We also include here the currents which generate $U^+(6)\otimes U^-(6)$. Thus, under (4) we have

$$\delta\mathcal{L}=\partial_\lambda\left[\frac{\partial\mathcal{L}}{\partial(\partial_\lambda\psi_L)}\delta\psi_L+\frac{\partial\mathcal{L}}{\partial(\partial_\lambda\psi_R)}\delta\psi_R\right]+\text{meson contributions}$$

$$=-\partial_\lambda[\bar\psi_L\gamma_\lambda(\tfrac{1}{2}i\alpha_{\mu\nu}{}^j\sigma_{\mu\nu}T^j+i\alpha^jT^j)\psi_L+\bar\psi_R\gamma_\lambda(\tfrac{1}{2}i\beta_{\mu\nu}{}^j\sigma_{\mu\nu}T^j+i\beta^jT^j)\psi_R+\cdots]$$

$$=-\tfrac{1}{2}i\alpha_{\mu\nu}{}^j\partial_\lambda J_{L\mu\nu,\lambda}{}^j-\tfrac{1}{2}i\beta_{\mu\nu}{}^j\partial_\lambda J_{R\mu\nu,\lambda}{}^j-i\alpha^j\partial_\lambda J_{L\lambda}{}^j-i\beta^j\partial_\lambda J_{R\lambda}{}^j,$$

where

$$J_{L\mu\nu,\lambda}{}^j=\bar\psi_L\gamma_\lambda\sigma_{\mu\nu}T^j\psi_L+\cdots,\quad J_{L\lambda}{}^j=\bar\psi_L\gamma_\lambda T^j\psi_L+\cdots$$

and similarly for J_R. Since the only change in the Lagrangian is that of the free part,

$$\partial_\lambda J_{L\lambda}{}^j=\partial_\lambda J_{R\lambda}{}^j=0,\quad j=0,1,\cdots,8$$

$$\left.\begin{aligned}\partial_\lambda J_{L\mu\nu,\lambda}{}^j&=2\bar\psi_L(\gamma_\nu\partial_\mu-\gamma_\mu\partial_\nu)T^j\psi_L+\cdots\\ \partial_\lambda J_{R\mu\nu,\lambda}{}^j&=2\bar\psi_R(\gamma_\nu\partial_\mu-\gamma_\mu\partial_\nu)T^j\psi_R+\cdots\end{aligned}\right\}j=1,\cdots,8$$

and

$$\partial_\lambda J_{R\mu\nu,\lambda}{}^0=\partial_\lambda J_{L\mu\nu,\lambda}{}^0=0\quad\text{if}\quad m=0.$$

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Properties of a Massive Neutral Gauge Particle*

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We have investigated the properties of a massive neutral vector particle which is assumed to be gauge particle associated with the baryon number and to bear strong interactions. The strong resemblance to quantum electrodynamics serves as a guide. Results are as follows: (i) A simple relation between the mass and wave-function renormalizations is obtained. (ii) The theory of mixing between two vector particles cannot be applied in its simplest form if one or both of them are gauge particles. (iii) There are some restrictions on the form of the interaction with mesons, which can be tested experimentally. The analysis of the production of two vector particles from a pion incident on a nucleon is proposed as an example.

I. INTRODUCTION

PROPOSALS have been made to introduce various kinds of vector particles which bear strong interactions, and are considered gauge particles associated with some conserved currents, i.e., baryon number,[1] isotopic spin, U spin, etc.[2] It is an interesting, but still open question whether the experimentally observed vector mesons are really these gauge particles or not. While the question of finite mass is also still open in the case of Yang–Mills-type mesons,[3] it is quite possible that a singlet neutral vector meson has a finite mass.[1,4–8] A strong resemblance to quantum electrodynamics is also expected in the latter case. In this paper we shall further investigate the properties of a massive neutral vector particle which we assume to couple to the baryon number, which seems to be conserved as strictly as the electric charge. By convention we call such a meson U; it may or may not be identified with the experimentally observed ω(780 MeV), or ϕ(1020 MeV).

It is convenient to describe such a particle in terms of the Stueckelberg formalism,[9] in which the conventionally defined field U_μ is decomposed into A_μ and B:

$$U_\mu=A_\mu+(1/m)\partial_\mu B\,, \tag{1}$$

with m the mass of U. The Lagrangian is given by[1,10]

$$\begin{aligned}&L=L_0+L_1+L_F+L_G\,,\\ &L_0=-\tfrac{1}{4}A_{\mu\nu}{}^2-\tfrac{1}{2}m^2A_\mu{}^2-mA_\mu\partial_\mu B-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}\chi^2\,,\\ &L_1=ig\textstyle\sum\bar\psi\gamma_\mu\psi U_\mu\equiv j_\mu U_\mu\,,\\ &L_F=-\textstyle\sum\bar\psi(\gamma_\mu\partial_\mu+M)\psi\,,\\ &L_G=\text{Lagrangian containing other fields,}\end{aligned} \tag{2}$$

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[1] Y. Fujii, Progr. Theoret. Phys. (Kyoto) **21**, 232 (1959).

[2] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); A. Salam and J. C. Ward, Nuovo Cimento **19**, 165 (1961).

[3] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1959); H. Umezawa and S. Kamefuchi, Nucl. Phys. **23**, 399 (1961).

[4] R. J. Glauber, Progr. Theoret. Phys. (Kyoto) **9**, 295 (1953).

[5] V. I. Ogievetskii and I. V. Polubarinov, Zh. Eksperim. i Teor. Fiz. **41**, 247 (1961) [English transl.: Soviet Phys.—JETP **14**, 179 (1962)].

[6] G. Feldman and P. T. Matthews, Phys. Rev. **130**, 1633 (1963).

[7] J. Schwinger, Phys. Rev. **125**, 397 (1962); **128**, 2425 (1962); O. Hara, Progr. Theoret. Phys. (Kyoto) **30**, 370 (1963); S. Bonometto, Nuovo Cimento **28**, 309 (1963); M. Lévy, Phys. Letters **7**, 36 (1963); S. Kamefuchi and H. Umezawa, Nuovo Cimento **32**, 448 (1964).

[8] Y. Fujii and S. Kamefuchi, Nuovo Cimento **33** 1639 (1964).

[9] E. C. G. Stueckelberg, Helv. Phys. Acta **11**, 299 (1938); H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Company, Amsterdam, 1956).

[10] The formalism can be generalized to the case in which the B field has a mass different from m, as already shown in the preceding paper (Ref. 8) (also in the Appendix of this paper). Such a generalization is indeed necessary in discussing the renormalization of the mass (Sec. II).

where

$$A_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu,$$
$$\chi\equiv\partial_\mu A_\mu+mB, \tag{3}$$

and the summation in L_1 and L_F extends over all the baryons. The equivalence of this formalism to the ordinary one can be inferred by rewriting L_0 in (2) as

$$L_0=-\tfrac{1}{4}U_{\mu\nu}^2-\tfrac{1}{2}m^2U_\mu^2-\tfrac{1}{2}\chi^2,$$

and by observing that the supplementary condition

$$\chi=0 \tag{3'}$$

should be imposed. The theory is invariant under the gauge transformation

$$A_\mu\rightarrow A_\mu+\partial_\mu\Lambda,$$
$$B\rightarrow B-m\Lambda, \tag{4}$$
$$(\square-m^2)\Lambda=0.$$

The interaction Lagrangian L_1 was obtained by the substitution

$$\partial_\mu\rightarrow\partial_\mu-igU_\mu, \tag{5}$$

in L_F. The part

$$(1/m)j_\mu\partial_\mu B$$

in L_1 can be dropped by partial integration and by making use of the conservation law

$$\partial_\mu j_\mu=0,$$

of the baryon-number current. In this representation[11] (5) is replaced by

$$\partial_\mu\rightarrow\partial_\mu-igA_\mu, \tag{5'}$$

and the transformations (4) should be supplemented by

$$\psi(x)\rightarrow e^{ig\Lambda(x)}\psi(x),\quad \bar\psi(x)\rightarrow\bar\psi(x)e^{-ig\Lambda(x)}, \tag{4}$$

so that the analogy to quantum electrodynamics is quite obvious.

It should be noted that many other types of interaction are possible if we consider only the general requirement of invariance under (4); for example, the so-called Pauli terms, and also any form which contains A_μ and B in the combination $U_\mu=A_\mu+(1/m)\partial_\mu B$, are gauge-invariant. In particular we cannot exclude the gauge-*invariant* interaction in which U_μ couples to some nonconserved current. Here we make a more restrictive assumption that the fundamental interaction is the one given by (2) derived from the formal substitution (5) or (5′). This is the analog of the principle of minimal interaction in quantum electrodynamics. In the latter case this seems to be supported by several facts, namely, that the magnetic moment of electron is the Bohr magneton plus the higher order correction, and that the magnitudes of many other quantities in the electromagnetic interaction, however complicated, are of order e, the electric charge.

Starting from the Lagrangian (2), the interaction Hamiltonian in the interaction representation is given by[4]

$$H=-j_\mu A_\mu. \tag{6}$$

The scalar part B no longer appears. Therefore the S-matrix elements, which involve only A_μ, take a form quite similar to that in quantum electrodynamics, and should be invariant under the first transformation of (4). This is the particular consequence of the strict conservation of the current to which the vector field couples. On this basis we should expect many features in common with quantum electrodynamics. Among them three points will be discussed in the following sections.

II. MASS RENORMALIZATION

The problem of the mass renormalization[12] of the neutral vector meson has been discussed by several authors,[5,13] but no correct result has been obtained. We shall show what the analog of the absence of mass renormalization in quantum electrodynamics is.

We start from the Lagrangian (2) in which m is replaced by the bare mass m_0. After the interaction is introduced, we add the renormalization term of the form

$$\delta L_0=-\tfrac{1}{2}m_0^2DA_\mu^2-\tfrac{1}{4}CA_{\mu\nu}^2, \tag{7}$$

where no B field appears, as shown by the preceding arguments. The first term on the right-hand side should be zero according to gauge invariance, but it is left, for the moment, in order to give another argument for its vanishing. The sum of L_0 in (2) (with m replaced by m_0) and (7),

$$\begin{aligned}\tilde L_0&=L_0+\delta L_0\\&=-\tfrac{1}{4}(1+C)A_{\mu\nu}^2-\tfrac{1}{2}m_0^2(1+D)A_\mu^2-m_0A_\mu\partial_\mu B\\&\qquad-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}\chi^2,\end{aligned} \tag{8}$$

should be the free Lagrangian for the renormalized fields $\tilde A_\mu$ and $\tilde B$, which may be defined by

$$\tilde A_\mu=(1+C)^{1/2}A_\mu\equiv Z^{-1/2}A_\mu,$$
$$\tilde B=B. \tag{9}$$

Substituting (9) into (8) we have

$$\tilde L_0=-\tfrac{1}{4}\tilde A_{\mu\nu}^2-\tfrac{1}{2}m^2(1+D)\tilde A_\mu^2-m\tilde A_\mu\partial_\mu\tilde B-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}Z\tilde\chi^2, \tag{10}$$

where

$$\tilde\chi=\partial_\mu\tilde A_\mu+(m_0^2/m)B, \tag{10'}$$

$$m=Z^{1/2}m_0. \tag{11}$$

[11] In quantum theory the partial integration corresponds to a change of representation by a unitary transformation.

[12] We confine ourselves to the case of a stable particle as the first approximation.

[13] O. Hara and H. Okonogi, Progr. Theoret. Phys. (Kyoto) **10**, 191 (1953); R. J. Glauber, *ibid.* **10**, 690 (1953).

This Lagrangian can be compared with that in the generalized Stueckelberg formalism[14] given by

$$L_0=-\tfrac{1}{4}A_{\mu\nu}{}^2-\tfrac{1}{2}m^2A_\mu{}^2-mA_\mu\partial_\mu B-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}\zeta\chi^2\,, \quad (12)$$

with

$$\chi=\partial_\mu A_\mu+(\kappa^2/m)B\,, \quad (12')$$

or after substituting (12′) into (12) and integrating by parts,

$$L_0=-\tfrac{1}{2}(\partial_\nu A_\mu)^2-\tfrac{1}{2}m^2A_\mu{}^2+\tfrac{1}{2}(1-\zeta)(\partial_\mu A_\mu)^2$$
$$-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}\zeta\left(\frac{\kappa^2}{m}\right)^2B^2+\left(m-\zeta\frac{\kappa^2}{m}\right)B\partial_\mu A_\mu\,. \quad (13)$$

The Lagrangian (12) or (13) describes, roughly speaking, the system of (the covariant transverse part of) A_μ with mass m, and B with mass κ; the negative energy coming from the component A_4 is completely canceled by the positive energy from B, leaving only three components with the observed mass m.

For $D=0$, we find that (10) is nothing but (12) with the particular set of parameters

$$m_0=\kappa\,,$$
$$\zeta=Z=m^2/m_0^2\,. \quad (14)$$

For these values the coupling between A_μ and B vanishes, as is easily seen in (13).

Equation (11) shows that the self-energy is completely determined by the Z factor. It is this fact that corresponds to the vanishing self-energy in the case of quantum electrodynamics. The observed mass is evidently lighter than the bare one since $Z<1$ in a consistent theory. It is also noted that the mass of B remains the bare mass, being compatible with the fact that the B field has no interaction. For $D\neq 0$, the corresponding Lagrangian must contain different m's in the second and third terms in (12), so that no consistent theory follows. This argument gives another support for a vanishing of the $A_\mu{}^2$ term in the self-energy part of a massive vector particle.

The above result can also be obtained in the method of the Green's function. The equation for the modified propagator $\Delta'(k)$ is given by

$$\Delta'_{\alpha\beta}(k)=\Delta_{\alpha\beta}(k)+\Delta_{\alpha\gamma}(k)\Pi^*{}_{\gamma\zeta}(k)\Delta'_{\zeta\beta}(k)\,, \quad (15)$$

where α, β, etc., run over $\mu(=1,\cdots,4)$ and B (corresponding to the A_μ and B fields, respectively), and

$$\Delta_{\alpha\beta}(k)=\delta_{\alpha\beta}(k^2+m_0^2)^{-1}.$$

Corresponding to (7) with $D=0$, we substitute

$$\Pi^*{}_{\alpha\beta}(k)=C(k^2)(k_\mu k_\nu-k^2\delta_{\mu\nu})\,,\quad \text{for}\quad (\alpha,\beta)=(\mu,\nu)\,,$$
$$=0\,,\quad \text{otherwise,} \quad (16)$$

into (15) and solve the equation for $\Delta'_{\alpha\beta}(k)$. Then we have a denominator given by

$$\det|\delta_{\alpha\beta}-\Delta_{\alpha\gamma}(k)\Pi^*{}_{\gamma\beta}(k)|=\left[\frac{[1+C(k^2)]k^2+m_0^2}{k^2+m_0^2}\right]^3,$$

which is zero at

$$k^2=-m^2\,,$$

where m^2 satisfies

$$[1+C(-m^2)]m^2=m_0^2\,. \quad (17)$$

Simply setting $C(k^2)$ equal to a constant $C=C(-m^2)$, we obtain the result:

$$\Delta'_{\mu\nu}(k)=Z\left[\left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{m^2}\right)\frac{1}{k^2+m^2}-\frac{k_\mu k_\nu}{m^2}\frac{1}{k^2+m_0^2}\right],$$
$$\Delta'_{\mu B}(k)=\Delta'_{B\mu}(k)=0\,, \quad (18)$$
$$\Delta'_{BB}(k)=1/(k^2+m_0^2)\,,$$

with the same definitions for Z and m as given by (9) and (11).[15] These are precisely the propagators for the Lagrangian (12) with the values (14). [See also (A10) in the Appendix.] Furthermore, the modified propagator $\boldsymbol{\Delta}'_{\mu\nu}(k)$ for $U_\mu(=A_\mu+m_0^{-1}\partial_\mu B)$ is obtained as follows:

$$\boldsymbol{\Delta}'_{\mu\nu}(k)=\Delta'_{\mu\nu}+(i/m_0)(k_\nu\Delta'_{\mu B}-k_\mu\Delta'_{\nu B})$$
$$+(1/m_0^2)k_\mu k_\nu\Delta'_{BB}$$
$$=Z(\delta_{\mu\nu}+k_\mu k_\nu/m^2)1/(k^2+m^2)\,,$$

which is the ordinary form for a vector field with mass m.

III. MIXING BETWEEN TWO VECTOR PARTICLES

Usually the mixing between two vector particles, say ω and ϕ, is discussed on the basis of an effective interaction term

$$L_{\text{mix}}=fU_\mu V_\mu\,, \quad (19)$$

where V_μ describes another vector field.[16] From the viewpoint stated before, however, the coupling will result from the interaction with a virtually created

[14] See the Appendix. By introducing an arbitrary constant ζ in the last term of (12), this form has been further generalized from the one considered in Ref. 8.

[15] We may conjecture that there is a spectral representation for $C(k^2)$ given by

$$C(k^2)=\frac{1}{\pi}\int_{\lambda_c^2}^\infty\frac{\gamma(\lambda^2)}{k^2+\lambda^2-i\epsilon}d\lambda^2,$$

with $\gamma(\lambda^2)>0$. If $m_0^2<\lambda_c^2$, then m^2 satisfying (17) is certainly found below m_0^2. Furthermore the term to be added to the first equation of (18) can be written as

$$\frac{1}{\pi}\int_{\lambda_c^2}^\infty\left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{\lambda^2}\right)\frac{\sigma(\lambda^2)}{k^2+\lambda^2-i\epsilon}d\lambda^2,$$

with

$$\sigma(\lambda^2)=\lambda^2\gamma(\lambda^2)\,|[1+C(-\lambda^2)]\lambda^2-m_0^2|^{-2},$$

if possible subtractions are neglected.

[16] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); Phys. Rev. **132**, 434 (1963); M. Ichimura and K. Yazaki, Phys. Letters **6**, 345 (1963).

baryon-antibaryon pair, and involve only the part A_μ. Then the matrix element takes the form

$$A_\mu(k)(k^2\delta_{\mu\nu}-k_\mu k_\nu)\beta(k^2)V_\nu(k),$$

again due to the gauge invariance applied to U.[17] We assume $\beta(k^2)$ to be constant as the simplest approximation. Then (19) is replaced by[18]

$$L_{\mathrm{mix}}=\tfrac{1}{2}fA_{\mu\nu}V_{\mu\nu}=\tfrac{1}{2}fU_{\mu\nu}V_{\mu\nu}. \quad (20)$$

By adding the term (20) to the free Lagrangian for U and V, we can derive the equations of motion. The supplementary conditions as given by (3′) can be consistently applied both to U and V, and we get

$$(\Box-m_U'^2)U_\mu-fm_V'^2V_\mu=0,$$
$$(\Box-m_V'^2)V_\mu-fm_U'^2U_\mu=0, \quad (21)$$

where

$$m_{U,V}'^2=m_{U,V}^2(1-f^2)^{-1},$$

with m_U, m_V the masses of the decoupled U and V, respectively. If we write (21) in the 2×2 matrix notation, the off-diagonal elements are different in general. Except in the trivial case $m_U=m_V$, the matrix is not Hermitian, so that the diagonalizing matrix cannot be unitary. Thus the mixing effect cannot be described by a single parameter, the so-called mixing angle, as is usually done.[16]

This conclusion seems to be quite strange at first sight. It has, however, been suggested[19] that if there is a matrix element between two particles through the interaction with other particles, say, baryon and antibaryon, and consequently it is generally energy-dependent, then the unitary diagonalizing matrix is obtained only if the intermediate state (baryon and antibaryon) is taken into account in addition to the two particles in question. In the present case, the matrix element is essentially energy dependent as given by (20) even in the simplest approximation. One may argue that the form (20) is quite phenomenological and is not satisfactory because it changes the nature of the propagation of the particles for $|f|\geq1$, as also seen in the expression of $m'_{U,V}{}^2$. But, in any event, the idea of ω-ϕ mixing cannot be applied in its simplest form, as long as ω or ϕ, or both of them, are the gauge particle.

[17] The V field may or may not be the gauge field.

[18] If V is a member of, say, the octet Yang-Mills fields, then $V_{\mu\nu}$ might be replaced by a "covariant" tensor which involves other octet fields through the structure constants. But the above form is sufficient as long as we consider only the two-particle interaction to discuss the mixing.

The difference between (19) and (20) can be seen most simply by comparing their matrix elements, $f\delta_{\mu\nu}$ for (19) and $f(k^2\delta_{\mu\nu}-k_\mu k_\nu)$ for (20). In the case of (20) we can always prove the equations $\partial_\mu U_\mu=\partial_\mu V_\mu=0$, permitting us to drop the second term $k_\mu k_\nu$. Owing to the factor k^2 in the first term, the matrix element of (20) takes different values on the mass shells of U and V, instead of the same values in the case of (19). This situation is closely connected with the difference between the "off-diagonal" terms in (21).

[19] K. Yazaki (private communication). Also T. Kaneko, Y. Ohnuki, and K. Watanabe [Progr. Theoret. Phys. (Kyoto) **30**, 521 (1963)] and G. Feldman and P. T. Matthews [Phys. Rev. **132**, 823 (1963)] gave general formulas in which fields are mixed by general linear transformations. Further discussions will be made elsewhere.

It is also worthwhile to note that we can avoid discussing the mixing between photon and ω, or photon and ρ^0, etc., because the form given by (20) also arises when the gauge invariance is applied to the photon.[20]

IV. RESTRICTIONS ON THE FORM OF THE INTERACTION

In quantum electrodynamics, the forms of various matrix elements are restricted to some extent by the requirement of gauge invariance. The same situation can be expected also in the present case of a massive vector particle. The form of the interaction with the baryon is simply the same as in the electromagnetic form factors of nucleon. We are here mainly interested in the interaction with a number of mesons.

For the interaction of $U(I=0, G=-1)$ and an odd number of pions we obtain, however, no restrictions which are of practical interest. To see this we first consider the interaction U-3π. From Lorentz invariance and charge independence, the matrix element has the form

$$\epsilon_{\mu\nu\lambda\sigma}k_\nu^{(+)}k_\lambda^{(0)}k_\sigma^{(-)}A_\mu(k)F(k^{(+)},k^{(0)},k^{(-)}), \quad (22)$$

where $k^{(+)}$, for example, stands for the momentum of the positive pion, $k=k^{(+)}+k^{(0)}+k^{(-)}$ is that of U, and F is a function of the invariant combinations of k's. Replacing $A_\mu(k)$ by k_μ gives zero, so that the form (22) is already gauge-invariant and no additional restrictions are obtained.[21] For the interaction of U and more than five pions, the matrix element should have the same form as in (22), where $k^{(+)}$, for example, is the *sum* of the momenta of all the positive pions. (Because of the Bose statistics, only the sum of the momenta enters, if the possible strong correlation among some of the pions is neglected.) Then we again have zero on replacing $A_\mu(k)$ by $k_\mu=k_\mu^{(+)}+k_\mu^{(0)}+k_\mu^{(-)}$.

One way to get around such a too restrictive condition may be to replace some of the pions by resonant "particles." We shall consider the interactions which involve (i) one gauge particle U, (ii) one meson, which is assumed to have zero strangeness, spin and isospin zero or one, and parity and G parity plus or minus, so that it may or may not be identified with some of the mesons already observed, and (iii) a least number of pions to get a form which satisfies various invariance principles.

In the first column of Table I, the types of such additional mesons are enumerated. In the second column the number of pions is shown. If both of the two kinds of forms are possible, one gauge-invariant and the other not, then the sign + is indicated in the third

[20] See, for example, Eq. (1) in the paper by R. W. Huff, Phys. Rev. **112**, 1021 (1958). If the form (19) is assumed for the photon case, and the usual diagonalization technique is applied for the *squared* mass operator, then the photon will get an imaginary mass.

[21] We have another well-known example for the interaction π^0-2γ. The form $\epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}\pi^0$, which is obtained from the Lorentz invariance, is already invariant under the gauge transformation applied to the photon fields.

TABLE I. Possible interactions between U and a number of mesons.

Type of meson: Spin parity	Type of meson: Isospin	Type of meson: G parity	Number of pions	Coexistence of gauge-invariant and noninvariant terms	Uniqueness of invariant terms	Distinction of the mass shells
0+	0	+	3	+	+	+
	1	−	2	+	−	
	0	−	2	+	+	−
	1	+	1	−		
0−	0	+	3	+	+	+
	1	−	2	−		
	0	−	2	−		
	1	+	1	+	+	−
1−	0	−	2[a]	+	+	+
			2[b]	+	−	
	1	+	1	−		
	0	+	3	+	−	
	1	−	2	+	+	+
1+	0	+	3	+	−	
	1	−	2	+	+	+
	0	−	2	−		
	1	+	1	+	−	

[a] Gauge particle.
[b] Nongauge particle.

G parity $+1$. The noninvariant and invariant forms are given by

$$(\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\eta}-\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\eta})U_\mu \tag{24a}$$

and

$$(\partial_\mu \boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\eta}-\partial_\nu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\eta})U_{\mu\nu}\,, \tag{24b}$$

respectively. By making use of the equation $\partial_\mu U_{\mu\nu} = m^2 U_\nu$, the latter form becomes, apart from a four divergence,

$$m^2(\partial_\mu \boldsymbol{\pi}\cdot\boldsymbol{\eta}-\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\eta})U_\mu\,,$$

which is equivalent to (24a). The sign $+$ in the fifth column shows that there is certainly a distinction between noninvariant and invariant terms even on the mass shell, so that we can expect a meaningful prediction from the gauge invariance.

Some of such examples will be shown:

(*i*) *Scalar meson* $\sigma(I=0, G=+1)$*:* This interaction may be related to a possible rare decay mode of ω or ϕ into 3π and the ABC "particle" (310 MeV). The general form is given by

$$\epsilon_{\mu\nu\lambda\sigma}[x(\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi})\cdot\partial_\nu\boldsymbol{\pi}\sigma+y(\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi})\cdot\boldsymbol{\pi}\partial_\nu\sigma]U_\mu\,, \tag{25a}$$

with arbitrary constants x and y. For a particular choice, $x=y$, the above form reduces to a gauge-invariant one:

$$\epsilon_{\mu\nu\lambda\sigma}(\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi})\cdot\boldsymbol{\pi}\sigma U_{\mu\nu}\,. \tag{25b}$$

(*ii*) *Pseudoscalar meson* $\eta(I=0\; G=+1)$*:* This meson can be identified with the observed η(550 MeV), and the interaction may be related to a possible rare decay mode of ϕ into 3π and η. The general form is given by

$$[x\partial_\nu(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi})\cdot\boldsymbol{\pi}\eta+y(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi})\cdot\boldsymbol{\pi}\partial_\nu\eta]U_\mu\,. \tag{26a}$$

For a particular choice, $x=y$, we have a gauge-invariant form

$$(\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi})\cdot\boldsymbol{\pi}\eta U_{\mu\nu}\,. \tag{26b}$$

(*iii*) *Vector meson* φ_μ ($I=0$, $G=-1$*; gauge particle*)*:* The general form is given by

$$\boldsymbol{\pi}^2(x\varphi_\mu U_\mu+y\varphi_{\mu\nu}U_{\mu\nu})\,. \tag{27a}$$

For a particular choice, $x=0$, the gauge-invariant form is obtained;

$$\boldsymbol{\pi}^2\varphi_{\mu\nu}U_{\mu\nu}\,. \tag{27b}$$

(*iv*) *Axial-vector meson* $\psi_\mu(I=1, G=-1)$*:* This meson may contribute to the axial-vector form factor of the weak interaction. The general form is given by

$$\epsilon_{\mu\nu\lambda\sigma}[x(\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi})\cdot\partial_\nu\boldsymbol{\psi}_\lambda+y(\partial_\nu\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi})\cdot\boldsymbol{\psi}_\lambda]U_\mu\,. \tag{28a}$$

For a particular choice, $x=y$, we obtain the gauge-invariant form

$$\epsilon_{\mu\nu\lambda\sigma}(\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi})\cdot\boldsymbol{\psi}_\lambda U_{\mu\nu}\,. \tag{28b}$$

It is possible in principle to test experimentally whether the form is really the one required by gauge invariance. In the next section, a more detailed discussion of example (iii) will be given.

column. The interaction between U and an odd number of pions, already considered, belongs to the category of "−." In some cases, there are many alternative forms which are all gauge invariant. An example will be shown in the case of the vector meson φ_μ which has $I=0$, $G=-1$, but is *not* a gauge particle.

There are two forms which are not equivalent to each other[22]:

$$\boldsymbol{\pi}^2\varphi_\mu U_\mu\,, \tag{23a}$$

$$\boldsymbol{\pi}^2\varphi_{\mu\nu}U_{\mu\nu}\,. \tag{23b}$$

The first form (23a) is not gauge invariant,[23] while the second (23b) is invariant under the gauge transformation for U as well as for φ. From these two forms we can construct another form

$$\boldsymbol{\pi}^2(2m^2\varphi_\mu U_\mu+\varphi_{\mu\nu}U_{\mu\nu})=2\partial_\mu\boldsymbol{\pi}^2\varphi_\nu U_{\mu\nu}\,, \tag{23c}$$

on the mass shell of U. This is invariant under the gauge transformation for U, but not for φ. Then an arbitrary linear combination of (23b) and (23c), and so of (23a) and (23b), can be allowed as long as the gauge invariance is required only for U. The form (23b) is uniquely determined only if φ is also a gauge particle (eventually φ is U itself). In such a case, "+" is indicated in the fourth column.

There are also some cases in which the invariant form is reduced to a noninvariant one on the mass shells of the particles. An example is shown in the case of a pseudoscalar meson $\boldsymbol{\eta}$ with $I=1$ and the "abnormal"

[22] For simplicity we give the expressions in the coordinate representation, neglecting possible occurrence of form factors. We neglect also the forms containing higher order derivatives.

[23] This is indeed gauge-invariant, because U_μ receives no change under the transformations (4). But such a term never appears if we assume the principle of minimal interaction, as discussed in Sec. I. In this sense the terms which involve U_μ not in the form $U_{\mu\nu}$, and do not give zero on replacing U_μ by $\partial_\mu\Lambda$, will hereafter be called not gauge invariant.

V. PROPOSED EXPERIMENTAL TEST

One of the most direct tests of gauge nature is to evaluate experimentally the coupling constants to various baryons and to see whether the universality is really valid. But, in addition to the practical difficulties, this test involves an extrapolation procedure from the observed value on the mass shell of the massive vector particle to the unphysical point corresponding to vanishing momentum transfer, the only point at which the theory predicts universality. Therefore it will be worthwhile to look for other kinds of experiments for which gauge invariance gives definite predictions. We shall propose the measurement of angular correlations in the momenta of decay products of two vector particles produced from a pion incident on a nucleon, as an application of example (iii) in the preceding section. The threshold momenta of the incident pion in the laboratory system is 2.86 BeV/c for 2ω, 3.53 BeV/c for $\omega\phi$, and 4.27 BeV/c for 2ϕ production. The one-pion-exchange process is relevant, as illustrated in Fig. 1.

By denoting one of the (gauge) vector particles by U with momentum q, and the other by V with p (they may be the same kind of particles), the matrix element of (27a) takes the form,

$$(x+2ymE)(\mathbf{V}\cdot\mathbf{U})-2y(m/E)(\mathbf{p}\cdot\mathbf{V})(\mathbf{p}\cdot\mathbf{U}), \quad (29a)$$

in the rest system of U, where m is the mass of U, E is the energy of V, $\mathbf{U}$, $\mathbf{V}$ stand for the polarization vectors of U and V, respectively. For the gauge-invariant form (27b), we have

$$E[(\mathbf{V}\cdot\mathbf{U})-(1/E^2)(\mathbf{p}\cdot\mathbf{V})(\mathbf{p}\cdot\mathbf{U})]. \quad (29b)$$

The polarization vectors are proportional to the appropriate relative momenta of the decay products; for example,

$$\mathbf{k}_K-\mathbf{k}_{\bar{K}}, \quad (30)$$

for the decay into K and $\bar{K}$ (as in ϕ), and

$$\mathbf{k}_{\pi+}\times\mathbf{k}_{\pi-}, \quad (31)$$

for the decay into 3π (as in ω). Thus we can predict the angular correlation among the relative momenta of the decay products and the momenta of the vector particles, as given by squaring (29b), with $\mathbf{U}$ and $\mathbf{V}$ replaced by (30) or (31).

It is necessary that the momentum of one of the vector particles in the rest system of another [denoted by $\mathbf{p}$ in (29b)] be of magnitude comparable to its energy, otherwise the important second term of (29b) is negligibly small. The total energy of the two vector particles with the masses m (assumed common for simplicity) in their center-of-mass system is denoted by κ, and is given by

$$\kappa^2=2m[m+(\mathbf{p}^2+m^2)^{1/2}].$$

The condition

$$|\mathbf{p}|\geqq m,$$

may be imposed to give

$$\kappa^2\geqq 2(1+\sqrt{2})m^2. \quad (32)$$

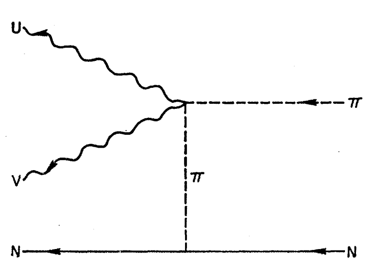

FIG. 1. One-pion-exchange process in the production of two vector particles U and V.

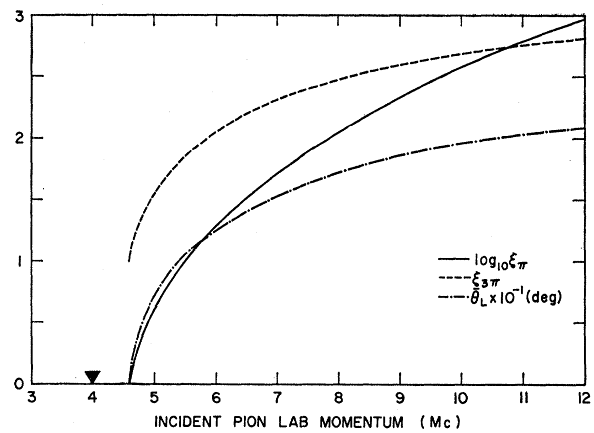

FIG. 2. ξ_π gives a measure of the "sharpness of the forward peak" in the cross section coming from the one-pion-exchange process, while $\xi_{3\pi}$ gives that from the three-pion-exchange process. $\bar{\theta}_L$ represents the opening angle of the forward cone of the direction of the center of mass of two vector particles in the laboratory system, and is shown in degrees, divided by 10. The estimate is made in a simplified case where two vector particles have common mass equal to the nucleon mass M, so that the conventional threshold momentum is 3.99 Mc (indicated by an arrow). The total mass κ of the two vector particles is chosen to be $[2(1+\sqrt{2})]^{1/2}M=2.20M$.

Another problem which requires special consideration is to distinguish the contribution of the particular process shown in Fig. 1 from those of various other processes. Since we have no definite idea about the magnitude of each contribution, we can only make use of the peripheral nature of the process under consideration. The cross section includes a factor

$$f(\theta)=\mathbf{t}^2/(t^2+m_\pi{}^2)^2, \quad (33)$$

where t is the momentum transferred to the exchanged pion, which depends on θ, the angle between the incident pion and the center of mass of two vector particles in the over-all center-of-mass system. Then, for sufficiently high energies, the center of mass of two vector particles is emitted predominantly in the forward direction (small θ). In Fig. 2, we plotted the ratio $\xi_\pi=f(0°)/f(90°)$ versus the incident pion momentum, as a simple measure of sharpness of the forward peak. We tentatively chose $\kappa^2=2(1+\sqrt{2})m^2$ as a lower limit in (32), and $m=M$, the nucleon mass. For the pion momentum around $8Mc$, the ratio is as large as 10^2, so that we may expect to separate the process in question well from the background to which other kinds of processes contribute.

Next to the one-pion state, the three-pion state will occur. The sharpness of its contribution may be estimated by calculating also the ratio $f(0°)/f(90°)$ where an effective mass of approximately the nucleon mass is substituted for the pion mass in (33). This is also plotted in Fig. 2 ($\xi_{3\pi}$). For momenta somewhat lower than $5Mc$, $\xi_{3\pi}$ is of magnitude comparable to ξ_π; therefore the higher momenta will be more convenient for separating the contribution considered from that of higher mass states. The former peak is 40 times sharper than the latter at around $8Mc$. Finally the opening angle of the forward cone of the direction of the center of mass of two vector particles in the laboratory system is plotted. This also favors higher momenta.

In all these respects, we may conclude, in spite of crudeness of the estimate, that the proposed experiment is possible for reasonable energies, namely, for pion momenta larger than, say, 7 BeV/c.

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APPENDIX

The generalized Lagrangian for the Stueckelberg formalism is given by (12) or (13). They are invariant under the gauge transformation (4), but with m^2 in the last equation replaced by κ^2. By introducing the field U_μ given by

$$U_\mu = A_\mu + (1/m)\partial_\mu B, \tag{A1}$$

Eq. (12) can be written in the form

$$L_0 = -\tfrac{1}{4}U_{\mu\nu}^2 - \tfrac{1}{2}m^2 U_\mu^2 - \tfrac{1}{2}\zeta\chi^2, \tag{A2}$$

which suggests that U_μ describes the vector field with mass m, because χ will be set equal to zero, as will be seen in the next paragraph.

The field equations are

$$\begin{aligned}&\partial_\nu A_{\nu\mu} - m^2 A_\mu - m\partial_\mu B + \zeta\partial_\mu\chi = 0,\\ &\Box B + m\partial_\mu A_\mu - \zeta(\kappa^2/m)\chi = 0.\end{aligned} \tag{A3}$$

By differentiating the first equation with respect to x_μ and combining the result with the second, we have

$$\zeta(\Box - \kappa^2)\chi = 0, \tag{A4}$$

which enables us to put

$$\chi = 0, \tag{A5}$$

if the initial condition $\chi = \dot{\chi} = 0$ at, say, $t=0$ is assumed. We have, then, from (12′)

$$B = -(m/\kappa^2)\partial_\mu A_\mu. \tag{A6}$$

On substituting this together with (A5) into (A3), we obtain

$$\begin{aligned}&\partial_\nu A_{\nu\mu}{}^{(\kappa)} - m^2 A_\mu{}^{(\kappa)} = 0,\\ &(\Box - \kappa^2)B = 0,\end{aligned} \tag{A7}$$

where

$$A_\mu{}^{(\kappa)} = (\delta_{\mu\nu} - \partial_\mu\partial_\nu/\kappa^2)A_\nu. \tag{A8}$$

The first equation is the same with Eq. (2.22) of Feldman and Matthews.[6] Using (A6) we find that $A_\mu{}^{(\kappa)} = U_\mu$ given by (A1), and that the first equation of (A7) is nothing but the ordinary Proca equation for the vector field. Equations (A3) can also be put into the form,

$$\begin{aligned}&(\Box - m^2)A_\mu - (1-\zeta)\partial_\mu\partial_\nu A_\nu - (m - \zeta\kappa^2/m)\partial_\mu B = 0,\\ &(\Box - \zeta\kappa^4/m^2)B + (m - \zeta\kappa^2/m)\partial_\mu A_\mu = 0,\end{aligned} \tag{A3′}$$

by using the explicit form of χ (12′).

The quantum-mechanical discussion, including the method of indefinite metric,[24] can be developed completely parallel to the case of $\zeta = 1$ in Ref. 8. The main changes are as follows:

In the left-hand side of the second equation of (3.1) in Ref. 8, π_4 is changed to $\zeta^{-1}\pi_4$. The terms $\frac{1}{2}\pi_4^2$ in (3.2) and (3.7) are multiplied by ζ^{-1}. The right-hand sides of (3.5) and (3.6) are multiplied by ζ^{-1}. Then, in the Hamiltonian, all the terms containing ζ are made to vanish by imposing the supplementary condition (3.8). In the commutation relations (3.16), the factor $m^2 - \kappa^2$ in front of $(\partial/\partial\kappa^2)\Delta_\kappa(x-x')$ is changed to $\zeta^{-1}m^2 - \kappa^2$, which vanishes for a particular choice

$$\zeta = m^2/\kappa^2, \tag{A9}$$

as in (14), to give a simple form

$$\begin{aligned}(1/i)[A_\mu(x), A_\nu(x')] &= (\delta_{\mu\nu} - \partial_\mu\partial_\nu/m^2)\Delta_m(x-x')\\ &\quad + \partial_\mu\partial_\nu/m^2\Delta_\kappa(x-x'),\\ (1/i)[A_\mu(x), B(x')] &= 0,\\ (1/i)[B(x), B(x')] &= \Delta_\kappa(x-x').\end{aligned} \tag{A10}$$

These equations are closely related to the Green's functions given by (18).

According to the change of the Lagrangian (4.1) to (13), the Hamiltonian becomes rather complicated. The right-hand sides of (4.12) and (4.14) are multiplied by ζ^{-1}. All the expressions for Ψ_n in Sec. V of Ref. 8 are unchanged. In (5.12), the only change necessary is to multiply all m^2 by ζ^{-1}, so that there is no change in the final result.

Note added in proof. It should be noted that the techniques of the ordinary Stueckelberg formalism developed, for example, in Umezawa's book (Ref. 9), can also be used to derive the interaction Hamiltonian (6) in our generalized formalism.

[24] S. N. Gupta, Proc. Roy. Soc. (London) **63**, 681 (1950); K. Bleuler, Helv. Phys. Acta **23**, 567 (1950).