

A Study of Two-Prong Events in Proton-Antiproton Annihilations at Rest

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(Received 19 October 1964)

Proton-antiproton annihilations at rest yielding two charged mesons have been analyzed to search for multimeson resonances. The results show evidence for appreciable production of the ρ , the K^* , and the φ mesons. The data on the ρ -meson production indicate that antiproton annihilation from the singlet S state is not negligible compared to the triplet S state.

I. INTRODUCTION

THE antiproton films which we analyzed were taken at the CERN proton synchrotron, using the 81-cm Saclay liquid-hydrogen bubble chamber. This chamber was placed in a 20-kG magnetic field and was exposed to a separated beam of slow antiprotons which stopped in the chamber. A total of about 300 000 pictures were taken. Some of these pictures have been analyzed by the Oxford-Padua¹ and CERN-Paris² collaborations. A similar analysis has been carried out in Cambridge. The work done in Cambridge on the two-prong events is discussed in this paper.

A total of about 2700 two-prong events were found within a chosen fiducial volume of the chamber. These events were measured with a digitized measuring projector, and were processed by the Cambridge set of data analysis programs, TTR, BUGG-GEOMETRY, and FANY. Each of the events we measured could be one of the following types:

$$\bar{p} + p \rightarrow \pi^+ + \pi^-, \quad (1)$$

$$\pi^+ + \pi^- + \pi^0, \quad (2)$$

$$\pi^+ + \pi^- + n\pi^0, \quad n > 1, \quad (3)$$

$$\pi^+ + \pi^- + K^0 + \bar{K}^0 + m\pi^0, \quad m \geq 0, \quad (4)$$

$$K^+ + K^-, \quad (5)$$

$$K^+ + \pi^- + \bar{K}^0, \quad (6)$$

$$K^- + \pi^+ + K^0, \quad (7)$$

$$K^+ + K^- + \pi^0, \quad (8)$$

$$K^+ + K^- + n\pi^0, \quad (9)$$

$$K^+ + \pi^- + \bar{K}^0 + n\pi^0, \quad (10)$$

$$K^- + \pi^+ + K^0 + n\pi^0. \quad (11)$$

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¹G. B. Chadwick, W. T. Davies, M. Derrick, J. H. Mulvey, D. Radojicic *et al.*, in *Proceedings of the Aix-en-Provence International Conference on Elementary Particles, 1961* (Centre d'Etudes Nucleaires de Saclay, Seine et Oise, 1961), Vol. I, p. 267; also *Phys. Rev. Letters* **10**, 62 (1963). *Proceedings of the Sienna International Conference on Elementary Particles, 1963*, edited by G. Bernadini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963).

²R. Armenteros, L. Montanet, D. R. O. Morrison, S. Nilsson, A. Shapira *et al.*, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki, (CERN, Geneva, 1962), pp. 295, 351; also *Proceedings of the*

These two-prong events were measured regardless of whether V 's were associated with them or not. In each case, we measured only the two charged tracks originating from the annihilation vertex.

II. IDENTIFICATION OF EVENTS

All the events were first classified into two broad groups. The first group consisted of reactions (1)–(4), involving two charged pions. The second group consisted of reactions (5)–(11) and involved at least one charged K meson. In the subsequent discussions we shall refer to events of the first group as pion events, and events of the second group as K events. Apart from those obvious cases where particles stopped or decayed in the chamber, the kaons were separated from pions by means of mean-gap-length measurements, made on all fast tracks up to about 900 MeV/c. In this way, a clear separation between kaons and pions was possible in the majority of cases. However, tracks dipping steeply down away from the cameras were difficult to identify. In addition, their momenta were subject to large measurement errors ($\geq 10\%$). To remove such tracks from the sample, an event selection criterion was used, based on the angle between the normal \mathbf{Z} to the camera plane and the normal \mathbf{n} to the plane containing the two tracks of an event. We found that identification was poor for events with $|\mathbf{n} \cdot \mathbf{Z}| < 0.25$. All such events were therefore rejected. It is important to note that this selection criterion is independent of the kinematics of the event, and therefore our sample is unbiased. After applying this criterion, we finally obtained 196 K events (types 5–11), and 1883 pion events (types 1–4). For these two classes of events, we have plotted the usual histograms to search for multimeson resonances. We discuss first the pion events.

III. PION EVENTS: THE REACTION

$$\bar{p} + p \rightarrow \pi^+ + \pi^- + X^0$$

In Fig. 1 we show the measured two-pion ($\pi^+\pi^-$) effective mass distribution. The two curves shown are covariant phase-space distributions for essentially two different values of the Fermi interaction volume. The derivation of the curves is explained at the foot of Fig. 1. The curves fit the experimental distribution fairly well,

Sienna International Conference on Elementary Particles, 1963, edited by G. Bernadini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963).

but there are significant deviations in the region near 750 MeV. This is probably due to the ρ meson. An interesting feature of the distribution is the peak around 780 MeV, the mass region of the ω^0 meson. It has been shown³ that the ω^0 meson can decay electromagnetically into a $\pi^+\pi^-$ system with violation of the G parity. In this case, the production of a $\pi^+\pi^-$ pair via ω^0 meson mode will interfere with ρ^0 formation. This interference will be observed as a splitting in an otherwise broad ρ^0 peak. The position of the ρ^0 peak could also be shifted on account of this interference. The two peaks near 750 and 780 MeV are believed to be due to this effect.

The bump around 1800 MeV in Fig. 1 is due to

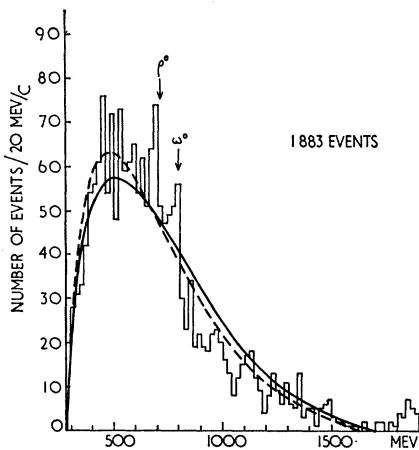


FIG. 1. Effective mass distribution of $\pi^+\pi^-$ in the reaction $\bar{p}+p \rightarrow \pi^++\pi^-+X^0$ at rest. The solid and the dashed curves are both covariant phase-space distributions. In calculating a curve to fit the observed $\pi^+\pi^-$ effective mass distribution from the two-prong events, one has to add, rather incoherently, phase-space curves for two-pion effective mass distributions from various $n\pi$ final states, with $3 \leq n \leq 8$. The relative proportions in which these curves have to be combined depends on the Fermi interaction volume used. The dashed curve is that predicted by the ordinary covariant statistical model with interaction volume $=8\Omega_0$, where $\Omega_0 = \frac{4}{3}(\hbar/\mu\pi c)^3$. The solid curve is that predicted by a modified statistical model [F. N. Ndili, Ph.D. thesis, University of Cambridge, 1964 (unpublished)], in which the possible occurrence of heavy bosons (e.g., ρ , ω , and K^*) was considered. The data, however, show a better agreement with the first model (dashed curve) than with the second.

collinear two-meson $\pi^+\pi^-$ events. From this figure we see that these collinear two-pion events are well resolved from the remaining two-prong events, and we estimate that there are 28 $\pi^+\pi^-$ events in the sample. A better method of estimating the number of $\pi^+\pi^-$ events in our sample is discussed in Sec. VI.

In Fig. 2, we show the missing-mass distribution, with a statistical-model curve fitted to the region from 400 to 1500 MeV. The agreement is satisfactory. As in the experiment of Chadwick *et al.*,¹ the missing-mass distribution shows no strong evidence for production of

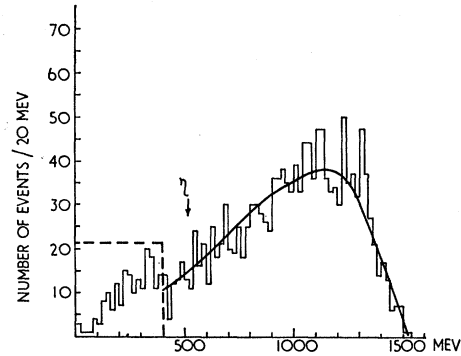
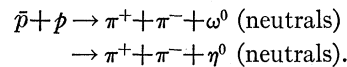


FIG. 2. Missing mass plot in $\bar{p}+p \rightarrow \pi^++\pi^-+X^0$. The solid curve is the ordinary covariant phase space distribution. See reference cited in Fig. 1 caption.

any heavy neutral boson. In particular, we note the complete absence of any significant deviation from phase space in the region of the η (550) meson. It seems that the η meson is not produced in any appreciable amount in proton-antiproton annihilations at rest. This conclusion is also borne out by the data of Chadwick *et al.*¹ on the six-prong events. In general, the significant conclusion from Fig. 2, is the absence of reaction channels of the type



It is not very clear why this should be the case, especially since the η^0 meson is known to decay predominantly via its neutral mode.

In Fig. 3, we show the charged pion (π^\pm) momentum distribution. The distributions for the π^+ and π^- were found to be similar, and have been combined in Fig. 3. The smooth and the dashed curves are again covariant phase-space distributions for essentially two different Fermi interaction volumes. There appears to be a significant deviation from phase space in the region from 200–400 MeV/c. One possible explanation of such

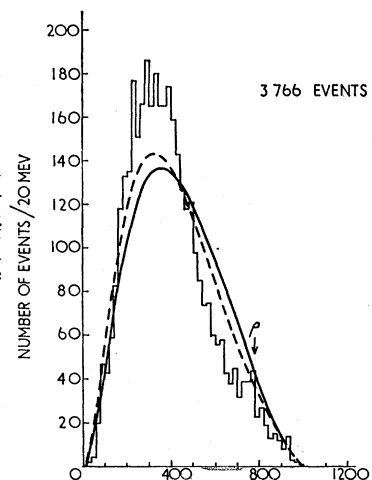


FIG. 3. Momentum distribution of π^\pm in $\bar{p}+p \rightarrow \pi^++\pi^-+K^0$. For explanation of the solid and dashed curves see reference cited in Fig. 1 caption.

³ W. D. Walker, E. West, A. R. Erwin, and R. H. March, in *Proceedings of the 1962 Annual International Conference on High Energy Physics at CERN*, edited by J. Prentki, (CERN, Geneva, 1962), p. 42.

an enhancement is that it is due to a two-body reaction of the type

$$\bar{p} + p \rightarrow \pi^\pm + X^\mp, \quad (12)$$

where X^\mp is an unstable heavy boson of mass around 1400 MeV, decaying into $\pi^\mp m\pi^0$ with $m \geq 1$. Another plausible explanation is that the enhancement is due to two-heavy-boson production

$$\bar{p} + p \rightarrow X_1^+ + X_2^-, \quad (13)$$

where X_1 and X_2 are two heavy bosons, or may be the same object, decaying into $\pi^\pm m\pi^0$ systems. If the mass of such heavy bosons is around 900 MeV, and if reactions of type (13) are appreciable, then as discussed in a similar problem in Sec. V(a), the observed π^\pm momentum distribution will show a broad peak in the region from 200–400 MeV/c. The enhancement in Fig. 3 may be due to such a mechanism. However, there is no evidence at present for such heavy bosons; the recently discovered H and A_1, A_2 mesons⁴ may of course be good candidates. In addition, since the enhancement which we are discussing occurs in the region where the normal phase-space background is expected to peak, it is not easy to see if the enhancement is real.

In Fig. 4, we show a plot of the opening angle between the charged pion tracks. The solid curve is that predicted by the statistical model. The agreement is good. The peaking of the distribution towards large opening angle (or negative $\cos \theta$) can easily be understood from the kinematics of multipion systems. In our distribution, the number of events with $\cos \theta \leq 0$ is 1397 compared with 487 events with $\cos \theta > 0$. This gives a ratio of 2.92 ± 0.15 . This may be compared with the result: 2.59 ± 0.13 obtained by Chadwick *et al.*¹

IV. THREE PION ANNIHILATIONS:

$$\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^0$$

The measured pion events were put through a least-squares kinematics-fitting program. Events fitted to reaction (2) with $\chi^2 \leq 5.0$ were accepted as $\pi^+ \pi^- \pi^0$

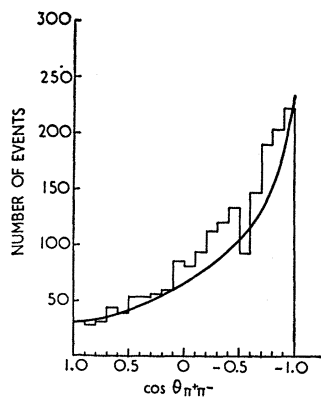


FIG. 4. Angular correlations ($\cos \theta_{\pi^+ \pi^-}$) in $\bar{p} + p \rightarrow \pi^+ + \pi^- + X^0$. The solid curve is ordinary covariant phase-space distribution.

⁴ M. Aderholz, L. Bondar, W. Brauneck, M. Deutschmann, H. Lengeler *et al.*, Phys. Letters **10**, 226 (1964).

events. In this way, we found 208 events of type (2). Possible contamination from $\pi^+ \pi^- 2\pi^0$ events was estimated as less than 10%.

An interesting feature of our result on the 3π events is the Dalitz plot which is shown in Fig. 5. This differs in one important respect from the results of Chadwick *et al.* While the data of Chadwick *et al.* show about the same number of ρ^+ , ρ^- , and ρ^0 , our distribution shows a significant shortage of ρ^0 mesons. By counting the number of points along the three bands we obtain the relative proportion of $\rho^+ : \rho^- : \rho^0$ as 98:90:70. For the same ratios, Chadwick *et al.* found 92:100:100. This difference in the two results may be due to the extra care with which we have separated fast K mesons from pions, and consequently, to the higher purity of our sample.

Now, if we assume that antiproton annihilations proceed mainly from the S state of protonium as is generally believed, then from considerations of isotopic spin, equality of ρ^+ , ρ^- , and ρ^0 implies that the annihilation proceeds from the triplet S state. Complete absence of ρ^0 from our data would imply that only the singlet S state is contributing to the observed $\pi + \rho$ production. If, however, both the triplet and singlet S states are contributing, each with its statistical weight, then the expected relative proportions of $\rho^+ : \rho^- : \rho^0$ will be 100:100:75. Our results are more consistent with the last hypothesis than with either of the other two. We conclude therefore that contribution from the singlet S state is not negligible. This conclusion is also suggested by the angular distribution shown in Fig. 6 for the ρ -meson decay, where one observes a small asymmetry, with a slight forward peaking. As shown by Bouchiat and Flammand,⁵ this is what one would expect if some of the ρ mesons are produced from the 1S_0 state. Also, we note that this asymmetry is present only in the charged ρ^\pm -decay angular distribution. This

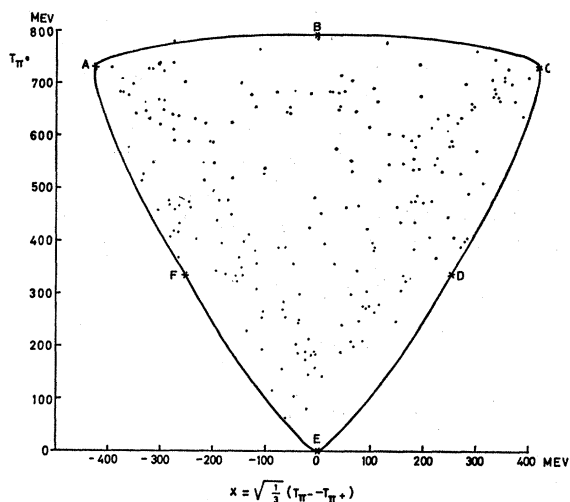


FIG. 5. Dalitz plot for the reaction $\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^0$ at rest.

⁵ C. Bouchiat and G. Flammand, Nuovo Cimento **23**, 13 (1963).

is to be expected from the fact that if the singlet S state contributes at all, only the charged ρ^\pm mesons can be produced from it. Thus, of the charged ρ^\pm mesons which we observe, about 25% are produced from the 1S_0 state and the others from the 3S_1 state. The ρ^\pm mesons from the 1S_0 state give rise to the observed small asymmetry in the angular distribution. All the observed ρ^0 mesons are produced from the 3S_1 state.

Finally, in Fig. 7 we show the two-pion effective mass distribution for the $\pi^+\pi^-\pi^0$ events. The solid curve is taken from Ref. 1, and is an incoherent sum of equal proportions of resonant and pure phase-space distributions. The fit in the high-mass region is not very good. This could again be attributed to the effect of 1S_0 annihilations which have been neglected in calculating the resonant curve. There appears to be some small enhancement in the regions around 680 and 1390 MeV. Both effects are, however, small (two standard deviations), and may simply be due to statistical fluctuations.

V. K-MESON EVENTS

We consider next the K events found in our sample. There was a total of 196 K events, made up as follows:

$$\bar{p} + p \rightarrow K^+ + K^- \quad (9 \text{ events}), \quad (14)$$

$$K^+ + K^- + m\pi^0, \quad m \geq 1 \quad (30 \text{ events}), \quad (15)$$

$$K^\pm + \pi^\mp + X^0 \quad (157 \text{ events}). \quad (16)$$

(a) The Reaction $\bar{p} + p \rightarrow K^\pm K^\mp X^0$

The only unambiguous result from the analysis of these events is the production of $K^*(880)$ in the $K^\pm\pi^\mp$

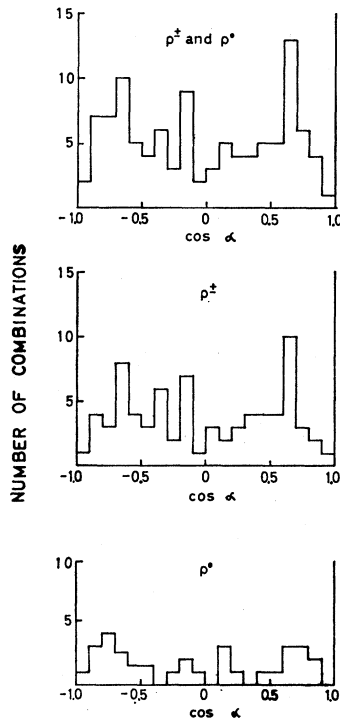


FIG. 6. Angular distribution in $\cos\alpha$ for ρ meson decay, where α is the angle in the ρ meson rest frame between the line of flight of the ρ meson and one of the decay products. For this plot we used events with $700 \leq M_{\pi\pi} \leq 820$ MeV.

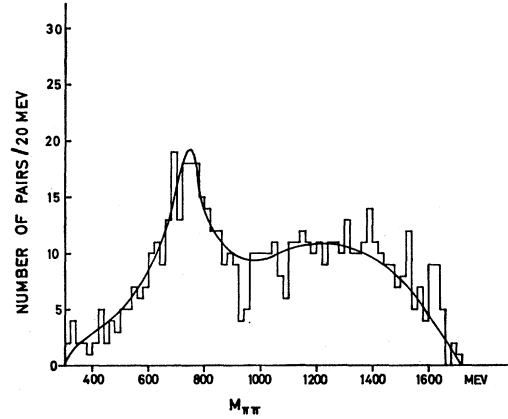


FIG. 7. Two-pion effective mass distribution for the reaction $\bar{p} + p \rightarrow \pi^+\pi^-\pi^0$ at rest. For explanation of the solid curve see text.

system. This can be seen in Fig. 8(a). There is some small evidence for associated K^*K^* production. There is no evidence for any E^\pm production, where the E^\pm mesons are possible charged counterparts of the $K\bar{K}\pi$ resonance reported by Armenteros *et al.*² If such charged multiplets exist, and if the enhancement observed by Armenteros *et al.* is due to the Pais mechanism as has been suggested by Oakes,⁶ then one would expect to observe the follow-

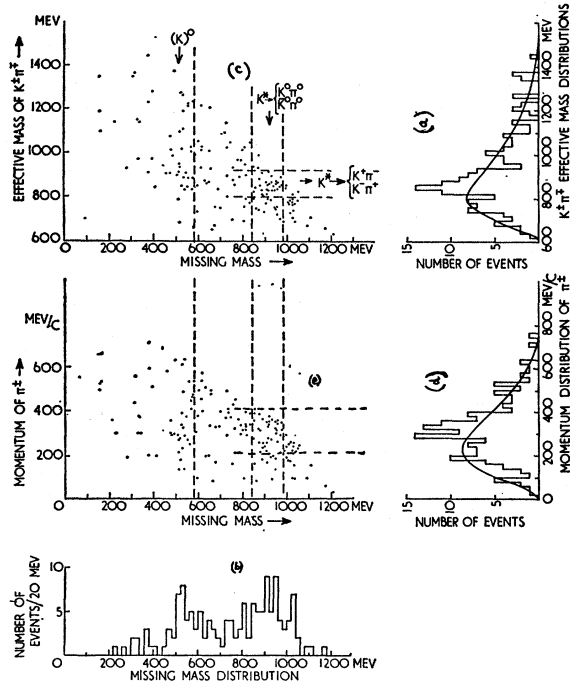
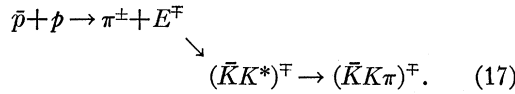


FIG. 8 (a) Effective mass distribution of the $K^\pm\pi^\mp$ system for events of the type $\bar{p} + p \rightarrow K^\pm\pi^\mp X^0$. (b) Missing-mass plot for the same events. (c) Scatter diagram of the $K^\pm\pi^\mp$ effective mass against the missing mass. (d) Momentum distribution of the π^\pm . (e) Scatter diagram of the π^\pm momentum against the missing mass. In (a) and (d) the solid curve is the ordinary covariant phase-space distribution.

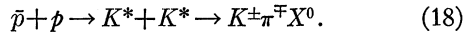
⁶R. J. Oakes, Phys. Rev. Letters 12, 134 (1964).

ing reaction sequence:



In such a two-body process, the π^\pm momentum will have a value around 390 MeV/c corresponding to a mass assignment of 1410 MeV to the E meson. If, therefore, reaction (17) were appreciable, the observed π^\pm momentum distribution for events of type (16) will exhibit a peaking around 390 MeV/c. This feature is, however, not seen in Fig. 8(d). The conclusion is that reaction channels of type (17) are absent from our data.

However, Fig. 8(d) shows a small enhancement in the region from about 200 to 400 MeV/c. Such a broad enhancement can be explained by the hypothesis of associated K^*K^* production through the channel



The way this arises can be seen from considerations of the kinematics of such a reaction. Thus, in the decays of the two K^* mesons, the laboratory momentum of the π^\pm will be kinematically limited to the range 180–410 MeV/c. Then the observed π^\pm momentum distribution will be expected to show a broad peak in this momentum region. Figure 8(d) exhibits just such an effect, and thus suggests a mechanism of double K^* production via channel (18).

We have searched for such associated K^*K^* production in our data. In Fig. 8(c) we show a scatter diagram of the effective mass of the $K^\pm\pi^\mp$ system against the missing mass. Projections on to the two axes are shown in Fig. 8(a) and 8(b), respectively, where some evidence for K^* formation in the $K^\pm\pi^\mp$ system and in the missing mass distribution can be seen. Our measured K^* mass appears fairly low, as can be seen from Fig. 8(a); consequently, the K^* enhancement in the missing-mass plot is slightly shifted towards higher mass value. This effect is probably due to a systematic error in our geometry program which we are now checking. The scatter diagram in Fig. 8(c) shows a small evidence for double K^* production. There are 27 events in the rectangular region enclosed by the dotted lines, where one would expect 10 events on the basis of uniform phase-space distribution.

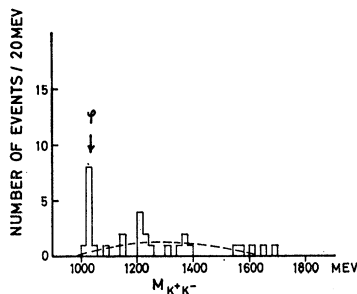


FIG. 9. Effective mass distribution of the K^+K^- system in the reaction $\bar{p} + p \rightarrow K^+K^-\pi^0$ at rest, $m \geq 1$. The dashed curve is phase space.

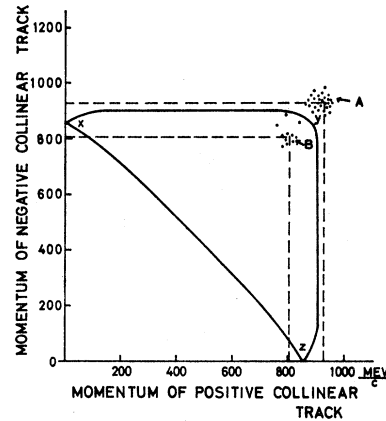


FIG. 10. Scatter diagram of the momenta of collinear two-meson annihilations.

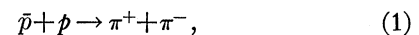
We conclude that a small fraction of the annihilations proceed via reaction (18). We observe also that 60% of the events in the rectangular region in the second scatter diagram in Fig. 8(e) are also in the rectangular region in Fig. 8(c). When such events are removed, the clustering of points in Fig. 8(e) and the small enhancement in Fig. 8(d) disappear. These distributions are therefore consistent with the hypothesis of double K^* production. However, this conclusion must be viewed against our limited statistics.

(b) The Reaction $\bar{p} + p \rightarrow K^+ + K^- + X^0$

These events show evidence for φ meson production. This can be seen from the effective-mass distribution of the K^+K^- system shown in Fig. 9. The mass of the φ meson is found to be 1020 MeV with a width which is compatible with our experimental resolution (≤ 10 MeV). An appreciable fraction of the $K^+K^-X^0$ events proceed via φ meson formation. Within our small statistics, the proportion is about 40%.

VI. ANTIPROTON ANNIHILATIONS AT REST INTO TWO MESONS

We consider next the two-meson channels



We observed 28 events of type (1) and 9 events of type (5) in our sample. We give details of how these events were selected.

The essential requirement for an event to be a two-meson annihilation at rest is that the two tracks should be collinear. In addition, the momentum of each track in channel (1) should be 928 MeV/c, within measurement errors, while that of channel (5) should be 801 MeV/c.

From the sample of the two-prong events within the restricted fiducial volume, we selected all events which looked approximately collinear. About 113 events satisfied this criterion. These events were then measured as

candidates for reactions (1) and (5). The geometry program computed the momentum of each track, and the included angle (θ) between the two tracks. From a study of the reconstruction errors on these angles (typical error ≈ 3 mrad), it was apparent that tracks with included angles $\leq 1^\circ$ (i.e., 17.41 mrad) might be considered as collinear. Of the 113 two-meson candidates, only 39 events satisfied this criterion. These 39 events were then accepted for further analysis.

In order to estimate the amount of possible $\pi^+\pi^-\pi^0$ contamination in this final sample, and to see how well the momenta of the selected events lie within the expected regions, we have plotted each event as a point in a momentum-space diagram. This is shown in Fig. 10. Genuine $\pi^+\pi^-$ events are expected to cluster around point A, while genuine K^+K^- events should cluster around point B. Three-pion ($\pi^+\pi^-\pi^0$) events in which the π^+ and π^- are collinear will lie at any point along the kinematics boundary curve XYZ .

From the distribution, we see that the events tend to cluster around the points A and B, thus indicating that almost all the 39 events are genuine two-meson annihilations. The contamination is therefore small. Also, there is a good separation between $\pi^+\pi^-$ and K^+K^- events. However, in the K^+K^- region, there are 3 events that could possibly be $\pi^+\pi^-\pi^0$ events; two of these events lie very close to the 3π boundary curve.

We have tried to resolve the ambiguities by measuring the mean gap lengths along the tracks of these 3 events. The result is that two of them are $\pi^+\pi^-\pi^0$ events while the third is a K^+K^- event.

The result of the entire analysis is that there are 28 $\pi^+\pi^-$ events and 9 K^+K^- events in the sample.

To see if we have lost some genuine two-meson events, and to justify the angle cutoff at 179° imposed initially on the two-meson candidates, we have plotted the distribution in angle φ for the 39 events, where $\varphi = (\pi - \theta)$

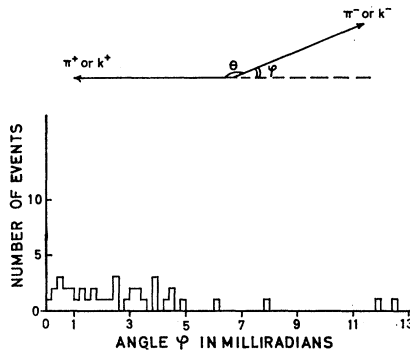


FIG. 11. Distribution in opening angle φ for the selected two-meson candidates.

TABLE I. Experimental estimates of the branching ratios of two-prong channels in antiproton annihilations.

Annihilation channel	% two-prong events (i)	% total antiproton annihilations (ii)	Results of other groups (i)	Results of other groups (ii)
$\pi^+\pi^-$	1.04 ± 0.19	0.389 ± 0.074	...	0.395 ± 0.038^a 0.27 ± 0.04^b
K^+K^-	0.33 ± 0.11	0.125 ± 0.042	...	0.131 ± 0.018^a 0.089 ± 0.023^b
Ratio $\pi^+\pi^-/K^+K^-$	3.2 ± 0.4	3.2 ± 0.4	...	3.02 ± 0.41^a 3.07 ± 0.03^b
$\pi^+\pi^-\pi^0$ (non-resonant)	11.8 ± 0.8 5.9	4.0 ± 0.23 2.1 ± 0.1	12.0(b) ...	5.4 ± 1.0^b 2.7 ± 0.6^b
$\pi^+\rho$ $\omega^0 + \pi^0$ $\pi^+\pi^-$	5.9 0.8	2.1 ± 0.1 0.17 ± 0.06	...	2.7 ± 0.6^b ...
$K^+\pi^0$ $K^0(K^0)m\pi^0$ $K^+\pi^0$ $K^0(K^0)m\pi^0$ (nonresonant)	7.5 ± 0.4 4.5 ± 0.5 2.9 ± 0.3	2.8 ± 0.1 1.7 ± 0.2 1.1 ± 0.1
$K^+K^-m\pi^0$ $\varphi + m\pi^0$ K^+K^- $K^+K^-m\pi^0$	1.4 ± 0.2 0.8 ± 0.1 0.6 ± 0.1	2.8 ± 0.1 0.3 ± 0.1 0.2 ± 0.1

^a Results of Armenteros *et al.* (1962).

^b Results of Chadwick *et al.* (1962).

in mrad. This distribution is shown in Fig. 11. We observe that all but three of the events have $\varphi \leq 6$ mrad. Of the 3 events with $\varphi > 6$ mrad, two have $\varphi \approx 12$ mrad, and these two events were those identified previously as $\pi^+\pi^-\pi^0$ events from ionization measurements. The distribution in Fig. 11 therefore justifies the initial cutoff at $\varphi > 17.41$ mrad.

From the analysis we deduce that the proportion of $\pi^+\pi^-$ events in two-prong annihilations is $(1.04 \pm 0.19)\%$. The proportion of K^+K^- is $(0.33 \pm 0.11)\%$. The ratio

$$\frac{\bar{p} + p \rightarrow \pi^+ + \pi^-}{\bar{p} + p \rightarrow K^+ + K^-} = 3.2 \pm 0.4.$$

A summary of the absolute rates of various identifiable two-prong channels is given in Table I.

ACKNOWLEDGMENTS

It is a pleasure to thank my colleagues Dr. J. A. Zoll, Dr. D. V. Bugg, and B. A. Westwood for assistance with some of the computer programs; also, Dr. A. G. Forson and D. M. Sendall and R. Marsh for maintenance of the measuring machines (TARA). Finally, I would like to thank the Cambridge team of measurers, particularly S. Katvars and M. A. Ali.

A research grant from the University of Ibadan, Nigeria, is gratefully acknowledged.