\frac{5}{2}-$	1.87	115	0.76	3.25
4	4.551	$\frac{3}{2}-$	1.99	90	0.55	1.24

[a] The experimental differential cross sections are integrated over the angular range from 10° to 170° c.m. to yield σ_{I}.

[b] The determination of the magnitude of $d\sigma_{\mathrm{CN}}$, the compound-nucleus differential cross section, is described in detail in the text. Its integral from 10 to 170° c.m. is designated σ_{CN}.

[c] The direct interaction cross section, σ_{DI}, is defined as $(\sigma_{\mathrm{I}}-\sigma_{\mathrm{CN}})$.

The present data have been analyzed using the two-nucleon pickup theory of Newns[18] as extended to include heavy-particle stripping by Manning and Aitken.[29] The isotropic contributions discussed above and given in Table I were subtracted from the experimental data before fitting. The expression used for the DI fits is

$$d\sigma/d\Omega \propto \sum_L \left(F^2+2DFG+G^2H\right), \qquad (1)$$

where:

$$F=B(L)\exp(-2\gamma^2R_1^2)\exp\left(\frac{-K^2}{16\gamma^2}\right)$$

$$\times\left[\frac{(2l_p+1)(2l_n+1)}{(2L+1)}\right]^{1/2}(l_p,0,l_n,0|L,0)j_L(kR_1),$$

$$D=(-1)^L\left[\frac{(2l_\alpha+1)}{4\pi(2L+1)}\right]^{1/2}(l_\alpha,0,l_d,0|L,0)Y_{l_d}{}^0(\hat{K}_0\cdot\hat{q}_0),$$

$$G=C(L)j_{l_d}(K_0R_2)j_{l_\alpha}(q_0R_2),$$

and

$$H=(-1)^L\left[\frac{(2l_\alpha+1)(2l_d+1)}{(4\pi)^{3/2}}\right]\sum_{L'}\left[(l_d,0,l_d,0|L',0)\right.$$

$$\left.\times(l_\alpha,0,l_\alpha,0|L',0)W(l_\alpha,l_d,l_\alpha,l_d;L,L')Y_{L'}{}^0(\hat{K}_0\cdot\hat{q}_0)\right];$$

L is the angular momentum transfer in the reaction; $B(L)$ and $C(L)$ are combinations of various constants such as fractional parentage coefficients, configuration mixing parameters, and radial integrals; R_1 and R_2 are the pickup and HPS interaction radii, respectively; γ is the width parameter associated with the Gaussian wave function used for the deuteron and is taken to be[8] 0.300 F^{-1}; l_p and l_n are the orbital angular momenta of the picked-up proton and neutron; and l_α and l_d are the orbital angular momenta of the alpha particle before, and the deuteron after, HPS. The vectors $\mathbf{K}$, $\mathbf{k}$, $\mathbf{K}_0$, and $\mathbf{q}_0$ are defined as follows:

$$\mathbf{K}\equiv\tfrac{1}{2}\mathbf{K}_\alpha-\mathbf{K}_d, \qquad \mathbf{K}_0\equiv\mathbf{K}_d+(M_d/M_F)\mathbf{K}_\alpha,$$

$$\mathbf{k}\equiv\mathbf{K}_\alpha-(M_F/M_I)\mathbf{K}_d, \quad\text{and}\quad \mathbf{q}_0\equiv(M_\alpha/M_I)\mathbf{K}_d+\mathbf{K}_\alpha.$$

The subscripts I and F refer to the initial and final nucleus, and $\mathbf{K}_d$ and $\mathbf{K}_\alpha$ are the wave vectors associated

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with the incident deuteron and the outgoing alpha particle. The selection rules for the use of Eq. (1) are $\mathbf{J}_F=\mathbf{J}_I+\mathbf{L}+\mathbf{S}$, $\mathbf{L}=\mathbf{l}_n+\mathbf{l}_p$ for pickup, and $\mathbf{L}=\mathbf{l}_d+\mathbf{l}_\alpha$ for HPS, $|\mathbf{S}|=1$. There is interference between the pickup terms, F^2, and the HPS terms, G^2H, in Eq. (1) only if the same L value is allowed for both processes. The two-nucleon pickup theory was formulated within the framework of the shell model and allows the simultaneous pickup of two nucleons from the outermost subshells of the initial nucleus. It is a basic assumption of the model that all nucleons of the initial nucleus except the picked-up pair form an inert core which takes no part in the reaction.

In Fig. 1 are presented the experimental angular distributions associated with the alpha-particle groups $(\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ corresponding to (d,α) reactions which leave the residual nucleus, O^{17}, in one of its lower five energy states. The best fitting theoretical differential cross section is represented by a solid curve which was normalized to the experimental data at the point indicated by the arrow. The parameter values used in the evaluation of Eq. (1) are also shown. The components of the theoretical cross section corresponding to the F^2 terms, the pickup contribution, and the G^2H terms, the HPS contribution, of Eq. (1) are indicated by the dashed curves. The assumed SCN contribution is shown by the broken line.

B. The α_0 and α_1 Angular Distributions

The ground and first excited states of O^{17} are known to have spins of $\frac{5}{2}$ and $\frac{1}{2}$, respectively, and positive parity.[30] These two states have been successfully described in terms of the shell model.[21] The ground state is a pure single-particle state consisting of an O^{16} core and a $1d_{5/2}$ neutron which is promoted to the $2s_{1/2}$ subshell to form the first excited state. The accepted shell-model description of the F^{19} ground state is three nucleons outside an O^{16} core in the mixed configuration $[12\%\text{-}(1d)^3,\ 59\%\text{-}(1d)^2(2s)^1,\ 29\%\text{-}(2s)^3]$.[31]

In Table II are presented the orbital angular momenta of the picked-up neutron and proton, and the allowed values of the angular momentum transfer associated with (d,α) reactions which connect the ground and first excited states of O^{17} with each of the F^{19} ground-state configurations. In the HPS model it was assumed that an alpha particle in a p state was coupled to a N^{15} core $(\frac{1}{2}^-)$ to form the ground state of $F^{19}(\frac{1}{2}^+)$. In the stripping process the deuteron was assumed to be captured into a p state by the N^{15} core, with the deuteron's spin and orbital angular momentum coupling to the spin of the core to yield the ground state $(\frac{5}{2}^+)$ or the first excited state $(\frac{1}{2}^+)$ of O^{17}. In both cases the allowed angular momentum transfer values are 0 and 2.

TABLE II. Orbital angular momenta of the picked-up neutron and proton, and the allowed values of the angular momentum transfer associated with (d,α) reactions which connect the ground and first excited states of O^{17} with each of the F^{19} ground-state configurations.

F^{19} shell-model configuration[a]	l_n	l_p	Allowed L values	O^{17} final state
$12\%\text{-}(1d)^3$	2	2	0, 2, 4	Ground
$59\%\text{-}(1d)^2(2s)^1$	2	0	2	Ground
	2	2	0, 2	1st
$29\%\text{-}(2s)^3$	0	0	0	1st

[a] See Ref. 31.

The experimental and theoretical α_0 angular distributions are shown in Fig. 1. In this case there is no interference between the pickup and HPS processes. The over-all agreement is quite good with the largest discrepancy at extreme backward angles. The large HPS interaction radius, $R_2=7.55$ F, was necessary to achieve a reasonable fit in the backward direction.

The experimental and theoretical α_1 angular distributions are shown in Fig. 1. The gross features of the experimental angular distribution are reproduced by the theory, but the magnitudes and angular positions of the experimental maxima could not be fitted any better. Just as in the case of the α_0 theoretical curve, a large HPS interaction radius, $R_2=7.70$ F, was required.

The corresponding α_0 and α_1 angular distributions in the present and previous investigations[9–11] exhibit pronounced back-angle peaking and show the same general character with slight differences in detail.

C. The α_2, α_3, and α_4 Angular Distributions

The second, third, and fourth excited states of O^{17} are known to have negative parity and spins of $\frac{1}{2}$, $\frac{5}{2}$, and $\frac{3}{2}$, respectively.[30,32–35] The formation of these negative-parity states within the shell-model formalism requires the excitation of a nucleon from the core. As discussed in Sec. III-A, the two-nucleon pickup model used in these analyses is not a valid description of a reaction proceeding via core excitations. An analysis[36] of the present data has been performed within the framework of a knockout model[24] which characterizes these low-lying states of O^{17} as a N^{15} core coupled to

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a deuteron. This is in a sense a less appealing description of the positive-parity states than afforded by the shell model, but it is consistent with the description of O^{17} which is used in the HPS process. The forward-angle fits to the data using the knockout model were very similar to those obtained with the pickup model, the F^2 terms of Eq. (1).

The experimental and theoretical α_2, α_3, and α_4 angular distributions are shown in Fig. 1. The fits to the α_2 and α_4 data are quite good, but the maximum at about 102° in the α_3 data could not be reproduced with reasonable values of the parameters. The wide spread in the interaction radii, both the R_1 and the R_2 values, required to fit these three angular distributions may in part be a manifestation of the oversimplifications of the theory.

D. The $(2I+1)$ Rule

The integrated cross section for a nuclear reaction resulting in a given final nuclear state is, within the framework of SCN theory, independent of the details of structure of the final state and only dependent on its energy and spin. Furthermore, under certain circumstances the integrated cross section for a specific type of reaction resulting in a final state of spin I is proportional to $(2I+1)$. Ericson[37] and MacDonald[38] have within the context of the SCN theory listed and/or discussed certain of the conditions under which the statistical factor $(2I+1)$ would dominate the behavior of the cross section. Even if extensive experimental evidence were found for the existence of a $(2I+1)$ rule, its origin could not in every instance be unambiguously ascribed to the presence of a SCN reaction mechanism since particular descriptions of certain DI processes, e.g., Newns' description of two-nucleon pickup,[18] yield expressions for the cross section involving a $(2I+1)$ factor. In view of the complexity of these latter expressions, it can be reasonably argued that the simple $(2I+1)$ dependence of the DI mechanism could be obscured by fluctuations arising from the complex dependence of the other factors on energy and angular momentum.

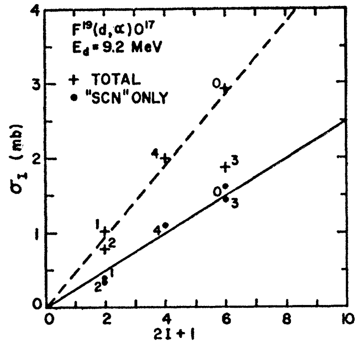

FIG. 2. The crosses represent the integrated $F^{19}(d,\alpha)O^{17}$ cross sections (10 to 170°) associated with the ground state and lowest four excited states of O^{17}. The numbers refer to the ordering of the O^{17} states according to increasing energy, starting with the ground state. The dashed line was required to pass through the origin and was least-squares fitted to the crosses 0, 1, 2, and 4. The solid circles represent the assumed statistical-compound-nucleus contributions to the cross sections. The solid line was required to pass through the origin and was least-squares fitted to all the points represented by the solid circles.

At the present time insufficient relevant experimental information is available to permit the formulation of any general conclusions concerning the $(2I+1)$ rule and its implications. Moreover, since almost all of the very small number of experimental studies dealing directly with this subject involve the (d,α) reaction, only a brief summary of those investigations will be presented here. The $Al^{27}(d,\alpha)Mg^{25}$ reaction has been studied at many deuteron energies between 1.5 and 10.5 MeV.[39–43] The limitations in scope and diversity in approaches of these studies preclude any summarizing conclusions other than perhaps a statement that some evidence for a $(2I+1)$ rule exists among the production cross sections associated with those Mg^{25} states below 4.5 MeV which have a spin less than $\frac{7}{2}$. The $Mg^{25}(d,\alpha)Na^{23}$ reaction has been systematically studied at a large number of energies in the ranges 3.35 to 3.70 MeV and 7.1 to 7.7 MeV.[42] The energy-averaged (higher energy range), integrated cross sections for the low-lying ($E<5$ MeV) states of Na^{23} of known spin were reported to satisfy the $(2I+1)$ rule. These investigators then assumed the general validity of the $(2I+1)$ dependence and made spin assignments for the remaining Na^{23} states below 5 MeV. They asserted that these latter assignments did not disagree with other existing data.

The experimental conditions and the reaction studied in the present investigation meet the various SCN energy and angular-momentum criteria for the applicability of the $(2I+1)$ rule as set down by MacDonald[38] and restated by Hansen *et al.*[42] As discussed in Sec. III-A, the angular distributions shown in Fig. 1 might be assumed to arise from the simultaneous action of SCN and DI mechanisms. Moreover, since the $(2I+1)$ dependence could arise from both mechanisms, the (d,α) cross sections corresponding to the lowest five states in O^{17} integrated from 10 to 170° might be expected to be proportional to $(2I+1)$. This analysis is presented graphically in Fig. 2 where the crosses represent the integrated $F^{19}(d,\alpha)O^{17}$ cross sections associated with the ground and lowest four excited states of O^{17}. The numbers refer to the ordering of the O^{17} states according to

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increasing energy, starting with the ground state. The dashed line was required to pass through the origin and was least-squares fitted to the crosses 0, 1, 2, and 4. The proportionality between $(2I+1)$ and these cross sections is excellent with a nominal rms deviation of 6.5%. The spin and parity of the third excited state were previously thought to be $\frac{7}{2}^-$,[30] but more recent studies strongly imply a $\frac{5}{2}^-$ assignment.[32–35] A direct application of the $(2I+1)$ rule would yield the apparently incorrect $\frac{3}{2}$ spin value. Under the assumption that the angular distributions are incoherent superpositions of isotropic SCN and nonisotropic DI contributions as described in detail in Sec. III-A, the proportionality between the SCN component of the cross sections and $(2I+1)$ may be investigated. This analysis is also shown in Fig. 2 where the solid circles represent the SCN contributions to the cross sections. The solid line was required to pass through the origin and was least-squares fitted to the five cross sections represented by the solid circles. The argreement is good for all cross sections (rms deviation 14.6%) including the one which had appeared anomalous in the previous treatment. While the validity of this type of decomposition can be seriously challenged, the over-all agreement with the $(2I+1)$ rule of SCN theory is improved considerably. It would be interesting to investigate whether such improvement would result for other (d,α) reactions under similar analysis.

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Elastic Electron Scattering from Tritium and Helium-3*

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The mirror nuclei of tritium and helium-3 have been studied by the method of elastic electron scattering. Absolute cross sections have been measured for incident electron energies in the range 110–680 MeV at scattering angles lying between 40 and 135° in this energy range. The data have been interpreted in a straightforward manner and form factors are given for the distributions of charge and magnetic moment in the two nuclei over a range of four-momentum transfer squared 1.0–8.0 F^{-2}. Model-independent radii of the charge and magnetic-moment distributions are given and an attempt is made to deduce form factors describing the spatial distribution of the protons in tritium and helium-3.

INTRODUCTION

ONE of the important questions in nuclear physics concerns a problem about which we are almost totally ignorant; this is the question of whether significant three-body nuclear forces exist.[1] The obvious place to search for evidence of such forces is in the simplest nuclei in which they can occur—tritium and helium-3. However, despite a growing body of experimental data on these nuclei, as well as on scattering and reactions of protons and neutrons with deuterons, we still do not have enough information to provide an insight into the details of the structure of the three-body systems. For example, Blatt[2,3] and his collaborators have made a determined effort to calculate the binding energy of the triton by a variational type of calculation in which the best-known parameters of the two-body nuclear forces were used. Their difficulty in obtaining reasonable agreement with the experimental binding energy can be ascribed partly to uncertainties in our knowledge of the two-body forces as well as to the lack of a suitable trial wave function. Thus, data that will improve our knowledge of the ground-state wave functions would be particularly helpful. If, when better

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