Ratios of Independent Yields of the Isomers Te^{131-131m} and Te^{133-133m} in Fission*

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The independent yield ratios of (30-h Te^{131m})/(25-min Te¹³¹) and of (53-min Te^{133m})/(12.5-min Te¹³³) were measured in the following types of fission: U²³⁵ with thermal neutrons, and Th²³² and U²³⁸ with 18-MeV deuterons and with 33-MeV He⁴ ions. The fractional independent chain yields of the isomers were measured by determining the growth curves of the daughter iodine activities extracted at successive time intervals from aliquots of samples containing tellurium activities quickly separated from fission induced in short irradiations, and correcting for contributions from precursors. The isomer-yield ratios for independent formation for 30-h Te^{131m}/25-min Te¹³¹ and for 53-min Te^{133m}/12.5-min Te¹³³, respectively, are as follows: 1.8 ± 0.4 and 1.55 ± 0.5 for U²³⁵ (n_{th},F) , 3.3 ± 0.5 and 2.8 ± 1.7 for Th²³² (α_{33},F) , 2.7 ± 0.5 and 1.7 ± 0.9 for Th²³²(d_{18} , F), 5.0±1.0 and 3.1±2.0 for U²³⁸(α_{33} , F), and 3.3±0.5 and 1.8±1.1 for U²³⁸(d_{18} , F). The fractional independent chain yield of Te^{131m}+Te¹³¹ was measured as 0.124 ± 0.014 for U²⁸⁵(n_{th},F). The fraction of decay of Sb¹⁸¹ to Te^{131m} was measured and found to be $6.8 \pm 1\%$. On the basis of the measured isomer ratios, the intrinsic spin of the primary fragment (leading by the emission of neutrons and γ rays to the secondary fragments studied) was estimated. The calculations indicate that in thermal-neutron fission an appreciably larger spin $(\sim 5\hbar)$ than that of the compound nucleus must be assigned to the heavy primary fragment in order to explain the observed isomer ratios in Te¹³¹ and Te¹³³. Similar reasoning in the 33-MeV He⁴-ion fission of Th²³² indicates that the primary fragment must be given an intrinsic spin value roughly 7*ħ*.

I. INTRODUCTION

D^{URING} the past few years efforts have been made to obtain information on the distribution of angular momentum in the fission process. A measure of the orbital angular momentum between the fission fragments is obtained from angular distributions of the fission fragments.¹⁻⁶ From these measurements it is indicated that, in general, a small fraction of the angular momentum of the compound nucleus is distributed as orbital angular momentum between the fragments for low or medium energy neutron, deuteron, or helium-ion fission. The rest of the angular momentum should appear as intrinsic spins of the two fragments.

A more direct measure of the intrinsic spin of the fragments can in principle be obtained from measurements of the ratios of yields of isomers of the fragments. Relatively few careful measurements of this type have been done.7-11 Most of the work reported refers to

medium- or high-energy fission. Due to the short β^{-} -decay chains in these cases, independent isomer ratios could in some cases be determined directly or inferred from cumulative-yield measurements. A trend of increasing high-to-low-spin isomer ratio with increasing energy of the bombarding particle is observed. This result is to be expected because the amount of angular momentum deposited in the compound nucleus increases with increasing energy and mass of the projectile. No independent yield ratios in thermal neutron fission have been published in detail.

In this work the isomer ratios of Te^{131m}/Te¹³¹ and Te^{133m}/Te¹³³ in thermal neutron fission of U²³⁵ were measured and compared with the results from mediumenergy charged-particle-induced fission. The chargedparticle reactions chosen¹² were Th²³²(α_{33} , F) [leading to the same compound system as $U^{235}(n_{\text{th}},F)$], Th²³²(d_{18},F), $U^{238}(\alpha_{33},F)$, and $U^{238}(d_{18},F)$. The fraction of β^- decay of Sb¹³¹ to Te^{131m} was measured because knowledge of this quantity is required for the calculation of the independent Te^{131m} and Te¹³¹ vields.

II. EXPERIMENTAL PROCEDURE

A. Irradiations and Bombardments

Targets of \sim 200 mg of natural uranium as ammonium uranyl carbonate were irradiated in the thermal neutron flux of the MIT reactor. Use of the pneumatic tube system made possible quick access to the irradiated samples, which were available for chemical operations within 15 to 25 sec after the end of the irradiation.

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¹ I. Halpern, Ann. Rev. Nucl. Sci. 9, 245 (1959). ² J. R. Huizenga and R. Vandenbosch, in *Nuclear Reactions*, edited by P. M. Endt and P. B. Smith (North-Holland Publishing Company, Amsterdam, 1962), Vol. 2, Chap. 2. ³ I. Halpern and V. M. Scrutting is *Description* 4 (1977).

⁸ I. Halpern and V. M. Strutinski, in *Proceedings of the Second* United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958), Vol. 15, p. 408.

 ⁴ V. M. Strutinski, Zh. Eksperim. i Teor. Fiz. 39, 781 (1960)
 [English transl.: Soviet Phys.—JETP 12, 546 (1961)].
 ⁵ R. Vandenbosch, H. Warhanek, and J. R. Huizenga, Phys. Rev. 124, 846 (1961).

⁶ R. Chaudhry, R. Vandenbosch, and J. R. Huizenga, Phys.

Rev. 126, 220 (1962)

⁷ H. G. Hicks and R. S. Gilbert, Phys. Rev. **100**, 1286 (1955). ⁸ R. A. Sharp and A. C. Pappas, J. Inorg. Nucl. Chem. **10**, 173 (1959)

⁹ S. M. Bailey, University of California Radiation Laboratory Report UCRL-8710, 1959 (unpublished).

For the charged-particle experiments, foils of natural

¹⁰ I. F. Croall and H. H. Willis, J. Inorg. Nucl. Chem. 24, 221

^{(1962).} ¹¹ H. Warhanek and R. Vandenbosch, J. Inorg. Nucl. Chem. 26, 669 (1964).

¹² F refers to fission and the subscript to the energy of the projectile in MeV or "th" for thermal.

uranium (137 mg/cm²) and thorium (70.9 mg/cm²) were bombarded in the external beam of the Brookhaven National Laboratory cyclotron. The reduction of the energy of the deuteron and He⁴-ion beams by the target wrapping and the target foils was determined from range-energy curves.¹³ The average energies of the deuterons and He⁴ ions causing fission in the reactions $U^{238}(d,F)$, $Th^{232}(d,F)$, $Th^{232}(\alpha,F)$, and $U^{238}(\alpha,F)$ were 18.1, 18.6, 33.3, and 33.4 MeV, respectively.

In the thermal neutron experiments, the irradiation times were 45 sec; in the deuteron experiments, 10 to 20 sec; and in the He⁴-ion experiments, 1 min.

B. Chemistry

Branching-Ratio Experiments

The branching fraction of Sb¹³¹ decaying to Te^{131m} was measured in experiments in which antimony was cleanly separated from tellurium, tin, and iodine activities of freshly made fission products. From the antimony in solution, the granddaughter I¹³¹ was extracted at appropriate growth times, and the growth curves were constructed after correction for iodine chemical yield.

The irradiated ammonium uranyl carbonate samples were dissolved in 40 ml of concentrated HCl containing $\sim 20 \text{ mg Sb(III)}$, 10 mg Te(IV), 10 mg Te(VI), 20 mg I⁻, and 20 mg Mo(VI) carriers in a separatory funnel. Oxidation was achieved by adding 5 ml of 5% NaOCl solution, followed by extraction with 20 ml of isopropyl ether saturated with concentrated HCl. The antimony was back-extracted as Sb(III) by shaking with 10 ml of 15% hydrazinium hydrochloride (N_2H_5Cl-HCl) solution containing 50 mg KSCN. The aqueous phase was washed with 10 ml of isopropyl ether; then 10 mg of mixed-valence tellurium carrier was added, and the solution was boiled. The precipitated tellurium was filtered and discarded, and the separation time was taken as the middle of the filtration. The filtrate containing the Sb(III) was collected in a 100-ml volumetric flask containing 200 mg I⁻ in alkaline solution. From aliquots of this solution, I_2 was extracted at appropriate times by the standard CCl₄ extraction methods¹⁴ and counted for I¹³¹.

Isomer-Ratio Experiments

The tellurium isomer ratios were determined in experiments in which tellurium was quickly separated from fission products, free from antimony and iodine activities. This was done by dissolving the target material in boiling 3M HCl solution (15 ml) containing 20 mg each of Te(IV), Te(VI), I⁻, and Sb(III). To the target solution, 5 ml of a 15% solution of N₂H₅Cl-HCl was added, and SO₂ was bubbled through to precipitate elemental tellurium. The reaction vessel, with fritted bottom, was connected with a bell-type vacuum-filtering apparatus in which a 100-ml volumetric flask collected the filtrate without intermediate transferring. The tellurium separation time was taken as the middle of filtration. The elemental tellurium was dissolved in aqua regia after three washings with 3M HCl solution. From suitable aliquots of this solution, containing 200 mg I⁻ carrier and kept alkaline, I₂ was extracted¹⁴ at appropriate times. These samples containing the iodine carrier as I⁻, were kept for about 50 h, then boiled to expel the xenon activities formed, and promptly counted for I¹³¹ and I¹³³. Data were taken for times of about 5, 10, 18, 35, 70, and 140 min, and at five more times up to 300 h, after tellurium separation.

In the thermal neutron experiments the tellurium separation times were between 1 and 1.5 min after the end of irradiation. In the charged-particle experiments, separation times were from 3 to 4 min, due to foil dissolution problems. The longer times in the latter case could be tolerated, however, because of the higher Te¹³¹ and Te¹³³ fractional chain yields in the charged-particle cases.

C. Counting

The method applied here was the construction of the integral growth curve of I¹³¹ and I¹³³ obtained in the decay of the two isomeric pairs Te^{131m}, Te¹³¹ and Te^{133m}, Te¹³³. The counting of the iodine samples was chosen to provide only two-component gross decay curves (without perturbation from 12-day Xe^{131m}, 5.3-day Xe¹³³, 2.3-day Xe^{133m}, and 9.1-h Xe¹³⁵). A few hours after the separation of iodine from tellurium, the main radiations come from 6.7-h I¹³⁵, 20.8-h I¹³³, and 8.05-day I¹³¹ (here the 2.3-h I¹³² and 52-min I¹³⁴ do not interfere), but at later times the xenon activities mentioned above build up. By allowing 50- to 60-h decay time and boiling the samples, the interference from 6.7-h I^{135} and 9.1-h Xe^{135} was eliminated. A survey of the decay schemes of the nuclides in question¹⁵ showed that by counting the γ rays falling in the energy band 170 to 2000 keV, all other interfering activities (in particular the abundant 80-keV γ ray from 5.3-day Xe¹³³) were eliminated.

The detectors used were 2.0×2.0 -in. well-type NaI(Tl) crystals. No counting efficiency corrections were needed as only relative disintegration rates were required. The total chain activities were measured by separating the iodine activities from an aliquot of the target solution after complete decay of the mass 133 and 131 chains to 20.8-h I¹³³ and 8.05-day I¹³¹. Thus, after appropriate corrections for growth-decay and chemical yields, the fractional chain yields were obtained as the ratio of the activity of the iodine from the quickly separated tellurium to that in the sample representing the total chain activity.

In the charged-particle experiments the samples were

 ¹³ M. Rich and R. Madey, University of California Radiation Laboratory Report UCRL-2301, 1954 (unpublished).
 ¹⁴ J. Kleinberg and G. A. Cowan, Nuclear Science Series Report NAS-NS 3005 (National Academy of Sciences-National Research DOC) Council, 1960), p. 31.

¹⁵ Nuclear Data Sheets, compiled by K. Way et al. (National Academy of Sciences-National Research Council, Washington, D. C., 1962).

simultaneously followed for decay by recording the γ rays in the bands 170 to 2000 keV and 380 to 2000 keV. The latter type of counting gave a higher 20.8-h I¹³³ component than the 8.05-day I¹³¹ component (which gives mostly 350-keV γ rays), which allowed for more accurate determination of the 20.8-h I¹³³. This procedure was necessary because the fractional chain yield of the Te¹³¹ increases more than that of the Te¹³³ as we go from thermal-neutron- to charged-particle-induced fission.

The analysis of the iodine decay curves into the 20.8-h and 8.05-day components was performed by the least-squares technique using the FRANTIC computer program.16

The counting of the iodine samples for the Sb¹³¹ branching to Te^{131m} was done using the same counting system discussed above.

III. RESULTS

A. Analysis of the Isomer-Ratio Data

In view of the fact that a large number of activities having similar half-lives is involved in this problem, the milking procedure outlined above was used in the determination of the fractional chain yields of the Te¹³¹ and Te¹³³ isomeric pairs. From the integral growth curves for I131 and I133, one can determine the activities of Te^{131m}, Te¹³¹ and Te^{133m}, Te¹³³ in the following way. Consider, as an example, the results from $U^{235}(n_{th}, F)Te^{131}$. In Fig. 1, curve B gives the growth curve for I^{131} , which was constructed from the I¹³¹ activities extrapolated to

the milking time after correction for iodine chemical yield. Curve A is the asymptote of B having a half-life of 8.05 days and is obtained from points that are off the abscissa scale in Fig. 1. Curve C, also plotted as D on the upper time scale is curve A minus B, and E is its short-lived component. It can be seen that the difference of B from A yields C or D, a two-component decay curve with half-lives of 30-h Te^{131m} (C) and 25-min $Te^{131}(E)$. The effective zero-time (tellurium separation) activities $A_m^{\text{eff}}(0)$, $A_q^{\text{eff}}(0)$ are related to the true Te^{131m} and Te¹³¹ zero-time activities $A_m(0)$, $A_g(0)$ through the relations given in Appendix A. Figure 2 gives typical growth curves obtained in $U^{235}(n_{th},F)Te^{131m,131}$, $Th^{232}(\alpha_{33},F)Te^{131m,131}$, $U^{238}(\alpha_{33},F)Te^{131m,131}$, and the hypothetical curve corresponding to Te^{131m}/Te¹³¹ produced in the relative amounts of secular equilibrium $\left[A_{q}^{\text{eff}}(0)=0\right]$. This shows that the relative amount of ground state falls on going from thermal fission to charged-particle fission.

Since the nuclides under study are not shielded, corrections must be applied for the contribution to the measured activities due to decay of precursors during irradiation and subsequent decay to the time of the tellurium separation. In order to avoid serious errors, the corrections were evaluated using the exact equations for growth during irradiation and subsequent decay to the separation time. The antimony and tin precursors for both mass numbers 131 and 133 were included in the calculations. These corrections were evaluated using a computer program¹⁷ written for

FIG. 1. Analysis of the I¹³¹ growth curve obtained in the decay of $Te^{131m} + Te^{131}$, which was quickly separated from the fission products in $U^{235}(n_{th},F)$. Curve A is the asymptote of the growth curve, and is obtained by extrapolation of points that are off scale (refer to bottom time scale). Curve B is the I¹³¹ growth curve (refer to bottom scale). Curve C is the decay curve of $Te^{131m} + Te^{131}$ obtained by subtracting B from A (refer to bottom scale). Curve D is the decay curve Cplotted in the upper time scale. Curve E is the decay curve of Te^{131g} plotted in the upper time scale.



¹⁶ P. C. Rogers, Massachusetts Institute of Technology Laboratory for Nuclear Science, Technical Report No. 76 (NYO-2303), 1961 (unpublished). ¹⁷D. G. Sarantites, Ph.D. thesis, Massachusetts Institute of Technology, 1963 (unpublished).





the Bateman equations in the general form, including possible branching in the chain. The use of this program made possible the investigation of the effects upon the corrected isomer-yield ratios of uncertainties in the decay properties of the nuclides involved and of uncertainties in choice of most probable charge Z_P for the two chains. Here a Gaussian charge dispersion was used for the fractional chain yield f^i , namely,

with C=0.94 (see, for example, Ref. 18). The values of the most probable charge, Z_P , in the charged-particle cases were taken from the data of Storms¹⁹ on the chains 135 and 133 by extrapolation to 131 and correction for the excitation energy according to the prescription given by Coryell, Kaplan, and Fink.²⁰

The necessary data and the results are presented in Table I. The fourth and fifth columns of Table I give $A_m^{\text{eff}}(0)$ and $A_g^{\text{eff}}(0)$, the effective activities for the

$$f^{i} = (\pi C)^{-1/2} \exp[-(Z - Z_{P})^{2}/C]$$
(1)

TABLE I. Experimental data and results on the independent yield ratios of Te^{131m,131} and Te^{133m,133}.

Reaction	Irrad. time (min)	Separ. time (min)	$A_m^{ m eff}(0)$ (10 ⁶ counts/h)	$A_{g^{\mathrm{eff}}(0)}$ (10 ⁶ counts/h)	Z_P	$\begin{array}{c} f_m^i + f_g^i \\ C = 0.94 \\ \text{(calc.)} \end{array}$	f_m/f_g (uncor-rected)	σ_m/σ_g
$\overline{{\rm U}^{235}(n_{\rm th},F){\rm Te}^{131}}$ No. 1	0.75	1.00 ± 0.10	3.10 ± 0.05	2.29 ± 0.05	50.75	0.110ª	1.097	1.65
$U^{235}(n_{th},F)$ Te ¹³¹ No. 2	0.75	1.45 ± 0.20	3.95 ± 0.10	2.55 ± 0.10	50.75	0.110ª	1.239	$2.00 > 1.8 \pm 0.4$
$U^{235}(n_{\rm th},F)$ Te ¹³¹ No. 3	1.00	1.25 ± 0.20	2.23 ± 0.15	1.40 ± 0.20	50.75	0.110ª	1.249	1.90
$Th^{232}(\alpha_{33},F)Te^{131}$	1.00	4.0 ± 0.5	2.40 ± 0.10	0.60 ± 0.10	52.23	0.545	2.970	3.3 ± 0.5
$Th^{232}(d_{18},F)Te^{131}$	0.167	4.0 ± 0.5	1.25 ± 0.05	0.38 ± 0.05	52.16	0.567	2.494	2.7 ± 0.5
$U^{238}(\alpha_{33},F)Te^{131}$	1.00	3.5 ± 0.5	1.70 ± 0.05	0.29 ± 0.05	52.24	0.545	4.371	5.0 ± 1.0
$U^{238}(d_{18},F)Te^{131}$	0.333	3.5 ± 0.5	1.37 ± 0.05	0.37 ± 0.05	52.01	0.582	2.800	3.3 ± 0.5
$U^{235}(n_{th},F)$ Te ¹³³ No. 1	0.75	1.00 ± 0.10	314 ± 12	144 ± 8	51.57	0.430	1.77	1.59]
$U^{235}(n_{th},F)$ Te ¹³³ No. 2	0.75	$1.45 {\pm} 0.20$	297 ± 23	162 ± 30	51.57	0.430	1.53	1.50
$Th^{232}(\alpha_{33},F)Te^{133}$	1.00	4.0 ± 0.5	5.1 ± 0.3	1.3 ± 0.8	52.79	0.300	2.7	2.8 ± 1.7
Th ²³² (d ₁₈ ,F)Te ¹³³	0.167	4.0 ± 0.5	4.7 ± 0.2	2.0 ± 1.0	52.59	0.400	1.7	1.7 ± 0.9
$U^{238}(\alpha_{33},F)Te^{133}$	1.00	3.5 ± 0.5	3.5 ± 0.3	0.8 ± 0.6	52.78	0.303	3.0	3.1 ± 2.0
$\mathrm{U}^{238}(d_{18},F)\mathrm{Te}^{133}$	0.333	3.5 ± 0.5	3.9 ± 0.2	1.6 ± 1.0	52.56	0.412	1.8	1.8 ± 1.1

* Value taken from Ref. 18.

¹⁸ A. C. Wahl, R. L. Ferguson, D. R. Nethaway, D. E. Troutner, and K. Wolfsberg, Phys. Rev. 126, 1112 (1962).

¹⁹ H. A. Storms, Ph.D. thesis, Massachusetts Institute of Technology, 1962 (unpublished).
 ²⁰ C. D. Coryell, M. Kaplan, and R. D. Fink, Can. J. Chem. 39, 646 (1961).

metastable and ground state at the tellurium separation time as obtained from the analysis of the iodine decay curves. The seventh column gives the values for the total independent chain yield for the tellurium isomers $f_{\text{Te}}^{i} = f_{\text{Tem}}^{i} + f_{\text{Teg}}^{i}$, obtained using the Z_{P} values of column six. The last two columns in Table I give the isomer yield ratios, uncorrected and corrected for precursor decay, respectively. The ratio of the independent yields $f_m i / f_g i$ is equal to the ratio of the cross sections σ_m/σ_g . The magnitude of corrections for precursor decay in the cases of the metastable and the ground state were about 1% and 40 to 50% in U²³⁵ (n_{th},F) Te¹³¹, respectively, and less than 1% and 15 to 25%, respectively, for the rest of the cases in Te¹³¹. In the Te¹³³ case the corrections were invariably less than 10%. The halflives and branching fractions used in the calculations are given in Table II.^{21,22,23} The corrected isomer ratios were found to be rather insensitive to small variations in the branching fraction of Sb¹³¹ to Te^{131m}, to the halflife of Sn^{131} , and to the assumed Z_P . In the thermal neutron case only, the ratio is rather sensitive to the half-life of Sb131. For example, in the worst case [Run No. 2 of $U^{235}(n_{\rm th},F)$ Te¹³¹ in Table I], using a half-life of 19.4 min for Sb¹³¹ increases σ_m/σ_g by 30% over the value obtained with the assumed 23-min half-life. The assigned errors in the results cover the changes which might arise from these uncertainties.

The fractional chain yield of $Te^{131m} + Te^{131}$ for thermal neutron fission was found to be 0.124 ± 0.014 , compared with 0.110 used for the Z_P calculations.¹⁸

The analysis of the growth curves of I¹³¹ for the Te^{131m-131} yields was straightforward, as shown in Fig. 1, and from three thermal neutron runs independent corroboration of the half-lives was also obtained: Te^{131m} , 30 ± 2 h, and Te¹³¹, 25 ± 2 min. The runs for Te^{133m-133} were more difficult to analyze because the half-lives are shorter and closer together. During the course of this

 TABLE II. Decay properties used in the calculation of the independent isomer ratios.

Nuclide	Half-life	Branching fraction to
Sn ¹³¹	3.4 min ^a	
SIL 131	30 sec	(D) 191m (0.07 ·
Sh133	25 min^{a} 2.64 minb	$1e^{101m}, 0.8\%$
Te ^{131m}	30 ha,c	$T_{e^{131}}, 72\%$
Te ¹³¹	25 min ^{a,c}	10, 19/0-
Te^{133m}	53 min ^a	Te133, 13%d
Te ¹³³	12.45 min ^e	1 0 , 1 0 /0
I ¹³¹	8.05 dav ^a	
\mathbf{I}^{133}	$20.8 \ h^{a}$	
See Ref. 21.	• This work.	• See Ref. 23.

²¹ S. Katcoff, Nucleonics 18, 201 (1960).

²² P. O. Strom, paper given at the Discussion on Nuclear Chemistry (Fision and Other Low Energy Nuclear Processes), University of Oxford, 1962 (unpublished). ²³ S. G. Prussin (private communication).



FIG. 3. Analysis of the growth curve of I¹³¹ obtained in the decay of Sb¹³¹ data from two independent experiments for the branching of Sb¹³¹ to the Te¹³¹ isomers. The asymptotic curve with slope of 8.05 days is based on two points that are off scale. ⊙ data from Experiment No. 1. \oplus data from Experiment No. 2

investigation, Dr. Prussin²³ informed us of his study of Te^{133g} which provides the half-life 12.45 ± 0.3 min, replacing the older value²¹ of 2 min. The growth curves for I^{133} were analyzed by computer¹⁶ for three components: a positive amount $A_{1}^{eff}(0)$ of I¹³³ decaying with half-life of 20.8 h, and negative amounts $-A_m^{\text{eff}}(0)$ and $-A_{g^{\text{eff}}}(0)$ decaying with half-lives of 53 min and 12.45 min. The computer program was also modified to introduce the constraint, $A_{I}^{eff}(0) - A_{m}^{eff}(0) - A_{g}^{eff}(0) = 0$, required by the decay laws. This modification reduces the degrees of freedom by one and seemed to give unsatisfactory results for the very limited number of points available. The values of $A_1^{\text{eff}}(0)$ and $A_m^{\text{eff}}(0)$ were rather well determined by the unmodified program. It was decided to take $A_g^{eff}(0)$ as the difference between these, assigning a rather large uncertainty in the value. The better runs gave independent evidence for the 12.45-min half-life of Te¹³³; Dr. C. E. Bemis of our laboratory has also confirmation of the species.

B. Sb¹³¹ Branching-Ratio Experiments

For the measurement of the branching of Sb¹³¹ to Te^{131m}, the growth curve of the granddaughter I¹³¹ was determined as discussed in Sec. IIB. Figure 3 shows the result of two experiments normalized to the same asymptotic activity. The asymptotic activity with half-life of

Target	Projectile	Projectile energy (MeV)	Isomer pair	Spin of isomers			σ_m
				m	g	σ_m/σ_g	$\sigma_m + \sigma_g$
U ²³⁸ a	α	42	Cs134	8	4	4.27	0.81
U^{238} a	α	30	Cs134	8	4	2.33	0.70
U^{235} a	α	42	Cs ¹³⁴	8	4	3.35	0.77
$U^{235 a}$	α	42	Cs134	8	· 4	1.38 ^d	0.58^{d}
$U^{235} a$	a	27	Cs ¹³⁴	8	4	1.27	0.56
U^{235} a	d	21	Cs ¹³⁴	8	4	1.63	0.62
U ²³³ a	α	42	Cs ¹³⁴	8	4	2.23	0.69
U ²³³ a	α	42	Cs ¹³⁴	8	4	1.94 ^d	0.66^{d}
U ²³³ a	γ	16	Cs ¹³⁴	8	4	0.75^{d}	0.43 ^d
$U^{235} b$	n	thermal	Te ¹³¹	11/2	3/2	1.80	0.64
U ²³⁵ c	n	thermal	Te ¹³¹	11/2	3/2	1.7	0.63
$\mathrm{Th^{232\ b}}$	α	33	Te ¹³¹	11/2	3/2	3.3	0.77
$\mathrm{Th^{232\ b}}$	d	18	Te ¹³¹	11/2	3/2	2.7	0.73
U ²³⁸ b	α	33	Te ¹³¹	11/2	3/2	5.0	0.83
U^{238} b	d	18	Te ¹³¹	11/2	3/2	3.3	0.77
U ²³³ c	n	thermal	Te^{131}	11/2	3/2	2.1	0.68
Pu ²³⁹ c	n	thermal	Te ¹³¹	11/2	3/2	3.4	0.77
$U^{235 b}$	n	thermal	Te ¹³³	11/2	3/2	1.55	0.61
$\mathrm{Th^{232\ b}}$	α	33	Te ¹³³	11/2	3/2	2.8	0.74
Th ^{232 b}	d	18	Te ¹³³	11/2	3/2	1.7	0.63
$\mathrm{U}^{238}~\mathrm{b}$	α	33	Te ¹³³	11/2	3/2	3.1	0.76
U ²³⁸ b	d	18	Te ¹³³	11/2	3/2	1.8	0.64

TABLE III. Independent isomer-yield ratios in fission.

^a See Ref. 11. ^b This work.

• See Ref. 24. d Isomer ratio determined by anticoincidence counting.

8.05 days is determined from points that are off the scale. Extrapolation of this asymptotic activity to the time of purification of antimony yields $A_1^{\text{eff}}(0)$. The difference of the later part of the growth curve (t>5 h) from its asymptote yields a decay curve with the half-life of 30-h Te^{131m} having an extrapolated value $A_m^{\text{eff}}(0)$. It is shown in Appendix B that in this particular case the fraction of the decay of Sb¹³¹ to Te^{131m} is given by

$$F_1 = 0.835 R / (1 - 0.152 R)$$
,

where $R = A_m^{\text{eff}}(0)/A_1^{\text{eff}}(0)$. The experimental values for $A_m^{\text{eff}}(0)$ and $A_1^{\text{eff}}(0)$ were 68 ± 10 and 840 ± 20 arbitrary units, giving $F_1 = 6.8 \pm 1.0\%$. This is in accord with $5.5 \pm 0.5\%$ communicated by Dr. Glendenin,²⁴ in contrast with the old value²¹ of 15%.

IV. DISCUSSION

The available data in the literature on the *independent* isomer-yield ratios in low-energy fission are summarized in Table III. It should be noted that there are many other reported isomer-yield ratios; however, these have been omitted for one of two reasons. On the one hand, for many of the ratios reported for low-energy fission, the yield of at least one of the isomers is partially a chain yield, set principally by β branching. On the other hand, at high energies, where Z_P has moved to the

isomer pair in question or even closer to the line of β stability, effects of precursor decay can essentially be eliminated. However, in these cases, the identity of the fissioning nuclides and the mechanisms for their formation are too ambiguous to permit interpretation of the results. An example of both problems is given by the studies of the Cd^{115m}/Cd¹¹⁵ yield ratio by Hicks and Gilbert⁷ and by Sharp and Pappas.⁸

Our data for Te¹³¹ in thermal neutron fission of U²³⁵ indicate the major fraction, 0.64 ± 0.05 , yields Te^{131m} of spin 11/2 – in preference to Te^{131g} of spin 3/2+. This result is in excellent agreement with the privately communicated²⁴ value of the Argonne group, 0.63 ± 0.03 , also in Table III. This group finds the isomer fraction to be about the same for U²³³($n_{\rm th}$,F) and higher for Pu²³⁹($n_{\rm th}$,F). Our data for Te¹³³ show a similar fraction, $0.61_{-0.10}^{+0.06}$, yielding the high-spin Te^{133m}.

The data on Cs^{134} indicate an increase in the high-spin fraction with increasing energy of the bombarding particle. An increase in the fraction for Te^{131} and Te^{133} is observed when the bombarding particle is changed from 18-MeV deuterons to 33-MeV He⁴ ions. (Note that the excitation energy of the initial compound nucleus is less than 2 MeV greater for the He⁴-ion bombardment than with the deuterons. Of course, the initial compound nucleus is greater by one neutron and one proton in the former case.) Similarly, for Te^{131} , the fraction increases in going from thermal-neutron fission to that induced by charged particles. In the case of Te^{133} isomers, the

²⁴ M. Talat-Erben, L. E. Glendenin, H. C. Griffin, and E. P. Steinberg (private communication).

trend is observed for the He⁴-ion fission and the expected increase may well be within the limits of error on the deuteron measurements (see Table I). The trends observed, that is, increase in the high-spin fraction with increasing energy and/or mass of the bombarding particle, are to be expected because of the increase in angular momentum deposited in the compound nucleus.

We have attempted to make a careful quantitative investigation of the dependence of the fragment isomervield ratios upon the assumed spin distribution of the primary fission fragments.²⁵ First, it is necessary to estimate the "primary parentage" of the given secondary fragment, i.e., the contribution to the yield of secondary species (Z, A) by emission of no neutrons from primary (Z,A), the contribution by emission of one neutron from primary (Z,A+1), etc. We have determined primary parentage and energies of emitted neutrons by means of Monte Carlo calculations similar to those of Dostrovsky et al.,26 except that we have ignored charged-particle emission.

In the model presented below, there are many possible reasonable assumptions. Unfortunately, in many cases, there are no experimental data upon which a choice can be made among the assumptions. The choices we have made below are reasonable, but not unique. We have tested the model by comparison of various results of the calculations with experimental data. Some of the comparisons are noted below and are discussed in more detail in Ref. 27.

The Monte Carlo calculations were performed for a wide range of fragment mass numbers around mass 132 and in the complementary light-mass region.²⁷ The primary yield-mass curve was taken from the time-offlight data of Milton and Fraser.²⁸ Primary fractional chain yields were assumed to be given by Eq. (1) with C=0.94. Original estimates of the primary Z_P values suggested by Wahl et al.18 were adjusted slightly to reproduce better the experimental secondary Z_P values.

In order to estimate the distribution of number of prompt neutrons emitted by the various primary species, the following procedure was used. The most-probable sum \overline{E} of the excitation energies of the complementary light and heavy primary species, (Z_L, A_L) and (Z_H, A_H) , respectively, was taken as

$$\vec{E} = \Delta M(\mathrm{U}^{236*}) - (\Delta M_L + \Delta M_H) - \langle \mathrm{KE} \rangle_{\mathrm{av}} (A_H / A_L), \quad (2)$$

where the mass excess ΔM of excited U²³⁶ was obtained

from Foreman and Seaborg's table,²⁹ those for the fragments from Seeger's mass formula,³⁰ and the average kinetic energy release, $\langle KE \rangle_{av}$, for divisions into mass ratio A_{H}/A_{L} was obtained from Milton and Fraser's results.²⁸ From the assumptions made in Eq. (2), we have for the dispersion of energies:

$$r_E = \sigma_{\rm KE},$$
 (3a)

and we further assume that

$$\sigma_{E_L}^2 + \sigma_{E_H}^2 = \sigma_E^2, \qquad (3b)$$

where σ_{E_L} and σ_{E_H} are dispersions of the excitation energies of the individual fragments. There are several implicit assumptions contained in Eqs. (2), (3a), and (3b). Instrumental studies give $\langle KE \rangle_{av}$ and σ_{KE} as a function of A_H/A_L , but for a given A_H/A_L the dependence of $\langle \text{KE} \rangle_{av}$ and σ_{KE} upon Z_H/Z_L are not known. The total energy release, $E_T = \Delta M (U^{236*}) - (\Delta M_L + \Delta M_H)$], for a given A_H/A_L has approximately parabolic dependence on Z_L , generally having its maximum near Z_{P_L} . As implied in Eq. (2), we have assumed that the KE distribution is the same for each charge division for a given mass division and equal to the KE distribution found by instrumental studies for the given mass division. Thus, changes in E_T with Z_H/Z_L require changes in \overline{E} . Presently, there are no experimental data upon which to make a choice between Eq. (2) and the opposite assumption, namely, that the change in E_T with Z_H/Z_L affects $\langle \text{KE} \rangle_{av}$ while leaving \overline{E} approximately constant. The truth probably lies somewhere between these extremes.

Equation (3b) implies that there is no correlation between the excitation energies received by the two fragments of a given event. Again we have made what we consider to be a reasonable assumption in the absence of experimental data. If there is a negative correlation between fragments, the standard deviations for the individual fragments will be larger than indicated by Eq. (3b). On the other hand, deviations from the assumption implied by Eq. (2), as discussed in the preceding paragraph, will decrease the standard deviations.

Values of σ_{KE} were taken from the semiconductordetector measurements by Gibson, Thomas, and Miller.³¹ We further assume that

$$(\sigma_{E_H}^2 / \sigma_{E_L}^2) = (\bar{E}_H / \bar{E}_L) \equiv F(A_H / A_L),$$
 (4)

where \bar{E}_L and \bar{E}_H are the most-probable excitation energies of the light and heavy complementary fragments, respectively. The relationship between σ 's and \vec{E} 's was assumed on general statistical grounds. In principle, one could obtain $F(A_H/A_L)$ from a complete theory of the fission process. In the absence of any

²⁵ By primary fragments, we denote fragments prior to promptneutron emission.

⁶ I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. 116, 683 (1959).

G. E. Gordon, N. K. Aras, and L. A. Wilson, Massachusetts Institute of Technology, Laboratory for Nuclear Science Progress Report, MIT-2098-No. 64, 1964 (unpublished); N. K. Aras, M. P. Menon, and G. E. Gordon (unpublished); G. E. Gordon and N. K. Aras, to be presented at the International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Salzburg, Austria, 1965. ²⁸ J. C. D. Milton and J. S. Fraser, Can. J. Phys. **40**, 1626 (1962).

²⁹ B. M. Foreman, Jr. and G. T. Seaborg, J. Inorg. Nucl. Chem. 7, 305 (1958).

 ³⁰ P. A. Seeger, Nucl. Phys. 25, 1 (1961).
 ³¹ W. M. Gibson, T. D. Thomas, and G. L. Miller, Phys. Rev. Letters 7, 65 (1962); also, W. M. Gibson (private communication).

Product		No. of	Contribution	Av. neutron kinetic energies (MeV)			Av. γ -ray
nuclide	Parent	emitted	to product yield	1st	2nd	3rd	(MeV)
	Te ¹³¹	0	0.378				4.0
Te ¹³¹	Te^{132}	1	0.468	0.71	•••		3.3
	Te ¹³³	$\overline{2}$	0.154	1.01	0.54	•••	2.6
	Te ¹³³	0	0.269	• • •	•••		4.3
Te ¹³³	Te^{134}	1	0.384	0.84			3.6
	Te ¹³⁵	2	0.286	1.57	0.73		3.2
	Te ¹³⁶	3	0.061	1.45	1.33	0.53	2.6

TABLE IV. Parentage and neutron energies in formation of Te¹³¹ and Te¹³³ secondary fragments.

satisfactory theory, we have left it as a parameter to be adjusted freely until it approximately reproduced the experimental value³² of the ratio $\bar{\nu}_{AH}/\bar{\nu}_{AL}$ of average number of prompt neutrons emitted by heavy fragments of mass A_H to that for the complementary light fragments of mass A_L . The implications of the assumptions made above are discussed in more detail in Ref. 27.

The first step in each iteration for a given species was the selection of an excitation energy from the distribution constructed as described above. The selections were made via the Monte Carlo rejection technique.³³ The number and kinetic energies of neutrons emitted were determined by the same technique as discussed in Ref. 26. The nuclear level density was assumed to be



FIG. 4. Calculated average spin for the species in the cascade $Te^{135} \rightarrow Te^{133}+2n$ for various *B* values. The notation $Te^{133}(0)$, $Te^{133}(1)$, \cdots refers to Te^{133} after emission of 0, 1, etc. γ rays. Except where noted for B = 10, the values of σ_n and σ_γ are 4 and 3, respectively.

proportional to $\exp[2(aE)^{1/2}]$, with $a=A/\gamma$, the value of γ being left as a free parameter. Neutron separation energies and δ values (to correct for odd-even effects on level density, see Ref. 26) were taken from Seeger's mass formula and the smooth curve of pairing energy as a function of A, respectively.³⁰

A number of calculations were performed with 50 to 200 iterations per nuclide in order to find the best values of the parameters, in particular the charge-dispersion parameter C, primary Z_P values, γ , and $F(A_H/A_L)$. Results were compared with various experimentally known quantities, notably secondary Z_P and $\bar{\nu}_A$ values, the gross neutron kinetic energy spectrum, and the average prompt γ -ray energy release. For the mass region of interest, the experimental quantities were well fitted using $\gamma = 13$ MeV, C = 0.94, and primary Z_P values close (within 0.3 charge units) to those suggested by Wahl et al.¹⁸ For example, the calculated fractional chain yield for Te^{131m}+Te¹³¹ is 0.109, in good agreement with the experimental value 0.124 ± 0.014 given above. Results of the calculations on primary parentage of Te¹³¹ and Te¹³³ are listed in Table IV.

It is not possible to determine the spin distribution of the primary fragments from the observed isomer-yield ratios of the secondary fragments. However, if we assume a functional form of the primary spin distribution, we can determine approximately the average primary fragment spin that corresponds to the observed isomer-yield ratio. As suggested by Warhanek and Vandenbosch on statistical grounds, we assume that the angular-momentum distribution of the primary fragments is given by

$$P(J_c) = (2J_c+1) \exp[-J_c(J_c+1)/B^2], \quad (5)$$

in which $P(J_e)$ is the probability for primary frgaments of spin J_e and B is a parameter somewhat analogous to a spin-cutoff factor.¹¹

We have used the statistical treatment of neutron emission developed by Vandenbosch and Huizenga³⁴ to determine the change in the spin distribution caused by the emission of neutrons. In these calculations we have used values for barrier-transmission coefficients, $T_l(E_n)$, obtained from optical-model calculations based on a

³² J. Terrell, Phys. Rev. 127, 880 (1962).

³³ H. Kahn, Atomic Energy Commission Report No. AECU-3259, 1954 (unpublished).

²⁴ R. Vandenbosch and J. R. Huizenga, Phys. Rev. 120, 1313 (1960).

potential of the type described by Bjorklund and Fernbach³⁵ and the neutron kinetic energies given in Table IV. The parameter σ_n of the spin distribution (see Ref. 34) was taken as constant and equal to 4. For the γ -ray de-excitation process, we assumed that only dipole radiations are emitted and that three γ rays are emitted in each cascade. The latter is based on the relationship given by Strutinski *et al.*³⁶ for the average number of γ rays emitted, \bar{N}_{γ} , from a nucleus of excitation energy E_{γ} :

$$\bar{N}_{\gamma} = (aE_{\gamma})^{1/2}/(l+1),$$
 (6)

where l is the assumed multipolarity, and a is the leveldensity parameter which we take to be A/13 MeV⁻¹. Under the assumption of dipole radiation, we find a range of \bar{N}_{γ} from 2.6 to 3.3 for the \bar{E}_{γ} values listed in Table IV. Of the distribution resulting from emission of two γ rays, we assume that all nuclei having J < 7/2decay to the lower isomer (3/2+), all with J > 7/2 decay to the upper isomer (11/2-), and those with J=7/2divide equally between the isomers upon emission of the third γ ray. From the work of Huizenga and Vandenbosch,³⁷ the value of the spin-cutoff factor, σ_{γ} , for γ -ray de-excitation is expected to be about 3 or 4. We have used both values and taken an average of the results.

Before comparing the calculations with experimental results, we first investigate the sensitivity of the calculations to changes in the values of the parameters for the typical reaction $Te^{135} \rightarrow Te^{133} + 2n$. In Fig. 4 we have plotted the average spin \bar{J} of the primary and succeeding species in the de-excitation chain for various values of B. We see that the average spin value does not change strongly with the emission of neutrons and γ rays, particularly for B values below 10. Also, in Fig. 4 we show the effects of variations in σ_n and σ_{γ} , the effect of the former being appreciably greater than that of the latter. In Fig. 5, we show the dependence of the calculated fraction of high spin for $Te^{135} \rightarrow Te^{133} + 2n$ as a function of neutron kinetic energy, here assumed to be the same for the first and second neutrons, for two values of B. In this calculation we have used neutron transmission coefficients taken from the calculations of Feld et al.³⁸ We see that the high-spin fraction is very insensitive to neutron kinetic energy, particularly for the lower B value. This is also rather insensitive to the number of γ rays emitted, N_{γ} ; for example, for $Te^{135} \rightarrow Te^{133}+2n$ and B=10, the calculated fraction varies from 0.83 to 0.80 for N_{γ} equal to 1 and 4, respectively. That the calculated fraction is not strongly dependent upon the number of neutrons emitted is demonstrated in Table V in which we list the calculated

	Product nuclide	Parent	No. of neutrons emitted	Contri- butions to product yield	$\sigma_m/(\sigma_m+\sigma_g)$
		Te ¹³¹	0	0.378	0.836
		Te^{132}	1	0.468	0.823
	$\mathrm{Te}^{131m \cdot g}$	Te ¹³³	2	0.154	0.814
		Weighted			
		average			0.826
		Te ¹³³	0	0.269	0.836
		Te^{134}	1	0.384	0.820
${ m Te}^{{}^{133m,g}}$	FF 199	Te ¹³⁵	2	0.286	0.809
	Te ¹³⁶	3	0.061	0.793	
		Weighted			
		average			0.820

TABLE V. Calculated high-spin fractions of Te¹³¹ and Te¹³³ resulting from the various primary parents.^a

^a Calculated with B=10, $\sigma_n=4$, average of $\sigma_\gamma=3$ and 4, $N_\gamma=3$, dipole radiation only, average neutron kinetic energies from Table IV, and transmission coefficients from optical-model calculations (see text).

fractions for Te¹³¹ and Te¹³³ resulting from the various primary parents, all having an assumed primary spin distribution given by Eq. (5) with B=10.

Taking into account the various uncertainties in the values of the parameters and the effects of such uncertainties upon the calculated fraction of high spin for a given B value, as discussed in the preceding paragraph, we estimate that for a given observed fraction, the uncertainty in determining the corresponding B value is about ± 1.5 units at the most. As the average initial spin varies more slowly than B (see Fig. 4), the uncertainty in average initial spin value is only about $\pm 1.2h$ in the range of interest here. Including the uncertainty in the experimental value of the high-spin fraction raises the uncertainty in the average initial spin value to 1.4 and $1.5\hbar$, respectively, for Te¹³¹ and Te¹³³. Thus, in spite of the complexity of the calculations and the uncertainties in the values of the parameters involved, we may obtain a meaningful estimate of the average spin of the primary fragments from the observed isomer-yield ratios. The root-mean-square average $\langle J^2 \rangle_{\rm av}^{1/2}$ is a little higher. In the limit of large B,

FIG. 5. Calculated high-spin fraction, $\sigma_m/(\sigma_m + \sigma_g)$, for Te¹³³ resulting from emission of two neutrons from Te¹³⁵, as a function of the average kinetic energies of the two neutrons (here assumed the same for both). Calculated using transmission coefficients from Feld *et al.* (Ref. 38), with parameters $\sigma_n = 4$, $\sigma_\gamma = 3$, $N_\gamma = 3$.



³⁵ F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 2951 (1958). ³⁶ V. M. Strutinski, L. V. Groshev, and M. K. Akimova, Nucl. Phys. **16**, 657 (1960).

³⁷ J. R. Huizenga and R. Vandenbosch, Phys. Rev. **120**, 1305 (1960).

³⁸ B. T. Feld, H. Feshbach, M. L. Goldberger, H. Goldstein, and V. F. Weisskopf, Atomic Energy Commission Report NYO-636, 1951 (unpublished).



FIG. 6. Calculated high-spin fraction, $\sigma_m/(\sigma_m + \sigma_g)$, for Te¹³³ as a function of *B*. The parameters are the same as those given in Table V, footnote a.

 $\bar{J} = \pi^{1/2} B/2$ and $\langle J^2 \rangle_{\rm av}^{1/2} = B$. For *B* equal 5 or 10, the most-probable *J* value is 3.16 or 6.85, \bar{J} is 3.94 or 8.37, and $\langle J^2 \rangle_{\rm av}^{1/2}$ is 4.58 or 9.56.

In Fig. 6 we have plotted the calculated Te¹³³ isomeryield ratio as a function of B, using our best estimates of the parameters. The corresponding curve for Te¹³¹ isomers is almost indistinguishable and has not been shown. The measured fractional yields of high spin, $\sigma_m/(\sigma_m+\sigma_g)$, of 0.64±0.05 and 0.61_{-0.10}+0.06 for production of Te¹³¹ and Te¹³³, respectively, in thermal neutron fission of U^{235} correspond to B values of 6.0 ± 1.5 and 5.9 ± 1.5 , or average initial spin values of $(5\pm1.5)h$ for both. These results are in agreement, within the limits of experimental error, with the value $(7\pm 2)\hbar$ found for gross fission fragments from thermal neutron fission of U²³⁵ by Hoffman,³⁹ who determined angular correlations between γ rays and fragments. Also, from the isomer-yield ratio Cs^{134m}/Cs¹³⁴ from photofission of U²³³, Warhanek and Vandenbosch¹¹ have inferred a root-mean-square angular momentum of the primary parents of about 7.4 h, corresponding to \bar{J} of about 6.5 h.

As the spins of the fissioning nuclei are 3 or $4\hbar$ (assuming *s*-wave neutron capture) to be distributed between the fragments, the high average spins of the primary fragments are at first rather surprising. Hoffman's results further indicate that the fragment spin vectors are directed preferentially perpendicular to



FIG. 7. Hoffman explanation (Ref. 39) for the origin of fragment spins. (a) Typical asymmetry of a type that can produce torques by electrostatic repulsion. (b) Orientation of resulting spin vectors.

the fragment direction of motion. He has made a reasonable explanation of both the magnitudes and orientations of the fragment spin vectors on the basis of torques applied to the fragments by electrostatic repulsion if, near the time of scission, the fragment system has an asymmetry of the type shown in Fig. 7(a). His calculation predicts an average primary fragment spin of 5.5 to $7\hbar$ for fragments in the region of A = 134, in good agreement with our results. On the basis of his model, it is easy to see how both complementary fragments can have spins J_L and J_H greater than that of the fissioning nucleus as there is a strong tendency for \mathbf{J}_L and \mathbf{J}_H to be approximately antiparallel, thus largely cancelling each other. In addition, it should be noted that some of the angular momentum of the fissioning system can be carried off as mutual orbital angular momentum, *l*; however, for thermal neutron fission, l would generally be quite small.³

In the discussion of charged-particle-induced fission we shall consider the case of $Th^{232}(\alpha_{33},F)Te^{131}$, with the same compound nucleus as for $U^{235}(n_{th},F)$. In this case we start with a compound nucleus having an average angular momentum of 12.7 h. This value was calculated using the empirical relations given by Halpern and Strutinski.³ Here the excitation energy is high enough to cause neutron emission prior to fission. The prefission neutrons carry away a relatively small fraction of the angular momentum of the compound nucleus. Still, the average angular momentum in the fission processes competing with neutron emission will be much higher than that in the thermal-neutron case. If we assume that the number of prompt neutrons emitted to yield the secondary Te¹³¹ is roughly two, as indicated by Britt and Whetstone,⁴⁰ we can estimate the average spin value of the primary fragment to be roughly equal to $7\hbar$ from the observed high-spin isomer fraction 0.74.

In this case we have some additional information about the fragment spins. From the anisotropy of the fission-fragment angular distribution,⁵ we can use Halpern and Strutinski's model³ to find the average value of K, the projection of the spin of the fissioning nucleus, J_f , along the symmetry axis of the fissioning nucleus. According to the model, the fragments are emitted in the direction of the symmetry axis; therefore, the component of J_f perpendicular to the symmetry axis be-

³⁹ M. M. Hoffman, Phys. Rev. 133, B714 (1964).

 $^{^{40}\,\}mathrm{H.}$ C. Britt and S. L. Whetstone, Jr., Phys. Rev. 133, B603 (1964).

comes the orbital-angular momentum of the fragments, and the fragment intrinsic spins add vectorially to give a vector of length K along the symmetry axis. The anisotropy, $[\sigma(0^{\circ})/\sigma(90^{\circ})]$, of about 1.6 for Th²³²(α_{30},F), gives an average K of about $8\hbar$. From the average value of $\bar{J}_H = 7\hbar$ found above, we see that, again, there is apparently some cancellation of \mathbf{J}_H by \mathbf{J}_L , assuming that J_L is also about $7\hbar$. There is little more that can be said. The Hoffman³⁹ model which successfully reproduces the observations on thermal-neutron fission of U²³⁵ is also plausible in the case of $Th^{232}(\alpha_{33},F)$; however, there is no evidence in the latter case that the fragments are aligned preferentially perpendicular to the direction of fragment motion. Perhaps the most interesting qualitative result is the observation that in going from $U^{235}(n_{\rm th},F)$ to Th²³²(α_{33}, F) the average spin of the fissioning nucleus goes from about 3.5 to 13h; however, the average spin of the primary fragments in the mass 134 region increases by only about $2\hbar$, from 5 to $7\hbar$. Thus, the spins of the two fragments apparently do not line up as nearly antiparallel as in the case of $U^{235}(n_{\rm th},F)$.

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APPENDIX A

Consider the chain of mass number 131:



It can be shown that the difference of the growth equation for I^{131} from its asymptote (growth equation for t > 10 days) is given by

$$A_{\mathrm{I}}(t) = A_{g}^{\mathrm{eff}}(0)e^{-\lambda_{g}t} + A_{m}^{\mathrm{eff}}(0)e^{-\lambda_{1}t}, \qquad (A1)$$

where

$$A_m^{\text{eff}}(0) = A_m^0 \frac{\lambda_{\mathrm{I}}(\lambda_g - \lambda_m)}{(\lambda_1 - \lambda_{\mathrm{I}})(\lambda_g - \lambda_{\mathrm{I}})}, \qquad (A2)$$

and

$$A_{g}^{\text{eff}}(0) = A_{g}^{0} \frac{\lambda_{\mathrm{I}}}{(\lambda_{g} - \lambda_{\mathrm{I}})} - A_{m}^{0} \frac{\lambda_{mg}\lambda_{g}\lambda_{\mathrm{I}}}{\lambda_{1}(\lambda_{g} - \lambda_{1})(\lambda_{g} - \lambda_{\mathrm{I}})}; \quad (A3)$$

 $A_m^{\text{eff}}(0)$, $A_g^{\text{eff}}(0)$ are the effective zero-time (tellurium purification time) activities of Te^{131m}, Te¹³¹, respectively; A_m^0 , A_g^0 are the true zero-time activities of Te^{131m}, Te¹³¹, respectively; and the λ 's are as indicated above.

Similar expressions hold for the chain of mass number 133.

APPENDIX B

$$F_{1} \xrightarrow{F_{1}} 30\text{-h Te}^{131m}(\lambda_{2}) \xrightarrow{I-F_{2}} 1-F_{2}$$
23-min Sb¹³¹(λ_{1})
$$F_{2} \xrightarrow{F_{2}} 8.05\text{-day I}^{131}(\lambda_{1}) \rightarrow,$$

$$1-F_{1} \xrightarrow{F_{2}} 25\text{-min Te}^{131}(\lambda_{g}) \xrightarrow{I-F_{2}}$$

where F_1 is the fraction of Sb¹³¹ decaying to Te^{131m}, and F_2 is the fraction of Te^{131m} decaying by isomeric transition. It can be shown that, for $\lambda_1 t$, $\lambda_q t > 15$, the activity of I¹³¹ grown from an initial activity $A_1(0)$ of Sb¹³¹ is given as

$$A_{\mathrm{I}}(t) = A_{\mathrm{I}}^{\mathrm{eff}}(0) \ e^{-\lambda_{\mathrm{I}}t} - A_{m}^{\mathrm{eff}}(0) \ e^{-\lambda_{2}t}, \tag{B1}$$

where

Consider the chain

$$A_{\mathrm{I}}^{\mathrm{eff}}(0) = A_{\mathrm{I}}(0) \left[F_{\mathrm{I}} \frac{\lambda_{2} \lambda_{\mathrm{I}}}{(\lambda_{2} - \lambda_{\mathrm{I}})(\lambda_{1} - \lambda_{\mathrm{I}})} \left(1 + \frac{\lambda_{\mathrm{I}}}{\lambda_{g} - \lambda_{\mathrm{I}}} F_{2} \right) + (1 - F_{\mathrm{I}}) \frac{\lambda_{g} \lambda_{\mathrm{I}}}{(\lambda_{g} - \lambda_{\mathrm{I}})(\lambda_{1} - \lambda_{\mathrm{I}})} \right], \tag{B2}$$

and

$$A_m^{\text{eff}}(0) = A_1(0)F_1 \frac{\lambda_2 \lambda_1}{(\lambda_2 - \lambda_1)(\lambda_1 - \lambda_2)} \left[1 + \frac{\lambda_2}{\lambda_g - \lambda_2} F_2 \right].$$
(B3)

As discussed in the text, $A_{I}^{eff}(0)$ and $A_{m}^{eff}(0)$ are obtained from the experiment.

Dividing (B2) by (B3) one has

$$\frac{A_{\mathrm{I}}^{\mathrm{eff}}(0)}{A_{m}^{\mathrm{eff}}(0)} \equiv \frac{1}{R} = \left(\frac{\lambda_{1} - \lambda_{2}}{\lambda_{1} - \lambda_{\mathrm{I}}}\right) \frac{B_{2}}{B_{1}} \left[1 + \frac{(1 - F_{1})\lambda_{g}(\lambda_{2} - \lambda_{\mathrm{I}})}{F_{1}B_{2}\lambda_{2}(\lambda_{g} - \lambda_{\mathrm{I}})}\right], \qquad (B4)$$
$$B_{1} \equiv 1 + \frac{\lambda_{2}}{\lambda_{g} - \lambda_{2}}F_{2} \quad \text{and} \quad B_{2} \equiv 1 + \frac{\lambda_{\mathrm{I}}}{\lambda_{g} - \lambda_{\mathrm{I}}}F_{2}.$$

where

Thus, 1/R is insensitive to λ_1 , F_2 , λ_g and depends linearly on $1/F_1$. Using the above half-lives and solving for F_1 , one finds $F_1 = 0.835R/(1-0.152R)$.

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Proton Polarization in the $Sr^{88}(d,p)Sr^{89*}(1.05-MeV)$ Reaction*

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The polarization of protons from the $Sr^{88}(d,p)Sr^{80*}$ reaction exciting the 1.05-MeV state in Sr^{89} has been measured at 14 laboratory angles from 12 to 131° using a deuteron bombarding energy of 11 MeV. In this reaction, which involves no orbital-angular-momentum transfer by the captured neutron, spin-orbit terms in the optical-model potentials describing the interactions of the incident and exit particles are considered to be responsible for the polarization. The measurements were made using a heavy-particle magnetic spectrometer to select the desired proton group and focus it on the carbon second scatterer. This scatterer was viewed to the "right" and the "left" by a pair of scintillation counters set at a mean laboratory angle of 40°. The sign of the polarization seems to correspond to the sign of the slope of the angular distribution, being negative at the forward angles with apparent sign changes at the subsequent minima and maxima. This behavior is in approximate agreement with the "derivative rule" for stripping reactions with $l_n=0$ orbital-angular-momentum transfer. The polarization changes most rapidly near the stripping minima and becomes largest at the backward angles $(-28\% \text{ at } 95^\circ)$.

I. INTRODUCTION

NGULAR distributions of protons emitted in deuteron stripping reactions have been successfully fitted by the distorted-wave Born approximation,¹ in which the waves describing the incoming deuterons and outgoing protons are distorted by an optical-model potential. While theoretical fits to the angular distributions yield useful information with regard to the reaction process, a more sensitive test of the theoretical formulation is obtained when the polarization of the outgoing nucleons is considered. Since polarized particles do not result if plane waves are assumed in the formulation, measurements showing nonzero proton polarizations from stripping reactions clearly emphasize the necessity for using a distorted-wave-Born-approximation (DWBA) treatment.

Newns² was the first to suggest a measurement of the proton polarization resulting from a deuteron stripping reaction. He postulated that the nucleus receives a net oriented pulse of orbital angular momentum (l_n) due to unequal absorption of the incoming and outgoing beams of particles. Hence, for a given angular-momentum transfer to the nucleus (j_n) , the emitted particle is expected to be polarized since the neutron and proton spins are coupled in the deuteron. These absorptive distortion effects alone are not sufficient to account for the polarizations observed experimentally, however. Neglecting spin-dependent interactions, distorted-wave calculations^{3,4} predict polarizations restricted in magnitude to $\leq 33\%$. The sign of the polarization is expected to conform to the rule that $P = \pm$ for $j_n = l_n \pm \frac{1}{2}$, assuming deuterons are more strongly absorbed than protons in nuclear matter. (The direction of positive polarization is taken to be $\mathbf{n} = \mathbf{k}_d \times \mathbf{k}_p$ in accordance with the Basel convention.) A further consequence of these calculations is the absence of polarization for $l_n = 0$ neutron transfers. Previousp olarization measurements have

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