Production of Σ^+K^+ by π^+-p Interactions at 1111, 1206, and 1265 MeV/c†

N. L. CARAYANNOPOULOS,* G. W. TAUTFEST, AND R. B. WILLMANN
Department of Physics, Purdue University, Lafayette, Indiana
(Received 14 December 1964)

The reaction $\pi^++p\rightarrow\Sigma^++K^+$ has been studied at incident pion momenta of 1111, 1206, and 1265 MeV/c in the Lawrence Radiation Laboratory 72-in. liquid-hydrogen bubble chamber. The cross section for this process was found to be 144 ± 10 μb at 1111 MeV/c, 214 ± 15 μb at 1206 MeV/c, and 278 ± 20 μb at 1265 MeV/c. The measured value of the mean lifetime of the Σ^+ was $(0.76\pm0.03)\times10^{-10}$ sec. Decay asymmetries gave values of $\alpha^+\bar{P}$ of -0.16 ± 0.15, 0.05 ± 0.14, and 0.30 ± 0.14, and values of $\alpha^0\bar{P}$ of 0.50 ± 0.13, 0.37 ± 0.13, and 0.68 ± 0.12 at 1111, 1206, and 1265 MeV/c, respectively. The angular distribution of the Σ^+ hyperons and α^0P as a function of the Σ^+ production angle are presented at each of the three momenta, along with the results of partial-wave analyses based on these distributions.

I. INTRODUCTION

THE purpose of this paper is to present data obtained in a study of the reaction $\pi^++p\rightarrow\Sigma^++K^+$ at incident laboratory momenta of the π^+ of 1111, 1206, and 1265 MeV/c. At each momentum the total cross section, the production angular distribution, and the angular distribution of the decay pion from each of the two decay modes $\Sigma^+\rightarrow p+\pi^0$ and $\Sigma^+\rightarrow n+\pi^+$ have been measured. In addition, a determination of the partial-wave amplitudes has been attempted.

It is hoped that these data, when combined with those of previous similar experiments,[1–7] will aid in a study of the processes responsible for Σ^+K^+ production. They may also be used with data on Σ^- and Σ^0 production to test the charge-independence hypothesis.

II. EXPERIMENTAL DETAILS

The Σ^+K^+ events were produced by a π^+ beam in the 72-in. liquid-hydrogen bubble chamber. The experimental beam setup has been described in Ref. 8. Because of the 32-MeV/c momentum loss due to ionization as the pions passed through the fiducial volume, each set of data represents an average over this momentum interval. The mean values of the momenta at the center of the chamber were 1111 ± 7, 1206 ± 10, and 1265 ± 9 MeV/c. These values were obtained from those Σ^+K^+ events which had a K^+ that stopped in the chamber.

† Supported in part by the U. S. Atomic Energy Commission.
* This work constituted part of a thesis submitted by N. L. Carayannopoulos to the Purdue Graduate School in partial fulfillment of the requirements for the Ph.D. degree.
[1] J. L. Brown, D. A. Glaser, D. I. Meyer, M. L. Perl, and J. Vander Velde, Phys. Rev. **107**, 906 (1957).
[2] A. R. Erwin, Jr., J. K. Kopp, and A. M. Shapiro, Phys. Rev. **115**, 669 (1959).
[3] W. H. Hannum, H. Courant, E. C. Fowler, H. L. Kraybill, J. Sandweiss, and J. Sanford, Phys. Rev. **118**, 577 (1960).
[4] A. Berthelot, A. Daudin, O. Goussu, F. Grard, M.-A. Jabiol *et al.*, Nuovo Cimento **21**, 693 (1961).
[5] C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. L. Stonehill, and H. D. Taft, Rev. Mod. Phys. **33**, 374 (1961).
[6] F. Grard and G. A. Smith, Phys. Rev. **127**, 607 (1962).
[7] F. S. Crawford, Jr., F. Grard, and G. A. Smith, Phys. Rev. **128**, 368 (1962).
[8] S. E. Wolf, N. Schmitz, L. J. Lloyd, W. Laskar, F. S. Crawford, Jr., *et al.*, Rev. Mod. Phys. **33**, 439 (1961).

A. Selection of Events

All stereo triads were scanned twice in two views for Σ^+K^+-type events. All events which occurred within the fiducial volume, and which were produced by a beam track that passed between the limiting marks of a template used with view 1 were accepted for measurement. For this experiment, the fiducial volume was defined as -68 cm$\leq y\leq68$ cm and -17.5 cm$\leq x\leq17.5$ cm, where y is the dimension parallel to the beam, and x is perpendicular to the beam and parallel to the top glass. The template was designed so that tracks passing within the boundaries would not leave and re-enter the fiducial volume.

After measurement, the events were analyzed using the PANG and KICK programs. They were considered acceptable if the production vertex gave a χ^2 of less than 25 for a four-constraint fit or 18 for a three-constraint fit, and the decay vertex gave a χ^2 of less than 12 for a one-constraint fit. There were two possible classes for the production fit because p, the measured momentum of the Σ, was not used whenever $\Delta p/p$ was greater than 0.50, and the constraint class was dropped to three. Since most of the Σ tracks were short, few vertex fits were of the four-constraint type.

To ensure that the momentum interval over which the data were taken was no larger than 50 MeV/c, events produced by pions whose momenta were more than two standard deviations from the mean were discarded. To insure that the beam was well collimated, the remaining events were used to determine the parameters p through u in the expressions

$$\varphi=p+qx+rz$$

and

$$\lambda=s+tx+uz\,,$$

where φ=azimuth of the beam track at $y=-89$ cm, λ=dip of the beam track at $y=-89$ cm, and z is the coordinate perpendicular to x and y. The dip and azimuth of each event were then compared with the values computed from the above equations. Those events having values which differed by more than 1 deg in either φ or λ were discarded.

B. Tests for Scanning Biases

A histogram of the number of events versus time of flight as measured in the Σ rest system was made for each of the two decay modes at each of the three momenta. The histograms indicated that events with short Σ tracks were missed in the scanning. Surprisingly, this bias did not seem to depend upon the decay mode. In order to correct for the scanning loss, a minimum length was chosen for each momentum, and events with Σ tracks shorter than this were discarded. The lengths used were 0.4 cm at 1111 MeV/c, 0.25 cm at 1206 MeV/c, and 0.25 cm at 1265 MeV/c.

In the subsequent calculations of the total cross sections, production angular distributions, and pion-decay distributions, each event which was retained was weighted by a factor equal to

$$+\exp(l_{\min}M_{\Sigma^+}/cp_{\Sigma^+}\tau_{\Sigma^+})$$

representing the inverse of the probability for the observation of a Σ track with length greater than $l_{\min}$. Here, M_{Σ^+} is the mass of the Σ^+ (1189.4 MeV/c^2),[9] c is the velocity of light, p_{Σ^+} is the momentum of the Σ^+ in the laboratory system, and τ_{Σ^+} is the mean lifetime of the Σ^+ (0.81×10^{-10} sec).[9]

There appeared to be no scanning losses correlated with the orientations of the production plane and decay plane in the bubble chamber.

If parity is conserved in the strong interactions there should be no forward-backward asymmetry in the distribution of the decay pion in the Σ^+ rest system. However, such an asymmetry was found at the highest momentum in the $\Sigma^+\to p+\pi^0$ decay mode. This was attributed to a scanning loss of events in which the proton was emitted at a small angle to the Σ^+ in the laboratory system. Support for this assumption was obtained from the observed branching ratio

$$(\Sigma^+\to p+\pi^0)/(\Sigma^+\to n+\pi^+)=0.84.$$

The observed ratio is very near one at the two lower momenta, as is predicted by the $\Delta I=\frac{1}{2}$ rule.

A correction for the small-angle decay loss was made by keeping only those events with $-0.70<\cos\psi<0.97$, where ψ is the angle between the π^0 and Σ^+ direction in the Σ^+ rest system. Each event which was retained was weighted by the factor

$$2.0/(0.70+0.97)=1.20$$

in the calculation of the total cross section and production angular distribution.

III. EXPERIMENTAL RESULTS

A. Σ^+ Mean Lifetime

The maximum-likelihood method was used to obtain τ_{Σ^+}, the mean lifetime of the Σ^+. For events with Σ tracks longer than a minimum length $l_{\min}$ and with a potential time of flight large compared to τ_{Σ^+}, this method reduced to the following equation:

$$\tau_{\Sigma^+}=\Big[\sum_{i=1}^{N}(t_i-M_{\Sigma^+}l_{\min}/p_i)\Big]/N.$$

Here τ_i is the observed lifetime for the ith decay, p_i is the momentum of the Σ^+, and N is the total number of decays. Results are presented in Table I. The value obtained from a combination of these data is $(0.76\pm0.03)\times10^{-10}$ sec.

TABLE I. Σ^+ lifetimes for the decay modes $\Sigma^+\to\pi^0+p$ at three incident momenta.

Incident momentum	Decay mode	N	τ (10^{-10} sec)
1111	$\Sigma\to\pi^++n$	127	0.81 ± 0.07
	$\Sigma\to\pi^0+p$	132	0.79 ± 0.07
1206	$\Sigma\to\pi^++n$	173	0.76 ± 0.06
	$\Sigma\to\pi^0+p$	167	0.73 ± 0.06
1265	$\Sigma\to\pi^++n$	161	0.72 ± 0.06
	$\Sigma\to\pi^0+p$	135	0.78 ± 0.08
Combined data			0.76 ± 0.03

B. Cross Sections

The total π^+ path length scanned was obtained from a count of the number of tracks which passed through the limiting marks on a template, as described in II. The fraction of these tracks which satisfied the criteria on p, λ, and φ was obtained from measurement of a sample of noninteracting beam tracks.

The proton contamination in the beam was obtained from measurements of two-prong events at 1111 and 1265 MeV/c. The number of entering protons was calculated from the number of proton-proton elastic scatterings which had a c.m. angle of the proton greater than 30°. At smaller angles elastic (p,p) events could not be separated from elastic (π^+,p) events. The elastic (p,p) cross sections and angular distributions were extrapolated to 1111 and 1265 MeV/c from the data in Ref. 10. At both of these momenta a value of $\sigma(p,p)_{\text{elastic}}=25\pm2$ mb was used. At these two momenta the proton contamination was found to be less than 0.05%, and has been neglected in the calculations of the Σ^+K^+ cross section. The proton contamination was not measured at 1206 MeV/c.

Leptonic contamination in the beam at 1111 and 1265 MeV/c was calculated from the number of beam tracks which produced high-energy δ rays. If p_1 is the upper limit of the δ-ray spectrum from pions of momentum p_i, and if p_2 is the upper limit of the δ-ray spectrum from muons of momentum p_i, δ rays with momenta greater than p_2 must be produced by posi-

[9] W. H. Barkas and A. H. Rosenfeld, UCRL-8030, 1963 (unpublished).

[10] L. W. Smith, A. W. McReynolds, and G. Snow, Phys. Rev. **97**, 1186 (1955).

trons. δ rays with momenta between p_1 and p_2 can be produced by either positrons or muons. The cross sections for δ-ray production, integrated over the appropriate momentum intervals, are given in Table II.[11] Events in which a high-energy δ ray was produced by the incident particle were measured and constrained to obtain the δ-ray momenta. The combined muon and positron contamination was found to be $(7.8\pm1.9)\%$ at 1111 MeV/c and $(7.8\pm2.2)\%$ at 1265 MeV/c. Dr. Margaret Foster generously supplied us with the value of the contamination at 1206 MeV/c [$(8.4\pm3.6)\%$] which she measured in the same film.

TABLE II. Cross sections used to determine the leptonic contamination. p_i is the incident momentum; p_1 is the upper limit of the δ-ray spectrum from pions; p_2 is the upper limit of the δ-ray spectrum from muons; σ_1 is the cross section for production of a δ ray with momentum greater than p_1 by a muon; σ_{12} is the cross section for production of a δ ray with momentum between p_1 and p_2 by a positron; σ_2 is the cross section for production of a δ ray with momentum greater than p_2 by a positron.

p_i (MeV/c)	p_1 (MeV/c)	p_2 (MeV/c)	σ_1 (mb)	σ_{12} (mb)	σ_2 (mb)
1111	61.1	102.6	0.373	1.55	1.55
1265	78.7	131.3	0.277	1.13	1.21

A summary of the information used to determine the (Σ^+,K^+) cross sections is given in Table III.[12–14] The values of the cross sections obtained were 0.144 ± 0.010 mb at 1111 MeV/c, 0.214 ± 0.015 mb at 1205 MeV/c, and 0.278 ± 0.020 mb at 1265 MeV/c.

C. Angular Distribution

The angular distributions of the Σ^+ in the production center of mass are shown in Fig. 1. The least-squares method was used to obtain best fits to polynomials of the form

$$d\sigma/d\Omega \propto 1+A_1\cos\theta+A_2\cos^2\theta+\cdots,$$

where θ is the angle of production of the Σ^+ in the interaction center of mass. At 1111 MeV/c, the fit was not improved when the order of the polynomial was increased above two. At 1206 and 1265 MeV/c the fit was not improved when the order was increased above four. The coefficients are given in Table IV.

D. Decay Asymmetries

If parity is not conserved in the decay process, the angular distribution of decay pions in the Σ^+ rest system must be of the form

$$d\sigma/d(\cos\theta)=1+\alpha\bar{P}\cos\theta,$$

where α is the decay asymmetry parameter, $\bar{P}$ is the average polarization of the Σ hyperons, and θ is the angle between the momentum vector of the decay pion and $\mathbf{n}$, the normal to the production plane. The last mentioned is defined by

$$\mathbf{n}=\mathbf{K}_1\times\mathbf{K}_2/|\mathbf{K}_1|\,|\mathbf{K}_2|\,\sin\theta_{12},$$

where $\mathbf{K}_1$ represents the momentum of the incident

TABLE III. Scanning information and total cross sections for $\pi^++p\rightarrow\Sigma^++K^+$.

	1111 MeV/c	1206 MeV/c	1265 MeV/c
(a) Mean momentum	1111 MeV/c	1206 MeV/c	1265 MeV/c
(b) Number of pictures scanned	36 458	34 985	24 765
(c) Number of incident beam tracks	$(6.318\pm0.031)\times10^5$	$(4.590\pm0.035)\times10^5$	$(3.418\times0.025)\times10^5$
(d) Lepton contamination			
Muons	$7.27\pm1.88\%$	$7.95\pm3.56\%$	$7.27\pm2.18\%$
Positrons	$0.58\pm0.18\%$	$0.47\pm0.21\%$	$0.48\pm0.20\%$
Total	$7.85\pm1.90\%$	$8.42\pm3.57\%$[a]	$7.75\pm2.20\%$
(e) Fraction of beam with acceptable p, λ, φ	0.852 ± 0.025	0.952 ± 0.010	0.935 ± 0.012
(f) Mean track length corrected for strong interactions	129.0 ± 2.5 cm	129.1 ± 2.5 cm	127.6 ± 2.6 cm
Total (π^+,p) cross sections used	25.80 mb[b]	30.8 ± 0.44 mb[c]	34.3 ± 0.52 mb[c]
(g) Observed number of events			
π^+n	128	173	161
π^0p	132	167	135
(h) Number of events corrected for scanning loss			
π^+n	162.4	198.9	184.0
π^0p	166.5	192.0	186.0
(i) Cross section based on a target density of 0.0595 g/cm³	0.144 ± 0.010 mb	0.214 ± 0.015 mb	0.278 ± 0.020 mb

[a] See Ref. 12.
[b] See Ref. 13.
[c] See Ref. 14.

[11] B. Rossi, *High-Energy Particles* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1952).
[12] Margaret Foster (private communication).
[13] G. W. Tautfest and R. B. Willmann (to be published).
[14] T. J. Devlin, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. **125**, 690 (1962).

TABLE IV. Coefficients obtained from least-squares fits to the production angular distributions.

Incident momentum	A_1	A_2	A_3	A_4	x^2	Degrees of freedom
1111 MeV/c	1.23 ± 0.13	$0.81\ \pm0.25$			2.56	7
1206 MeV/c	1.93 ± 0.24	$0.45\ \pm0.75$	-1.92 ± 0.41	-0.23 ± 0.88	6.42	5
1265 MeV/c	1.78 ± 0.21	0.045 ± 0.70	-2.14 ± 0.32	-0.51 ± 0.78	6.93	5

pion, $\mathbf{K}_2$ represents the momentum of the Σ^+, and θ_{12} is the angle between them.

The product $\alpha\bar{P}$ was determined for each of the two decay modes from a histogram of the number of events (each weighted because of the loss of short Σ tracks) versus $\cos\theta$. An additional correction was applied to the $\Sigma^+\rightarrow\pi^0+p$ decay mode at 1265 MeV/c because of the loss of events with small decay angles. This was done in the following way: A coordinate system was defined in the Σ^+ rest system with the z axis along $\mathbf{n}$, the y axis along the Σ^+ direction, and the x axis perpendicular to the two. Each event is to be represented by a unit vector along the decay-pion direction. The projection along the z axis is $\cos\theta$ and the projection along the y axis is $\cos\psi$. The tips of the vectors fall on a sphere of unit radius. Because of scanning loss only events with $-0.70<\cos\psi<0.97$ were retained. This is shown geometrically in Fig. 2. Those events which would fall in the shaded area were lost. If parity is conserved in the production process, the density of points on the sphere should show circular symmetry about the z axis. Hence, the correction to the histogram was made by dividing the number of events in a given range of $\cos\theta$ by the fraction of the area of the annular surface which was unshaded.

The angular distributions of the decay pions in the Σ^+ rest system for the decay mode $\Sigma^+\rightarrow\pi^++n$ are shown in Fig. 3, and for $\Sigma^+\rightarrow\pi^0+p$ in Fig. 4.

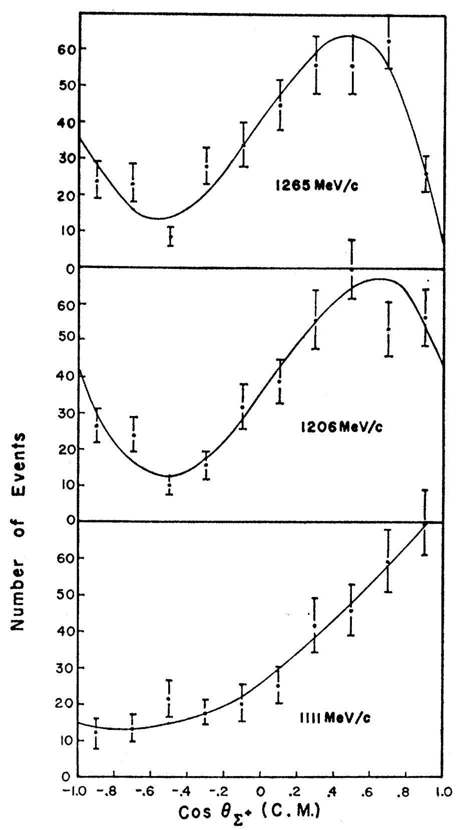

FIG. 1. The angular distributions of Σ hyperons in the center-of-mass system at incident-pion momenta of 1265, 1206, and 1111 MeV/c. The curves are the results of least-squares fits to polynomials of the form $1+A_1\cos\theta+A_2\cos^2\theta+\cdots$. The coefficients are given in Table IV.

E. Partial-Wave Analysis

The expressions for the differential cross section $d\sigma/d\Omega$ and the sigma polarization P as functions of the partial-wave amplitudes have been given by Nauenberg and Pais,[15] and have been written out explicitly for s, p, and d waves by Crawford *et al.*[7] As stated in the latter

$$d\sigma/d\Omega=[g(\theta)]^2+[h(\theta)]^2,$$
$$P(d\sigma/d\Omega)=2\,\mathrm{Im}h^*(\theta)g(\theta),$$

where $g(\theta)$ is the non-spin-flip amplitude, $h(\theta)$ is the

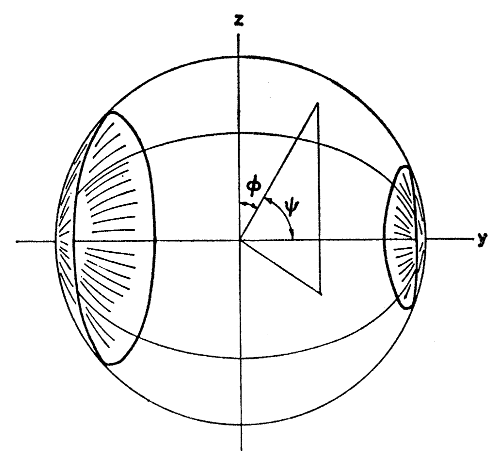

FIG. 2. The coordinate system used to correct for scanning loss of Σ hyperons produced by 1265-MeV/c pions and decaying by the (π^0p) mode. The z axis is along the normal to the production plane and the y axis is along the Σ^+ direction in the Σ^+ rest system. Each possible event is represented by a unit vector from the origin along the decay-pion direction. Those events for which the tips of the vectors would fall in the shaded area were missed in scanning.

[15] M. Nauenberg and A. Pais, Phys. Rev. **123**, 1058 (1961).

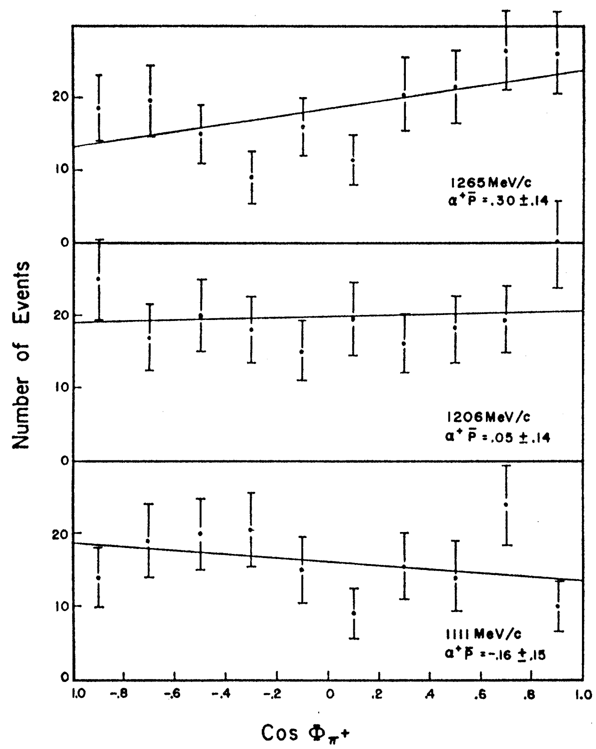

FIG. 3. Angular distributions of the pions from the decay mode (π^+n) in the Σ^+ rest system. The angle between the normal to the production plane and the pion direction is represented by φ_{π^+}. The solid lines are the results of least-squares fits to $1+\alpha^+\bar{P}\times\cos\varphi_{\pi^+}$.

spin-flip amplitude, and θ is the angle of production of the Σ^+ in the center of mass. In terms of the s-, p-, and d-wave amplitudes,

$$g(\theta)=S_{1/2}+(2P_{3/2}+P_{1/2})\cos\theta+\tfrac{1}{2}(3D_{5/2}+2D_{3/2})\times(3\cos^2\theta-1),$$

$$h(\theta)=(P_{3/2}-P_{1/2})\sin\theta+3(D_{5/2}-D_{3/2})\sin\theta\cos\theta,$$

where the subscript refers to the total angular momentum of the state. The probability that a Σ^+ is produced at an angle θ and decays to a pion which makes an angle φ with the normal to the production plane in the Σ rest system is

$$f=A_0+A_1\cos\theta+A_2\cos^2\theta+A_3\cos^3\theta+A_4\cos^4\theta+\alpha\cos\varphi\sin\theta(A_5+A_6\cos\theta+A_7\cos^2\theta+A_8\cos^3\theta),$$

where α is the decay asymmetry parameter of the Σ^+ (α^0 for $p\pi^0$ and α^+ for $n\pi^+$). Here,

$$A_0=a^2+d^2/4-ad\cos\chi_d+c^2,$$
$$A_1=2ab\cos\chi_b-bd\cos(\chi_d-\chi_b)+6ce\cos(\chi_c-\chi_e),$$
$$A_2=b^2-c^2+9e^2-3d^2/2+3ad\cos\chi_d,$$
$$A_3=3bd\cos(\chi_d-\chi_b)-6ce\cos(\chi_c-\chi_e),$$
$$A_4=9d^2/4-9e^2,$$
$$A_5=-2ac\sin\chi_c+cd\sin(\chi_c-\chi_d),$$
$$A_6=-2cb\sin(\chi_c-\chi_b)-6ea\sin\chi_e+3ed\sin(\chi_e-\chi_d),$$
$$A_7=-3cd\sin(\chi_c-\chi_d)-6ed\sin(\chi_e-\chi_b),$$
$$A_8=-9ed\sin(\chi_e-\chi_d),$$

and

$$S_{1/2}=a,$$
$$2P_{3/2}+P_{1/2}=b\exp(i\chi_b),$$
$$P_{3/2}-P_{1/2}=c\exp(i\chi_c),$$
$$3D_{5/2}+2D_{3/2}=d\exp(i\chi_d),$$
$$D_{5/2}-D_{3/2}=e\exp(i\chi_e).$$

The values of a through e were determined by the maximum likelihood method from the likelihood function

$$L=\prod_{i=1}^{N}\left[f^{(i)}k^{(i)}\Big/\int_{-1}^{1}\int_{-1}^{1}fkd(\cos\theta)d(\cos\varphi)\right],$$

where N is the number of events, and k is given by

$$k=\exp[-l_{\min}M_{\Sigma^+}/cp_{\Sigma^+}\tau_{\Sigma^+}].$$

The quantity k is the probability that a Σ^+ produced at an angle θ would be observed in the sample of events used. The decay asymmetry parameters α^0 and α^+ have been taken equal to $+1$ and zero, respectively.

At 1111 MeV/c, a second-order polynomial was

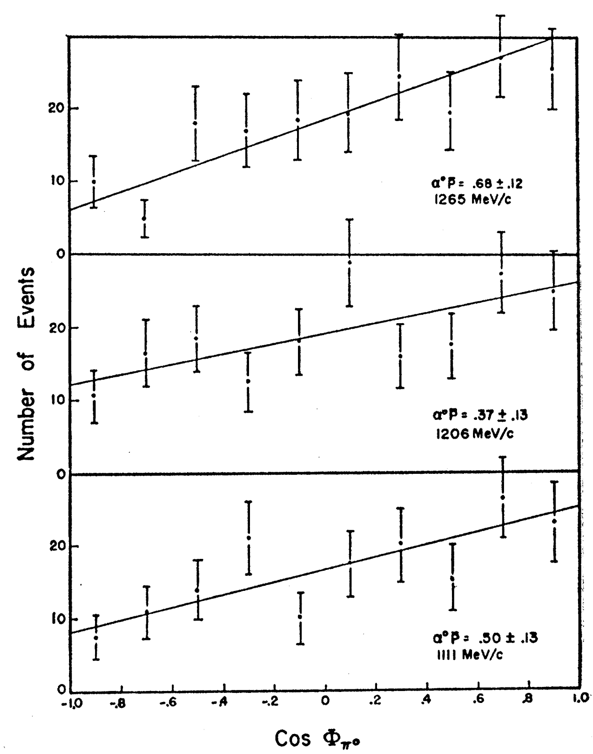

FIG. 4. Angular distributions of the pions from the decay mode (π^0p) in the Σ^+ rest system. The angle between the normal to the production plane and the pion direction is represented by φ_{π^0}. The solid lines are maximum-likelihood fits to $1+\alpha^0\bar{P}\cos\varphi_{\pi^0}$.

TABLE V. Maximum-likelihood solutions to the partial-wave amplitudes assuming $\alpha^0=1$ and $\alpha^+=0$. Angles are expressed in radians. The error matrix for the 1111-MeV/c data is given at the bottom.

Incident momentum	1111 MeV/c	1206 MeV/c	1265 MeV/c
b/a	1.38 ± 0.40	2.25 ± 0.35	2.55 ± 0.50
χ_b	0.76 ± 0.12	0.90 ± 0.15	1.60 ± 0.30
c/a	0.83 ± 0.35	2.05 ± 0.30	3.65 ± 0.55
χ_c	3.85 ± 0.25	4.00 ± 0.30	4.50 ± 0.20
d/a		0.75 ± 0.40	1.50 ± 0.70
χ_d		4.80 ± 0.50	3.95 ± 0.45
e/a		0.50 ± 0.15	1.45 ± 0.27
χ_e		4.30 ± 0.55	5.10 ± 0.20
a	0.074 ± 0.013 (mb/sr)$^{1/2}$	0.054 ± 0.010 (mb/sr)$^{1/2}$	0.038 ± 0.008 (mb/sr)$^{1/2}$

Error matrix

b/a	0.1601	0.1248	-0.02075	-0.03915
c/a		0.1239	-0.02434	-0.03669
χ_b			0.01510	0.00947
χ_c				0.06482

sufficient to fit the angular distribution, so it was assumed that only s and p waves were present. Therefore, the likelihood function was simplified by setting d and e equal to zero. As in two previous experiments[6,7] setting α^0 equal to $+1$ gave a better fit to the data than setting α^0 equal to 0.8. At 1206 and at 1265 MeV/c, fourth-order polynomials were required to fit the angular distributions, and it was assumed that s, p, and d waves were present.

At 1265 MeV/c, the likelihood function had to be changed to account for the loss of small-angle decays in the $\pi^0 p$ decay mode. The following likelihood functions were used in the different ranges of $\cos\varphi$:

$$|\cos\varphi|<[1-(0.97)^2]^{1/2},$$

$$L=\prod_{i=1}^{N_j}\frac{f^{(i)}k^{(i)}\left[\cos^{-1}\left(\dfrac{-0.7}{\sin\varphi_i}\right)-\cos^{-1}\left(\dfrac{0.97}{\sin\varphi_i}\right)\right]\Big/\pi}{1.67\displaystyle\int_{-1}^{1}fkd(\cos\theta)},$$

$$[1-(0.97)^2]^{1/2}\leq|\cos\varphi|<1-(0.7)^2,$$

$$L=\prod_{i=1}^{Nj}\frac{f^{(i)}k^{(i)}\left[\cos^{-1}\left(\dfrac{-0.7}{\sin\varphi_i}\right)\right]\Big/\pi}{1.67\displaystyle\int_{-1}^{1}fkd(\cos\theta)},$$

$$[1-(0.7)^2]^{1/2}\leq|\cos\varphi|,$$

$$L=\prod_{i=1}^{Nj}\left[f^{(i)}k^{(i)}\Big/1.67\int_{-1}^{1}fkd(\cos\theta)\right],$$

where N_j is the number of events observed in the range of $\cos\varphi$. The complete likelihood function for 1265 MeV/c was obtained from the product of these three functions and the likelihood function for the (π^+n) decay mode.

The results of the partial-wave analysis are presented in Table V, along with the complete error matrix for the 1111-MeV/c data. These results should not be inter-

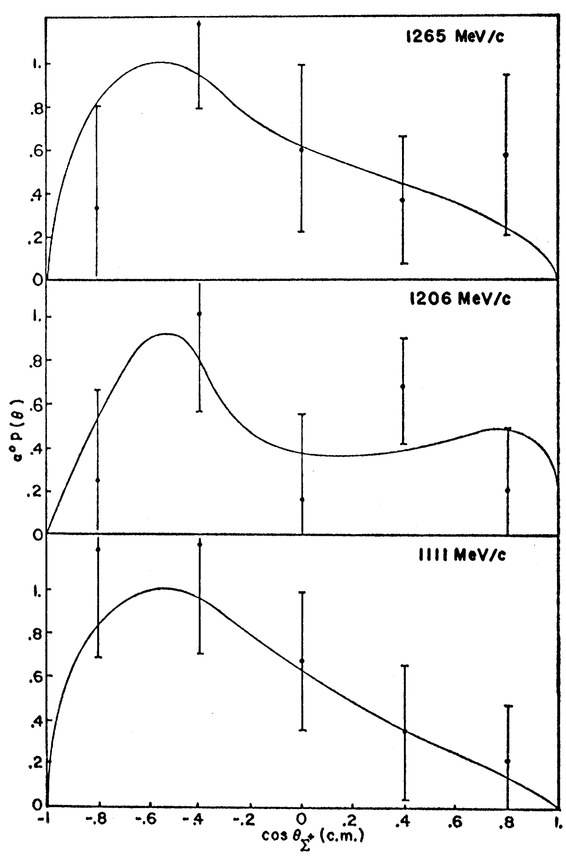

FIG. 5. The decay asymmetry of the decay pions from the $(\pi^0 p)$ decay mode with respect to the normal to the production plane. The solid curves represent $P(\theta)$ computed from the coefficients in Table VI, under the assumption that $\alpha^0=1$.

TABLE VI. Table of A_n computed from the partial-wave amplitudes given in Table V. All values are in units of mb/sr.

Incident momentum	1111 MeV/c	1206 MeV/c	1265 MeV/c
A_0	0.00925 ± 0.00099	0.0147 ± 0.0055	0.0241 ± 0.0093
A_1	0.01090 ± 0.00135	0.0278 ± 0.0108	0.0429 ± 0.0181
A_2	0.00671 ± 0.00230	0.0061 ± 0.0086	0.0079 ± 0.0146
A_3		-0.0273 ± 0.0126	-0.0520 ± 0.0222
A_4		-0.0023 ± 0.0054	-0.0200 ± 0.0144
A_5	0.00594 ± 0.00177	0.0054 ± 0.0050	0.0153 ± 0.0072
A_6	-0.00070 ± 0.00315	0.0053 ± 0.0103	0.0139 ± 0.0126
A_7		0.0147 ± 0.0140	-0.0016 ± 0.0167
A_8		0.0050 ± 0.0071	0.0272 ± 0.0193

preted as giving the individual partial-wave amplitudes unambiguously, however. Two types of ambiguities exist which give the same angular distribution and polarization. The first alternate solution may be obtained from the solutions presented by replacing χ_b by $-\chi_b$, χ_c by $\pi-\chi_c$, χ_d by $-\chi_d$, and χ_e by $\pi-\chi_e$. A second and a third alternate solution can be obtained from these two solutions by letting the amplitudes given for $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, and $d_{5/2}$ represent the amplitudes for $p_{1/2}^*$, $s_{1/2}^*$, $d_{3/2}^*$, $p_{3/2}^*$, and $f_{5/2}^*$, respectively.[16]

It was hoped that some of these solutions could be eliminated by comparing them to those at neighboring energies. However, at the two higher momenta, the errors were too large. At 1111 MeV/c, it appears that the set of amplitudes shown in Table V is a better choice than the two Minami solutions. First, this set agrees well with the values extrapolated from the threshold experiment, assuming the energy dependence used in that experiment. Second, the angular distribution can be fitted quite well with a second-order polynomial, which can be obtained with only $s_{1/2}$, $p_{1/2}$, and $p_{3/2}$ nonzero, or with $s_{1/2}$, $p_{1/2}$, and $d_{3/2}$ nonzero. It seems unlikely that the $d_{3/2}$ amplitude is nonzero if the $p_{3/2}$ amplitude is zero.

The A_n were computed from the partial-wave amplitudes and are given in Table VI. The angular distributions based on the A_n agree very well with the observed distributions. As another comparison, the polarization $P(\theta)$, computed from the A_n is shown in Fig. 5 along with the data from the decay mode $\Sigma^+\rightarrow p+\pi^0$. The average value of $\bar{P}$ computed from the coefficients is 0.41 at 1111 MeV/c, 0.44 at 1206 MeV/c, and 0.55 at 1265 MeV/c.

[16] S. Minami, Progr. Theoret. Phys. (Kyoto) **11**, 213 (1954).

ACKNOWLEDGMENTS

The authors wish to thank Professor Luis Alvarez, Professor Frank Crawford, and Professor Myron Good for making the film available. We also want to thank Professor Good and his collaborators at the University of Wisconsin for their assistance throughout this work.

The contributions of the scanning and measuring staff are gratefully acknowledged. In particular we appreciate the conscientious work of Barbara Kelly, Donna Kress, Wayne Lockwood, Judy Strasburger, Loretta Wagner, and Phyllis White. Also, we want to thank Eugene Durr for maintaining the equipment and handling all technical problems.