isotopes have maxima about 4° after those for elastic scattering in all cases.) A possible explanation may be excitation of the 5^- level in Zr^{90} at 2.315 MeV, which would not be resolved from the 2^+ level at 2.182 MeV in this experiment. No strong $l=2$ transition was seen in Zr^{91}.

Possible two-phonon levels were seen in Zr^{92} and Zr^{94}. The peak at 1.5 MeV in Zr^{92} is presumably due to the levels at 1.38 and 1.50 MeV seen by Jolly *et al.*,[2] and the peak at 1.9 MeV is presumably their 1.84-MeV peak. The peak at 1.42 MeV in Zr^{94} is presumably due to the levels at 1.31 and 1.47 MeV, found by Jolly *et al.*, while the 1.60-MeV peak is presumably their 1.68-MeV level.

In a previous experiment, peaks were seen in six even-even nickel and zinc isotopes at nearly exactly 1.5 times the excitation energy of the $l=3$ peak. In the present experiment, only one similar case is seen—the in-phase 3.55-MeV peak in Zr^{92}. However, it is possible that better resolution might find such a peak in the 3-MeV region of Zr^{94}.

Table I summarizes all excitation energies found in this experiment.

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Determination of Neutron Reduced Widths by the $^{14}N(^{14}N,^{13}N)^{15}N$ Reaction*

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Measurements have been made of cross sections for the neutron transfer reaction $^{14}N(^{14}N,^{13}N)^{15}N$ at energies below the Coulomb barrier. It is shown that the tunneling theory for neutron transfer is consistent with the angular distribution function, $d\sigma(\theta)/d\Omega$, at the center-of-mass energy of 6.62 MeV. By using the tunneling theory, the single-particle reduced width θ_0^2 is found to be $(4.5\pm1.0)\times10^{-2}$ for the transferred neutron at a radius of 5 F by assuming that θ_0^2 is the same for ^{14}N and ^{15}N. This value is to be compared to $\theta_0^2=(4.0\pm1.0)\times10^{-2}$ as determined from (d,p) stripping and $\theta_0^2=(5.2\pm1.0)\times10^{-2}$ as determined from shell-model calculations. Good agreement is thus obtained between reduced widths determined by neutron transfer and by (d,p) reactions.

I. INTRODUCTION AND SUMMARY

THE possibility of using the neutron-transfer reaction for the determination of neutron reduced widths in nuclei was demonstrated in 1956 by Breit and Ebel.[1] However, analysis[1] of the $^{14}N(^{14}N,^{13}N)^{15}N$ neutron-transfer data of Reynolds and Zucker[2] showed that the neutron tunneling process considered by Breit and Ebel was not able to account for the experimental data. Because of this disagreement as well as later similar ones, neutron reduced widths have heretofore not been determined with a very high degree of accuracy by means of neutron-transfer reactions.[3]

Since the time of the early experimental and theoretical work on neutron transfer reactions, a growing body of experimental data and theoretical calculations has indicated the importance of nuclear absorption in modifying the transfer cross sections.[4,5] Such absorption of the projectile nuclei has, of course, no place in the simple tunneling theory. For this reason, an experimental investigation has been made of the $^{14}N(^{14}N,^{13}N)^{15}N$ reaction at a center-of-mass energy [E(c.m.)$=6.62$ MeV] below the Coulomb-barrier energy where the effects of nuclear absorption would presumably be reduced to a small amount.

The results of these experiments[6] have been most gratifying in that the experimental data and the theo-

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† Present address: Hiram College, Hiram, Ohio. This material has been submitted by L. C. Becker as part of a dissertation for the degree of Doctor of Philosophy at Yale University.
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[1] G. Breit and M. E. Ebel, Phys. Rev. **103**, 679 (1956).
[2] H. L. Reynolds and A. Zucker, Phys. Rev. **101**, 166 (1956).
[3] K. S. Toth and E. Newman, Phys. Rev. **130**, 536 (1963) have calculated neutron reduced widths from (^{14}N,^{13}N) and (^{19}F,^{18}F) transfer data. The results obtained are internally consistent to about a factor of 3.
[4] F. C. Jobes and J. A. McIntyre, Phys. Rev. **133**, B893 (1964). This paper gives a discussion of the recent experimental and theoretical investigations concerning absorption.
[5] J. A. Polak, D. A. Torchia, and H. G. Wahsweiler, Bull. Am. Phys. Soc. **9**, 429 (1964).
[6] Preliminary reports of different aspects of this work have been made by L. C. Becker, F. C. Jobes, and J. A. McIntyre, in *Proceedings of the Third Conference on Reactions between Complex Nuclei*, edited by A. Ghiorso, R. Diamond, and H. Conzett (University of California, 1963), p. 106; J. A. McIntyre and L. C. Becker, Bull. Am. Phys. Soc. **9**, 67 (1964); and J. A. McIntyre, in *Nuclear Spectroscopy with Direct Reactions*, edited by F. E. Throw (Argonne National Laboratory Report ANL-6848, 1964), p. 160.

retical tunneling angular distributions[7,8] were found to be in good agreement (see Fig. 2). Unless this agreement is fortuitous or insensitive to other transfer processes such as virtual Coulomb excitation,[9] the tunneling process can be assumed to account for the transfer cross sections. It therefore becomes possible, by using the tunneling theory, to determine the product of the neutron reduced widths for ^{14}N and ^{15}N.

In order to obtain experimental cross-section values with better statistical accuracy, total-cross-section measurements have also been made. By determining the total-cross-section values at a number of energies it has been possible to compare experimental results again with the predictions of tunneling theory. The total-cross-section results are shown in Fig. 3, the tunneling curve[1,8] being normalized to the high-energy experimental data where the angular-distribution measurements had shown agreement with the tunneling predictions. A least-squares fit to the theoretical-tunneling curve yielded a reasonable χ^2 fit to the data over the entire range of energies. This fit to the total-cross-section tunneling predictions constitutes further evidence for the validity of the tunneling description of the neutron-transfer reaction.

The normalization of the tunneling curve in Fig. 3 determines the product of the reduced widths, $(1/\lambda_{14})(1/\lambda_{15})$ for ^{14}N and ^{15}N. The quantity $1/\lambda$ is defined by Breit and Ebel to be the probability that the neutron is to be found between a radius r and $r+dr$ of the nucleus, i.e.,

$$1/\lambda = r^2 R^2(r)\,, \tag{1}$$

where $R(r)$ is the radial wave function for the neutron. Since only the product $\lambda_{14}\lambda_{15}$ is determined by the normalization of the tunneling curve in Fig. 3, the assumption will be made in the following that the reduced widths for ^{14}N and ^{15}N are equal, i.e., that $\lambda_{14}=\lambda_{15}=\lambda$. This assumption is a quite reasonable one since the last neutron in both ^{14}N and ^{15}N is a $p_{1/2}$ neutron and both neutrons have essentially the same binding energy. In addition, the reduced widths found from (d,p) stripping interactions for ^{14}N and ^{15}N are found to be essentially the same.[10] Following this procedure then, the value for λ for nitrogen has been determined at a radius of 5 F.

For comparison with other values that have been obtained for the reduced width, the single-particle reduced width $\theta_0^2 = r^3R^2(r)/3$ will be used.[10] θ_0^2 may then be expressed as

$$\theta_0^2 = r/3\lambda\,.$$

[7] K. R. Greider, Phys. Rev. **133**, B1483 (1964).

[8] G. Breit, K. W. Chun, and H. G. Wahsweiler, Phys. Rev. **133**, B404 (1964).

[9] The importance of virtual Coulomb excitation at low energies has been emphasized by Breit. See, e.g., G. Breit, *Proceedings of the Second Conference on Reactions Between Complex Nuclei*, edited by A. Zucker, E. C. Halbert, and F. T. Howard (John Wiley & Sons, Inc., New York, 1960), pp. 1–15.

[10] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

TABLE I. Values of the neutron reduced width θ_0^2 at a radius of 5 F.

Method of measurement	θ_0^2
Neutron transfer[a]	$(4.5\pm1.0)\times10^{-2}$
Butler stripping[b]	$(4.0\pm1.0)\times10^{-2}$
Shell model[c]	$(5.2\pm1.0)\times10^{-2}$

[a] This paper.
[b] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).
[c] See Ref. 13.

The value of θ_0^2 for ^{14}N and ^{15}N was then found to be $(4.5\pm0.3)\times10^{-2}$ at a radius of 5 F. The uncertainty which is given results from experimental factors. An estimate of 10–20% accuracy for the tunneling theory has been made by Breit.[11] Thus, the value[12] of the reduced width as determined by neutron transfer is given in Table I as $(4.5\pm1.0)\times10^{-2}$, the uncertainty including then both theoretical and experimental factors. This value is to be compared to the θ_0^2 value of $(4.0\pm1.0)\times10^{-2}$ obtained from (d,p) stripping studies[10] and the θ_0^2 value of $(5.2\pm1.0)\times10^{-2}$ obtained[13] for a $1p_{1/2}$ neutron bound by the appropriate energy in a Woods–Saxon potential with reasonable parameters (see Table I). There is, therefore, good agreement between reduced width values obtained from neutron-transfer reactions and (d,p) stripping reactions.

II. EXPERIMENTAL PROCEDURE

A. General

The experimental methods have been described before.[14] Briefly, the ^{13}N nuclei in the $^{14}N(^{14}N,^{13}N)^{15}N$ reaction are collected on foils distributed around the target and the radioactivity of the foils is measured to determine the angle and range of the radioactive ^{13}N ions.

In this experiment, the "prestripper" 14-MeV ^{14}N beam from the Yale Heavy Ion Accelerator was used The energy of the beam was varied by placing thin carbon or nickel foils in the beam. The energy determination of the beam was made by comparing the experimental total-cross-section data with similar data obtained at Oak Ridge.[15] The energy calibration is considered to be reliable to $\pm0.2\%$.

[11] G. Breit, Comments at Session GC 3 of the 1964 New York meeting of the American Physical Society (unpublished).

[12] The θ_0^2 values that were published in the latter two papers in Ref. 6 are incorrect owing to an error in the energy calibration for the total-cross-section measurements.

[13] B. Buck, Nuclear Physics Computing Group Report No. 15, Oxford University 1960 (unpublished). Reduced-width value supplied by E. C. Halbert.

[14] J. A. McIntyre, T. L. Watts, and F. C. Jobes, Nucl. Instr. Methods **21**, 281 (1963).

[15] After the experimental work was completed, Hiebert, McIntyre, and Couch repeated the total-cross-section measurements using the Oak Ridge Tandem Van de Graaff. A discrepancy between their data and those being discussed here indicated that our original energy calibration was in error. We have therefore normalized our energy scale to the Oak Ridge scale by comparing the two sets of total-cross-section data.

The nitrogen targets used were made of adenine ($C_5H_3N_4 \cdot NH_2$) which had been evaporated onto a thin carbon backing (15–20 μg/cm^2). The thicknesses of the targets were measured by weighing the targets and also by using an alpha-particle thickness gauge which was calibrated for adenine thickness through the weighing process. The alpha-particle thickness gauge was also used to determine the energy loss of the ^{14}N beam particles in the target and in the energy-absorbing foils. The gauge was calibrated for this purpose by observing the shift in energy (as detected by a solid-state detector) of a low-intensity beam of ^{14}N particles when a foil or target was placed in the beam path. The gauge was further used to determine the "thicknesses" of the foils used in the range measurements. For these measurements the alpha-particle gauge was calibrated with weighed aluminum foils. The gauge measurements were used to convert the foil thicknesses into equivalent aluminum thicknesses so that the energy loss of the ^{13}N nuclei could be determined from Northcliffe's range-energy curves.[16]

B. Range Measurements

Three types of measurements were made in these experiments: range measurements of the ^{13}N nuclei, angular-distribution determinations, and total-cross-section measurements. The range measurements of the ^{13}N from the reaction are important in that they can be used to determine whether or not the ^{15}N product nucleus was left in its ground state. At angles larger than 27.5° (lab), neutron-transfer reactions which leave ^{15}N in an excited state are kinematically forbidden because of the large negative Q value for exciting even the first level of ^{15}N ($Q=5.0$ MeV) and the low center-of-mass energy for the experiment (6.62 MeV). Consequently, stacked perlodian foils[14] (equivalent in thickness to about 400 μg/cm^2 of aluminum) were used at 20° (lab) to collect the ^{13}N ions and to check the fraction of the transfer proceeding via the ground state of ^{15}N.

C. The Angular-Distribution Measurements

Angular-distribution measurements were made by placing plastic vinyl Scotch Tape collectors at twelve different scattering angles simultaneously. Under these conditions, destruction of the target by the ^{14}N beam was not an important matter since the relative number of ^{13}N ions at the various angles would be unaffected by the target thickness. This remark must be qualified, of course, to the extent that the mean energy of the reaction was changed by the target destruction. However, the targets used were thin enough so that the effects of energy change on the angular distribution were small. The angular resolution of the experiment was 5° in the center-of-mass system.

[16] L. C. Northcliffe, unpublished curves: See also Ann. Rev. Nucl. Sci. 13, 67 (1963).

D. Total-Cross-Section Measurements

Each total-cross-section measurement was made by placing the target inside a specially fabricated Faraday cup. A foil was put immediately behind the target to collect the ^{13}N leaving the target.[17] The radioactivity in the collector foil and in the target was then measured to determine the number of ^{13}N produced in the reaction.

Since the radioactivity detector[18] is sensitive to positron decay, it is possible for other positron emitters that are produced in the ^{14}N+^{14}N interaction to be detected also. Such an emitter was indeed found,[19] namely ^{15}O, which has a 2-min half-life in comparison to the 10-min half-life of ^{13}N. Decay curves were taken of the radioactivity, and a computer program[20] used to fit the decay curves with 2-min and 10-min contributions. The magnitude of the 10-min component, extrapolated back to the initial time when the beam was turned off, was then considered to be the number of ^{13}N nuclei produced during the bombardment.

Because of the destruction of the adenine target by the ^{14}N beam, care was taken during each measurement to minimize, as well as to measure, any change in target thickness. The ^{14}N beam was therefore defocused to cover an area of the target of $\frac{1}{2}$-in. diameter and the target thickness was measured before and after each bombardment with the alpha-particle thickness gauge. No data were used if the target thickness changed by more than 20% during the measurement. Actually, of the 28 runs taken, only 8 had target losses of over 10%.

III. RESULTS

A. The Range Measurements

The range measurement was taken at a laboratory angle of 20°; the results are shown in Fig. 1. The amount of radioactivity collected in the various foils is plotted against the depth of the foil in the stack. At the top of the figure the arrows indicate the depth of penetration into the stack expected for the ^{13}N and ^{15}O nuclei. The various arrow positions are determined by the energy levels of the residual ^{15}N and ^{13}C nuclei, G. S. denoting the ground state and E. S., the lowest excited states. Higher energy excited states are not considered because the excitation of such states are not allowed kinematically at 20°.

Most, if not all, of the activity, is seen to correspond to ^{13}N ions associated with unexcited ^{15}N nuclei. A curve

[17] Because of the kinematics of the reaction a negligible number of ^{13}N nuclei escape the target in the back direction.

[18] F. C. Jobes, J. A. McIntyre, and L. C. Becker, Nucl. Instr. Methods 21, 304 (1963).

[19] In agreement with the results of Reynolds and Zucker in Ref. 2.

[20] We are indebted to Dr. T. L. Watts for the development of this program which is similar to the FRANTIC program [P. C. Rodgers, FRANTIC Program for Analysis of Exponential Growth and Decay Curves, MIT Laboratory for Nuclear Science, Technical Report No. 76, June 1962 (unpublished)].

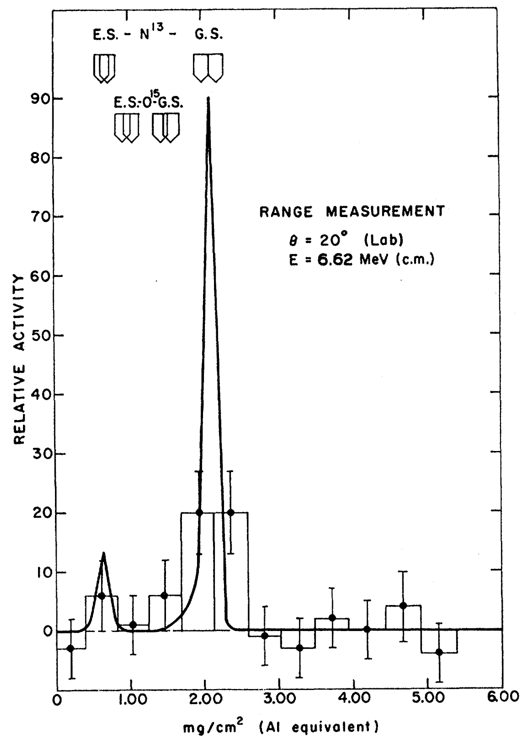

FIG. 1. Results of the range measurements of the reaction products at the laboratory angle of 20° with $E(\text{c.m.}) = 6.62$ MeV. The width of each histogram represents the aluminum equivalent thickness of the catching foils. The solid-line curve is drawn so that the full width at half-maximum of each peak is equal to the expected range spread of the particles while the area under the peak is equal to the area of the corresponding histograms. The indicator pairs at the upper left show the expected range (1) for ^{13}N (upper set) from a reaction which leaves ^{15}N in its ground state (G.S.) or first doublet excited state (E.S.) and (2) for ^{15}O (lower set) from a reaction which leaves both products in their ground states or ^{13}C in its first excited state. The width of each indicator represents the expected range spread of the reaction products while the separation of the points of each pair represents the uncertainty in the depth of the foil stacks.

having the expected range spread (~1 MeV at half-maximum) has been drawn such that its enclosed area is equal to that under the experimental histogram. There also may be, as indicated, a small fraction (~13%) of the ^{13}N ions which correspond to excited ^{15}N nuclei; however, this contribution is hardly statistically significant. This predominance of the ground-state transfer is also confirmed by the angular-distribution measurements (see next section) where there is no indication that the differential cross sections for center-of-mass angles less than 55° are larger than those at angles greater than 55° even though the latter include, because of kinematics, only ground-state transfers. Furthermore, this result is consistent with the 9.0-MeV data of Jobes and McIntyre[4] and the 8.15-MeV data of Reynolds and Zucker[2] which also showed predominantly ground-state transfers.

The absence of 2-min activity corresponding to ^{15}O ions in Fig. 1 is to be expected because of the time delay involved (~8 min) in loading the stacked foils into the counting system.

B. Angular-Distribution Measurements

Three independent runs were made on angular distributions. Each run consisted of a number of independent bombardments, with 12 detectors (plastic vinyl tapes) simultaneously covering 12 angles during each bombardment. Because of the partial destruction of the adenine target during each bombardment, no absolute cross sections were measured in these runs. The total angular range was covered therefore by measuring relative cross sections for overlapping groups of 12 angles. The different groups in each run were then normalized to one another through their common set of angles (usually 4 of the 12 angles were overlapping). The data from the different runs were finally combined by a least-squares adjustment. Small differences in target thicknesses and bombarding energies were accounted for by making appropriate corrections.[21] The final data thus obtained[22] are plotted in Fig. 2. The differential-cross-section scale shown in Fig. 2 was obtained by integrating over the experimental differential cross section values and by comparing this result with the value of the measured total cross section.

It should also be noted here that the experimental interference pattern appearing in Fig. 2 is somewhat washed out because of the geometrical spread of the target and ^{13}N collectors in the experiment. For the experimentally used angular spread of 5° for the target and collectors, a 20% reduction of the ratio of maximum to minimum in the interference structure is to be expected.[23]

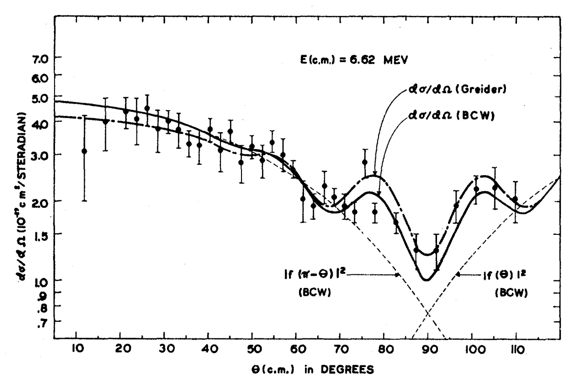

FIG. 2. Differential-cross-section results at $E(\text{c.m.}) = 6.62$ MeV plotted in the center-of-mass system against $\Theta(\text{c.m.})$. The experimental data are plotted as points with error bars. The theoretical-tunneling curves are from Greider (Ref. 7) and Breit, Chun, and Wahsweiler (Ref. 8) denoted by BCW. Also plotted are the absolute squares of the scattering amplitudes $|f(\Theta)|^2$ and $|f(\pi-\Theta)|^2$.

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[21] L. C. Becker, Ph.D. dissertation, Yale University, 1964 (unpublished).

[22] These data may be obtained in tabular form from the authors.

[23] See, e.g., S. D. Baker, dissertation, Yale University, 1964 (unpublished).

C. Total-Cross-Section Measurements

The results of the total-cross-section measurements[22] are shown in Fig. 3. Also shown in Fig. 3 are the earlier results of Reynolds and Zucker,[2] the cross-hatched area indicating the uncertainty in their experimental results. The agreement between these earlier data and the present results is good at the high-energy values. The semiclassical tunneling theory as modified by Breit, Chun, and Wahsweiler[8] has been normalized to the high-energy points of the experimental data and is indicated by the curve.

Because of the thickness of the targets and the sharply rising dependence of cross section with energy, a correction was made to convert the energy at the center of the targets to the mean effective energy in the targets. For the thickest targets this correction was 0.05 MeV, for the thinnest 0.02 MeV.

In addition to the statistical factors shown on the points in Fig. 3, the accuracy of the total-cross-section measurements is limited by uncertainties in (1) target thickness, $\pm 2\%$, and (2) in the efficiency of the positron detector, $\pm 3\%$. These two effects combine in quadrature to give a $\pm 4\%$ error for the total-cross-section scale.

The uncertainty in the energy scale in Fig. 3 depends on several factors. As noted earlier[15] the energy scale was determined by comparing the experimental data in Fig. 3 with similar experimental data obtained by Hiebert, McIntyre, and Couch at Oak Ridge. The accuracy of the comparison of these two sets of data depends on the statistical fluctuation of the data which is about $\pm 3\%$ over-all for an averaging curve drawn through the cross-section data points. Since $(dE/E)/(d\sigma/\sigma)$ is 0.07 at 6.6 MeV, the energy uncertainty is $\pm 0.2\%$ or ± 13 keV due to the intercomparison of the Yale and Oak Ridge data. In addition, the energy scale of the Oak Ridge data has an uncertainty of ± 5 keV introduced by the energy calibration of the Van de Graaff beam. The total uncertainty in the energy scale is thus ± 14 keV.

Finally, the accuracy of the normalization of the tunneling-theory curve in Fig. 3 to the experimental data must be determined. Since $(dE/E)/(d\sigma/\sigma)=0.07$ the ± 14-keV error at 6.6 MeV (0.21% error) introduced a $\pm 3\%$ uncertainty into the normalization of the theory to the cross-section data. In addition, a $\pm 5\%$ error is introduced through normalizing the theoretical curve to the statistically uncertain experimental data between 6 and 7 MeV where the theoretical curve is expected to be valid. The final uncertainty in the normalization of the tunneling theory curve is, therefore, made up of cross section nonstatistical uncertainties, $\pm 4\%$, energy calibration uncertainty, $\pm 3\%$, and statistical uncertainties, $\pm 5\%$. The normalization of the theory is therefore considered to have a total uncertainty of $\pm 7\%$.

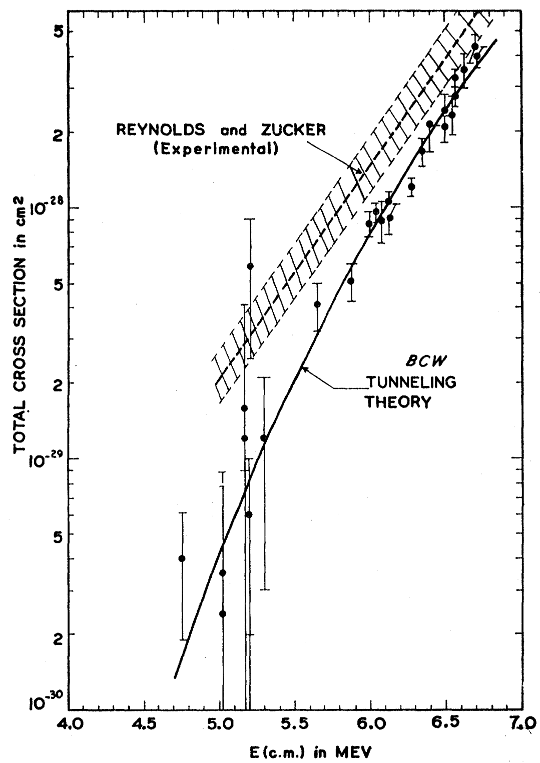

FIG. 3. Total-cross-section results plotted in the center-of-mass system against E(c.m.). The experimental data are plotted as points with error bars. The theoretical tunneling curve is from Breit, Chun, and Wahsweiler (Ref. 8). Also plotted as a cross-hatched area are the earlier experimetnal results of Reynolds and Zucker (Ref. 2).

IV. DISCUSSION OF RESULTS

A. Demonstration of the Validity of the Tunneling Description of the Transfer Process

As noted in the Introduction, the neutron-transfer reaction has not been used heretofore for the determination of neutron reduced widths because it was clear that for the $^{14}N(^{14}N,^{13}N)^{15}N$ reaction, at least, the transfer process was not proceeding via the tunneling mechanism.[24] The two tests that have been used[1] in the past to determine the nature of the transfer process have been a comparison between the tunneling theory and the experimental data for (1) the angular distribution function $d\sigma(\theta)/d\Omega$ and (2) the excitation function $\sigma(E)$. For this reason, the data reported here have been compared to the tunneling theory for the two functions, $d\sigma(\theta)/d\Omega$ and $\sigma(E)$.

1. Angular Distribution

The theoretical angular-distribution functions for the tunneling theory as determined by Greider[7] and by Breit, Chun, and Wahsweiler[8] are plotted in Fig. 2 along

[24] References 2 and 4 and K. S. Toth, Phys. Rev. **121**, 1190 (1961); K. S. Toth, *ibid.* **123**, 582 (1961).

with the experimental data. These two theories differ in the approximations made in the tunneling calculation. In Fig. 2, the theoretical curves have each been normalized to the experimental data by a least-squares analysis. Aside from this single arbitrary adjustment, each theoretical curve is completely determined by the tunneling theory. The physical reason for the absence of the adjustable parameters in the theory is, of course, that the transfer of the neutron takes place when the nitrogen nuclei are well enough separated so that only the known Coulomb forces affect the transfer process. This situation is to be contrasted to the more common (d,p) neutron-transfer reaction where several nuclear-force parameters must be introduced to determine the theoretical curve. The one adjustable parameter that does appear in the tunneling theory is related to the overlap of the neutron wave function in the projectile before transfer and the wave function in the target nucleus after transfer. Thus, as discussed in the Introduction, the product of the reduced widths $(1/\lambda_{14})(1/\lambda_{15})$ is determined by the normalization of the theory to the experiment.

Also shown in Fig. 2 are the squares of the scattering amplitudes for neutron transfer from the projectile, $|f(\Theta)|^2$, and neutron transfer to the projectile, $|f(\pi-\Theta)|^2$. Because of the semiclassical features of the tunneling process these quantities are useful for understanding the details of the transfer mechanism. Since Rutherford scattering trajectories are associated with the projectile, distant collisions correspond to the cross sections at small angles. As would be expected, then, since $f(\Theta)$ depends on the overlap of the wave functions of the ^{14}N and ^{15}N, $|f(\Theta)|^2$ is small at the small angles while $|f(\pi-\Theta)|^2$ is small at the large angles. The dependence of the cross section, $d\sigma/d\Omega = |f(\Theta)+f(\pi-\Theta)|^2$, on angle is then easily understood, there being the most pronounced interference structure at 90°, where $f(\theta)=f(\pi-\Theta)$, and little effect of interference at the small angles where $f(\theta)$ becomes negligible.

A χ^2 test of the fit for the Breit-Chun-Wahsweiler (BCW) theory to the data in Fig. 2 yielded the probability factor of 0.30 while a χ^2 test of a straight-line fit to the data yielded a probability factor of 0.24. Thus, while the data agree with the tunneling theoretical curve with its interference pattern, they agree essentially as well with a straight line and so do not verify the presence of the pattern.

A χ^2 test of the fit for the Greider theory yielded a very small probability factor. However, this poor fit would be considerably improved if the experimental interference pattern were increased 25% in magnitude as should be done to take into account the angular resolution of the experiment. Also, the poor fit is almost entirely due to the three points between 70 and 80° while the mirror data between 100 and 110° fit the theory well. It would appear, therefore, that the Greider theory also is in agreement with the data.

It should also be noted here that Polak, Torchia, and Wahsweiler[5] have shown that an even better fit to the data can be obtained by introducing a reasonable amount of nuclear absorption into the BCW theory. The absorption manifests itself in the close collisions and therefore reduces the theoretical cross section at the small angles.

The important feature about Fig. 2 in this discussion is the good agreement between the angular dependence of the theoretical curves and the experimental data. As already discussed in the Introduction, this agreement is in strong contrast to the disagreements found at higher energies.[24] Since corrections to the tunneling theory would be expected to appear at higher energies because of the nuclear absorption of the ^{14}N projectiles, it seems reasonable to believe that such absorption accounts for the discrepancies between tunneling theory and experiment[4] encountered at these energies. And, indeed, recent experiments[25] at energies between the low 6.62-MeV energy of this paper and the higher energies of the earlier work show clearly how the absorption becomes a dominating factor at the energies previously studied.

The good agreement between the tunneling theory and the experimental data in Fig. 2 constitutes the chief piece of evidence for the reliability of the tunneling theory as a description of the neutron-transfer process. Corroborative evidence is also found in the dependence of the transfer process on the energy of the reaction as will be discussed in the next section. Nevertheless, caution is still advisable in making this conclusion because of the expected influence of the virtual Coulomb excitation process for neutron transfer at these energies.[9] It is possible, of course, that the angular distribution function is not a sensitive test for the presence of virtual Coulomb excitation, and that the agreement between theory and experiment in Fig. 2 is not strong evidence for the dominance of the tunneling process in the neutron transfer reaction.

2. *Excitation Function*

The theoretical curve, normalized to the high-energy points in Fig. 3, follows the experimental data over the entire range of energies. A χ^2 test gave a value for the probability factor of 0.15. This agreement between the theory and experiment constitutes additional evidence for the validity of the tunneling description of the transfer process at these low energies.

B. Determination of Nitrogen Reduced Widths

Since the tunneling theory describes the neutron transfer reaction, the reduced widths can be extracted from the total cross section measurement by using the results of Breit and Ebel[1] as modified by Breit, Chun,

[25] J. C. Hiebert, J. A. McIntyre, and J. G. Couch, following paper, Phys. Rev. 138, B346 (1965).

and Wahsweiler.[8] In Eq. (25.2), Breit and Ebel[1] give for the semiclassically calculated total cross section, σ_{SC},

$$\sigma_{SC}=\frac{\pi^2}{2}\left(\frac{\hbar}{Mv}\right)^2\left(\frac{1}{\alpha^2\lambda_1\lambda_2}\right)\left(\frac{\alpha a_1}{1+\alpha a_1}\right)^2\left(\frac{\alpha a_2}{1+\alpha a_2}\right)^2 \times\exp\{-2\alpha(2a'-a_1-a_2)\}\,. \quad (2)$$

Here, σ_{SC} is the total cross section for the transfer of a neutron both to the target nucleus and to the projectile; $M^2=M_1M_2$ where M_i is the neutron reduced mass in the ith nucleus ($i=1$ or 2); $v=\{2E(\text{lab})/M_p\}^{1/2}$ is the relative velocity of the projectile and the target nucleus, M_p being the projectile mass and E(lab) the laboratory energy; $\alpha_i=(2M_iE_i/\hbar^2)^{1/2}$, E_i being the neutron separation energy in the ith nucleus; $\alpha=(\alpha_1+\alpha_2)/2$; a_i is the radius r_i at which the reduced width $1/\lambda_i$ is evaluated; and $a'=Z_1Z_2e^2/2E(\text{c.m.})$ is half the distance-of-closest-approach with E(c.m.) being the reaction energy in the center-of-mass system. In Eq. (4.3), Breit, Chun, and Wahsweiler[8] give for the quantum-mechanically corrected value of the total cross section σ_{QM},

$$\sigma_{QM}=\sigma_{SC}\exp[8\eta(u_0-\tan^{-1}u_0)]\,. \quad (3)$$

Here, $\eta=Z_1Z_2e^2/\hbar v$ and $u_0=\alpha/2k$, where $k=\eta/a'$ is the wave number.

All quantities are known in Eqs. (2) and (3) except λ_1 and λ_2. The following values were used in determining the product $\lambda_1\lambda_2$. From Ref. 8, $\alpha=(\alpha_1+\alpha_2)/2=0.69267$ F^{-1} while the nuclear masses for ^{14}N, ^{13}N, and the neutron are 14.008, 13.010, and 1.009 amu, respectively. These values give, then, for the reduced masses, $M_1=0.9364$ amu, $M_2=0.9405$ amu, and $M^2=M_1M_2$ 0.8807 amu^2. Other values used were $E(\text{lab})=2E(\text{c.m.})=13.60$ MeV, $Z_1=Z_2=7$, and $a_1=a_2=5$ F. From the experimental curve, the total cross section at $E(\text{c.m.})=6.80$ MeV was found to be 4.30×10^{-2} F^2. This value is considered to be σ_{QM} in calculating the value for $1/\lambda_1\lambda_2$. The value for $1/\lambda_1\lambda_2$ is found then to be 7.39×10^{-4} F^{-2}.

In principle, the neutron transfer experiment can give only the product of the reduced widths, $1/\lambda_1$ and $1/\lambda_2$. However, for the reaction under study, $1/\lambda_1$ is related to the last neutron bound to ^{14}N while $1/\lambda_2$ is related to the last neutron bound to ^{15}N. Both of these neutrons are $1p_{1/2}$ neutrons according to the single-particle model and have very nearly the same binding energies (10.55 and 10.84 MeV, respectively). To a quite good approximation then the reduced widths for these two neutrons should be the same. Other evidence from (d,p) stripping investigations[10] corroborates this point of view. In the following, it will be assumed therefore that $1/\lambda_1=1/\lambda_2=1/\lambda$. The value for $1/\lambda$ at a radius of 5 F then is found to be 2.72×10^{-2} F^{-1}.

In order to compare this result to other determinations of neutron reduced widths it is convenient to introduce the parameter θ_0^2, called the single-particle reduced width, and defined as[10]

$$\theta_0^2(r)=r^3R^2(r)/3\,. \quad (4)$$

Here r is the nuclear radius and $R(r)$ is the radial part of the neutron wave function. Thus, the variable r in Eq. (4) is the same as the variable a in Eq. (2). Since $1/\lambda$ is defined as

$$1/\lambda=r^2R^2(r)\,, \quad (5)$$

θ_0^2 may be determined from $1/\lambda$ through the expression

$$\theta_0^2=r/3\lambda\,. \quad (6)$$

Substitution of the experimentally determined value of $1/\lambda$ into Eq. (6), yields a value of θ_0^2 of 4.5×10^{-2} at $r=5$ F.

As discussed in Sec. III.C, the experimental uncertainty of the cross-section measurement has been assigned to be $\pm7\%$. However, since the reduced width varies as the square root of the experimental cross section, the error in the reduced width should be only about $\pm5\%$. In addition to the experimental uncertainty, the inaccuracies in the theory[11] are probably in the range of 10–20%. The neutron-transfer value[12] given in Table I is therefore listed as $(4.5\pm1.0)\times10^{-2}$.

C. Comparison to Other Data

Determinations of neutron reduced width values have been made also by other methods. By applying the Butler stripping theory to a comprehensive group of (d,p) stripping data, Macfarlane and French[10] have calculated the reduced width for ^{14}N and ^{15}N to be $(4.0\pm1.0)\times10^{-2}$ at a radius of 5 F. A θ_0^2 value has also been obtained for a $1p_{1/2}$ neutron bound by the appropriate energy in a Woods–Saxon potential.[13] Using optical-model parameters determined from elastic scattering and (d,p) experiments a neutron wave function yielding a θ_0^2 value of $(5.2\pm1.0)\times10^{-2}$ at 5 F was calculated. The three different values for θ_0^2 are listed in Table I and are seen to be in good agreement.

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