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Collective Excitations in the Zirconium Isotopes*

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Angular distributions have been derived for the scattering of 43-MeV alpha particles by Zr^{90}, Zr^{91}, Zr^{92}, and Zr^{94}. Angular distributions for elastic scattering for the $l=2$ transitions, for the $l=3$ transitions, and for many other peaks are presented. The collective excitations in Zr^{92} and Zr^{94} are quite similar, and resemble those of the Ni and Zn isotopes. However, the $l=3$ transition is more prominent than the $l=2$ transition in all four zirconium isotopes.

INTRODUCTION

A PREVIOUS study of inelastic scattering from six even-even isotopes of Ni and Zn revealed interesting and unexpected relations among the various peaks seen.[1] In particular, in-phase peaks were seen in all six targets at slightly more than 1.5 times the energy of the collective $l=3$ level. Peaks were seen at an excitation energy close to the sum of the excitation energies of the collective octupole and collective quadrupole levels. Peaks believed to arise from the excitation of two $l=2$ phonons were seen in all the Ni and Zn isotopes. In the even-even Ni isotopes the angular distributions of these two-phonon groups reached maxima about 0.2 cycle later than did the elastic-scattering angular distribution, but the two-phonon groups in the even-even Zn isotopes reached maxima 0.2 cycle earlier than elastic scattering. These similarities raise the problem of whether or not such nuclear-reaction systematics might also be found in a widely different region of the table of nuclides. The Zr isotopes seemed an interesting locale in which to study this problem, since the vibrational model is often applied to Zr^{92} and Zr^{94} while Zr^{90} exhibits some of the features of a closed-shell nucleus. In particular, Zr^{90} has a 0^+ first excited state, and its first 2^+ level is at a much higher excitation energy than the first 2^+ levels in Zr^{92} and Zr^{94}. On the other hand, Zr^{92} and Zr^{94} have the familiar 2^+ first excited levels at an excitation energy of about 0.92 MeV.

Experimentally the zirconium region is more difficult than the nickel-zinc region because of the higher level density and stronger Coulomb force, but nevertheless many peaks were seen. These zirconium isotopes were studied recently by Jolly, Lin, and Cohen[2] by inelastic deuteron scattering. These authors found that the Blair phase rule[3] is obeyed fairly well in inelastic deuteron scattering from zirconium isotopes. This rule also appears to be obeyed in inelastic alpha scattering, except for excitation of two-phonon levels.

The elastic and inelastic scattering of alpha particles from Zr^{90} was observed at angles from 17 to 47° (lab) at 2° intervals. Data from Zr^{91}, Zr^{92}, and Zr^{94} were taken subsequently at angles from 21 to 53° (lab), and also at 2° intervals. The large amount of numerical analysis was performed by computer techniques.[4] The experimental procedure and detection methods were similar to the ones described in Ref. 1.

ELASTIC SCATTERING

Figure 1 shows the ratio of elastic scattering to Rutherford scattering for the four isotopes. Regular diffraction patterns are found. The Zr^{90} cross section has maxima at 25, 34, and 43° (in the c.m. system). The diffraction pattern has maxima every 9° instead of at the 10° intervals which prevail in the Ni-Zn region.

SPECTRUM FROM Zr^{90}

Figure 2 shows the spectrum from Zr^{90} observed at 41° (lab). The peak at channel 72 is due to the (α,He^3) reaction. The next two peaks are probably statistical fluctuations since they do not show up consistently at other angles. The fourth peak is probably real.

The elastic peak at channel 188 is two orders of magnitude larger than the inelastic peaks. The letter A marks the position where the 0^+ first excited state at 1.752 MeV is expected. There is no sign of this level in any of the spectra. In the deuteron-scattering experiment of Jolly, Lin, and Cohen,[2] this level was assigned an intensity equal to 1/9 of that of the first 2^+ level. In the present alpha-scattering experiment, this level has less than 1/60 of the intensity of the first 2^+ level. A similar result is found in our unpublished data on Ca^{40}: we have found on excitation of the 0^+ first excited states of Ca^{40} and Zr^{90}. Hence there seems to be a rule that 0^+ first excited states can be observed in deuteron inelastic scattering but in alpha inelastic scattering these states have a cross section too small to be observed. However, 0^+ higher excited states can be seen in alpha inelastic scattering. A well-known example is the 0^+ second excited state of C^{12}. Another example is the 0^+ level in Si^{28} at 4.97 MeV.[5]

Peak B has a Q value of -2.22 ± 0.03 MeV and thus is mainly due to the 2^+ second excited level at 2.182 MeV

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* Work performed under the auspices of the U. S. Atomic Energy Commission.

† Now at Bell Telephone Laboratories, Whippany, New Jersey.

[1] H. W. Broek, Phys. Rev. **130**, 1914 (1963).

[2] R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **128**, 2292 (1962).

[3] J. S. Blair, Phys. Rev. **115**, 928 (1959).

[4] H. W. Broek, Argonne National Laboratory Report ANL-6718 1963 (unpublished).

[5] J. C. Cramer and W. W. Eidson, Bull. Am. Phys. Soc. **8**, 317 (1963).

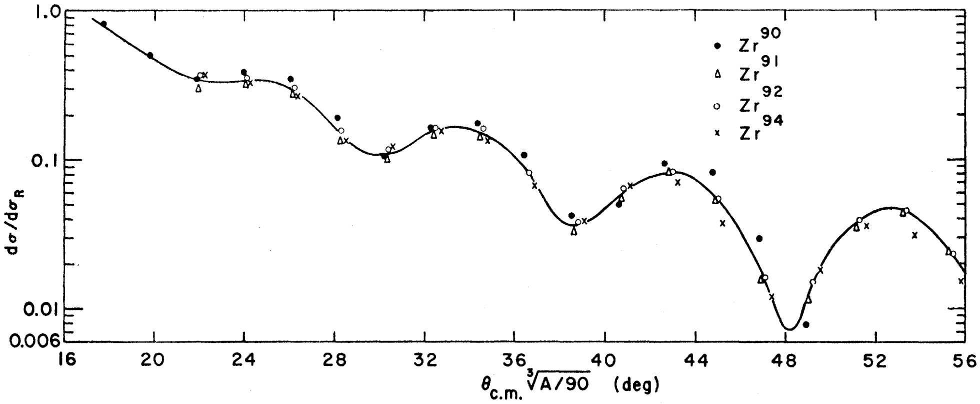


FIG. 1. Ratio of the elastic cross section to the Rutherford cross section for the four zirconium isotopes.

rather than the 5^- third excited level at 2.315 MeV. The angular distribution is out of phase with elastic scattering and has been fitted quite well by a distorted-wave calculation done by Satchler and Bassel at Oak Ridge.[6]

Group C at 2.78±0.03 MeV is the strongest inelastic group, and is undoubtedly the collective 3^- vibration.

Groups D, E, and F at 3.32±0.04, 3.92±0.05, and 4.42±0.07 MeV, respectively, are weak and out of phase with elastic scattering. It is not obvious which of these is the two-phonon group. Peak F lies at exactly twice the energy of the first 2^+ level. The weak group (peak F') at 5.15±0.05 MeV is probably in phase with elastic scattering.

Peak G is a strong in-phase group at 5.7 MeV which resembles the $l=1$ and $l=3$ curves calculated by Satchler and Bassel. Peak G lies at exactly twice the energy of the 3^- collective vibration (peak C). The next four groups (H, I, J, and K) also resemble the $l=1$ and $l=3$ curves obtained from the distorted-wave theory. The excitation energies are 6.4, 6.7, 7.2, and 7.5 MeV, respectively. These four peaks are not quite resolved from one another, and one cannot be sure how many peaks lie in this region.

The linewidth was about 300 keV in the Zr^{90} data.

ANGULAR DISTRIBUTIONS FROM Zr^{90}

The upper part of Fig. 3 shows the angular distribution to peak B, which is primarily due to the 2^+ second excited level at 2.182 MeV. A distorted-wave calculation by Satchler has given a good fit to the data. The calculation used a Saxon-Woods potential with a 50-MeV real well depth, a 20-MeV absorptive well depth, a radius parameter of 1.52 F, and a surface thickness parameter of 0.56 F. By way of comparison, these quantities were 47 MeV, 14 MeV, 1.58 F, and 0.55 F, respectively, in a potential well for alpha scattering[1] (at the same beam energy) by Ni^{58}.

FIG. 2. Spectrum of alpha particles scattered at 41° by Zr^{90}.

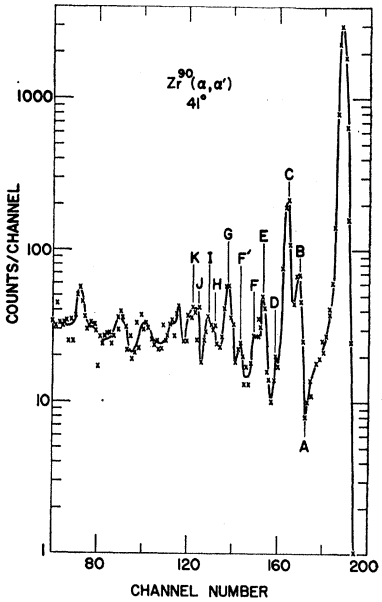
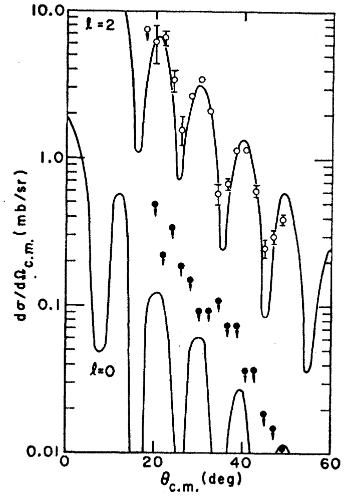


FIG. 3. Angular distribution for scattering to the 2^+ collective level of Zr^{90}. In the lower part of the drawing are the upper limits for excitation of the 0^+ first excited states of Zr^{90}. The curves were calculated by the distorted-wave Born approximation by Satchler and Bassel.



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[6] G. R. Satchler and R. H. Bassel (private communication).

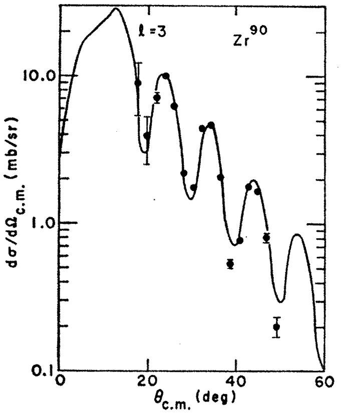

FIG. 4. Angular distribution for scattering to the 3^- collective level of Zr^{90}. The curve was calculated by the distorted-wave Born approximation by Satchler and Bassel.

The lower part of Fig. 3 shows the upper limits for excitation of the 0^+ first excited state, and also shows a theoretical curve calculated for this transition under the assumption that the level represents a monopole vibration. No evidence for this transition was seen at any angle.

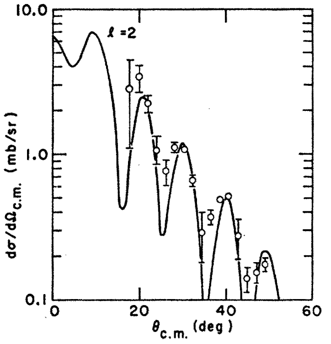

FIG. 5. Angular distribution for scattering to peak *E* of Zr^{90} at an excitation energy of 3.92 ±0.05 MeV. The curve was calculated by the distorted-wave Born approximation by Satchler and Bassel.

Figure 4 shows the angular distribution to the collective 3^- level in Zr^{90} at 2.78±0.03 MeV. A pronounced diffraction pattern gives maxima at 24, 34, and 43° in the center-of-mass system. The data are in very good agreement with the distorted-wave calculation.

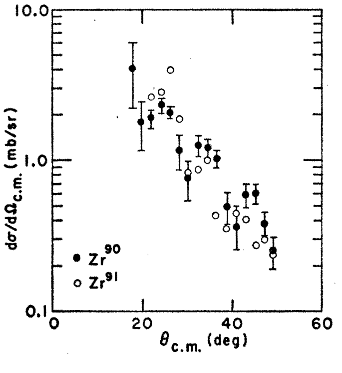

FIG. 6. Angular distributions for scattering to a peak at 5.7 MeV which occurs in both Zr^{90} and Zr^{91}. The experimental uncertainties in the Zr^{91} data are somewhat larger than in the Zr^{90} data.

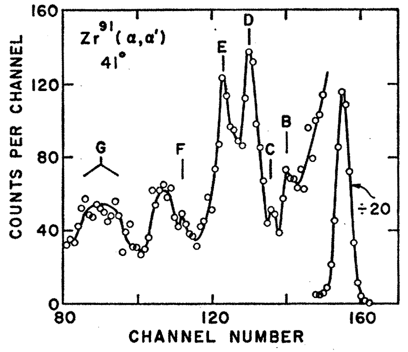

FIG. 7. Spectrum of alpha particles scattered at 41° by Zr^{91}.

Figure 5 gives the cross section for excitation of a peak in Zr^{90} at 3.92±0.05 MeV (peak *E* of Fig. 2). It exhibits a pronounced diffraction effect and a strong resemblance to the $l=2$ distorted-wave curve. Peaks *D* and *F* at 3.32 and 4.42 MeV, respectively, have a similar but less pronounced diffraction pattern. Peak *G* at 5.7 MeV is a strong and somewhat broad peak whose angular distribution resembles the $l=1$ and $l=3$ distorted-wave curves. Figure 6 compares this angular distribution with that of a level at the same excitation energy in Zr^{91}.

The peaks above 5.7 MeV in Zr^{90} all seem to have an in-phase angular distribution. It is not clear how many such groups exist near 7 MeV; better resolution is needed to probe this region. These angular distributions are all in phase with elastic scattering for any reasonable choice of background level. In this regard, Zr^{90} resembles the even-even Ni and Zn isotopes where in-phase groups predominate at the higher excitation energies.

ZIRCONIUM 91

Figure 7 gives the observed spectrum from Zr^{91} at an angle of 41° (lab). At the lower excitations, three weak peaks are found: peak *A* at 0.98±0.05 MeV (not seen at 41°), peak *B* at 1.4±0.1 MeV, and peak *C* at 1.9±0.1 MeV. All three of these peaks are significant in most of the spectra and are undoubtedly real. Peak *B* is probably in phase with elastic scattering and peak *C* is probably out of phase.

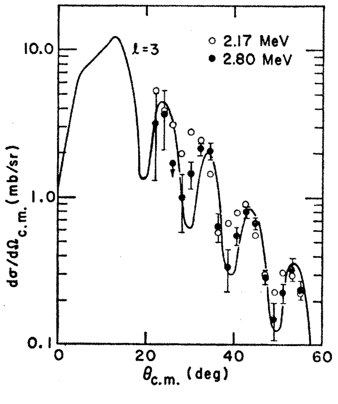

FIG. 8. Angular distributions for scattering to the two prominent $l=3$ peaks in Zr^{91}. The curve was calculated for the case of Zr^{90} by the distorted-wave Born approximation by Satchler and Bassel. The experimental uncertainties in the two angular distributions are about the same.

Peaks *D* and *E* are the strong in-phase peaks presumably due to the sextet of levels produced by coupling a 3^- phonon to the $\frac{5}{2}^+$ ground state of Zr^{91}. One of the peaks is at 2.80±0.08 MeV, which is (within experimental uncertainty) the same as the 3^- level in Zr^{90}; the other is at 2.17±0.06 MeV. The angular distribution to these levels is compared with the $l=3$ distorted-wave curve in Fig. 8. Peak *E* (at 2.80 MeV) gives a very good fit to the theory. However, peak *D* has maxima that consistently fall at slightly smaller angles than the maxima from the theoretical result.

A small peak *F* at 3.7 MeV is seen in some of the data. Its existence is not fully established. The broad structure seen in Fig. 7 at channel 108 is from oxygen contamination of the target.

Peak *G* is from a group of levels between 5.3 and 5.9 MeV. It probably is in phase with elastic scattering. The angular distribution is illustrated in Fig. 6.

In the data on Zr^{91}, Zr^{92}, and Zr^{94}, the linewidth was 400 keV.

ZIRCONIUM 92

The spectrum of alpha particles scattered at 39° from Zr^{92} is presented in Fig. 9.

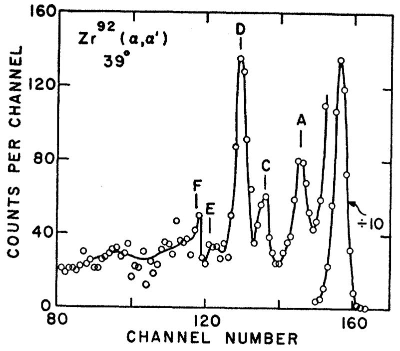

FIG. 9. Spectrum of alpha particles scattered at 39° by Zr^{92}.

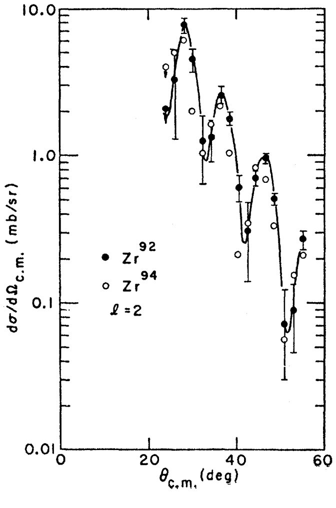

FIG. 10. Angular distributions to the 2^+ collective first excited levels of Zr^{92} and Zr^{94}. The experimental uncertainties in the Zr^{94} data are about the same as in the Zr^{92} data.

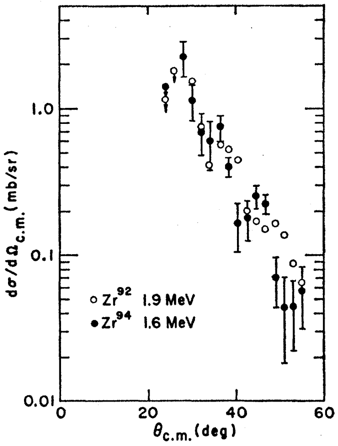

FIG. 11. Angular distributions to the 1.9-MeV peak in Zr^{92} and the 1.6-MeV peak in Zr^{94}. Both peaks are believed to be two-phonon excitations. The experimental uncertainties in the Zr^{92} data are about the same as in the Zr^{94} data.

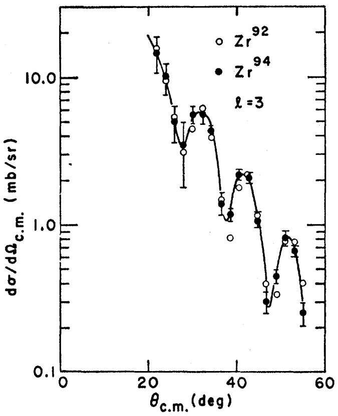

FIG. 12. Angular distributions to the 3^- collective levels of Zr^{92} and Zr^{94}. The experimental uncertainties in the two sets of data are about the same.

Peak *A* is the 2^+ first excited state at 0.91±0.03 MeV. Its proximity to the very strong elastic peak made the derivation of its angular distribution more difficult and less accurate than was the case for the first excited states of the Ni and Zn isotopes. Nevertheless the angular distribution was found and is plotted in Fig. 10, where it is compared with the first excited state in Zr^{94}.

Two peaks were found in the two-phonon region. Peak *B* (which does not appear at 39° lab) is a weak peak at 1.5±0.1 MeV. Peak *C* at 1.9±0.1 MeV is shown in Fig. 11. It is definitely out of phase with elastic scattering.

Peak *D*, the strongest inelastic group in Zr^{92}, is the well-known collective 3^- level at 2.35±0.03 MeV. Its angular distribution (Fig. 12) clearly shows diffraction structure with maxima at 33, 42, and 52° in the c.m. system.

Peaks *E* and *F* are weak, but show up in most of the spectra and are undoubtedly real. The energies are 3.2±0.1 and 3.55±0.10 MeV, respectively. Peak *F* is in phase with elastic scattering.

ZIRCONIUM 94

In general the spectra of alpha particles scattered from Zr^{94} bore a rough similarity to those from Zr^{92}. A

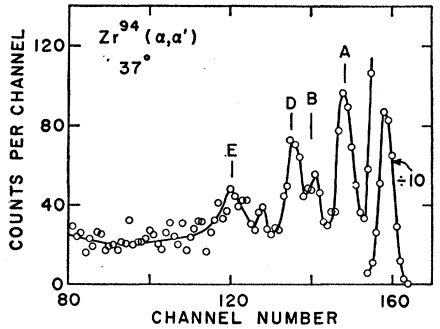

FIG. 13. Spectrum of alpha particles scattered at 37° by Zr^{94}.

detailed analysis verifies the similarities, which are primarily ascribed to the similar energies of the 2^+ collective level and of the 3^- collective level.

The spectrum from Zr^{94} at 37° lab is shown in Fig. 13. The large elastic peak occurs at channel 158. Peak A is the first excited level (the 2^+ collective level at 0.92 ± 0.02 MeV) which is illustrated in Fig. 10. Peak B changes its shape somewhat from angle to angle and can be broken into two peaks at $Q_B = -1.42 \pm 0.05$ MeV and $Q_C = -1.60 \pm 0.06$ MeV. Peak C at 1.60 MeV comes presumably from the known level at 1.68 MeV; a pronounced diffraction effect exists in its angular distribution (Fig. 11).

Peak D is the 3^- collective level whose angular distribution may be seen in Fig. 12. The small peak seen at channel 128 (i.e., at 2.7 MeV) is visible at rather few angles, and hence its significance is not firmly established.

Peak E is a broad structure extending from 2.9 to 3.5 MeV of excitation. Better resolution is required to explore this region.

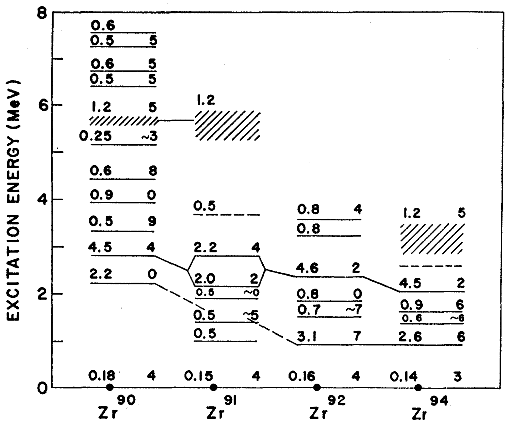

FIG. 14. Diagram of the observed peaks. The number on the left at each level is the differential cross section (mb/sr) at 35° on the "envelope" of the angular distribution. For elastic scattering the quoted value of $d\sigma/d\sigma_R$ is likewise the value at 35° on the "envelope." The number on the right at each level is $(\theta_{\max} - 30°)$, where $\theta_{\max}$ is the position of the maximum found between 30 and 39° in the angular distribution.

DISCUSSION

In the zirconium isotopes the strongest inelastic peak is that of the $l=3$ collective transition, whereas in nickel and zinc the $l=2$ excitation always is the largest. The cross section for the $l=3$ transition has the same value (to within experimental error) in all four zirconium isotopes. Theoretically the $l=3$ transition in Zr^{91} should be broken into six peaks by coupling to the ground state, but only two of these peaks could be resolved. A superficial resemblance between the Zr^{90} data and the Zr^{91} data is occasioned by the fact that the two biggest peaks in each spectrum occur at nearly the same energy, although peak B in Zr^{90} is primarily an $l=2$ transition while the peak at the same energy in Zr^{91} is believed to be primarily $l=3$.

TABLE I. Level energies (keV) in the zirconium isotopes.

Isotope	Level energy (keV)	Spin
Zr^{90}	2220 ± 30	2^+
	2780 ± 30	3^-
	3320 ± 40	
	3920 ± 50	
	4420 ± 70	
	5150 ± 60	
	5600 to 5800	
	6420 ± 60	
	6700 ± 40	
	7260 ± 50	
	7540 ± 50	
Zr^{91}	980 ± 50	
	1400 ± 100	
	1900 ± 100	
	2170 ± 60	$l=3$
	2800 ± 80	$l=3$
	3700(?)	
	5300 to 5900	
Zr^{92}	910 ± 30	2^+
	1500 ± 100	4^+
	1900 ± 100	2^+
	2350 ± 30	3^-
	3200 ± 100	
	3550 ± 100	
Zr^{94}	920 ± 20	2^+
	1420 ± 50	
	1600 ± 60	
	2040 ± 20	3^-
	2900 to 3500	

The second strongest peak in Zr^{90}, Zr^{92}, and Zr^{94} is the $l=2$ collective transition. The intensities are given on the left of each level in Fig. 14. The intensity is greatest in Zr^{92} and is least in Zr^{90} where the 2^+ level lies at considerably higher energy. In Zr^{92} and Zr^{94}, the angular distributions for the $l=2$ peak have maxima about 3° after the maxima in elastic scattering. However, in Zr^{90} the $l=2$ peak has its maxima about 5° after the elastic-scattering maxima. The reason for this difference in angular distributions is not understood. (By way of comparison, the $l=2$ transitions in the Ni and Zn isotopes have maxima about 4° after those for elastic scattering in all cases.) A possible explanation may be excitation of the 5^- level in Zr^{90} at 2.315 MeV, which would not be resolved from the 2^+ level at 2.182 MeV in this experiment. No strong $l=2$ transition was seen in Zr^{91}.

Possible two-phonon levels were seen in Zr^{92} and Zr^{94}. The peak at 1.5 MeV in Zr^{92} is presumably due to the levels at 1.38 and 1.50 MeV seen by Jolly *et al.*,[2] and the peak at 1.9 MeV is presumably their 1.84-MeV peak. The peak at 1.42 MeV in Zr^{94} is presumably due to the levels at 1.31 and 1.47 MeV, found by Jolly *et al.*, while the 1.60-MeV peak is presumably their 1.68-MeV level.

In a previous experiment, peaks were seen in six even-even nickel and zinc isotopes at nearly exactly 1.5 times the excitation energy of the $l=3$ peak. In the present experiment, only one similar case is seen—the in-phase 3.55-MeV peak in Zr^{92}. However, it is possible that better resolution might find such a peak in the 3-MeV region of Zr^{94}.

Table I summarizes all excitation energies found in this experiment.

ACKNOWLEDGMENTS

The authors wish to express their thanks to Dr. G. R. Satchler and Dr. R. H. Bassel for performing distorted-wave calculations on the zirconium isotopes, and for permission to include their results in this paper. The authors are also grateful to W. Ramler and the cyclotron crew for their help and cooperation.

PHYSICAL REVIEW VOLUME 138, NUMBER 2B 26 APRIL 1965

Determination of Neutron Reduced Widths by the $^{14}\mathrm{N}(^{14}\mathrm{N},^{13}\mathrm{N})^{15}\mathrm{N}$ Reaction[*]

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Measurements have been made of cross sections for the neutron transfer reaction $^{14}\mathrm{N}(^{14}\mathrm{N},^{13}\mathrm{N})^{15}\mathrm{N}$ at energies below the Coulomb barrier. It is shown that the tunneling theory for neutron transfer is consistent with the angular distribution function, $d\sigma(\theta)/d\Omega$, at the center-of-mass energy of 6.62 MeV. By using the tunneling theory, the single-particle reduced width θ_0^2 is found to be $(4.5\pm1.0)\times10^{-2}$ for the transferred neutron at a radius of 5 F by assuming that θ_0^2 is the same for $^{14}\mathrm{N}$ and $^{15}\mathrm{N}$. This value is to be compared to $\theta_0^2=(4.0\pm1.0)\times10^{-2}$ as determined from (d,p) stripping and $\theta_0^2=(5.2\pm1.0)\times10^{-2}$ as determined from shell-model calculations. Good agreement is thus obtained between reduced widths determined by neutron transfer and by (d,p) reactions.

I. INTRODUCTION AND SUMMARY

THE possibility of using the neutron-transfer reaction for the determination of neutron reduced widths in nuclei was demonstrated in 1956 by Breit and Ebel.[1] However, analysis[1] of the $^{14}\mathrm{N}(^{14}\mathrm{N},^{13}\mathrm{N})^{15}\mathrm{N}$ neutron-transfer data of Reynolds and Zucker[2] showed that the neutron tunneling process considered by Breit and Ebel was not able to account for the experimental data. Because of this disagreement as well as later similar ones, neutron reduced widths have heretofore not been determined with a very high degree of accuracy by means of neutron-transfer reactions.[3]

Since the time of the early experimental and theoretical work on neutron transfer reactions, a growing body of experimental data and theoretical calculations has indicated the importance of nuclear absorption in modifying the transfer cross sections.[4,5] Such absorption of the projectile nuclei has, of course, no place in the simple tunneling theory. For this reason, an experimental investigation has been made of the $^{14}\mathrm{N}(^{14}\mathrm{N},^{13}\mathrm{N})^{15}\mathrm{N}$ reaction at a center-of-mass energy [E(c.m.)$=6.62$ MeV] below the Coulomb-barrier energy where the effects of nuclear absorption would presumably be reduced to a small amount.

The results of these experiments[6] have been most gratifying in that the experimental data and the theo-

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[*] Supported by the U. S. Atomic Energy Commission.

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[1] G. Breit and M. E. Ebel, Phys. Rev. **103**, 679 (1956).

[2] H. L. Reynolds and A. Zucker, Phys. Rev. **101**, 166 (1956).

[3] K. S. Toth and E. Newman, Phys. Rev. **130**, 536 (1963) have calculated neutron reduced widths from (^{14}N,^{13}N) and (^{19}F,^{18}F) transfer data. The results obtained are internally consistent to about a factor of 3.

[4] F. C. Jobes and J. A. McIntyre, Phys. Rev. **133**, B893 (1964). This paper gives a discussion of the recent experimental and theoretical investigations concerning absorption.

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