Effect of Deformation Vibrations on $E2$ Branching Ratios in Deformed Even-Even Nuclei*

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(Received 15 June 1964; revised manuscript received 18 December 1964)

Ratios of electric-quadrupole reduced transition probabilities are calculated using a collective model. For transitions between positive-parity states the asymmetric-core model with β vibrations is used, while for negative-parity states a similar model with ζ vibrations is used. The reduced transition probability can be divided into two factors, the first an adiabatic term, and the second due to the vibration on the deformation parameter. For small values of the stiffness parameter μ, the vibrational contribution to the ratios is found to be a factor close to unity. However, this factor can be large for greater μ. Experimental branching ratios of several deformed even-even nuclei are compared with theory.

I. INTRODUCTION

LOW-LYING energy levels of positive and negative parity in deformed even-even nuclei have been explained with some success by the work of Davydov[1,2] and Davidson,[3,4] respectively, and co-authors. Reduced-transition-probability ratios predicted by the adiabatic or pure rotational models[1,3] are in reasonable agreement with experiment at least for transitions within and between what are usually called the ground-state and "γ-vibrational" bands for the positive-parity systems and their analogs in the negative-parity systems (although in the asymmetric-rotator models used here, both of these bands merge into a common rotational-level sequence). Discrepancies between theory and experiment are usually accounted for by the perturbation mixing of the various bands both for energy differences and for ratios of reduced transition probabilities. However, recent experimental investigations of the high spin levels of deformed even-even nuclei indicate that such a perturbation approach will not account for the observed level structure[5] and that it is necessary to take the beta (or deformation) vibrations into account more exactly as is done in Refs. 2 and 4. Other observations of level structure and gamma-ray branching ratios both in the rare-earth deformed region[6] and in the actinide deformed region[7] suggest that the influence of the beta-band mixing is an order of magnitude greater than that of the gamma-band mixing. In particular in Sm^{152} the experimental branching ratio[8] from the beta band to the ground state, $B(E2\colon 212 \rightarrow 211)/B(E2\colon 212 \rightarrow 011)$, is greater by a factor of two than predicted by the simple collective model.[6] This in itself would suggest that a perturbation approach to handling these vibrations is probably not adequate. Since the effects of γ-band admixtures are much smaller, at least in the ground-state band, perturbation methods will be more nearly adequate to describe any deviations from theory for them. It is the purpose of this paper to examine the effects on gamma-ray branching ratios of the beta vibrations in deformed even-even nuclei by a more exact method than perturbation theory. We deal here principally with $E2$ transitions both within the positive-parity and the negative-parity bands; however, the analysis is sufficiently general that the numerical calculations reported can be easily extended to other electric transitions.

It is worthwhile saying a word about the principal approximations used in this paper. As mentioned above the deformation vibrations are treated exactly, but the asymmetry degree of freedom (γ or η) is removed by considering this parameter as fixed (although not necessarily zero). Asymmetry vibrations are not included at all since their effects are apparently less important than those of the deformation vibrations.[6] The octupole treatment, while not complete, is probably as detailed as the present state of experimental information justifies. It is known that the inclusion of more degrees of freedom into this particular problem increases the complexity of the theory a great deal.[9] A further discussion of this truncation of the octupole theory is made in the next section. Two further approximations have been used. One is the form of the moments of inertia which is taken to be the hydrodynamic one[10] propor-

* This work was done under the auspices of the U. S. Atomic Energy Commission and supported in part by the National Science Foundation.

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[2] A. S. Davydov and A. A. Chaban, Nucl. Phys. **20**, 499 (1960).

[3] J. P. Davidson, Nucl. Phys. **33**, 664 (1962).

[4] S. A. Williams and J. P. Davidson, Can. J. Phys. **40**, 1423 (1962).

[5] F. S. Stephens, N. Lark, and R. M. Diamond, Phys. Rev. Letters **12**, 225 (1964).

[6] J. S. Greenberg, G. C. Seaman, E. V. Bishop, and D. A. Bromley, Phys. Rev. Letters **11**, 211 (1963).

[7] F. S. Stephens, B. Elbek, and R. M. Diamond, *Proceedings of the Third Conference on Reactions Between Complex Nuclei*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, California, 1963), p. 303.

[8] The notation here is $B(E2, INn \rightarrow I'N'n')$, where I is the spin of the state, N the ordinal of that spin, and n the ordinal of the beta-vibrational band. Thus the state 212 is the first $I=2$ level in the first *excited* beta-vibrational band. This is the notation of Refs. 3 and 4.

[9] M. G. Davidson, Ph.D. thesis, Rensselaer Polytechnic Institute, 1964 (unpublished).

[10] A. Bohr, Kgl. Danske. Videnskab. Selskab, Mat. Fys. Medd. **26**, 14 (1952).

tional to $B_\lambda \beta_\lambda^2$. The mass parameter B_λ is assumed constant for each nucleus, and the effect of centrifugal stretching is accounted for by the quadratic dependence in β. The form of the moments of inertia has recently been discussed by Diamond, Stephens, and Swiatecki,[11] and they show that, at least in the deformed region $150 \lesssim A \lesssim 190$, this approximation is quite reasonable. The other approximation is in the potential-energy term which is taken proportional to $C_\lambda(\beta_\lambda - \beta_{\lambda 0})^2$ with C_λ constant. This potential function has been shown to be a close approximation to empirical-mass-formula potentials at least for spins up to 10–14.[11] Further the experimental investigation of Ref. 5 shows that the ground-state band can be quite closely accounted for by using these forms of the potential and moment-of-inertia functions. Thus these calculations should be expected to apply to those regions of nuclear deformation where the even-even nuclei are known to have well-developed rotational spectra (i.e., $150 \lesssim A \lesssim 190$, $A \gtrsim 224$) and to levels whose nature is largely collective and which have moderate angular momenta, say $I \lesssim 16$.

In Sec. II, we outline the vibrational treatment of the problem and describe the resulting state functions both as a review and to fix the notation, while in Sec. III we examine the reduced transition probabilities and appropriate electric quadrupole operators for both parities. Section IV is a comparison of theory and experiment.

II. THE VIBRATION PROBLEM

We begin by expanding the nuclear surface in the laboratory coordinate system

$$R(\theta,\phi) = R_0[1 + \textstyle\sum_\mu \alpha_{\lambda\mu}{}^* Y_{\lambda\mu}(\theta,\phi)], \qquad \text{(1L)}$$

λ being 2 for π^+ and 3 for π^- states. Applying small oscillation theory to such a surface yields a Hamiltonian of the form[10]

$$H_\lambda = \tfrac{1}{2} B_\lambda \textstyle\sum_\mu |\dot{\alpha}_{\lambda\mu}|^2 + \tfrac{1}{2} C_\lambda \sum_\mu |\alpha_{\lambda\mu}|^2, \quad H = \sum_\lambda H_\lambda. \quad (2)$$

We now transform to the body-fixed reference system where the surface (1L) is given by

$$R(\theta',\phi') = R_0[1 + \textstyle\sum_\nu a_{\lambda\nu}{}^* Y_{\lambda\nu}(\theta',\phi')] \qquad \text{(1B)}$$

and the expansion coefficients are related by

$$\alpha_{\lambda\mu} = \textstyle\sum_\nu D_{\mu\nu}{}^{\lambda *}(\theta_i) a_{\lambda\nu},$$

the $D_{\mu\nu}{}^\lambda(\theta_i)$ being components of the $(2I+1)$-dimensional representaton of the rotation group[12] and are functions of the Euler angles θ_i. It is helpful if the body expansion coefficients $a_{\lambda\mu}$ are parameterized as[4]

$$a_{\lambda\mu} = \beta_\lambda \sigma_{\lambda\mu}, \qquad (3)$$

the β_λ being the λth-order deformation parameter and the asymmetry parameters $\sigma_{\lambda\mu}$ can be subjected to the further requirement

$$\textstyle\sum_\mu \sigma_{\lambda\mu}{}^2 = 1. \qquad (4)$$

For the quadrupole and octupole cases the $\sigma_{\lambda\mu}$ have the familiar form[10,3]

$$\sigma_{20} = \cos\gamma, \quad \sqrt{2}\sigma_{2\pm 2} = \sin\gamma, \quad \lambda = 2,$$
$$\sigma_{30} = \cos\eta, \quad \sqrt{2}\sigma_{3\pm 2} = \sin\eta, \quad \lambda = 3,$$

the others in each case being zero. (For $\lambda = 2$ this is a consequence of the degrees of freedom available, while for $\lambda = 3$ it is a sufficient condition to diagonalize the inertial tensor.[4] While this latter reduction of the degrees of freedom may appear arbitrary, it is supported by a recent calculation of Soloviev *et al.*,[13] which shows that the states associated with the $\lambda = 3$, $\mu = \pm 3$ degrees of freedom in deformed nuclei are almost pure two-quasiparticle states and thus not to be associated with a collective model.)

The Hamiltonian (2) so transformed consists, as is well known, of three terms: one representing rigid rotations, one vibrations, and the third a rotation-vibration cross term. For the cases $\lambda = 2$, 3 the latter term vanishes identically. In keeping with the desire to treat the deformation vibrations exactly leaving the effects of asymmetry vibrations to be treated as a perturbation, the β_λ are taken to be variables while the $\sigma_{\lambda\mu}$ remain as fixed fitting parameters. Thus the generalized curvilinear coordinate space with respect to which the system is quantized contains the four variables θ_i, β_λ. The transformed Hamiltonian is[4]

$$H = -\frac{\hbar^2}{2B_\lambda}\left\{\frac{1}{\beta_\lambda{}^3}\frac{\partial}{\partial\beta_\lambda}\left(\beta_\lambda{}^3 \frac{\partial}{\partial\beta_\lambda}\right) \right.$$
$$\left. \times (1/4\beta_\lambda{}^2) \sum_k \mathbf{I}_k{}^2/\mathcal{g}_k{}^\lambda \right\} + \tfrac{1}{2} C_\lambda (\beta_\lambda - \beta_{\lambda 0})^2,$$

where the potential term of the Hamiltonian (2) has been generalized to permit oscillations about a nonspherical deformation specified by $\beta_{\lambda 0}$, $\mathbf{I}$ is the angular-momentum operator in the body, and $\mathcal{g}_k{}^\lambda = 4B_\lambda \beta_\lambda{}^2 \mathfrak{J}_k{}^\lambda$ are the principal moments of inertia whose form is known.[10,3]

The Schrödinger equation separates into a rotational part

$$[\tfrac{1}{2} \textstyle\sum_k (I_k{}^2/\mathcal{g}_k{}^\lambda) - \epsilon_{IN}{}^\lambda] \phi_{IN}(\theta_i) = 0,$$

where the rotational eigenvalues have been given in tabular form for various values of the spin as a function of γ for the quadrupole case[14] and as a function of η for the octupole case.[4]

[11] R. M. Diamond, F. S. Stephens, and J. Swiatecki, Phys. Letters **11**, 315 (1964).

[12] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[13] V. G. Soloviev, P. Vogel, and A. A. Korneichuk, Izv. Akad. Nauk. SSSR, Ser. Fiz. (to be published).

[14] R. B. Moore and W. White, Can. J. Phys. **38**, 1149 (1960).

The vibrational Schrödinger equation is

$$\left[-\frac{\hbar^2}{2B_\lambda}\frac{1}{\beta_\lambda{}^3}\frac{\partial}{\partial\beta_\lambda}\left(\beta_\lambda{}^3\frac{\partial}{\partial\beta_\lambda}\right)+\frac{\hbar^2}{4B_\lambda\beta_\lambda{}^2}\epsilon_{IN}{}^\lambda+\tfrac{1}{2}C_\lambda(\beta_\lambda-\beta_{\lambda 0})^2\right]\times\Phi_n{}^{IN}(\beta_\lambda)=E_{INn}{}^\lambda\Phi_n{}^{IN}(\beta_\lambda). \quad (5)$$

By expanding the second and third terms of (5) about the new equilibrium position $\beta_\lambda(IN)$, keeping only the harmonic term for the new potential and defining a new independent variable y by

$$y=Z_1[\beta_\lambda-\beta_\lambda(IN)]/\beta_\lambda(IN),$$

and dependent variable $D_\nu(\sqrt{2}y)$ by

$$D_\nu(\sqrt{2}y)=\beta_\lambda{}^{3/2}\Phi_n{}^{IN}(\beta_\lambda),$$

then (5) can be placed in the form

$$[d^2D_\nu(\sqrt{2}y)/dy^2]+(2\nu+1-y^2)D_\nu(\sqrt{2}y)=0,$$

which is just Weber's equation.[15] Here ν is a real, but not necessarily integral, quantum number determined by the boundary condition

$$\Phi_n{}^{IN}(\beta_\lambda=0)\neq\infty,$$

and Z_1 is a known function of ν.[4]

III. REDUCED $E2$ TRANSITION PROBABILITIES

For the reduced transition probability we use[16]

$$B_l(INn\rightarrow I'N'n')=[1/(2I+1)]\times\textstyle\sum_{MM'}|\langle I'N'n'M'|Q_{l\mu}{}^{\text{Lab}}|INnM\rangle|^2, \quad (6)$$

where B_l is for the lth-order transition from the state INn to the state $I'N'n'$ and $Q_{l\mu}{}^{\text{Lab}}$ is the appropriate transition operator as seen in the space-fixed or laboratory coordinate system. The state vectors used are products of the rotation and vibration functions discussed above. The laboratory transition operator is related to the body-fixed operator by

$$Q_{l\mu}{}^{\text{Lab}}=\textstyle\sum_\nu Q_{l\nu}{}^{\text{body}}D_{\mu\nu}{}^{l*}(\theta_i).$$

The electric-quadrupole operator in the body-fixed coordinate system is

$$Q_{2\mu}=\left[\frac{4\pi}{5}\right]^{1/2}\int r^2Y_{2\mu}(\theta,\phi)\rho_e(r)d\tau, \quad (7)$$

where ρ_e is the static charge density and the integration is over the nuclear volume, or $\rho_e=3Ze/4\pi R_0{}^3$, and integrating from zero to the nuclear surface in r and using (1B), Eq. (7) becomes

$$Q_{2\mu}=(3ZeR_0{}^2/4\pi)(4\pi/5)^{1/2}\{a_{\lambda\mu}\delta_{2\lambda}+(5/\pi)^{\frac{1}{2}}\textstyle\sum_{\mu'}a_{\lambda\mu'}a_{\lambda\mu'-\mu}\times C(2\lambda\lambda;\mu,\mu'-\mu,\mu')C(2\lambda\lambda;000)\}, \quad (8)$$

where the $C(I_1I_2I;\mu_1\mu_2\mu)$ coefficients are Clebsch-Gordan coefficients.[12] For the positive-parity case, (8) becomes

$$Q_{2\mu}{}^{(2)}=(3ZeR_0{}^2/4\pi)[4\pi/5]^{1/2}a_{2\mu}, \quad (9a)$$

while for the negative-parity case

$$Q_{2\mu}{}^{(3)}=-(\tfrac{3}{5})^{1/2}(ZeR_0{}^2/\pi)\textstyle\sum_{\mu'}a_{3\mu'}a_{3\mu'-\mu}\times C(233;\mu,\mu'-\mu,\mu'). \quad (9b)$$

The $a_{\lambda\mu}$ are taken as real and written in the form (3). Substituting the quadrupole operator (9a) or (9b) into Eq. (6),

$$\begin{aligned}B(E2{:}\,INn\rightarrow I'N'n')&=[1/(2I+1)]\sum_{MM'}|\sum_\nu\langle I'N'M'|D^2{}_{\mu\nu}g_\nu(\sigma_{\lambda\rho})|INM\rangle|^2\\&\quad\times|\langle\phi_{n'}{}^{I'N'}|f(\beta_\lambda)|\phi_n{}^{IN}\rangle|^2\\&\equiv B_a(E2{:}\,IN\rightarrow I'N')S_{\nu\nu'}.\end{aligned} \quad (10)$$

Here $B_a(E2{:}\,IN\rightarrow I'N')$ is the adiabatic or pure-rotational reduced transition probability, and $S_{\nu\nu'}$ is the vibration contribution. The functions $g_\nu(\sigma_{\lambda\rho})$ and $f(\beta_\lambda)$ are those functions of the asymmetry and deformation parameters, respectively, which result from expressing the quadrupole operator in terms of the collective parameters: in particular $f(\beta_2)=\beta$ and $f(\beta_3)=\beta_3{}^2=\zeta^2$.

The rotational contribution is well known as a sum over Clebsch-Gordan coefficients in each case[1,3] and has been machine-calculated for numerous sets $(IN,I'N')$ as a function of the appropriate asymmetry parameters.

The vibrational contribution can be written in the form

$$S_{\nu\nu'}{}^{1/2}=N_\nu N_{\nu'}\int_0^\infty D_\nu\left(\sqrt{2}\left[\frac{Z_1}{\beta_\nu}\beta_\lambda-Z_1\right]\right)\times\beta_\lambda{}^MD_{\nu'}\left(\sqrt{2}\left[\frac{Z_1}{\beta_{\nu'}}\beta_\lambda-Z_1'\right]\right)d\beta_\lambda,$$

where

$$\begin{aligned}M&=1,\quad\lambda=2,\quad\pi^+\\&=2,\quad\lambda=3,\quad\pi^-.\end{aligned}$$

N is a normalization constant and can be written in terms of these same parameters and a normalization integral I_ν as

$$N_\nu{}^2=Z_1/\mu\beta_0ZI_\nu,$$

μ being the stiffness parameter of the nucleus, being (apart from a factor $\sqrt{2}$) the ratio of the deformation of a pure vibrator to that of a rotor-vibrator and Z is the positive real root of

$$Z^4-(1/\mu)Z^3-\tfrac{1}{2}(\epsilon_{IN}{}^\lambda+\tfrac{3}{2})=0,$$

and $\beta_\nu=\beta_\nu(IN)$ the new equilibrium deformation. By defining the ratio $R_z\equiv Z_1'Z/Z_1Z'$, we can rewrite the

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vibrational contribution as

$$S_{\nu\nu'}=(Z_1Z_1'/ZZ'I_\nu I_\nu')(Z/Z_1)^{2M+2}(\mu\beta_0)^{2M}I_{\nu\nu'},$$

with

$$I_{\nu\nu'}=\int_0^\infty D_\nu(\sqrt{2}[y-Z_1])y^M D_{\nu'}(\sqrt{2}[R_z y-Z_1'])dy.$$

In actual practice we calculate only the ratios of the reduced matrix elements so that we need evaluate only

$$\frac{S_{\nu\nu'}}{S_{\nu\nu''}}=\left(\frac{Z_1'Z''}{Z_1'Z_1''}\right)\frac{I_{\nu''}}{I_{\nu'}}\frac{I_{\nu\nu'}{}^2}{I_{\nu\nu''}{}^2}.$$

Since the Weber functions are in general not available in tabular form, these integrals have been calculated numerically by computer.

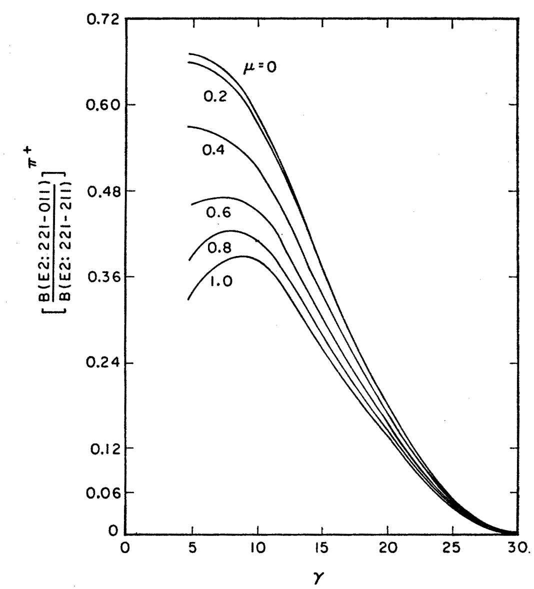

FIG. 1. Ratio of reduced $E2$ transition probabilities for the two π^+ transitions 221 to 011 and 221 to 211 plotted against the asymmetry parameter γ for various values of μ.

IV. DISCUSSION

In Figs. 1, 2, 3, and 4 are displayed the ratio of complete reduced matrix elements for $E2$ transitions both within a vibrational band ($\Delta n=0$) and between two adjacent bands ($\Delta n=1$) both for positive- and negative-parity states. They are plotted as a function of γ and μ for transitions between positive-parity states and as a function of η and μ for transitions between negative-parity states. For μ zero the curves represent the ratios for a rigid nucleus.[1,3] Figure 1 is the ratio of the reduced matrix elements $B(E2{:}\,221\rightarrow011)/B(E2{:}\,221\rightarrow211)$ from the "gamma" to the ground-state band. It is a strong function of γ but shows only a slight μ dependence. It is plotted only to $\gamma=5°$ since the 221 energy becomes infinite as γ vanishes. Figure 2 is the transition

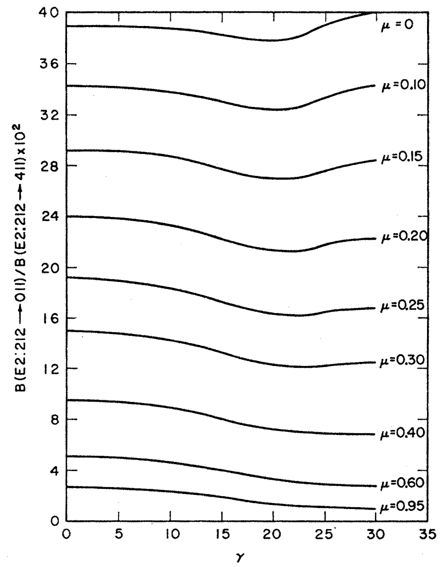

FIG. 2. Ratio of reduced $E2$ transition probabilities for the two π^+ interband transitions 212 to 011 and 212 to 411 plotted against the asymmetry parameter γ for various values of μ.

ratio $B(E2{:}\,212\rightarrow011)/B(E2{:}\,212\rightarrow411)$ from the beta-vibrational band to the ground-state band. This ratio shows only a slight γ dependence but a strong μ dependence. This is a general feature of transition ratios between the beta and ground bands.

Figures 3 and 4 are similar ratios of $E2$ transitions between negative-parity bands. Figure 3 is the ratio $B(E2{:}\,321\rightarrow311)/B(E2{:}\,321\rightarrow111)$ from the negative-parity analog of the "gamma" band (sometimes called the "g" band) to the ground-state negative-parity band.

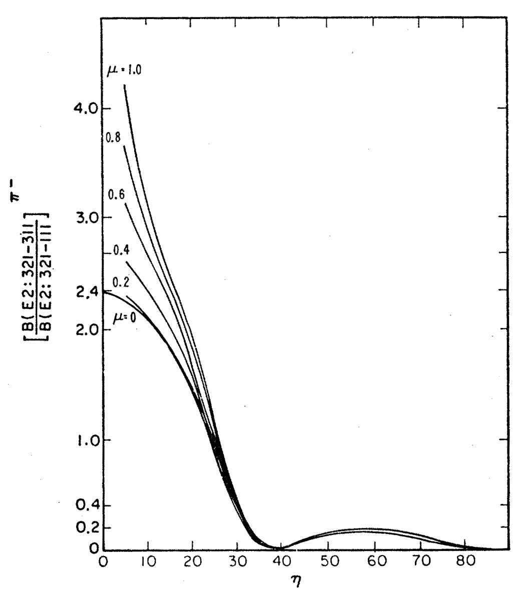

FIG. 3. Ratio of reduced $E2$ transition probabilities for the two π^- transitions 321 to 311 and 321 to 111 plotted against the asymmetry parameter η for various values of μ.

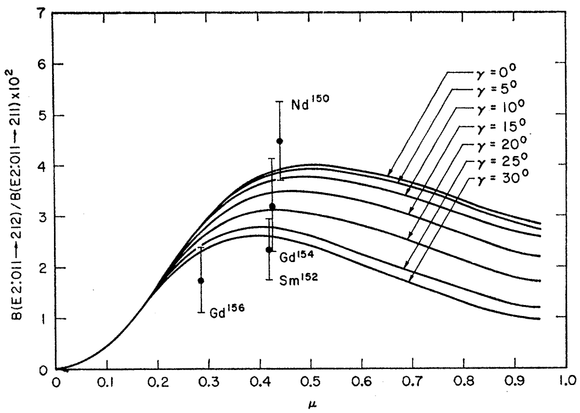

FIG. 4. Ratio of reduced $E2$ transition probabilities for the interband transitions 312 to 111 and 312 to 112 plotted against the stiffness parameter μ for different values of η.

This ratio is a strong function of the octupole asymmetry parameter η but shows only a slight μ dependence. The opposite situation is shown in Fig. 4, which gives the interband transition ratio $B(E2\colon 312 \to 111)/B(E2\colon 312 \to 112)$, that is, for transitions from the zeta-vibrational band (the octupole analog of the beta-vibrational band) to the ground-state negative-parity band. A strong μ and a slight η dependence is evident.

Figure 5 represents the ratio of Coulomb excitation from the ground state to the first 2^+ states in the beta- and ground-state bands, that is, the ratio $B(E2\colon 011 \to 212)/B(E2\colon 011 \to 211)$. As with other interband transitions, the μ dependence is much more marked than is the γ dependence. This figure also shows several recently measured Coulomb-excitation ratios and quoted errors[17] for nuclei near the lower edge of the rare-earth deformed region. The values of μ have been assigned in each case from the ratio of the energy of the beta band to the energy of the first excited state [i.e., from $E(012)/E(211)$].

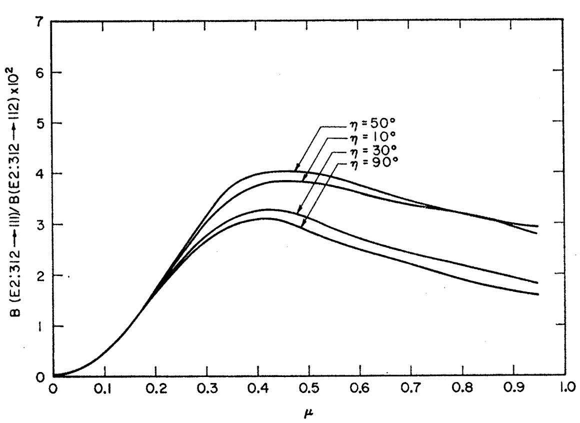

FIG. 5. Ratio of Coulomb-excitation transition probabilities for the excitation of the lowest 2^+ states in the ground and beta-vibrational bands from the 0^+ ground state plotted as a function of μ for different values of γ. The experimental values and errors for Nd^{150}, Sm^{152}, Gd^{154}, and Gd^{156} are from Ref. 17. The value of μ has been assigned from the energy ratio $E(012)/E(211)$.

In Table I we have compared this theory with experiment[6,17–21] and the adiabatic ratios for several $E2$ transitions in both positive- and negative-parity bands in W^{182} and Th^{228} and interband transitions in Sm^{152}. In W^{182} there are two high-lying 2^+ states below the first 3^+ state either of which could be identified with the second 2^+ state of the ground-state vibrational band. Choosing the lower 2^+ state as the (212) level and the upper as the (221) state, which satisfies the model criterion

$$E(21)+E(22) \geq E(31)\,, \qquad (11)$$

yields a better fit to the level energies than the opposite choice; however, the fit to the $E2$ branching ratios, especially the ratios $B(E2\colon 221 \to 211)/B(E2\colon 221 \to 411)$ and $B(E2\colon 212 \to 211)/B(E2\colon 212 \to 411)$, is very poor. Thus we have chosen the lower 2^+ state as the (221) state, which violates (11), and the other as the (212) state and obtained only a slightly poorer fit to the level energies while bringing the branching-ratio predictions into line with the experimental data. The level designated here as the (221) level has been interpreted as a 1^- level[22]; however, this assignment does not fit into the negative-parity, collective-model systematics.[3] Also Harmatz *et al.*[18] have made the assignment as we have and for similar reasons.

In Os^{186}, it has been noted,[23] one can fit the level energies at gamma of about 16.5°, but the fit to the $E2$ branching ratios is quite poor. On the other hand, one can obtain reasonable branching ratio values including the vibrational effects with γ between 10 and 13°, but then the fit to the energy levels is very poor. Unfortunately, μ has been obtained only from the ground and gamma-band energies which is the poorest method of determining this parameter. It is more uniquely fit from a knowledge of the (012) or (212) levels which are not identified experimentally. Until this is done, it is only possible to state that for this end of the rare-earth deformed region the model is not consistent with experiment. However, for the other end of this same region it is as is seen from the comparison between theory and

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TABLE I. Comparison between experiment and theory for $E2$ branching ratios of positive- and negative-parity bands of W^{182} and Th^{228} and for positive-parity bands of Sm^{152}. The first column gives the initial and final states for the transitions where $I_iN_in_i - I_fN_fn_f$; I is the spin of a level, N the ordinal of the level, and n the ordinal of the vibration band. The second and third columns give experimental values and references for Sm^{152}, W^{182}, and Th^{228}; the fourth column, the adiabatic ratio, while the fifth column gives the ratio including the vibrational contribution.

	Experiment		Theory	
Transitions	$B(E2)$	Ref.	$B_a(E2)$	$B(E2)$
Sm^{152}, π^+, $\gamma=11.3°$, $\mu=0.396$				
$011\to212$ / $011\to211$	0.023 ± 0.006	17	1.0	0.036
$212\to411$ / $212\to211$	$6.7\ \pm1.8$	6	1.81	4.22
$212\to011$ / $212\to411$	0.048 ± 0.015	6	0.386	0.088
$221\to011$ / $221\to211$	$0.44\ \pm0.02$	21	0.502	0.453
W^{182}, π^+, $\gamma=10.93°$, $\mu=0.28$[a]				
$221\to011$ / $221\to211$	0.69 / 0.70	18 / 19	0.522	0.507
$221\to211$ / $221\to411$	1.[b]	18	11.52	10.96
$212\to011$ / $212\to211$	0.57	18	0.708	0.600
$212\to211$ / $212\to411$	0.833	18	0.547	0.405
$421\to211$ / $421\to411$	0.2	18	0.158	0.150
$421\to411$ / $421\to611$	1.[b]	18	5.21	4.98
$621\to411$ / $621\to611$	0.17	18	0.068	0.066

	Experiment		Theory	
Transitions	$B(E2)$	Ref.	$B_a(E2)$	$B(E2)$
W^{182}, π^-, $\eta=83.5°$, $\mu=1.0$				
$411\to211$ / $411\to311$	0.883 / 0.556	18 / 19	0.631	0.563
$421\to211$ / $421\to311$	1. / 1.28	18 / 19	6.86	6.11
Th^{228}, π^+, $\gamma=9.1°$, $\mu=0.30$				
$221\to411$ / $221\to211$	$0.09\ \pm0.02$	20	0.073	0.080
$221\to211$ / $221\to011$	$2.32\ \pm0.28$	20	1.72	1.81
$421\to611$ / $421\to411$	≤0.25	20	0.151	0.165
$421\to411$ / $421\to211$	$6.25\ \pm0.8$	20	4.66	5.12
$311\to411$ / $311\to211$	$0.66\ \pm0.08$	20	0.600	0.658
Th^{228}, π^-, $\eta=12.3°$, $\mu=0.258$				
$211\to311$ / $211\to111$	<0.3	20	0.212	0.215
$321\to111$ / $321\to311$	$0.36\ \pm0.04$	20	0.502	0.495
$411\to511$ / $411\to311$	$0.75\ \pm0.02$	20	0.371	0.379

[a] Experimental errors are not given for the transition ratios of W^{182}.
[b] Unobserved.

experiment for Sm^{152} in Table I, and the Coulomb excitation data[17] shown in Fig. 5. It is clear then that an adequate test of these collective models must include the vibrational contributions both to the energy-level systematics and the electromagnetic transition probabilities.

ACKNOWLEDGMENTS

We wish to thank Dr. S. A. Williams for discussions and suggestions concerning the programming of the numerical calculations during the early phases of this work and Dr. Diamond and Dr. Stephens for several interesting discussions concerning the experimental situation particularly in the actinide region. We also wish to thank the Trustees of Rensselaer Polytechnic Institute for a grant which made possible some of the necessary computer calculations. Finally, one of us (J.P.D.) wishes to express his appreciation to Professor Perlman and Professor Rasmussen of the Lawrence Radiation Laboratory for their kind hospitality during his stay there.