SUMMARY AND CONCLUSIONS

It has been found that the decay of I^{128} leads, with a probability of about 6×10^{-3} %, to a Xe^{128} excited state at 1570 keV. This state then decays primarily by way of the first excited state at 450 keV. The log *ft* value associated with the low-energy beta-ray transition is found to be 8.0. This is most probably indicative of a first forbidden transition, which implies an odd parity and a spin between 0 and 3 for the Xe level. Jha *et al.*,[1] on the other hand, have reported a beta-ray transition from Cs^{128} to a Xe^{128} level at 1560 keV with a log *ft* value of about 5, indicating an allowed transition. Assuming the Xe level to be the same in each case, it thus appears that either the transition from I^{128} is somewhat slow, or the transition from Cs^{128} is somewhat fast.

ACKNOWLEDGMENTS

We wish to thank the staff of the Research Reactor, and especially Douglas S. Vonada, for their generous assistance with this work.

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Phase-Shift Analysis for $He^3(p,p)He^3$†

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A phase-shift analysis based on the scattering and polarization data for the reaction $He^3(p,p)He^3$ has been made for proton energies between 1.0 and 11.5 MeV. The singlet and triplet *S*-wave phase shifts are found to be negative and approximately equal; they are in good agreement with the resonating-group calculations of Bransden and Robertson. The *P*-wave phase shifts are positive and increase with energy; a single-level parametrization of these phase shifts indicates the possible presence of triplet states with $J^\pi=2^-$, 1^-, 0^- at center-of-mass energies 4.74, 6.15, and 7.94 MeV above the $p+He^3$ threshold. All three levels have reduced widths that are comparable with the Wigner limit. The presence of a broad singlet 1^- level at 9.8 MeV is indicated with somewhat less confidence.

I. INTRODUCTION

The discovery of a 0^+ excited state of He^4, 20 MeV above the ground state, has renewed interest in the study of the $A=4$ nuclei,[1,2] and has caused the question of whether H^4 is particle-stable to be reconsidered.[3] If, indeed, the 0^+ state of He^4 had $T=1$, then H^4 would probably be particle-stable and Li^4 would have a low-lying 0^+ resonance above the $p+He^3$ threshold. The work that will be reported herein concerns the presence of such $T=1$ resonances that would appear in the scattering of protons from He^3. Previous work has demonstrated that particle-stable states of Li^4 lying below this threshold are unlikely[4] and the problem to be considered concerns the position and spacing of the low-lying states of the continuum.

In a previous paper[5] the experimental and theoretical work that has been done on the scattering of protons from He^3 has been summarized. Since that time two groups have greatly increased the experimental data available, and it is upon this foundation that the present paper is based. The first of these papers, by Clegg *et al.*, presents precision measurements of the differential cross section for proton energies between 4.5 and 11.5 MeV[6]; the other, by McDonald *et al.*, consists of both proton-polarization data and precision differential-cross-section measurements for proton energies between 4.0 and 12.8 MeV.[7] As summarized previously it has been demonstrated for this reaction that it is impossible to extract a unique set of scattering phase shifts on the basis of the differential cross sections alone, and all previous attempts have consisted of simple, spin-independent analyses involving only a few phase shifts.[5,6,8,9] That such an analysis is not valid is immediately obvious from the large proton polarization observed by McDonald. It was, therefore, these polarization data that allowed the present analysis to be undertaken.

In addition to the experimental results just described, recent data of Kavanagh *et al.*, for proton energies between 0.13 and 1.5 MeV allow the proper set of *S*-wave phase shifts to be determined at very low energies and

† Supported by the U. S. Office of Naval Research [Nonr-220(47)].

[1] Y. G. Balashko, I. Y. Barit, and Y. A. Goncharov, Zh. Eksperim. i Teor. Fiz. **36**, 1937 (1959) [English transl.: Soviet Phys.—JETP **9**, 1378 (1959)].

[2] C. Werntz, Phys. Rev. **128**, 1336 (1962).

[3] C. Werntz and J. G. Brennan, Phys. Letters **6**, 113 (1963).

[4] S. Bashkin, R. W. Kavanagh, and P. D. Parker, Phys. Rev. Letters **3**, 518 (1959); H. A. Grench, W. L. Imhof, and F. J. Vaughn, Bull. Am. Phys. Soc. **7**, 268 (1962).

[5] T. A. Tombrello, C. M. Jones, G. C. Phillips, and J. L. Weil, Nucl. Phys. **39**, 541 (1962).

[6] T. B. Clegg, A. C. L. Barnard, J. B. Swint, and J. L. Weil, Nucl. Phys. **50**, 621 (1964).

[7] D. G. McDonald, W. Haeberli, and L. W. Morrow, Phys. Rev. **133**, B1178 (1964).

[8] R. W. Lowen, Phys. Rev. **96**, 826 (1954).

[9] R. M. Frank and J. L. Gammel, Phys. Rev. **99**, 1406 (1955).

thus allow a choice between the two solutions that appear in the present analysis.[10]

The result of the phase-shift analysis has been to indicate the existence of at least three P-wave levels of Li^4 and together with the data of Kavanagh to eliminate the possibility of a low-lying $T=1$, $J^\pi=0^+$ state.

II. METHOD OF ANALYSIS

For the scattering of two spin-$\frac{1}{2}$ particles, several separate experiments are necessary at each energy to allow the phase shifts to be uniquely specified. If no reaction channels are open, one such set of experiments is:

(1) The measurement of the differential cross section.

(2) The measurement of the spin correlation of the out going particles; when these particles are distinguishable this reduces to the measurement of the polarization of each separately.

(3) Triple scattering experiments—the measurement of the change in polarization when one of the particles is initially polarized.

Table I gives the sources of data used in the present analysis of the scattering of protons from He^3. It is obvious that there is at present insufficient data to guarantee the unique determination of the scattering parameters. However, at worst one result of the present analysis could be to determine which additional experiments are most necessary and what are the optimum conditions (angles, energies, etc.) under which they should be performed.

The phase-shift analysis was accomplished in the following way:

(1) Smooth curves were drawn through the polarization data so that it could be interpolated at all energies above 4 MeV where Clegg *et al.* and Tombrello *et al.* had taken angular distributions. Thus, at each energy both the polarization and the cross section could be considered simultaneously.

(2) The polarization and cross section were then calculated using a trial set of nine scattering parameters: singlet and triplet S-wave phase shifts; one singlet and three triplet P-wave phase shifts plus a channel-spin-mixing parameter for $J^\pi=1^-$; and one phase shift each for the singlet and triplet D waves. The analysis neglects the coupling between S and D waves for 1^+ states but does not neglect the coupling between the two-channel spin states having the same angular momentum and parity. The triplet D-wave phases are assumed to be unsplit, but as it turns out this phase shift is shown to be quite small. The effect of the reaction cross section has not been considered though the three-body reaction $He^3(p,2p)D$ becomes possible at proton energies above 7.3 MeV and the four-body reaction cross section for $He^3(p,3p)n$ above 10.3MeV. (Recent measurements have shown that at $E_p=10$ MeV the three-body differential cross section is less than 1% of the elastic-scattering cross section for laboratory angles of 30 and 45°.[11])

(3) The quantity $\mathfrak{M}$ is formed for the polarization and cross section in the following way:

$$\mathfrak{M}=\frac{a}{N_\sigma}\sum_{i=1}^{N_\sigma}\left(\frac{\sigma_{\text{calc}}(\theta_i)-\sigma_{\text{exp}}(\theta_i)}{\Delta\sigma_{\text{exp}}(\theta_i)}\right)^2+\frac{(1-a)}{N_p}\sum_{i=1}^{N_p}\left(\frac{P_{\text{calc}}(\theta_i)-P_{\text{exp}}(\theta_i)}{\Delta P_{\text{exp}}(\theta_i)}\right)^2$$
$$=a\mathfrak{M}_\sigma+(1-a)\mathfrak{M}_p,$$

where N_σ and N_p are the number of angles at which the cross section and the proton polarization were measured; σ_{calc} and P_{calc} are the calculated values of the cross section and proton polarization using the trial set of phase shifts; σ_{exp} and P_{exp} are the experimental values of the cross section and polarization with rms errors $\Delta\sigma_{\text{exp}}$ and ΔP_{exp}, respectively. The quantity a is used for the relative weighting of the two types of data and was taken to be $N_\sigma/(N_\sigma+N_p)$.

Partial derivatives of $\mathfrak{M}$ with respect to all nine parameters are calculated numerically—yielding a gradient of the $\mathfrak{M}$ surface. The computer program then moves a fixed distance along this vector in the direction of decreasing $\mathfrak{M}$, thus generating a new trial set of phase shifts. This procedure is repeated until a minimum of $\mathfrak{M}$ is reached.

In previous attempts at analysis based only upon the angular distributions it was found that the data could be explained in terms of a single phase shift for each of the S, P, and D partial waves.[5,8] These parameters were in quite good agreement with theoretical calculations based on the resonating-group-structure method —in which the spin-orbit and spin-spin forces were neglected.[12] It might, therefore, be expected that several solutions would result for the nine scattering parameters even when the polarization data are considered. Most

TABLE I. The sources of the experimental data used in the analysis.

Energy range (lab) (MeV)	Quantity measured	Number of angles measured	Reference
1.0–3.5	$\sigma(\theta)$	9	13
2.0–4.5	$\sigma(\theta)$	20	5
4.5–11.5	$\sigma(\theta)$	17–22	6
4.0–10.8	$\sigma(\theta)$	16	7
4.0–12.8	$P_p(\theta)$	4–6	7

[10] R. W. Kavanagh, P. D. Parker, and G. D. Symons, Bull. Am. Phys. Soc. 8, 597 (1963); R. W. Kavanagh (private communication).

[11] T. A. Tombrello, A. D. Bacher, and M. R. Dwarankanath (unpublished).

[12] B. H. Bransden, H. H. Robertson, and P. Swan, Proc. Phys. Soc. (London) A69, 877 (1956); B. H. Bransden and H. H. Robertson, *ibid.* A72, 770 (1958).

TABLE II. The values of the phase shifts, $\delta_{S,L}{}^{J}$, for solution I. (Where values of $\mathfrak{M}_p$ are in parentheses, interpolated polarization values were used.)

Reference	E_p (MeV)	$l=0$ $\delta_{0,0}{}^0$	$\delta_{1,0}{}^1$	$l=1$ $\delta_{0,1}{}^1$	$\delta_{1,1}{}^0$	$\delta_{1,1}{}^1$	$\delta_{1,1}{}^2$	$l=2$ ϵ	$\delta_{0,2}$	$\delta_{1,2}$	$\mathfrak{M}_\sigma$	$\mathfrak{M}_p$
13	1.01	−15.5°	−18.0°	2.5•	1.5°	3.9°	4.9°	0.0°	0.1°	0.3°	1.01	...
13	1.60	−25.3°	−27.9°	5.4°	2.7°	7.4°	10.8°	0.0°	0.0°	−0.3°	0.29	...
5	2.01	−29.9°	−33.4°	8.0°	3.2°	10.6°	17.0°	0.0°	−0.2°	−0.5°	0.54	...
13	2.25	−32.5°	−36.1°	9.4°	4.0°	12.2°	19.7°	0.0°	−0.5°	−0.6°	0.15	...
5	3.01	−39.2°	−41.5°	17.1°	6.4°	18.9°	31.3°	0.0°	−0.3°	−0.7°	0.73	...
13	3.52	−43.7°	−44.0°	20.5°	7.3°	23.1°	36.0°	0.0°	−0.5°	−0.6°	0.51	...
5	3.99	−45.5°	−46.4°	22.4°	10.1°	28.9°	43.1°	5.2°	6.5°	−2.3°	0.30	0.63
7	4.00	−47.0°	−51.5°	22.7°	9.2°	28.6°	40.0°	1.5°	0.5°	−2.0°	0.44	0.42
6	4.55	−52.6°	−53.7°	23.2°	13.7°	31.8°	47.0°	−0.1°	−1.8°	−1.8°	0.52	(0.44)
6	5.51	−56.4°	−59.2°	24.2°	14.3°	38.1°	55.7°	−1.5°	−5.5°	−1.5°	0.48	0.94
7	5.51	−53.2°	−59.9°	24.3°	13.8°	38.6°	54.4°	−0.9°	−5.4°	−1.2°	0.49	0.48
6	6.52	−61.7°	−64.9°	24.4°	15.1°	44.0°	60.2°	−3.2°	−10.3°	−0.4°	0.75	(1.92)
7	6.82	−59.0°	−67.5°	23.1°	16.7°	46.2°	59.8°	−4.7°	−9.9°	−0.3°	0.42	1.16
6	7.51	−68.0°	−70.7°	25.8°	18.5°	48.0°	62.4°	−5.3°	−11.4°	−0.3°	0.71	(0.64)
6	8.51	−72.2°	−75.8°	24.9°	21.8°	52.7°	63.8°	−6.3°	−13.2°	0.5°	0.45	(0.83)
7	8.82	−75.5°	−77.3°	23.7°	26.4°	52.7°	63.3°	−7.9°	−12.6°	0.0°	0.37	0.60
6	9.51	−80.9°	−79.0°	28.0°	30.3°	50.5°	64.4°	−7.7°	−15.0°	1.2°	0.55	(0.69)
6	10.38	−84.1°	−82.3°	28.4°	36.5°	49.6°	66.2°	−9.3°	−16.3°	1.9°	0.77	(0.84)
7	10.77	−84.7°	−84.4°	27.4°	36.9°	50.5°	65.6°	−13.8°	−15.4°	1.3°	0.37	0.84
6	11.48	−84.6°	−88.8°	21.4°	44.3°	49.4°	66.7°	−11.2°	−18.6°	2.5°	0.45	(1.20)

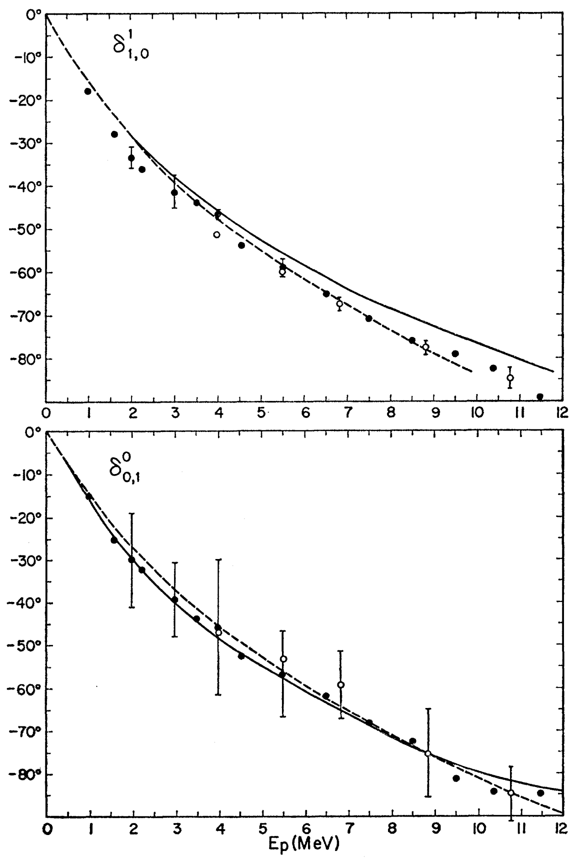

FIG. 1. The S-wave phase shifts for the singlet ($\delta_{0,0}{}^0$) and triplet ($\delta_{1,0}{}^1$) channel spins. The solid curves are the predictions of the resonating-group calculation of Bransden and Robertson. The dashed curves are hard-sphere phase shifts for 3.05 F in the singlet case and 3.15 F in the triplet case. (The open circles correspond to the data of Ref. 7; the solid points correspond to Refs. 5, 6, and 13.)

of these solutions may be discarded on the basis of the behavior of the phase shifts as a function of the energy, where either a lack of continuity or a violation of the causality requirement are demonstrated for any one of the phase shifts. After eliminating all such solutions there remain two possible sets of phase shifts that produce acceptable fits to the angular distributions and

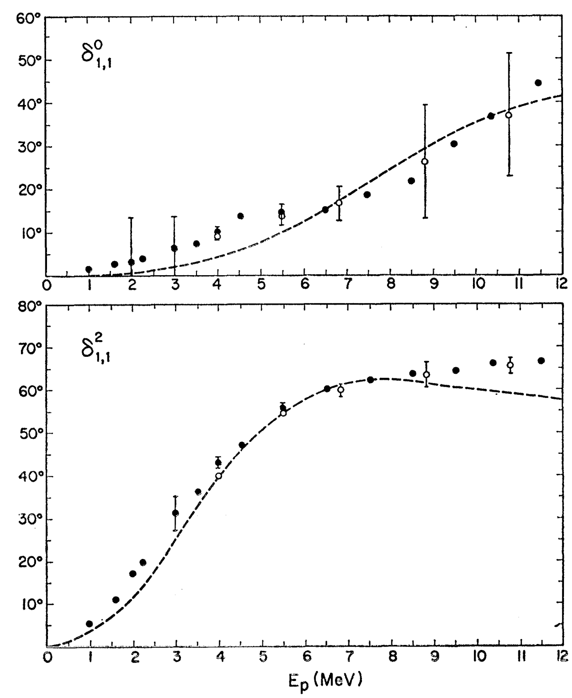

FIG. 2. The P-wave phase shifts for $J^\pi=2^-(\delta_{1,1}{}^2)$ and $J^\pi=0^-(\delta_{1,1}{}^0)$. The dashed line is the single-level fit using the parameters of Table IV.

TABLE III. The values of the phase shifts, $\delta_{S,L}{}^{J}$, for solution II. (Where values of $\mathfrak{M}_p$ are in parentheses, interpolated polarization values were used.)

Reference	E_p (MeV)	$l=0$ $\delta_{0,0}{}^{0}$	$\delta_{1,0}{}^{1}$	$l=1$ $\delta_{0,1}{}^{1}$	$\delta_{1,1}{}^{0}$	$\delta_{1,1}{}^{1}$	$\delta_{1,1}{}^{2}$	ϵ	$l=2$ $\delta_{0,2}$	$\delta_{1,2}$	$\mathfrak{M}_\sigma$	$\mathfrak{M}_p$
13	1.60	93.9°	−19.0°	−2.8°	2.2°	6.9°	7.8°	0.0°	0.7°	−0.9°	0.25	...
5	2.01	91.8°	−24.4°	−6.9°	3.7°	11.2°	15.3°	0.0°	−0.2°	−0.6°	0.64	...
13	2.25	90.3°	−27.3°	−8.4°	4.2°	13.3°	18.6°	0.0°	0.1°	−0.4°	0.08	...
5	3.01	85.1°	−39.1°	−6.8°	7.0°	21.5°	28.5°	0.0°	4.2°	−0.2°	0.77	...
13	3.52	81.4°	−43.6°	−9.5°	8.5°	25.5°	33.5°	0.0°	5.2°	0.0°	0.38	...
5	3.99	79.1°	−45.6°	−10.0°	12.5°	30.8°	42.3°	−0.6°	−1.1°	−1.4°	0.76	0.86
7	4.00	78.4°	−50.4°	−12.0°	10.3°	28.4°	39.7°	0.7°	7.2°	−0.3°	0.31	0.52
6	4.55	79.0°	−54.1°	−12.4°	14.7°	32.0°	46.4°	0.0°	7.1°	−0.9°	0.87	(0.42)
6	5.51	75.4°	−61.5°	−22.5°	18.2°	34.0°	54.0°	−0.5°	8.2°	−0.9°	0.76	0.71
7	5.51	71.8°	−61.2°	−17.9°	15.1°	37.7°	54.1°	0.2°	9.4°	0.0°	0.24	0.24
6	6.52	69.7°	−68.9°	−28.7°	20.2°	35.4°	60.3°	−1.9°	10.5°	−0.3°	0.73	(2.05)
7	6.82	66.5°	−71.1°	−25.5°	20.5°	39.6°	61.3°	−2.4°	10.9°	0.3°	0.11	0.97
6	7.51	67.4°	−75.5°	−29.8°	20.1°	38.2°	65.5°	−3.5°	11.0°	−0.8°	0.72	(0.65)
6	8.51	67.8°	−82.1°	−31.2°	28.0°	39.0°	69.0°	−6.5°	12.6°	−0.1°	0.50	(0.84)
7	8.82	68.9°	−84.2°	−30.6°	29.6°	39.2°	69.4°	−6.4°	12.7°	−0.1°	0.26	0.76
6	9.51	72.1°	−88.7°	−34.4°	33.2°	35.6°	70.7°	−6.9°	15.2°	0.8°	0.53	(0.95)
6	10.38	79.3°	−91.6°	−31.0°	38.5°	37.3°	73.4°	−7.0°	16.5°	1.5°	0.76	(1.37)
7	10.77	87.4°	−90.7°	−28.1°	42.2°	37.5°	71.8°	−5.4°	16.5°	1.2°	0.36	1.57
6	11.48	87.4°	−97.5°	−28.2°	43.3°	37.4°	74.3°	−6.3°	18.2°	2.7°	0.43	(0.94)

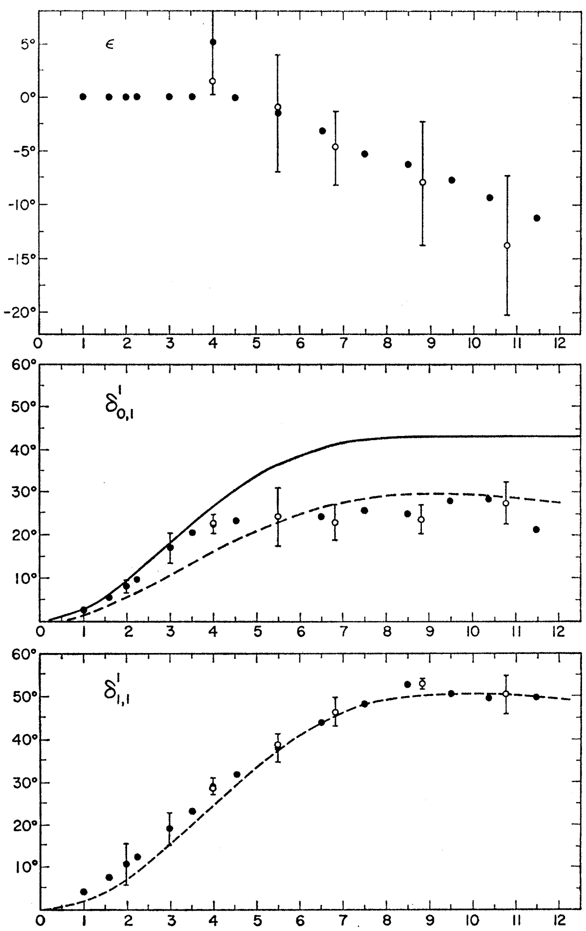

FIG. 3. The P-wave phase shifts and the channel-spin-mixing parameter (ϵ) for the singlet ($\delta_{0,1}{}^{1}$) and triplet ($\delta_{1,1}{}^{1}$) channel spins. The solid line is the resonating group prediction. The dashed lines correspond to the single-level parameters given in Table IV and neglect the coupling of the two channel spins.

proton polarization over the energy range between 4.0 and 11.5 MeV. These phase shifts were then extrapolated to lower energies (where no polarization data are available) and fits were obtained for the angular distributions of Famularo *et al.*[13] and of Tombrello *et al.*[5]

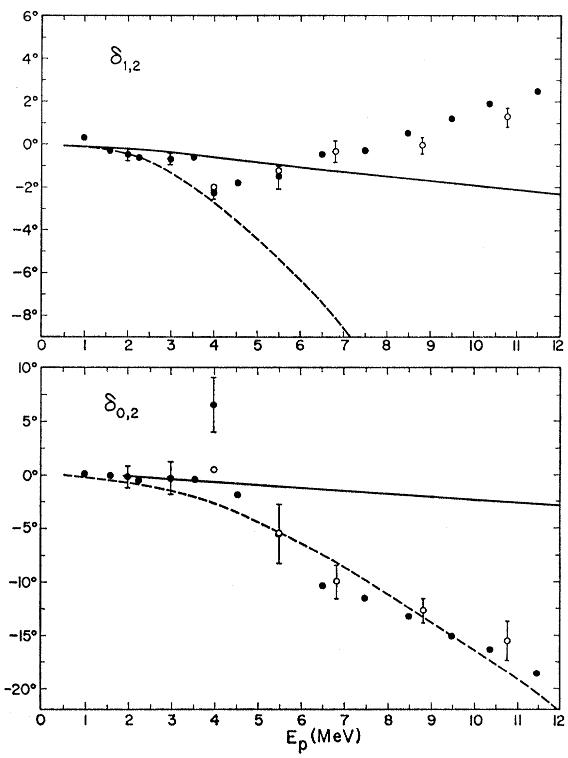

FIG. 4. The D-wave phase shifts for the singlet ($\delta_{0,2}$) and triplet ($\delta_{1,2}$) channel spins. The solid lines are the resonating-group predictions, and the dashed lines are hard-sphere phase shifts for 4.10 F.

[13] K. F. Famularo, R. J. S. Brown, H. D. Holmgren, and T. F. Stratton, Phys. Rev. 93, 928 (1954).

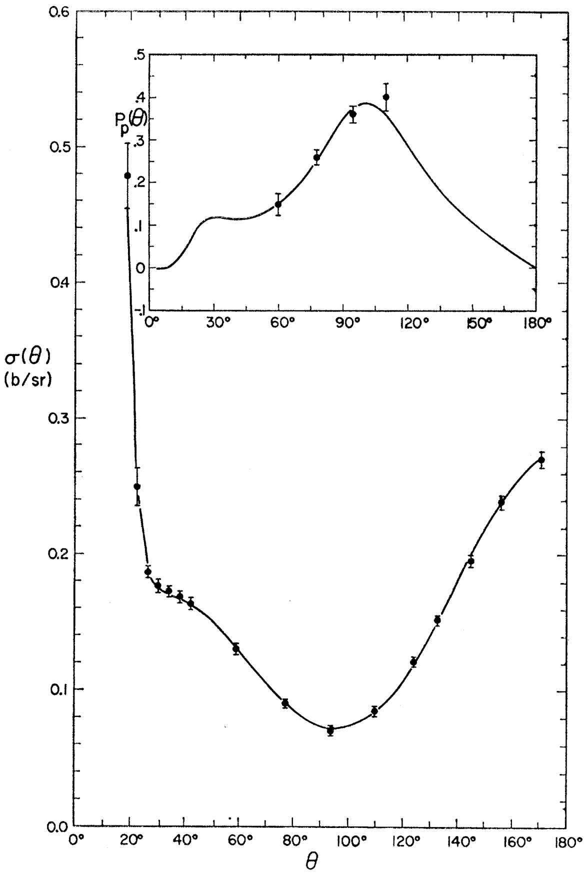

FIG. 5. The scattering and polarization data of Ref. 7 together with the fit corresponding to solution I at a proton energy of 4.00 MeV.

The determination of the phase shifts in this region below 4 MeV is, of course, subject to much greater inaccuracy than at higher energy; however, by requiring the extrapolation to be continuous it was found that the errors on the derived phase shifts could be held within reasonable bounds.

The error $\Delta\delta_k$ in the derived phase shift δ_k was calculated in the following way[14]:

$$\Delta\delta_k = [(H^{-1})_{kk}]^{1/2},$$

where

$$H_{ij} = \tfrac{1}{2}(N_\sigma + N_p)(\partial^2\mathfrak{M}/\partial\delta_i\partial\delta_j)\ \text{evaluated at the minimum of } \mathfrak{M}.$$

III. SOLUTIONS

Since the resonating-group calculation based on a Serber exchange mixture for the nucleon-nucleon interaction produced an excellent fit to the angular-distribution data even though the triplet phase shifts were assumed unsplit, it was a logical starting point for the present phase-shift analysis. (The solution obtained from this starting point will be designated as solution I.) These initial values had both S-wave phase shifts negative, all P-wave phase shifts positive, and both D-wave phase shifts negative. Considerable variation in both splitting and magnitude of each phase shift was tried and the requirement of continuity in the energy for each phase shift was used to reject local minima in $\mathfrak{M}$.

The final values of the phase shifts and the errors for solution I are given in Table II and in Figs. 1–4. Calculation of the errors was impractical for the angular distributions of Famularo *et al.*,[13] because they each consist of only nine data points. It was also found to be impractical to compute the error in the channel-spin-mixing parameter ϵ at those points where no polarization data were available.

The resulting S-wave phase shifts are seen to be in good agreement with the prediction of the resonating group structure method. The P-wave phase shifts are all positive; the behavior of these parameters in terms of levels of Li^4 will be discussed in the next section. The singlet D-wave phase shift is negative and does not agree with the resonating-group prediction. The triplet D-wave phase shift is quite small, and it is thus demonstrated that the assumption that these parameters were unsplit will introduce no appreciable error. It is im-

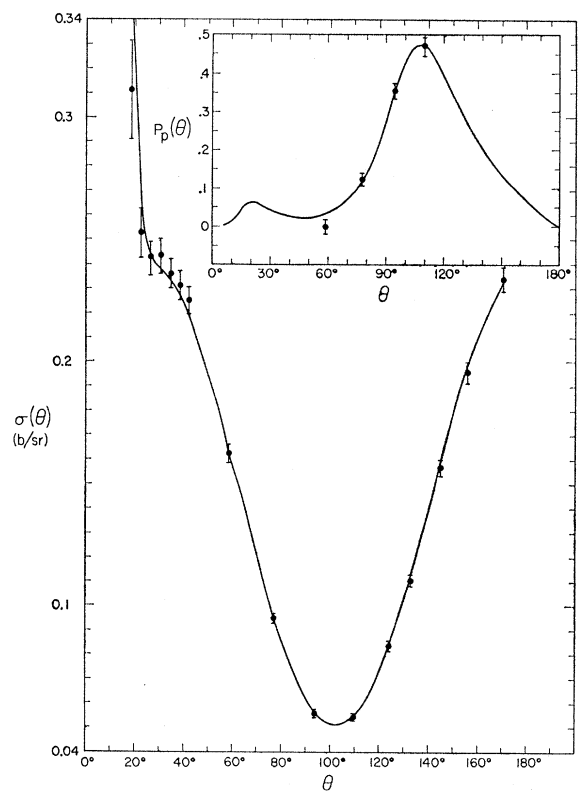

FIG. 6. The scattering and polarization data of Ref. 7 together with the fit corresponding to solution I at a proton energy of 5.51 MeV.

[14] Jay Orear, University of California Radiation Laboratory Report UCRL-8417, 1958 (unpublished).

portant to note that the S-wave phase shifts for this solution I match smoothly onto the solution found by Kavanagh.[10]

A second solution II was also found corresponding to the assumption of a low-lying 0^+ resonance. For this solution (shown in Table III) the triplet phase shifts and ϵ remain essentially unchanged—probably because they tend to be fixed by the requirement that the polarization data be reproduced. The singlet phase shifts all have sign opposite to those of solution I: the S-wave phase shift is positive; the P-wave is negative with approximately a hard-sphere variation with energy; and the D-wave phase shift is positive and slowly increasing. The fits to the data produced by this solution are in no way inferior to those of solution I; however, the singlet S-wave phase shift does not agree with that determined by Kavanagh, and thus this solution must be discarded.

Attempts to find other solutions that did not violate either the continuity or the causality conditions were unsuccessful. For example, it was not possible to find a solution with the triplet S-wave phase shift positive or a solution with the triplet P-wave phase shifts negative.

Because no other physical solutions were found that would reproduce the data, and because solution II did not agree with the analysis at low energies, we are left with no choice but solution I. Therefore, this solution will be designated as the "real" solution and all subsequent remarks will be concerned with it alone. The resulting fit to the data of McDonald *et al.* are shown in Figs. 5–9.

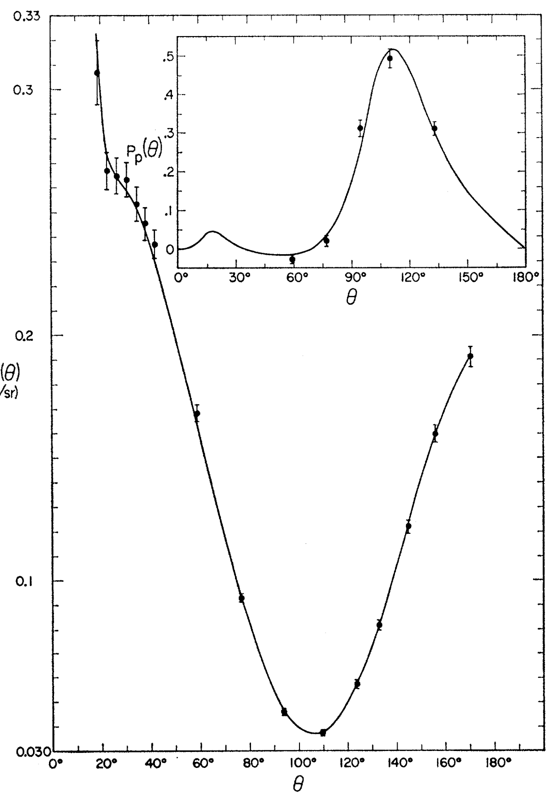

FIG. 7. The scattering and polarization data of Ref. 7 together with the fit corresponding to solution I at a proton energy of 6.82 MeV.

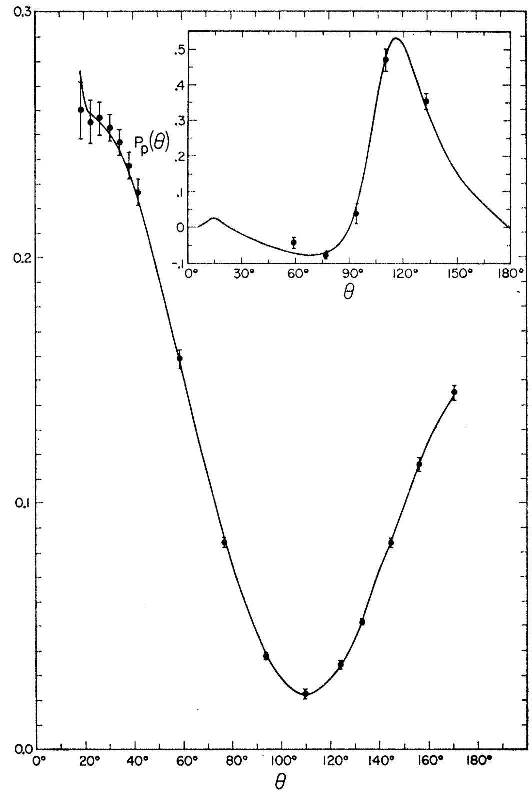

FIG. 8. The scattering and polarization data of Ref. 7 together with the fit corresponding to solution I at a proton energy of 8.82 MeV.

IV. SINGLE-LEVEL ANALYSIS

The S-wave phase shifts are adequately parametrized by hard-sphere phase shifts corresponding to radii of 3.05 F for the singlet channel spin and 3.15 F for the triplet channel spin. At energies below 3 MeV a somewhat larger radius (3.4–3.6 F) would give a better fit but would not reproduce the behavior at higher energies.

The singlet D-wave phase shift can also be interpreted as scattering from a charged impenetrable sphere but requires a somewhat larger radius of 4.10 F.

The P-wave phase shifts were parametrized in terms of the single-level formula using a radius of 4.0 F. The fits obtained are shown by the dashed curves in Figs. 2 and 3, and the resonance parameters are listed in Table IV. It was found to be impossible to reproduce the behavior of $\delta_{S=1,\,L=1}^{J=1}$, $\delta_{0,1}^{1}$, and ϵ in terms of a single resonance. Since ϵ remains small, $\delta_{1,1}^{1}$ and $\delta_{0,1}^{1}$ were

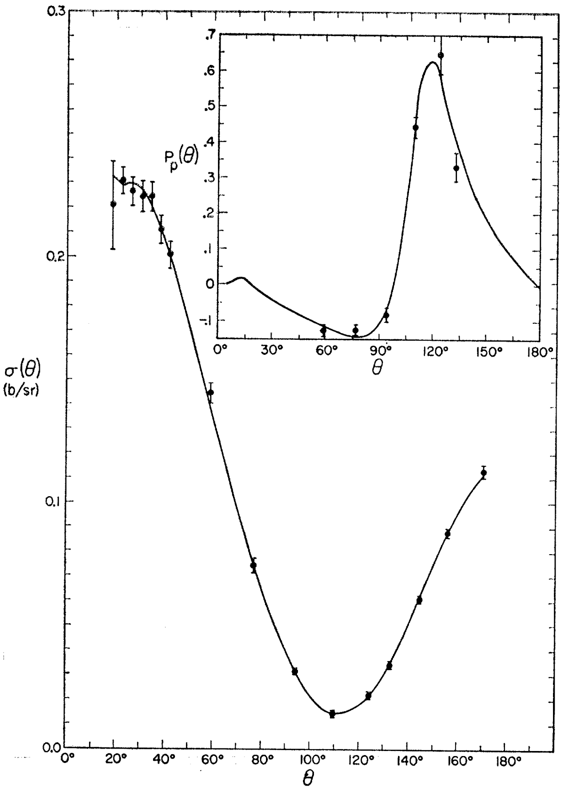

FIG. 9. The scattering and polarization data of Ref. 7 together with the fit corresponding to solution I at a proton energy of 10.77 MeV.

treated as if they were uncoupled in extracting the single-level parameters. This approximation together with the large reduced width obtained in this way for $\delta_{0,1}{}^1$ permits considerable doubt to exist concerning the presence of a singlet 1^- resonance; however, the three triplet resonances are on a somewhat sounder footing. These resonances have the ordering that one would expect from a simple *L-S* coupling model, and their widths and spacing are similar to those expected on the basis of the known widths and spacing of the low-lying states of Li^5.

TABLE IV. The single-level parameters for the $l=1$ phase shifts corresponding to a radius of 4.0 F.

Channel spin S	Angular momentum J^π	Resonance energy (c.m.) E_R(MeV)	Reduced width γ^2(MeV)	Ratio to the Wigner limit, θ^2
1	2^-	4.74	5.5	1.0
1	1^-	6.15	5.5	1.0
1	0^-	7.94	3.3	0.62
0	1^-	9.79	8.8	1.7

V. POLARIZATION

The expressions for the various amplitudes connecting initial and final magnetic quantum numbers have been given in a previous paper[5] and need not be repeated here. In the following expressions the identical notation will be followed, however, so that the previously given formulas may be used.

The scattering cross section for unpolarized protons and He^3 particles is

$$\sigma(\theta)=(1/2k^2)(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+|f|^2+|g|^2+|h|^2)\,.$$

The polarization of the scattered protons is

$$\mathbf{P}_p(\theta)=-\left[\frac{2\,\mathrm{Re}(ae^*+bh^*+cg^*+df^*)}{|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+|f|^2+|g|^2+|h|^2}\right](\hat{k}_{\rm in}\times\hat{k}_{\rm out})$$

and the polarization of the recoil He^3's is

$$\mathbf{P}_{\rm He^3}=-\left[\frac{2\,\mathrm{Re}(ad^*+bg^*+ch^*+ef^*)}{|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+|f|^2+|g|^2+|h|^2}\right](\hat{k}_{\rm in}\times\hat{k}_{\rm out})\,.$$

The difference of these two polarizations is zero unless there are terms in the scattering matrix that connect the singlet and triplet channel spin states. In the analysis described in this paper this difference is produced entirely by ϵ, the mixing parameter for $J^\pi=1^-$. The elements of the scattering matrix $U_{L=1,\,S;\;L'=1,S'}{}^{J=1}$ are given by

$$U_{1,0;\;1,0}{}^1=\cos^2\epsilon\,\exp(2i\delta_{0,1}{}^1)+\sin^2\epsilon\,\exp(2i\delta_{1,1}{}^1)\,,$$

$$U_{1,1;\;1,1}{}^1=\sin^2\epsilon\,\exp(2i\delta_{0,1}{}^1)+\cos^2\epsilon\,\exp(2i\delta_{1,1}{}^1)\,,$$

and

$$U_{1,0;\;1,1}{}^1=U_{1,1;\;1,0}{}^1=\tfrac{1}{2}\sin^2\epsilon[\exp(2i\delta_{0,1}{}^1)-\exp(2i\delta_{1,1}{}^1)]\,.$$

Values for $\mathbf{P}_p$ and $\mathbf{P}_{\rm He^3}$ calculated on the basis of smooth curves drawn through the derived phase shifts of solution I are presented in contour maps in Figs. 10 and 11.

Because both polarized proton beams and polarized He^3 targets are now available, it is possible to obtain more easily information that was previously obtainable only by triple-scattering experiments. Suppose we consider a beam of protons incident along the z axis with polarization P_p in the x direction. Let the target have polarization P_3 along the z axis of a new coordinate system obtained by rotating the original coordinate

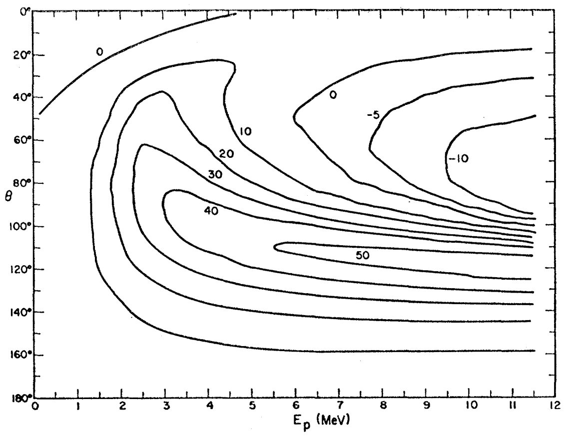

FIG. 10. The percent spin polarization of the scattered protons as a function of the proton energy and the center-of-mass scattering angle. These contours were calculated using the values of the phase shifts taken from smooth curves drawn through the derived values.

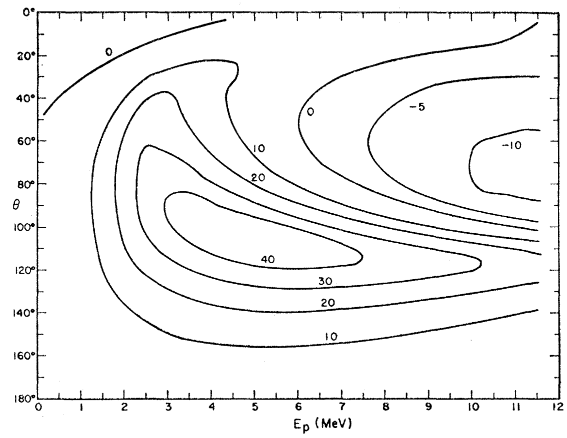

FIG. 11. The percent spin polarization of the recoil He^3's as a function of the proton energy and the center-of-mass scattering angle.

axes through the Euler angles α,β,γ.[15] Then the expression for the cross section for scattering to polar and azimuthal angles θ and φ is given by

$$\begin{aligned}\sigma(\theta,\varphi)=\sigma(\theta)\{&1-P_3\mathcal{P}_3\sin\beta\sin(\varphi-\alpha)-P_p\mathcal{P}_p\sin\varphi\\&+P_3P_p\cos\beta\cos\varphi(\mathrm{Im}(S)/\sigma(\theta))\}\\&+\tfrac{1}{2}P_3P_p\sin\beta\cos\alpha U\\&-\tfrac{1}{2}P_3P_p\sin\beta\cos(2\varphi-\alpha)T,\end{aligned}$$

where $\mathcal{P}_3$ and $\mathcal{P}_p$ are the polarizations of the scattered protons and recoil He^3's in the direction of $\hat{k}_{\mathrm{in}}\times\hat{k}_{\mathrm{out}}$ where both the beam and the target are unpolarized. The other symbols are defined by

$$S=(1/k^2)(ae^*+gc^*+hb^*+fd^*)\,,$$
$$U=(2/k^2)\,\mathrm{Re}(de^*+bc^*)\,,$$
$$T=(2/k^2)\,\mathrm{Re}(af^*+gh^*)\,,$$

and $\sigma(\theta)$ is the cross section where neither particle is polarized.

If we consider several possible measurements, we obtain

$$\frac{\sigma(\theta,\pi/2)-\sigma(\theta,-\pi/2)}{2\sigma(\theta)}=-(P_3\mathcal{P}_3\sin\beta\cos\alpha+P_p\mathcal{P}_p)\,,$$

$$\frac{\sigma(\theta,\pi/2)+\sigma(\theta,-\pi/2)}{2\sigma(\theta)}=1+\frac{P_3P_p\sin\beta\cos\alpha}{2\sigma(\theta)}(U+T)\,,$$

$$\frac{\sigma(\theta,0)-\sigma(\theta,\pi)}{2\sigma(\theta)}=P_3\mathcal{P}_3\sin\beta\sin\alpha+\cos\beta\,P_3P_p\frac{\mathrm{Im}(S)}{\sigma(\theta)}$$

and

$$\frac{\sigma(\theta,0)+\sigma(\theta,\pi)}{2\sigma(\theta)}=1+\frac{P_3P_p\sin\beta\cos\alpha}{2\sigma(\theta)}(U-T)\,.$$

The quantities $(U+T)/[2\sigma(\theta)]$ and $(U-T)/[2\sigma(\theta)]$ also occur if protons completely polarized in the x direction are scattered from unpolarized He^3's. The polarization along the x axis for the recoil He^3 in this case is

$$P_{\mathrm{He}^3,x}=\left(\frac{U-T\cos2\varphi}{2\sigma(\theta)}-\mathcal{P}_3\sin\varphi\right)\bigg/(1-\mathcal{P}_p\sin\varphi)\,.$$

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[15] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 48 ff.