Neutral Weak Interaction Currents

F. CURTIS MICHEL
Space Science Department, Rice University, Houston, Texas
(Received 10 December 1964)

The consequences of postulating that the strangeness-conserving weak interactions are symmetric in isotopic spin space (i.e., the leptons paired in isotopic spin doublets) are traced, with special attention to the resultant neutral weak currents and possible experiments to detect such currents. Far from being excluded experimentally, this theory in fact leads generally to only very difficult-to-detect effects such as elastic neutrino-nucleon scattering (order G in amplitude). Circular dichroism (microwave frequencies) of first excited hydrogen is expected. Other possible experimental subjects are also discussed.

I. INTRODUCTION

THE apparent success in assigning symmetries[1,2] to the strongly interacting particles emphasizes the paradoxical lack of success (or at least, nonacceptance) of similar assignments for the weakly interacting particles, the leptons. It is for the weak interactions, together with electromagnetism and gravitation, that apparently accurate calculations can be performed with perturbation theory; yet this computational ability has not revealed any apparent unifying symmetries for the leptons. The spin ($\frac{1}{2}$) and charge assignments apparently exhaust the electromagnetic properties (the anomalous magnetic moment being in experimental agreement[3] with quantum electrodynamics alone). The weakness of the gravitational coupling precludes (at least at present) any sophisticated experiments, and no direct involvement in the strong interactions has been indicated for the leptons. For the weak interactions, the success of the "Puppi[4] triangle" (Fig. 1) in elucidating the decay schemes together with the quantitative success of the couplings[5] $(\bar{e}\gamma_\mu a\nu_1)$ and $(\bar{\mu}\gamma_\mu a\nu_2)$ at the lepton vertices, requires any symmetry scheme to yield these couplings. From our standpoint, the symmetry schemes are descriptions of the couplings, or vice versa. Thus, any symmetries, in addition to those required in allowing $(\bar{e}\nu_1)$, $(\bar{\mu}\nu_2)$ and forbidding $(\bar{e}\nu_2)$, $(\bar{\mu}\nu_1)$, should result in additional observable (at least in principle) effects, otherwise all sets of symmetries satisfying the above criteria are indistinguishable. We will frequently suppress, as above, the explicit structure $(\gamma_\mu a)$ of the universal Fermi interaction and write out only the particles involved.

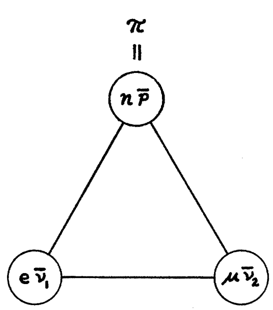

FIG. 1. The Puppi triangle. A schematic representation of the fermion pairs that interact via the (strangeness-conserving) weak interactions.

It is our intent here to suggest and discuss several experiments which appear promising in the search for additional observable effects by means of which the over-all lepton symmetries might be determined. We select here a very naive working hypothesis for the lepton symmetries, which we describe and discuss below.

II. PROPOSED LEPTON SYMMETRIES

A. This Theory

Let us first simply state the assignment and then examine the reasoning behind this scheme. The leptons e and ν_1 are taken to be an isotopic spin doublet as are μ and ν_2, and the two lepton doublets are distinguished by a quantum number λ. The isotopic spin assignment is made in such a way as to preserve isotopic spin in the strangeness-conserving weak interactions. Thus the beta-decay coupling is assumed to be $[a=\frac{1}{2}(1+i\gamma_5)]$

$$2^{1/2}G(\bar{l}_\lambda\gamma_\mu a\boldsymbol{\tau}l_\lambda)\cdot(\bar{N}\gamma_\mu a\boldsymbol{\tau}N) \tag{1}$$

and conservation of charge then requires the isotopic spinor to be

$$N=\begin{pmatrix}p\\n\end{pmatrix},\quad l_0=\begin{pmatrix}\nu_1\\e\end{pmatrix},\quad\text{and}\quad l_1=\begin{pmatrix}\nu_2\\\mu\end{pmatrix}, \tag{2}$$

where the subscripts to l indicate the leptonness, λ.

The assignment $\lambda=0$ to electrons and $\lambda=1$ to muons is somewhat arbitrary, in much the same way that the specific values of strangeness are arbitrary. That is, any value of strangeness could have been chosen for the nucleons, and any numerical difference could have been chosen for the strangeness difference between the Σ-particles and the nucleons. Equations (1) and (2) give the known strangeness-conserving beta-decay processes. In addition, these equations yield processes hitherto undetected, and thereby provide the test of this symmetry assignment. Since the isotopic spin assignments of Eq. (2) are equivalent to charge conservation, they

[1] C. N. Yang and H. Mills, Phys. Rev. **96**, 192 (1954).
[2] M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] G. Shapiro and L. M. Lederman, Phys. Rev. **125**, 1022 (1962).
[4] G. Puppi, Nuovo Cimento **6**, 194 (1949).
[5] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

can be justified only if the additional couplings predicted are found, and found with the correct strength.

B. Other Theories

Numerous authors have examined the possibility of assigning quantum numbers (other than charge and spin) to the leptons. Several such schemes[6–8] now lack force in that they assume three leptons, while at present there are apparently at least four.[9] The isotopic spin assignments by Bludman[10] fall in this category, but otherwise are quite similar to the proposal made here. His isotopic spin doublets are (ν,e) and (ν,μ) and, other than the unusual idea of associating a single particle (ν) as being a projection (in isotopic spin space) of two distinct particles (e,μ), the assignments are identical to Eqs. (1) and (2). A mathematical consequence of setting $\nu_1=\nu_2=\nu$ is complete destructive interference between the interactions $(\bar{e}e)(\bar{\nu}\nu)^+$ and $(\bar{e}\nu)(\bar{\nu}e)^+$, a prediction of some importance to "neutrino astronomy," since the latter assigns special significance to the process $e+\bar{e}\rightarrow\nu+\bar{\nu}$ in stellar interiors[11] (at temperatures of order 10^9 °K, the equilibrium concentration of electron-positron pairs becomes significant in producing an appreciable neutrino luminosity). In the present work we do not accept the possibility of interference between ν_1 and ν_2, with the result that the amplitude for $e+\bar{e}\rightarrow\nu_1+\bar{\nu}_1$ is reduced by $\frac{1}{2}$ (therefore, the cross section by $\frac{1}{4}$), instead of complete interference. From Eqs. (1) and (2), the process $e+\bar{e}\rightarrow\nu_2+\bar{\nu}_2$ should enjoy the same amplitude (with phase -1) as $e+\bar{e}\rightarrow\nu_1+\bar{\nu}_1$, where the former proceeds only via the neutral current $-\frac{1}{2}(\bar{e}e)(\bar{\nu}_2\nu_2)^+$ while the latter results from $-\frac{1}{2}(\bar{e}e)(\bar{\nu}_1\nu_1)^++(\bar{e}\nu_1)(\bar{e}\nu_1)^+$. The total cross section for neutrino production would, therefore, be half that derived from the charged current theory alone, and equal numbers of ν_1 and ν_2 are produced. In Bludman's theory, the neutrinos are not distinguished, and the cancellation is therefore complete.

Yet others have attempted to assign both strangeness and isotopic spin quantum numbers. Such assignments[12,13] usually appeal to the $|\Delta S|=1$ and $|\Delta I|=\frac{1}{2}$ rules inferred from the nonleptonic decays of the strange particles. In general, these authors have not been able to exclude, in a natural way, unobserved processes such as $K^0\rightarrow\bar{\mu}+e$ or $\mu\rightarrow 2e+\bar{e}$. Furthermore, it is difficult to assign symmetry only on the basis of interactions that violate that symmetry. A symmetry has little meaning unless preserved by some interaction. For example, if the neutron and proton entered only into the (isotopic spin violating) electromagnetic interactions, then the assignment of isotopic spin would be no more than a bookkeeping technique. The implicit assumption of symmetry assignments from the above quantum number violations is, therefore, that a new interaction exists[14] conserving S and I analogous to (but distinct from) the strong interactions. This note is less ambitious in that the symmetry assignments made here are simply descriptions of the weak interactions themselves.

Lee and Yang[15] propose neutral currents (via neutral intermediate bosons) among the baryons. In order to forbid the interaction $n+n\rightarrow\Lambda+\Lambda$, they were forced to postulate two neutral currents; interference between the two then eliminates $n+n\rightarrow\Lambda+\Lambda$. The same difficulty would be present in the present scheme, if mediated by neutral vector bosons coupled to $(\bar{\Lambda}n)$, since $n+W^0=\Lambda$ and $\bar{W}^0=W^0$ (e.g., $\bar{\pi}^0=\pi^0$), therefore, $n=\Lambda+\bar{W}^0=\Lambda+W^0$ and altogether $n+n=\Lambda+\Lambda$. Two neutral bosons can be arranged such that each leads to $n+n\rightarrow\Lambda+\Lambda$ except with opposite sign for the amplitude. Since the quantum numbers of the particles generating the neutral currents (considered here) do not change, the above considerations do not rise. The above examples do not exhaust the possibilities,[16–20] but for our purposes we will remain content with Eq. (2).

C. Leptonness

In the present scheme, the quantum number λ (leptonness) is conserved at each vertex, and this conservation serves to forbid decays such as $\pi^-\rightarrow e+\bar{\nu}_2$, while allowing $\pi^-\rightarrow e+\bar{\nu}_1$. More importantly, the decays $\mu\rightarrow 2e+\bar{e}$ and $\mu\rightarrow e+\gamma$ violate leptonness and, therefore, cannot enter via any order of perturbation theory. We could formally avoid referring to isotopic spin by inventing a special name for that symmetry when applied to the leptons; however, Eq. (1) explicitly identifies this symmetry as isotopic spin, since the scalar product of two vectors is essentially meaningless unless both vectors lie in the same Hilbert space. It is perhaps worthwhile to note that an assignment of sorts has already been implicitly made when the strangeness-conserving decays (e.g., $n\rightarrow p+e+\bar{\nu}_1$) are referred to as being "$\Delta I=0$, $\Delta I_z=\pm1$" (i.e., the leptons are assumed to be coupled into an $I=1$ state).

[6] O. Klein, Arkiv Fysik **16**, 191 (1959). A. Gamba, R. E. Marshak, and S. Okubo, Proc. Nat. Acad. Sci. (U. S.) **45**, 881 (1959).
[7] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).
[8] E. Johnson, Nucl. Phys. **36**, 688 (1962).
[9] G. Danby, J-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).
[10] S. A. Bludman, Nuovo Cimento **9**, 433 (1958), see also V. N. Baier and I. B. Khriplovich, Zh. Eksperim. i Teor. Fiz. **39**, 1374 (1960) [English transl.: Soviet Phys.—JETP **12**, 959 (1961)].
[11] F. Hoyle and W. A. Fowler, Nature **197**, 533 (1963).
[12] T. Okabayashi, S. Nakamura, and Y. Nisiyama, Progr. Theoret. Phys. (Kyoto) **25**, 701 (1961); T. Okabayashi, *ibid.* **20**, 583 (1958).
[13] W. Kròlikowski, Nucl. Phys. **33**, 261 (1962).
[14] This interaction cannot be the weak interaction (assumed in these schemes to violate isotopic spin and strangeness), the electromagnetic interaction (violates isotopic spin), the strong interactions (no direct coupling), nor the gravitational interaction (independent of these symmetries).
[15] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).
[16] P. Roman, Nucl. Phys. **2**, 651 (1957).
[17] I. Saavedra, Nucl. Phys. **10**, 6 (1959).
[18] A. Ramakrishnan, A. P. Balachandran, N. R. Ranganathan, and N. G. Deshpande, Nucl. Phys. **26**, 52 (1961).
[19] R. W. King, Phys. Rev. **121**, 1201 (1961).
[20] G. Feinberg and F. Gürsey, Phys. Rev. **128**, 378 (1962); G. Feinberg, F. Gürsey, and A. Pais, Phys. Rev. Letters **7**, 208 (1961).

Leptonness, as used here, is to the weak interactions as strangeness is to the strong interactions (e.g., the associated production of strange particles is taken to be analogous to the production of $\bar{\nu}_1$ with e). We have explicitly associated isotopic spin with the leptons, via their charge, in assuming charge symmetry of the beta-decay interaction. The analogous association of leptonness with strangeness can be made confidently only if strangeness-changing (but conserving) isotopic-spin violating couplings exist [e.g., $(\bar{\Lambda},n)(\bar{\mu},e)^+$]. Since no interaction is known to have this property [e.g., the decay $K^0 \to \mu+\bar{e}$ should occur at a rate 0.26×10^8/sec if coupled as in Eq. (1), and therefore should have been observed in $K_2{}^0$ decay], the above association requires the postulation of a new interaction rather than extension [i.e., Eq. (1)] of known interactions and is therefore beyond the intended scope of the present work.

We must also set aside discussion of the known strangeness-violating interactions, since the extension of the postulated isospin-conserving interaction to the manifestly isotopic-spin violating interactions (e.g., $\Lambda\to\pi^0+n$) cannot be prescribed on the basis of the extended isotopic spin symmetry of Eq. (1). Consider the phenomenological coupling $(\bar{\Lambda},p)$ which, together with $(\bar{n},p)$, qualitatively describes the strangeness-violating interactions. Does the transformation in isotopic spin space $(\bar{n},p)\to(\bar{n},n)$ lead to $(\bar{\Lambda},p)\to(\bar{\Lambda},n)$? Since the latter couplings violate the symmetry properties under which the transformation is defined, the question cannot be answered in the absence of a deeper understanding of the mechanism for violation. Pontecorvo[21] argues that "symmetry" forbids $(\bar{\Lambda},n)$ currents; however, this symmetry does not forbid the (charged) strangeness-violating weak decays, and therefore has less force than might be desired.

Experimentally the coupling of $(\bar{\Lambda},n)$ to the neutral lepton currents appears to require a very much smaller coupling than the other weak interactions, since the processes

$$\begin{aligned}\Lambda &\to n+e+\bar{e}\,,\\ K_1{}^0, K_2{}^0 &\to e+\bar{e}\,,\\ &\to \mu+\bar{\mu}\,,\end{aligned}\tag{3}$$

have not been reported. In particular, the decay $K_2{}^0\to\mu+\bar{\mu}$ can be estimated from the $K^-\to\mu+\bar{\nu}_2$ rate to have a lifetime of the order 0.5×10^{-8} sec while the observed $K_2{}^0$ lifetime[22] is 5.6×10^{-8} sec and, therefore, the decay should have been observed. We are then forced to assume (as discussed above) that the coupling $(\bar{\Lambda},n)$ [or equivalently $(\bar{\Sigma}^0,n)$] is not required by the postulation of Eq. (1).

[21] B. Pontecorvo, Phys. Letters **1**, 287 (1962).

[22] A. H. Rosenfeld, A. Barbara-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and Matts Ross, Rev. Mod. Phys. **36**, 977 (1964). The experimental lifetimes for elementary particle decays quoted herein are taken from this reference.

D. Lepton Masses

Finally, we should mention the large μ-e mass difference. Again, unlike the strongly interacting particles, the masses of the leptons are far from suggestive as far as symmetry groupings are concerned. We offer no speculation here to resolve this question, but merely note that the *charged* weak interactions are affected only kinematically. It is implicitly assumed here that the *neutral* weak interactions (if any) are similarly independent of the lepton mass.

E. Extended Theory

For the strangeness-conserving weak interactions we write

$$L_{\text{int}} = 2^{-1/2}G\mathbf{J}_\mu\cdot\mathbf{J}_\mu{}^+\,,\tag{4}$$

where

$$\mathbf{J}_\mu = (\bar{l}\gamma_\mu a\boldsymbol{\tau}l)+(\bar{N}\gamma_\mu a\boldsymbol{\tau}N)+\text{pionic terms}\tag{5}$$

and thereby further extend the theory. Thus, for example, the coupling $(\bar{e}\gamma_\mu ae)(\bar{e}\gamma_\mu ae)^+$ is anticipated, and the photon coupling to the electron charge should be corrected as shown in Fig. 2. The strangeness-changing currents enter into Eq. (5) asymmetrically, apparently coupled only to the charged currents.

III. ELECTROMAGNETIC EFFECTS

A. Electromagnetic Coupling Corrections

The most general gauge invariant $(q_\mu J_\mu{}^{(\text{em})}=0)$ electromagnetic current of the electron is

$$\begin{aligned}J_\mu{}^{\text{em}} = (4\pi)^{1/2}e(\bar{e}|&F_1(q^2)\gamma_\mu+F_2(q^2)\sigma_{\mu\nu}q_\nu\\ &+F_3(q^2)\gamma_5\sigma_{\mu\nu}q_\nu+F_4(q^2)i\gamma_5(q^2\gamma_\mu-2mq_\mu)|e)\,,\end{aligned}\tag{6}$$

where F_1, F_2 are the charge and magnetic moment form factors. The remaining couplings violate parity, as is permitted by the corrections in Fig. 2. We expect, however, that $F_3\equiv0$ since this coupling leads to time-reversal noninvariant effects, and such are not expected from this coupling of Eq. (2).

The coupling with coefficient F_4 does not seem to be excluded by the conservation laws; it appears to be rather artificial, however, since the relative coefficients of $i\gamma_\mu\gamma_5$ and $iq_\mu\gamma_5$ must be in the fixed ratio $q^2/2m$ if gauge invariance is to be maintained. In any event one can show by direct calculation that the lowest order terms in Fig. 2 give connections (of order $Gm^2\approx3\times10^{-12}$) only to F_1 and F_2. We can, therefore, regard the fundamental electromagnetic coupling to be unaltered

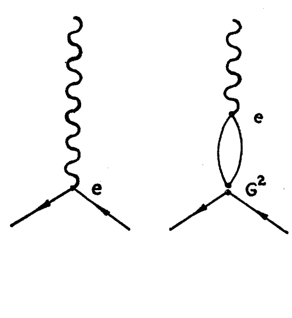

FIG. 2. Weak interaction correction of the electromagnetic interaction. A correction analogous to vacuum polarization in which the weak neutral currents produce an electron-positron pair (not possible to order G^2 in the conventional charged theory).

by Eq. (4). The neutrinos would also have electromagnetic properties via corrections shown in Fig. 2, as discussed by Bernstein *et al.*[23] Zel'dovich[24] has referred to the interaction $(\boldsymbol{\sigma}\cdot\nabla\times\mathbf{H})\rightarrow(\boldsymbol{\sigma}\cdot\mathbf{j})$ as being due to an "anapole" moment if $\mathbf{j}$ is the electromagnetic current. The above interaction was apparently invented as a counter-example to the conjecture[25] that CP invariance plus gauge invariance lead to conservation of spatial parity in the electromagnetic interactions. The conjecture is indeed incorrect as can be seen explicitly: The coupling Eq. (6) with coefficient $F_4(q^2)$ leads to a term in the interaction between two particles, A and B, of the form

$$8\pi^2 F_1(q^2)F_4(q^2)(\bar{A}|i\gamma_5\gamma_\mu q^2|A)1/q^2(\bar{B}|\gamma_\mu|B)$$
$$=8\pi e F_1(q^2)F_4(q^2)(\bar{A}|i\gamma_5\gamma_\mu|A)(\bar{B}|\gamma_\mu|B) \quad (7)$$

and the nonrelativistic expression for the spatial product $\mu(=x,y,z)$ of Eq. (7) is just $\boldsymbol{\sigma}\cdot\mathbf{j}^{(\mathrm{em})}$.

B. Electromagnetic Competition

The neutral currents predicted from Eq. (2) have the property that the total number of particles (particles minus antiparticles) of given quantum numbers (e.g., protons) is unchanged. For example, beta decay of the neutron

$$n\rightarrow p+e+\bar{\nu}_1 \quad (8)$$

is a readily detectable process since (1) the neutron (neutral) becomes a proton (charged), (2) an electron is created where none was present before, and (3) the process is exoergic. The analogous process involving neutral currents would be

$$N\rightarrow N+e+\bar{e}. \quad (9)$$

Although an electron-positron pair is created,[26] reaction (9) is endoergic. Furthermore, reaction (9) is almost always in competition with the reactions

$$N\rightarrow N+\gamma\rightarrow N+e+\bar{e}, \quad (10)$$

which enjoy much larger probabilities in general.

The reaction

$$N\rightarrow N+\nu+\bar{\nu} \quad (11)$$

must (*a posteriori*) result from the neutral weak interactions; however, detection of the emitted neutrinos (the absence of subscripts on ν indicates that either neutrino may be emitted) is at present beyond experimental capabilities unless reaction (11) is extraordinarily intense.

The reactions (9) and (11) can be exoergic if the nucleons make up an excited state of a nucleus. Assuming the excited state stable to fission or particle emission, the possible decay modes are

$$A^*\rightarrow A+\gamma, \quad (12a)$$
$$\rightarrow A+\nu+\bar{\nu}, \quad (12b)$$

and

$$\rightarrow A+e+\bar{e}. \quad (12c)$$

Reaction (12b) can always compete with (12a) while (12c) contributes, of course, only if $Q>1.02$ MeV. The radiative de-excitation (12a) typically has a lifetime of the order 10^{-15} sec for $E1$ transitions, while neutrino pair production will at best be of the order of allowed beta decay, or about 10^3 sec. The possibility is, therefore, quite remote that neutrino pair production ever competes importantly, and, in addition, the resultant small branching fraction [roughly $2\times10^{-18}E^2$ (MeV)] apparently precludes direct detection of the neutrinos.

It might be possible to detect the recoil from a nuclear decay $A^*\rightarrow A+\nu+\bar{\nu}$, which could be distinguished from $A^*\rightarrow A+\gamma$ or other decay modes via anticoincidence techniques. Due to the small branching ratio (order 10^{-18}) for the neutrino processes, both high detection efficiency (for the normal decay mode) and extraordinarily long runs would be required. It is rather easy to estimate the branching fraction, since the decay rate for $(\bar{\nu}\nu)$ is just the beta-decay rate [for neutrino pair emission, the Fermi factor f is simply $(E^5/30m^5)$]. We must then search for states with large "beta-decay" rates but small gamma-ray decay rates; explicit nuclear models are required for such a search and a very similar problem is discussed in Ref. 27.

IV. NEUTRAL WEAK CURRENT EFFECTS

A. Nuclear

For the $J=0\rightarrow J=0$ transitions in nuclei, reaction (12c) may be allowed although (10) is forbidden. In transitions of the type $0^\pm\rightarrow0^\pm$, the pair production must compete with an electromagnetic process giving the same final particles and any effects will be small (order 10^{-5}). Consider for definiteness the prototype decay

$$O^{16*}(6.06\text{ MeV})\rightarrow O^{16}\text{ (ground)}+e+\bar{e}, \quad (13)$$

which occurs at the rate 1.4×10^{10} sec^{-1}. From Eq. (1) we see that the lepton pairs must be emitted in a $(T,T_z)=(1,0)$ state; however, the above states of O^{16} are both predominantly (99%) $T=0$, hence the decay must involve the admixed $T=1$ states. Furthermore, the allowed Fermi matrix element[28] $\langle1\rangle$ is identically zero between two distinct states of the same system (O^{16}) via orthogonality. That the transition matrix element for reaction (13) tends to vanish is essentially due to the

[23] J. Bernstein, M. Ruderman, and G. Feinberg, Phys. Rev. **132**, 1227 (1963).

[24] Ya. B. Zel'dovich, Zh. Eksperim. i Teor. Fiz. **33**, 1531 (1957) [English transl.: Soviet Phys.—JETP **6**, 419 (1958)].

[25] V. H. Solov'ev, Zh. Eksperim. i Teor. Fiz. **33**, 537 (1957) [English transl.: Soviet Phys.—JETP **6**, 419 (1958)].

[26] We will frequently write virtual processes as if they could proceed spontaneously; thus (9) as it stands cannot occur, but this reaction can occur for bound nucleons as shown in Eq. (12c).

[27] F. C. Michel, Phys. Rev. **133**, B329 (1964).

[28] The conventional beta-decay terminology may be used, since correction for the mass and charge differences among the leptons is straightforward, and the nuclear matrix elements are essentially unchanged.

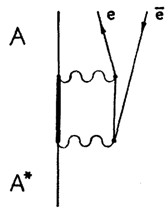

FIG. 3. The electromagnetic decay $0^- \rightarrow 0^+ + e + \bar{e}$.

amplitude for proton decay being nearly equal and opposite to the amplitude for neutron decay. The decay must, therefore, take place either via the second-forbidden matrix elements $i(\boldsymbol{\alpha}\cdot\mathbf{r})\mathbf{q}/3$ (relativistic-vector) or $-r^2q^2/6$ (ordinary vector). Note that a small parity impurity of the nuclear states plays no role (a little 0^- in one of the 0^+ states leads to an interfering $e+\bar{e}$ process only in much higher order, as is discussed below). We can estimate roughly the transition value of $(-\frac{1}{6}r^2t_z)$ as $\approx\frac{1}{6}(\langle r^2\rangle_{\text{neutrons}}-\langle r^2\rangle_{\text{protons}})$. Due to the Coulomb forces, the proton shells are more widely spaced than the neutron shells (in violation of isotopic spin symmetry), and for a harmonic oscillator potential $\langle r^2\rangle \sim 13/6\beta$, where $\beta = 2ME_0$ and E_0 is the spacing between the s and p states (≈ 16 MeV). Thus we obtain

$$\langle -\tfrac{1}{6}r^2t_z\rangle \approx 13\delta E_0/72ME_0^2\,, \tag{14}$$

where δE_0 = proton spacing minus the neutron spacing (~ 2 MeV). For the 6.06-MeV Q value, the transition is therefore weaker by a factor of 4×10^{-5} than that of a corresponding allowed Fermi transition ($\log ft \sim 3.5$); hence, a lifetime ($f\approx 4\times10^3$)

$$\tau \approx 1.5\times10^4 \text{ sec}. \tag{15}$$

Any interference effects (e.g., longitudinal polarization of the electrons) will have amplitudes of the order

$$F \approx [\tau(\text{em})/\tau(\text{nwc})]^{1/2} \sim 7\times10^{-8} \tag{16}$$

(em≡electromagnetic, nwc≡neutral weak current) and, hence, be quite difficult to detect.

In a nuclear transition $0^- \rightarrow 0^+$ (or vice versa), the electron-positron pair must be in a 0^- state (i.e., the ${}^1S_0{}^-$ state, electrons and positrons having opposite intrinsic parity), and CP invariance forbids a single intermediate photon from producing such a pair. The decay to an electron-positron pair must proceed in higher order[29] as shown in Fig. 3 and will, therefore, be much slower than the electromagnetic decay $0^+ \rightarrow 0^+ + e + \bar{e}$ which requires only one virtual photon. The experimental difficulty is to find a nucleus with a 0^- excited state displaced from a lower 0^+ state by more than 1 MeV, that does not decay almost exclusively to states other than the 0^+ state (or vice versa).

B. Elementary Particle Decay

An alternative to examining nuclei in a nucleus is to consider "bound systems" such as the pion. Since the charged pion has isotopic spin projection ±1, it follows from the isotopic-spin symmetric theory [Eq. (2)] that only the conventional charged current theory is involved in the (lowest order) decays.

On the other hand, the neutral pion can decay via

$$\left.\begin{aligned}\pi^0 &\rightarrow \gamma+\gamma\,,\\ &\rightarrow e+\bar{e}+\gamma\,,\\ &\rightarrow 2e+2\bar{e}\,,\\ &\rightarrow e+\bar{e}\,,\end{aligned}\right\} \text{electromagnetic} \tag{17}$$

and

$$\pi^0 \rightarrow e+\bar{e}\,, \quad \text{neutral weak}.$$

The neutral weak interaction decay $\pi^0 \rightarrow \mu+\bar{\mu}$ is energetically forbidden, and $\pi^0 \rightarrow \nu+\bar{\nu}$ is forbidden by helicity requirements.[30] The decay rate for $\pi^- \rightarrow e+\bar{\nu}_1$ is about 1.24×10^{-4} that for $\pi^- \rightarrow \mu+\bar{\nu}_2$, or 4.9×10^4/sec. Rotating the former interaction in isotopic spin space gives a factor of $\frac{1}{2}$, while the helicity restriction is relaxed by a factor of 2. The reduced phase space is negligible for our purposes, giving

$$\lambda(\pi^0 \rightarrow e+\bar{e};\,\text{nwc}) \sim 4.9\times10^4/\text{sec}. \tag{18}$$

The competing electromagnetic decay computed from Fig. 4 is logarithmically divergent, but can be argued[31] to have a branching ratio

$$\lambda(\pi^0 \rightarrow e+\bar{e};\,\text{em})/\lambda(\pi^0 \rightarrow \gamma+\gamma) \approx 10^{-7}. \tag{19}$$

Adopting $\lambda(\pi^0 \rightarrow \gamma+\gamma) \approx 5.6\times10^{15}$/sec gives

$$\lambda(\pi^0 \rightarrow e+\bar{e};\,\text{nwc})/\lambda(\pi^0 \rightarrow e+\bar{e};\,\text{em}) \approx 0.9\times10^{-4}. \tag{20}$$

The effective coupling is $\Gamma_{\pi e\bar{e}}(\bar{e}\gamma_5 e)\varphi_\pi$ for both decays,[32] and therefore no observable interference effects (e.g., electron polarization) result.

As has been discussed earlier, the analogous decay

$$K_2{}^0 \rightarrow \mu+\bar{\mu} \tag{21}$$

is not seen experimentally and cannot be predicted from the isotopic-spin symmetric theory.

The decay

$$\Sigma^- \rightarrow \Lambda+e+\bar{e} \tag{22}$$

does not have the limitation that the effective coupling is the same for the electromagnetic transition as for the neutral weak current transition. We can derive the admixing from the estimate given at the end of Sec. III B to be of the order

$$F \approx 1.1\times10^{-7}. \tag{23}$$

Alternatively, we can estimate the rate for

$$\Sigma^0 \rightarrow \Lambda+e+\bar{\nu} \tag{24}$$

[29] J. Eichler, Z. Physik **160**, 333 (1960).

[30] In the rest system of the pion, the outgoing neutrino (spin antiparallel to its momentum vector) and antineutrino (spin parallel to its momentum vector, hence parallel to the neutrino momentum vector) form a spin-1 system into which the spin-zero pion cannot decay.

[31] S. D. Drell, Nuovo Cimento **11**, 693 (1959).

[32] The most general scalar plus pseudoscalar coupling is $\langle\bar{e}|\Gamma_1(q^2)\gamma_5+\Gamma_2(q^2)\mathbf{q}\gamma_5+\Gamma_3(q^2)+\Gamma_4(q^2)i\mathbf{q}|e\rangle$ which reduces to $(\Gamma_1+2m\Gamma_2)\langle\bar{e}|\gamma_5|e\rangle+\Gamma_3\langle\bar{e}|1|e\rangle$, and time-reversal invariance forbids both couplings from appearing together.

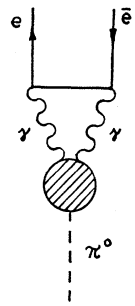

FIG. 4. The decay $\pi^0 \rightarrow e+\bar{e}$ showing the intermediate 2γ state.

to be roughly 6×10^4/sec; thus, reaction (22) should proceed via the neutral weak currents at the rate 3×10^4/sec. For the decay

$$\Sigma^0 \rightarrow \Lambda+\gamma\,, \tag{25}$$

we adopt the rate[33] 0.9×10^{19}/sec. The branching ratio for electron-positron pair production is roughly[34] one in 184 giving altogether a rate 5×10^{16}/sec for reaction (22); hence,

$$F\approx 7\times 10^{-7}. \tag{26}$$

Comparing (23) and (26), we see that the possibility for detection of the neutral weak interaction current effects in reaction (22) is quite remote. See Ref. 35 for a discussion of the decay (22) for charged weak interactions.

C. Parity Impurity of Atomic States

Several authors[27,36–39] have examined theoretically the expected parity impurity of nuclear states due to a (charged) current coupling $(\bar{n}p)(\bar{n}p)^+$. The weak interactions "remind" the nucleons of their intrinsic handedness, and thereby lead to slight parity impurities. Such impurities do not affect the static electromagnetic multiple moments of the nucleus,[27] and since the electrons, to an excellent approximation, interact only with the static electromagnetic moments of the nucleus, the electronic states should themselves remain parity pure (in the absence of external fields). On the other hand, parity impurity of the atomic states may arise from neutral currents[40]; therefore, detection of such impurity would indicate the presence of neutral weak currents. Interaction (1) also has a parity conserving part, but the (tiny) effects from that part of the interaction should be difficult to detect in the presence of the enormously stronger electromagnetic forces.

Quantitatively, we can take over the results of Eq. (6) of Ref. 27 with the replacements

$$2(\tau_+{}^1\tau_-{}^2+\tau_-{}^1\tau_+{}^2) \rightarrow \tau_z{}^1\tau_z{}^2\,,\quad \langle\tau_z{}^1\tau_z{}^2\rangle=-1\,,$$

(for electron-proton interaction), and

$$V_{\text{int}}=+\frac{8^{1/2}G}{16}\left[(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\left\{\left(\frac{\mathbf{p}_1}{M_1}-\frac{\mathbf{p}_2}{M_2}\right),\delta(1,2)\right\}_+ +i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\left[\left(\frac{\mathbf{p}_1}{M_1}-\frac{\mathbf{p}_2}{M_2}\right),\delta(1,2)\right]_-\right], \tag{27}$$

where $\delta(1,2)\equiv\delta^3(\mathbf{r}_1-\mathbf{r}_2)$ and the subscripts 1 and 2 label an electron and a proton respectively. The curly bracket with a subscript (+) is the anticommutator while the square bracket with the subscript (−) is the commutator. For an electron and a neutron the sign of the interaction changes. Since the electrons repel electromagnetically, the probability density for two electrons being within the same spatial volume $\rightarrow 0$ as the volume $\rightarrow 0$. The weak interaction (27) of electrons with electrons, is, therefore, different from zero only in that the delta function potential is only approximate. However, the range of the weak interaction is expected[27,37] to be only of the order 10^{-13} cm and therefore negligibly small on an atomic scale ($\sim 10^{-8}$ cm).

The neutral weak interaction is, therefore, important only between an electron and the nucleus, giving

$$V=\sum_n \frac{8^{1/2}G}{16m}[(\boldsymbol{\sigma}-\boldsymbol{\sigma}_n)\cdot\{\mathbf{p},\delta^3(\mathbf{r})\}_+ +i(\boldsymbol{\sigma}\times\boldsymbol{\sigma}_n)\cdot[\mathbf{p},\delta^3(\mathbf{r})]_-]\tau_n\,, \tag{28}$$

where $\tau_n=+1$ (protons), -1 (neutrons). The coordinates refer to the electron relative to the nucleus (approximated as being fixed: $\mathbf{p}_n=0$). For a symmetrical nucleus having filled proton and neutron shells (e.g., O^{16}) the interaction (28) becomes

$$V=\frac{8^{1/2}G}{16m}(Z-N)\boldsymbol{\sigma}\cdot\{\mathbf{p},\delta^3(\mathbf{r})\}_+. \tag{29}$$

The atomic levels of hydrogen would be parity impure, while those of deuterium would be pure[40]; thus quite different impurities are expected for very nearly identical atomic states, a situation useful in eliminating systematic experimental effects.

2S State of Hydrogen

An atomic state likely to have a comparatively large parity impurity is the $2S$ state of hydrogen. This state is nearly degenerate with the $2P$ state, and, provided $\langle 2S|V|2P\rangle$ is not anomalously small, these two states should be more strongly intermixed than the other atomic states of hydrogen.

Since $i(\boldsymbol{\sigma}_p\times\boldsymbol{\sigma})=P^\sigma(\boldsymbol{\sigma}_p-\boldsymbol{\sigma})$, where P^σ is the spin exchange operator, and $P^\sigma|{}^1S_0\rangle=-|{}^1S_0\rangle$, the transition

[33] J. Dreitlein and B. W. Lee, Phys. Rev. **124**, 1274 (1961).
[34] G. Feinberg, Phys. Rev. **109**, 1019 (1958).
[35] I. V. Lyagin and E. Kh. Ginzburg, Zh. Eksperim. i Teor. Fiz. **41**, 914 (1961) [English transl.: Soviet Phys.—JETP **14**, 653 (1962)].
[36] L. Krüger, Z. Physik **157**, 369 (1957).
[37] R. J. Blin-Stoyle, Phys. Rev. **118**, 1605 (1960).
[38] R. J. Blin-Stoyle, Phys. Rev. **120**, 181 (1960).
[39] R. J. Blin-Stoyle and R. M. Spector, Phys. Rev. **124**, 1199 (1961).
[40] Ya. B. Zel'dovich and A. M. Perelomov, Zh. Eksperim. i Teor. Fiz. **39**, 1115 (1960) [English transl.: Soviet Phys.—JETP **12**, 777 (1961)].

matrix element $\langle 2^1S_0|V|2^3P_0\rangle$ becomes

$$V_{21}=-\frac{8^{1/2}G}{8m}\langle{}^1S_0|\delta^3(\mathbf{r})(\boldsymbol{\sigma}_p-\boldsymbol{\sigma})\cdot\mathbf{p}|{}^3P_0\rangle. \quad (30)$$

Further making use of the relation

$$(\boldsymbol{\sigma}_p-\boldsymbol{\sigma})\cdot\mathbf{p}=i\left(\frac{\partial}{\partial r}+\frac{l+1}{2}\right)(\boldsymbol{\sigma}_p-\boldsymbol{\sigma})\cdot\hat{r},\quad \hat{r}=\mathbf{r}/r, \quad (31)$$

where $(\boldsymbol{\sigma}_p-\sigma)\cdot\hat{r}|2^3P_0\rangle=2|2^1S_0\rangle$ (except for the radial spin dependence, which is still that for $|2^3P_0\rangle$), thus further reducing Eq. (30) to

$$V_{21}=-\frac{8^{1/2}G}{4m}\langle R_s|\delta^3(\mathbf{r})i\left(\frac{\partial}{\partial r}+\frac{l+1}{r}\right)|R_p\rangle \quad (32)$$

with

$$R_s(n=2)=\frac{1}{\sqrt{2}}e^{-r/2}(1-\tfrac{1}{2}r);\quad R_p(n=2)=\frac{1}{2\sqrt{6}}re^{-r/2}$$

in units of $(m\alpha)^{-1}$. The integration is trivial since

$$\delta^3(\mathbf{r})=\frac{1}{4\pi r^2}\delta(r),$$

giving

$$V_{21}=-\frac{6^{1/2}}{32\pi}Gm^3\alpha^4=(-i)4.84\times10^{-16}R_y. \quad (33)$$

The observed splitting between these levels is 1057 Mc/sec giving

$$iF\approx V_{21}/\Delta E=(i)1.51\times10^{-10}. \quad (34)$$

Since angular correlations, photon polarization, etc., are proportional to F, detection of such a small impurity requires considerable experimental sensitivity. The lifetime of the $2S$ state, due to the parity nonconserving admixture, is

$$t_{2S}(\text{pnc})\approx t_{2P}/|F|^2=7\times10^{10}\ \text{sec}. \quad (35)$$

Indeed quite long, this decay mode (single proton) is already much longer than the $M1$ decay lifetime[41] (via small components of the Dirac wave functions) of $\sim2\times10^5$ sec; furthermore, the two-photon decay lifetime is estimated[41] to be 0.14 sec and, therefore, the total decay rate is negligibly affected.

If a magnetic field is present, the Zeeman splitting of the magnetic substates brings the $2S_{1/2-1/2}$ and $2P_{1/2-1/2}$ states together at $B\approx1300$ G. The value of F does *not* increase unrestrictedly however, due to the finite width of the level ($\Gamma=h/t=99.8$ Mc/sec). Thus,

$$|F|\leq V/\Gamma=1.60\times10^{-9}, \quad (36)$$

and, in principle, the value of F can be increased an order of magnitude, hence reducing the t_{2S}(pnc) lifetime to 20 yr. Although the reduction in the total lifetime of the $2S$ state is negligible, the enhancement of the distinctive single photon decay is considerable. It seems appropriate at this point to recall the Stark splitting from the random thermal motion in the magnetic field. The electric field seen is given from $\mathbf{E}=\mathbf{v}\times\mathbf{B}$ where $v\approx300$ m/sec in a gas at room temperature, $B=1.3\times10^{-1}$ W/m², and, therefore, $E\approx40$ V/m; the Stark interaction is then of the order

$$\langle r\rangle eE\approx a_0eE\approx2\times10^{-9}\ \text{eV}, \quad (37)$$

hence

$$V(\text{Stark})/V(\text{pnc})\approx2\times10^6. \quad (38)$$

The Stark interaction therefore imposes rather stringent monochromatization requirements on, say, an experiment employing atomic-beam techniques even if a compensating electric field is used to cancel the $-\mathbf{v}\times\mathbf{B}$ field at the central velocity.

Circular Dichroism in Hydrogen

Consider the $M1$ transition $2P_{3/2}\rightarrow2P_{1/2}$; such an $M1$ transition is extremely weak, while an $E1$ transition between these states is parity forbidden (of course, the $E1$ transition $2P_{3/2}\rightarrow1S_{1/2}$ is allowed and has a lifetime of $\sim1.6\times10^{-9}$ sec). The $E1$ transition matrix element is nonzero, however, if the $2P_{1/2}$ state contains an admixture of the $2S_{1/2}$ state. The photon from the transition $2P_{3/2}\rightarrow2P_{1/2}(2S_{1/2})$ will then be circularly polarized (via interference between the $M1$ and $E1$ amplitudes) with

$$P=2RF, \quad (39)$$

where

$$R=\frac{\langle 2S_{1/2\ 1/2}|E1|2P_{3/2\ 1/2}\rangle}{\langle 2P_{1/2\ 1/2}|M1|2P_{3/2\ 1/2}\rangle} \quad (40)$$

using $E1=z$, $M1=(\sigma_z+l_z)/2m$, and the tabulated wave functions, we find

$$R=-6(6\pi)^{1/2}/\alpha=-357 \quad (41)$$

or

$$P=-1.1\times10^{-7}. \quad (42)$$

Since the transition is in the experimentally convenient microwave region (~3 cm), perhaps this small but significant polarization can be detected (e.g., in the differential attenuation of right- versus left-hand circularly polarized waves). If we now recognize that the Stark splitting due to thermal motion in a magnetic field cannot give such circular dichroism, then it becomes clear that a magnetic field can be applied to increase that dichroism by an order of magnitude.

D. Scattering

In general it is difficult to isolate very small effects in scattering experiments (i.e., experiments with incident and outgoing free particles) unless the other interactions are either absent or easily distinguishable. Since neu-

[41] G. Breit and E. Teller, Astrophys. J. 91, 215 (1940).

trinos interact only via the weak interactions, several interactions falling into the above classification are the following:

$$\nu+N \to N+\nu\,, \tag{43a}$$

$$\nu_1+\mu \to \mu+\nu_1\,, \tag{43b}$$

$$\nu_2+e \to e+\nu_2\,, \tag{43c}$$

and

$$\nu+\nu \to \nu+\nu\,, \tag{43d}$$

while the interactions

$$n+e \to e+n \tag{44a}$$

and

$$n+\mu \to \mu+n \tag{44b}$$

are somewhat marginal (there being, of course, the electromagnetic interaction of the electron or muon with the neutron's magnetic moment). The cross section for interactions such as

$$\nu_1+e \to e+\nu_1 \tag{45}$$

is reduced by a factor of 2 from that expected if only charged currents contribute; however, this cross section is rather difficult to measure directly. Note that the cross sections quoted in Ref. 5 refer to unpolarized neutrinos; since only left-handed neutrinos interact and since only left-handed neutrinos are produced in nature, the cross sections for the latter are twice as large as those given in Ref. 5.

The neutrino-nucleon scattering could be detected directly either via the nucleon recoil or via the excitation of a complex nucleus. Since usefully intense beams of high energy neutrinos (ν_2) are apparently available,[9] an experiment designed around reaction (43a) might be feasible. The cross sections are of the order [calculated for $J_\mu{}^N=(\bar{N}|\gamma_\mu a|N)$]

$$\sigma(\nu+N)=\sigma_0\frac{\omega^2}{1+2\omega};\quad \frac{d\sigma}{d\epsilon}=\sigma_0 \tag{46}$$

and

$$\sigma(\bar{\nu}+N)=\sigma_0\frac{\omega}{6}\left[1-\frac{1}{(1+2\omega)^3}\right];\quad \frac{d\sigma}{d\epsilon}=\sigma_0\left(\frac{1+\omega-\epsilon}{\omega}\right)^2, \tag{47}$$

where $\sigma_0=2G^2M^2/\pi=2.8\times10^{-38}$ cm^2, ω = neutrino energy/M, ϵ is the nucleon recoil energy/M, and M is the nucleon mass. At present the experimental data[42] suggest a cross section somewhat smaller than that given by Eq. (46).

V. OTHER CONSEQUENCES

Feinberg *et al.*[43] have proposed an experiment to detect the transition in muonium

$$(\bar{\mu}e) \to (\mu\bar{e}) \tag{48}$$

which could be detected via the high-energy electrons given off in the decay of the latter system. We do not expect this transition on the basis of Eq. (4) (leptonness is violated), and therefore a definitive experiment on this system may reflect directly on the validity of Eq. (4) and indirectly on Eq. (1).

The nucleon-nucleon self-interaction terms give rise to parity impurity of nuclear states via $(\bar{n}p)(\bar{n}p)^+$. Some uncertainty arises in calculating with such an interaction[27] because the isotopic spin dependence $(\tau_+{}^1\tau_-{}^2+\tau_-{}^1\tau_+{}^2)$ is not symmetric in isotopic spin space: Pionic corrections should lead to a more complicated isotopic spin dependence. If this weak internucleon interaction contains, additionally, couplings such as $(\bar{p}p)(\bar{n}n)^+$ and *is* in fact symmetric in isotopic spin space (i.e., $\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2$), then pionic corrections only alter the magnitude of the coupling.

The conserved vector current (CVC) theory remains unchanged under the above so far as its predictions for the charged weak-current interactions are concerned. Since the CVC theory can be written symmetrically in isotopic spin space,[44] there is no ambiguity in extending it to include neutral weak interactions; however, there do not seem to be any consequences of the CVC theory that lead to exceptionally sensitive experiments.

VI. SUMMARY

We have analyzed several possible experiments to test the idea that the leptons are arranged in isotopic spin doublets coupled such that the strangeness-conserving weak interactions are charged symmetric. These experiments are difficult and the effects are rather small.

The weak neutral interaction could be observed directly via elastic scattering experiments; on the other hand, this interaction seems to have little role in the decay of the elementary particles. Experiments involving atomic nuclei are complicated via the parity-nonconserving features of the charged weak interactions [unless self-self terms such as $(\bar{p}n)(\bar{p}n)^+$ are absent]: Only for $0\to0$ transitions can parity nonconserving interference effects be unambiguously attributed to the neutral currents. Parity violating effects are expected in several experiments involving the $n=2$ states of hydrogen, although these experiments appear to be quite difficult.

The major weak points in the assignment of isotopic spin to leptons, as in Eq. (2), are: (1) no way is offered to extend the analysis to strangeness-violating interactions, and (2) the idea that leptons have isotopic spin should, if combined naively with the Yang-Mills theory[1] and/or the variations popular today [e.g., $SU(3)$], predict that the leptons are strongly interacting. Thus experimental verification or rejection of lepton isotopic spin may lead to theoretical consequences far beyond the possible existence of neutral weak interaction currents. On the other hand, it would be equally curious if no other interaction with the leptons existed, thereby leaving their symmetries indeterminate and ambiguous.

[42] J. S. Bell, J. Løvseth, and M. Veltman, *Proceedings of the Sienna International Conference on Elementary Particles, 1963*, edited by G. Bernadini and G. P. Puppi (Societá Italiana di Fisica, Bologna, 1963).

[43] G. Feinberg and S. Weinberg, Phys. Rev. **123**, 1439 (1961).

[44] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).