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Vector-Pseudoscalar Resonances

MAHIKO SUZUKI
Department of Physics, University of Tokyo, Tokyo, Japan
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Higher meson resonances are investigated in simple dynamical models based on the $SU(3)$ symmetry with and without recourse to the A-conservation rule. The present models consider the vector-pseudoscalar systems as their main configurations. The predicted level schemes are rather sensitively dependent on the trilinear self-coupling of the octuplet vector mesons. One of the main results is that there should exist a low-lying 1^+ singlet and a 1^+ octuplet probably above it. The existing higher meson resonances are tentatively assigned to the levels predicted here.

1. INTRODUCTION

THE octuplet version of unitary symmetry[1,2] has achieved remarkable success in classifying the strongly interacting particles. It predicts that the pseudoscalar mesons $(\pi,K,\bar{K},\eta)$ and the vector mesons $(\rho,K^*,\bar{K}^*,\phi)$ should belong to the octuplet representation of $SU(3)$, and that ω should be a unitary singlet. From the dynamical viewpoint, the so-called bootstrap calculations[3-9] have thrown light on the mechanism of producing these mesons as composite particles and have led us to a qualitative understanding.

Quite recently, invariant mass observations of mesonic matter have revealed new resonances one after another. A remarkable feature of the recent observations is evident in the many-boson resonances. They began with the $\pi\omega$ resonance called B,[10,11] then the $K\bar{K}\pi$ resonance,[12] the $\pi\rho$ resonance by Goldhaber *et al.*,[13] the $\eta\pi\pi$ resonance X,[14,15] the $K\pi\pi$ resonance around 1175 MeV,[16] the other $K\pi\pi$ resonance C around 1230 MeV,[17] the two $\pi\rho$ resonances[18,19] which may be a precise version of the single peak observed by Goldhaber *et al.*, and probably others. Except for A_2 (the $\pi\rho$ resonance around 1310 MeV), the isospin has not yet been determined for them. Neither spin nor parity has yet been determined for A_1 (the $\pi\rho$ resonance around 1090 MeV), B, or X. However, we should notice that most of the resonances listed above appear to be interpretable in terms of vector-pseudoscalar resonances.

As for B, several discussions[20-22] were presented according to simple versions of bootstrapping without recourse to unitary symmetry. There are also some discussions[23] based on unitary symmetry. Their spin-parity assignments are not always in agreement with each other.

In the present paper we shall investigate these resonances from the viewpoint of unitary symmetry by means of a simple version of bootstrapping calculations. We shall construct dynamical models of the vector-pseudoscalar resonances and derive general features of the resonance levels on the basis of unitary symmetry. Quantitative results will also be given through a numerical estimate. After predicting degenerate resonance levels of $SU(3)$ supermultiplets, we shall include a symmetry-breaking interaction in a simple way and deduce the signs of physical parameters in the Gell-Mann–Okubo formulas for predicted supermultiplets. Finally we shall try to identify the existing resonances with the members of the predicted supermultiplets.

2. MODELS

We shall investigate the vector-pseudoscalar resonances in two dynamical models. Throughout the present paper all the mesons in input diagrams will be treated as stable particles, since we do not know an appropriate way of taking account of instability of the particles. We shall hereafter abbreviate the meson

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families as follows:

$$P_8:\quad (\pi,K,\bar{K},\eta)\,,$$
$$V_8:\quad (\rho,K^*,\bar{K}^*,\phi^0)\,,$$
$$V_1:\quad (\omega^0)\,.$$

V_1 and the eighth component of V_8, namely ω^0 and ϕ^0, may not be identified with the observed ω and ϕ, owing to the $\phi\omega$ mixing.[24,25] We consider both the V_8P_8 resonances and the V_1P_8 resonances in the following.

Some of the existing VP_8 resonances are not coupled with the P_8P_8 channel, since the invariant-mass plots do not exhibit any peak in the P_8P_8 channel. For the A_2 meson, peaks were also observed in the $K\bar{K}$ and $\pi\rho$ systems, but the branching ratios appear not to be very large. In order to avoid complications, we shall provisionally neglect the channel coupling between VP_8 and P_8P_8. According to the principle of taking lower configurations, the channels of two-vector mesons will also be neglected in the present models. Another configuration which is expected to be coupled in is the baryon-antibaryon channel. However, we do not take account of it, following the same principle. A unitary symmetric treatment of the Fermi-Yang model[26] will be discussed in a separate paper. Thus we shall consider the V_8P_8 and V_1P_8 channels only. One-meson exchange processes will be taken exclusively as input forces. Let us further specify our models with and without recourse to the A-conservation rule proposed by Barton, and Bronzan, and Low.[27]

A. Model A

Since we assume that forces between V_8 and P_8 arise from one-meson exchange processes, they are due to P_8 exchange, V_1 exchange and V_8 exchange in the u channel, and V_8 exchange in the t channel. Among them the P_8 exchange produces the force of longest range, or the nearest left-hand singularity in a dispersion-theoretical term. The V_1 exchange also produces a sizable force. In the V_8 exchange in the u channel, the $V_8V_8P_8$ vertex appears in the Born amplitude. If we identify the observed ϕ with the eighth member of V_8, neglecting $\phi\omega$ mixing, we can deduce the $V_8V_8P_8$ coupling constant from the partial width of ϕ into $\rho\pi$. By comparing it with $V_1V_8P_8$ coupling, which is estimated from the width of ω by use of an $\omega\rightarrow\rho\pi$ model, we find that the squared $V_8V_8P_8$ coupling is smaller almost by two orders of magnitude than the squared $V_1V_8P_8$ coupling. The A-conservation rule recently proposed by Bronzan and Low[27] predicts that the $V_8V_8P_8$ coupling should be zero. It acquires a nonzero value owing to the coupling with the $B\bar{B}$ states. Thus, neglecting the $V_8V_8P_8$ coupling is consistent with A conservation. Model A assumes A conservation. It reduces to a single-channel problem. For the V_8 exchange in the t channel, we cannot estimate the $V_8V_8V_8$ coupling constant appearing in it in any reliable way. It is a far-away singularity in comparison with the P_8 exchange. As will be seen later, however, we have reason to expect that the V_8 exchange in the t channel contributes a considerable amount, owing to the dynamical structure of the corresponding Born amplitude. By assuming V_8 to be the unitary gauge mesons, we can connect the $V_8V_8V_8$ coupling to the $V_8P_8P_8$ coupling by the use of gauge theory.[28–30]

In this model the V_1P_8 channel will not be coupled with the V_8P_8 channel. It reduces to a single-channel problem in the V_1P_8 resonances, as long as one neglects the P_8P_8, the V_8V_8, and the $B\bar{B}$ channels.

B. Model B

This model does not invoke the A-conservation rule. Since the observed ϕ and ω are mixtures of the pure component of the octuplet ϕ^0 and the pure singlet ω^0, it does not necessarily follow from the smallness of the $\phi\rho\pi$ coupling that the $V_8V_8P_8$ coupling should be small. Actually one can estimate the $V_8V_8P_8$ coupling from $g_{\phi\rho\pi}=0$, if one knows the mixing angle of the $\phi\omega$ mixing. Several authors have deduced the mixing angle from different viewpoints. If one takes $\theta=30°$, as has often been done, the squared $V_8V_8P_8$ coupling turns out to be nearly one-fifth of the squared $V_1V_8P_8$ coupling with appropriate normalizations. When one adopts such a large mixing angle, the channel coupling of V_8P_8 with V_1P_8 can no longer be neglected. We shall therefore investigate the two-channel problem of the VP_8 resonances in model B.

In these models we shall look for the VP_8 resonances as outputs of the one-meson exchange forces. A complete bootstrap prescribes that the output resonances should again be put in as origins of force in the left-hand cuts, or should be bootstrapped. However, we shall stop at the first stage of bootstrapping to see only general tendencies of the level shifts of the predicted resonances when outputs other than P_8, V_8, and V_1 are put in as origins of force.

As is known, numerical values of the levels and widths are dependent on the cutoff energies in dispersion integrals and especially on the coupling constants involved. Moreover, since the breaking of unitary symmetry is not taken into account, predicted levels are something like level centers of $SU(3)$ supermultiplets. Thus, precise evaluation of the levels may not be of much importance.

3. BORN AMPLITUDES

We shall give the Born amplitudes, following the helicity representation by Jacob and Wick.[31] Relevant

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interactions will be written down as follows:

$$(F/2)3^{-1/2}f_{ijk}P_i\overleftrightarrow{\partial}_\mu P_j V_{k,\mu} \tag{3.1}$$

for the $P_8P_8V_8$ vertex;

$$(G/m)8^{-1/2}\epsilon_{\mu\nu\kappa\lambda}P_i\partial_\mu V_{i,\nu}\partial_\kappa V_\lambda \tag{3.2}$$

for the $P_8V_8V_1$ vertex;

$$(H/2)3^{-1/2}f_{ijk}V_{i,\mu}\overleftrightarrow{\partial}_\nu V_{j,\mu}V_{k,\nu} \tag{3.3}$$

for the $V_8V_8V_8$ vertex; and

$$(K/2m)(3/5)^{1/2}d_{ijk}\epsilon_{\mu\nu\kappa\lambda}P_i\partial_\mu V_{j,\nu}\partial_\kappa V_{k,\lambda} \tag{3.4}$$

for the $P_8V_8V_8$ vertex. In these expressions the Latin and the Greek subscripts refer to the unitary spin and the Lorentz variable, respectively. We have neglected the $\partial_\mu V_\nu \partial_\nu V_\kappa \partial_\kappa V_\mu$ coupling for $V_8V_8V_8$. If one requires that V_8 be the unitary gauge mesons, then H is related to F as $H=F$. (See Fig. 1.)

We have four independent helicity amplitudes among the Born amplitudes between eigenstates of helicity. First we shall give the results in the case of the V_8P_8 channel. Straightforward calculations lead us to the following:

$$(T_{\beta,\alpha})_{il,jm}=(f_{imk}f_{jlk}/3)T_{\beta,\alpha}, \tag{3.5}$$

$$\begin{aligned}
T_{00}&=(F^2/4\pi)(2p^3/m^2W)\{(E_1\cos\theta+E_2)^2\}/(\mu^2+u),\\
T_{+0}&=(F^2/4\pi)(2^{1/2}p^3/mW)\\
&\quad\times\{(E_1\cos\theta+E_2)\sin\theta\}/(\mu^2+u),\\
T_{++}&=-(F^2/4\pi)(p^3/W)\sin^2\theta/(\mu^2+u),\\
T_{+-}&=(F^2/4\pi)(p^3/W)\sin^2\theta/(\mu^2+u),
\end{aligned} \tag{3.6}$$

for P_8 exchange in the u channel;

$$(T_{\beta,\alpha})_{il,jm}=(\delta_{im}\delta_{jl}/8)T_{\beta,\alpha}, \tag{3.7}$$

$$\begin{aligned}
T_{00}&=-(G^2/4\pi)(p^3/2W)\sin^2\theta/(M^2+u),\\
T_{+0}&=(G^2/4\pi)(p^3/8^{1/2}mW)\\
&\quad\times\{(E_1\cos\theta+E_2)\sin\theta\}/(M^2+u),\\
T_{++}&=(G^2/4\pi)(p^3/4m^2W)\\
&\quad\times\{(E_1\cos\theta+E)^2+(E_1^2+E_2^2)\cos\theta\\
&\qquad+2E_1E_2+p^2\sin^2\theta\}/(M^2+u),\\
T_{+-}&=(G^2/4\pi)(p^3/4m^2W)\\
&\quad\times\{-(E_1\cos\theta+E_2)^2+(E_1^2+E_2^2)\cos\theta\\
&\qquad+2E_1E_2+p^2\sin^2\theta\}/(M^2+u),
\end{aligned} \tag{3.8}$$

for V_1 exchange in the u channel;

$$(T_{\beta,\alpha})_{il,jm}=(f_{ijk}f_{lmk}/3)T_{\beta,\alpha}, \tag{3.9}$$

$$\begin{aligned}
T_{00}&=(FH/4\pi)(p/2m^2W)\\
&\quad\times\{(E_1^2\cos\theta-p^2)(2p^2(1+\cos\theta)+4E_1E_2)\\
&\quad+4p^2E_1(1-\cos\theta)(E_1(1+\cos\theta)+2E_2)\}/(m^2+t),\\
T_{+0}&=(FH/4\pi)(p/2^{1/2}mW)\\
&\quad\times\{E_1(p^2(1+\cos\theta)+2E_1E_2)+2p^2(E_1+E_2)\\
&\qquad-4p^2(E_1\cos\theta+E_2)\}\sin\theta/(m^2+t),\\
T_{++}&=(FH/4\pi)(p/W)\\
&\quad\times\{(1+\cos\theta)(2p^2(1+\cos\theta)+4E_1E_2)\\
&\qquad+4p^2\sin^2\theta\}/(m^2+t),\\
T_{+-}&=(FH/4\pi)(p/W)\\
&\quad\times\{(1-\cos\theta)(2p^2(1+\cos\theta)+4E_1E_2)\\
&\qquad-4p^2\sin^2\theta\}/(m^2+t),
\end{aligned} \tag{3.10}$$

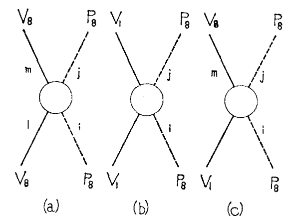

FIG. 1. (a) The V_8P_8 scattering. (b) The V_1P_8 scattering. (c) The channel coupling between V_8P_8 and V_1P_8.

for V_8 exchange in the t channel; and

$$(T_{\beta,\alpha})_{il,jm}=(3d_{imk}d_{ljk}/5)T_{\beta,\alpha}, \tag{3.11}$$

$$T_{\beta,\alpha}=\text{Eq. (3.8)}\ (G^2\to K^2) \tag{3.12}$$

for V_8 exchange in the u channel. Throughout all the cases

$$\begin{aligned}
T_{++}&=T_{--},\\
T_{+-}&=T_{-+},\\
T_{+0}&=T_{-0}=T_{0+}=T_{0-}.
\end{aligned} \tag{3.13}$$

In these expressions α and β denote the helicities of V_8 in the initial and the final state, respectively. W and p are the energy and momentum in the c.m. system, θ being the scattering angle in the same system. In addition

$$\begin{aligned}
E_1&=(p^2+m^2)^{1/2},\\
E_2&=(p^2+\mu^2)^{1/2},\\
u&=2p^2(1+\cos\theta)-(E_1-E_2)^2,\\
t&=2p^2(1-\cos\theta),
\end{aligned}$$

where μ, m, and M are the masses of P_8, V_8, and V_1, respectively. Since we do not take account of broken effects in unitary symmetry, they are represented by the respective mean values.

For the V_1P_8 channel, the Born amplitudes are given simply by

$$(T_{\beta,\alpha})_{i,j}=(\delta_{ij}/8)T_{\beta,\alpha}, \tag{3.14}$$

$$T_{\beta,\alpha}=\text{Eq. (3.8)}(m\rightleftarrows M)\times(M/m)^2. \tag{3.15}$$

The Born amplitudes for $V_8P_8\leftrightarrow V_1P_8$ are given by

$$(T_{\beta,\alpha})_{i,jm}=[(3/40)^{1/2}d_{imk}\delta_{jk}]T_{\beta,\alpha}. \tag{3.16}$$

$T_{\beta,\alpha}$, although a little complicated in a precise form, is given by Eq. (3.8) (in which G^2 is replaced by GK) in the approximation that neglects the V_8-V_1 mass difference. This suffices for the numerical estimate in the following sections.

Finally, we give the Born amplitudes in the s channel. From the viewpoint of the bootstrap philosophy in

S-matrix theory, it is desirable to produce all the particles, including the input ones, in a unified manner. However, a once-subtracted dispersion relation is, in general, not capable of predicting all the resonances correctly when masses of particles to be produced range widely from low to high. Indeed, if one chooses a subtraction point so that the mass of P_8 may fit the experimental value, one will not be able to obtain correct values for higher VP_8 resonances, and vice versa. Since the present purpose is to investigate the higher VP_8 resonances, we have to give up reproducing P_8, V_8, and V_1 at the numerically correct places. Therefore we put in the s-channel Born amplitudes. They are given by

$$(T_{\beta,\alpha})_{il,jm}=(f_{ilk}f_{jmk}/3)T_{\beta,\alpha}\,, \tag{3.17}$$

$$\begin{aligned}T_{00}&=(F^2/4\pi)(2p^3/m^2W)\{s/(\mu^2-s)\}\,,\\ T_{+0}&=T_{++}=T_{+-}=0\,,\end{aligned} \tag{3.18}$$

for P_8 in the V_8P_8 channel;

$$(T_{\beta,\alpha})_{il,jm}=(3d_{ilk}d_{jmk}/5)T_{\beta,\alpha}\,, \tag{3.19}$$

$$\begin{aligned}T_{++}&=(K^2/4\pi)(p^3/4m^2W)\{s(1+\cos\theta)/(m^2-s)\}\,,\\ T_{+-}&=-(K^2/4\pi)(p^3/4m^2W)\\ &\qquad\times\{s(1-\cos\theta)/(m^2-s)\}\,,\\ T_{00}&=T_{+0}=0\,,\end{aligned} \tag{3.20}$$

for V_8 in the V_8P_8 channel;

$$(T_{\beta,\alpha})_{il,jm}=(\delta_{il}\delta_{jm}/8)T_{\beta,\alpha}\,, \tag{3.21}$$

$$T_{\beta,\alpha}=\text{Eq. (3.20)}\ (K^2\to G^2) \tag{3.22}$$

for V_1 in the V_8P_8 channel;

$$(T_{\beta,\alpha})_{i,j}=(\delta_{ik}\delta_{jk}/8)T_{\beta,\alpha}\,, \tag{3.23}$$

$$T_{\beta,\alpha}=\text{Eq. (3.20)}\ (K^2\to G^2) \tag{3.24}$$

for V_8 in the V_1P_8 channel; and

$$(T_{\beta,\alpha})_{i,jm}=((3/40)^{1/2}d_{jmk}\delta_{ik})T_{\beta,\alpha}\,, \tag{3.25}$$

$$T_{\beta,\alpha}=\text{Eq. (3.20)}\ (K^2\to GK) \tag{3.26}$$

for V_8 in the $V_1P_8\leftrightarrow V_8P_8$ channel.

According to the general prescriptions[31] we can extract the partial-wave amplitudes with the definitions

$$t_{\beta,\alpha}(J)=\int d\cos\theta\; T_{\beta,\alpha}(\cos\theta)d_{\alpha,\beta}{}^J(\cos\theta)\,, \tag{3.27}$$

$$J^P=0^-:\quad t_{00}(0)\,, \tag{3.28}$$

$$J^P=1^-:\quad t_{++}(1)-t_{+-}(1)\,, \tag{3.29}$$

$$J^P=1^+:\quad \begin{pmatrix} t_{00}(1) & 2^{1/2}t_{+0}(1)\\ 2^{1/2}t_{+0}(1) & t_{++}(1)+t_{+-}(1)\end{pmatrix}, \tag{3.30}$$

$$J^P=2^+:\quad t_{++}(2)-t_{+-}(2)\,, \tag{3.31}$$

$$J^P=2^-:\quad \begin{pmatrix} t_{00}(2) & 2^{1/2}t_{+0}(2)\\ 2^{1/2}t_{+0}(2) & t_{++}(2)+t_{+-}(2)\end{pmatrix}, \tag{3.32}$$

$$J^P=3^+:\quad \begin{pmatrix} t_{00}(3) & 2^{1/2}t_{+0}(3)\\ 2^{1/2}t_{+0}(3) & t_{++}(3)+t_{+-}(3)\end{pmatrix}, \tag{3.33}$$

and so on. Since the observed resonances are situated not too high above the thresholds, we shall hereafter concentrate our attention only on s, p, and d waves, or 0^-, 1^-, 1^+, 2^-, 2^+, and 3^+.

4. LEVEL SCHEMES

With the expressions for the Born amplitudes derived above, we could carry out detailed numerical calculations. There are, however, many ambiguities associated with the coupling constants, cutoff parameters for dispersion integrals, and the like. Among others we make the approximation of neglecting all the kinds of deviations from unitary symmetry: mass splitting in a single supermultiplet, and the deviation of coupling constants. In such a crude approximation, it is more advisable to look for as many qualitative features as possible, than to give the results of detailed numerical calculations.

First, we estimate the three coupling constants appearing in the preceding expressions. From the width of ρ into 2π,

$$(F^2/4\pi)\sim 6\,. \tag{4.1}$$

As for the $V_1V_8P_8$ coupling, we have

$$(G^2/4\pi)\sim 100 \tag{4.2}$$

from the ω decay width in the $\omega\to\rho\pi$ model,[32] where it should be noted that G is in units of the V_8 mass, not in pion mass units. This is equivalent to $(g_{\omega\rho\pi}{}^2/4\pi)=0.35$ in pion mass units. For H we resort to the gauge meson theory to get

$$(FH/4\pi)=(F^2/4\pi)\,. \tag{4.3}$$

We give here the crossing matrix elements of the unitary spins. The results are tabulated in Table I. The elements are trivial for V_1P_8 and $V_8P_8\leftrightarrow V_1P_8$.

From numerical calculations by many authors we know that the effective cutoff parameters in dispersion integrals are not very large. Let us, therefore tentatively make a low-energy approximation to the Born amplitudes, to see in which channels forces are strong. This is consistent with the present models in which VV, $B\bar{B}$, and higher configurations are neglected. It should be remarked here that the P_8-exchange amplitudes have imaginary parts since some of the V_8 can actually decay into some of the P_8P_8, $m>2\mu$. That is, the left-hand cut comes above the branch point of the elastic right-hand cut. Accordingly, we must define the P_8-exchange force as the principal part of the corresponding Born amplitude. Then a low-energy part of the P_8-exchange force is effectively cut off owing to the principal-value integral in the dispersion relation. We

[32] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

should replace the denominator μ^2+u in the P_8-exchange amplitude by some effective-force range parameter μ^{*2}. Since in $V_8 \to P_8P_8$ the Q value is small on the average, μ^{*2} will be rather small.

In such a low-energy approximation, and with $\mu/m = 1/2$ and $M=m$, the amplitudes projected onto each partial wave $t(J^P)$ is given, apart from the unitary crossing factors, by

(i) V_8P_8 *channel*

$$\begin{aligned}
J^P=0^-:&\quad t(0^-)\sim(14/9)(F^2/4\pi)(p^3/m\mu^{*2}),\\
J^P=1^-:&\quad t(1^-)\sim-(8/9)(F^2/4\pi)(p^3/m\mu^{*2}),\\
J^P=1^+:&\quad t(1^+)\sim\begin{pmatrix}8/9 & 4(2^{1/2}/9)\\ 4(2^{1/2}/9) & 0\end{pmatrix}(F^2/4\pi)(p^3/m\mu^{*2}),\\
J^P=2^-:&\quad t(2^-)\sim\begin{pmatrix}16/45 & 8(6^{1/2}/45)\\ 8(6^{1/2}/45) & 8/15\end{pmatrix}(F^2/4\pi)(p^3/m\mu^{*2}),\\
J^P=2^+:&\quad t(2^+)\sim O(p^5),\\
J^P=3^+:&\quad t(3^+)\sim O(p^5)
\end{aligned}\tag{4.4}$$

for P_8 exchange in the u channel;

$$\begin{aligned}
J^P=0^-:&\quad t(0^-)\sim-(16/27)(G^2/4\pi)(p^3/m^3),\\
J^P=1^-:&\quad t(1^-)\sim(4/9)(G^2/4\pi)(p^3/m^3),\\
J^P=1^+:&\quad t(1^+)\sim\begin{pmatrix}0 & 4(2^{1/2}/27)\\ 4(2^{1/2}/27) & 16/27\end{pmatrix}(G^2/4\pi)(p^3/m^3),\\
J^P=2^-:&\quad t(2^-)\sim\begin{pmatrix}16/135 & 8(6^{1/2}/135)\\ 8(6^{1/2}/135) & 8/45\end{pmatrix}(G^2/4\pi)(p^3/m^3),\\
J^P=2^+:&\quad t(2^+)\sim O(p^5),\\
J^P=3^+:&\quad t(3^+)\sim O(p^5)
\end{aligned}\tag{4.5}$$

for V_1 exchange in the u channel; and

$$\begin{aligned}
J^P=0^-:&\quad t(0^-)\sim 0,\\
J^P=1^-:&\quad t(1^-)\sim(4/3)(FH/4\pi)(p^3/m^3),\\
J^P=1^+:&\quad t(1^+)\sim(4/9)\begin{pmatrix}1 & 2^{1/2}\\ 2^1 & 2\end{pmatrix}(FH/4\pi)(p/m),\\
J^P=2^-:&\quad t(2^-)\sim(4/15)\begin{pmatrix}2 & 6^{1/2}\\ 6^{1/2} & 3\end{pmatrix}(FH/4\pi)(p^3/m^3),\\
J^P=2^+:&\quad t(2^+)\sim O(p^5),\\
J^P=3^+:&\quad t(3^+)\sim O(p^5)
\end{aligned}\tag{4.6}$$

for V_8 exchange in the t channel. For V_8 exchange in the u channel, all the amplitudes t are the same as those in Eq. (4.5).

(ii) V_1P_8 *channel*

We have only to substitute M for m in Eq. (4.5) and multiply by $(M/m)^2$ since $(G^2/4\pi)$ is now in the unit of m. Since, after all, we shall take $M=m$, the partial-wave amplitudes in the low-energy limit are given by Eq. (4.5) with $M=m$.

(iii) $V_1P_8 \leftrightarrow V_8P_8$ *channel*

Again in the approximation of $M=m$, the partial-wave Born amplitudes t are given by Eq. (4.5) with $M=m$.

We have only to multiply these amplitudes by the

TABLE I. Unitary-crossing-matrix elements for the V_8P_8 scattering.

	s channel				
	1	8_F	8_D	(10,10*)	27
$P_8(8_F)$ exchange in the u channel	-1	1/2	$-1/2$	0	1/3
$V_1(1)$ exchange in the u channel	1/8	$-1/8$	1/8	$-1/8$	1/8
$V_8(8_F)$ exchange in the t channel	1	1/2	1/2	0	$-1/3$
$V_8(8_D)$ exchange in the u channel	1	$-1/2$	$-3/10$	2/5	1/5
27 exchange in the u channel	27/8	9/8	27/40	9/40	7/40

factors due to unitary symmetry to obtain the full amplitudes in the irreducible channels decomposed according to $SU(3)$. It should be remarked that the 1^+ amplitudes due to the one-particle exchanges in the u channel tend to zero as $\mu/m \to 0$. As is easily seen in the above expressions, the 2^+ and the 3^+ amplitudes are too weak to produce a bound state or a low-lying resonance. VP_8 resonances are expected to appear in the 0^-, 1^-, 1^+, and 2^- channels. We shall calculate the level scheme without the low-energy approximation below.

Model A. Let us discuss the level scheme on the assumption of the A conservation rule. First consider the V_8P_8 scattering. It is true that the effective mass μ^* in the P_8 exchange may be quite small in the low-energy region, but when the P_8-exchange amplitudes are integrated over the squared c.m. energy in the D functions, we find that the smallness of μ^* is masked by the threshold behavior, which is like p^3. Thus all the kinds of Born amplitudes contribute to the dispersion integrals by comparable amounts. The predicted level scheme is sensitively dependent on the $V_8V_8V_8$ coupling constant. We have estimated it by assuming V_8 to be the unitary gauge mesons, since we can hardly know anything about it experimentally. We shall therefore give the results in two typical cases—V_8 being the gauge mesons, and vanishing $V_8V_8V_8$ coupling—to illustrate how much the level scheme depends on the $V_8V_8V_8$ coupling constant.

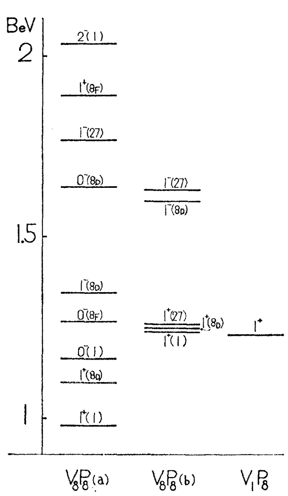

FIG. 2. The level schemes of the V_8P_8 and V_1P_8 resonances in Model A ($s_{\max}=8m^2$). V_8 is assumed to be the unitary gauge mesons in (a), while the $V_8V_8V_8$ coupling is zero in (b). The entry in the bracket denotes the representation of $SU(3)$.

We have used the once-subtracted dispersion relations for the partial-wave amplitudes, namely,

$$D_{ab}(s)=\delta_{ab}-\frac{s-s_0}{\pi}\int_{(m+\mu)}^{s_{\max}}\frac{\rho(s')N_{ab}(s')}{(s'-s)(s'-s_0)}ds', \quad (4.7)$$

where $\rho(s)N(s)$ is approximated by the Born amplitudes given in the preceding section, $\rho(s)$ being the phase-space p/W. The masses of bound states or resonances are given by the real parts of the roots of

$$\det D_{ab}(s)=0. \quad (4.8)$$

As is easily seen, the mass of any bound state that is produced is necessarily higher than $s_0^{1/2}$ for a simply behaved $N_{ab}(s)$. Since the present aim is directed at the VP_8 resonances above $\sim$1 BeV, not at the self-consistency of the (P_8,V_8,V_1) bootstrap, we have chosen $s_0=m^2$. Accordingly the level intervals near $s=m^2$ are considerably distorted. As for the cutoff parameter, we have chosen $s_{\max}=8m^2$ or about (2.5 BeV)^2 and by comparison $s_{\max}=6m^2$ or about (2 BeV)^2. The level scheme is insensitive to the cutoff energy. Indeed, the level shifts caused by a change of the cutoff energy are within a hundred MeV for lower levels and not larger than 200 MeV for higher levels. We have drawn the level schemes in the two typical cases side by side: the case in which the V_8 mesons are the gauge particles, and the case of vanishing $V_8V_8V_8$ coupling (see Fig. 2).

When V_8 is the gauge family, the V_8 exchange in the t channel dominates the other processes in the V_8P_8 scattering. The effect is especially strong in the 1^+ channels. Therefore, $1^+(1)$ and $1^+(8_D)$ produce bound states. One of the eigenchannels of $1^+(27)$ also becomes weakly attractive. In Model A, $1^-(8_D)$ should not be identified with the channel producing V_8. According to the A-conservation rule, V_8 must be produced in the 1^- channel of V_1P_8. When V_8 is the gauge meson family, $0^-(1)$ becomes attractive enough to produce a bound state. $0^-(8_D)$ also produces a resonance. It is the V_8 exchange in the t channel that is responsible for the attractive forces in $0^-(1)$ and $0^-(8_D)$. Therefore the bound state and the resonance in these channels disappear when the $V_8V_8V_8$ coupling becomes weaker. Since $0^-(8_F)$, which is the channel producing P_8, is

TABLE II. The bootstrap effects due to low-lying supermultiplet exchanges in the u channel in Model A. In the above entries, + and − denote an attractive and a repulsive force, respectively.

			Exchanged meson family								
			$0^-(1)$	$0^-(8_F)$	$0^-(8_D)$	$1^+(1)$	$1^+(8_D)$	$1^+(27)$	$1^-(8_D)$	$1^-(27)$	$1^+(V_1P_8)$
V_8P_8	0^-	1	+	−	+	−	−	−	−	−	
		8_F	−	+	−	+	+	+	+	−	
		8_D	+	−	−	−	+	−	+	−	
	1^+	1	+	−	+	+	+	+	+	+	
		8_D	+	−	−	+	−	+	−	+	
		27	+	+	+	+	+	+	+	+	
	1^-	8_D	−	+	+	−	+	−	−	+	
		27	−	−	−	−	−	−	+	+	
	2^-	1	+	−	+	+	+	+	+	+	
V_1P_8	1^+										+

strongly attractive, we have another $0^-(8_F)$ supermultiplet above P_8 if the $V_8V_8V_8$ coupling is strong. It is desirable for $0^-(8_F)$ to be strongly attractive so that the $V_8V_8V_8$ coupling is not very weak compared to the value predicted from the gauge meson theory.

Unfortunately $1^-(1)$ and $1^-(V_1P_8)$ are not so strongly attractive as $1^+(1)$. Since they are the main configurations of V_1 and V_8 in the VP_8 models, they must be more strongly attractive than $1^+(1)$. We can justify this by arguing that V_8 is composed mainly of $1^-(8_A)$ of P_8P_8, and that the $B\bar{B}$ channel is responsible for the tight binding of V_1, since the coupling of ω with the baryonic current is quite strong. $0^-(10,10^*)$ resonances, even if they exist, appear to be situated at least above 2 BeV. Among the 2^- channels $2^-(1)$ is most attractive but does not produce a low-lying resonance.

As for the V_1P_8 resonances, 1^+ is the lowest level, which is of course independent of the $V_8V_8V_8$ coupling.

We should like to discuss the effects due to the mesons predicted here other than P_8, V_8, and V_1. Although dependent on the strength of the coupling constants, the amplitudes due to the axial-vector mesons are sizable. The effective interaction for AVP_8 will be

$$mLA_\mu V_\mu P_8\,, \tag{4.9}$$

where the d-wave interaction $A_\mu\partial_\mu V_\nu\partial_\nu P_8$ is neglected and the unitary spin indices are suppressed. It will be a permissible approximation to neglect the d-wave interaction. Then the Born amplitudes for the A-exchange processes are written, apart from the unitary spin structure, as

$$\begin{aligned}
T_{00}&=(L^2/4\pi)(p/2W)\{E_1^2\cos\theta-p^2\\
&\qquad+(p/m)^2(E_1\cos\theta+E_2)^2\}/(m_A^2+u)\,,\\
T_{+0}&=(L^2/4\pi)(pm/2^{1/2}W)\\
&\qquad\times\{[E_1+(p/m)^2(E_1\cos\theta+E_2)]\sin\theta\}/(m_A^2+u)\,,\\
T_{++}&=(L^2/4\pi)(p/4W)\\
&\qquad\times\{(1+\cos\theta)-(p/m)^2\sin^2\theta)\}/(m_A^2+u)\,,\\
T_{+-}&=(L^2/4\pi)(p/4W)\\
&\qquad\times\{(1-\cos\theta)+(p/m)^2\sin^2\theta)\}/(m_A^2+u)\,.
\end{aligned}\tag{4.10}$$

The crossing factors due to unitary symmetry may be found in Table I. Projecting onto each partial wave, we find the effects on those channels for which resonances are predicted. We have tabulated qualitative tendencies of level shifts in Table II. As is seen there, $1^+(1)$ supports itself and $1^+(8_D)$, while $1^+(8_D)$ supports $1^+(1)$ but not itself. $2^-(1)$ is supported by both of them. $1^+(27)$ supports all the 1^+ resonances. The 1^+ composed of V_8P_8 is also self-supporting. In this way $1^+(1)$ still remains low-lying, while the other 1^+ supermultiplets may be pushed upwards by some amount. The $0^-(1)$ meson is not supported by $1^+(1)$ or $1^+(8_D)$. We feel that $0^-(1)$ may be considerably pushed up. In contrast $0^-(8_F)$ is supported by all the 1^+ resonances. Even if a vector-pseudoscalar resonance with $T=2$ or $|Y|=2$ is found, it is questionable whether it belongs to $1^-(27)$ or not.

Model B. In this model we do not assume the A-conservation rule. Instead we use the $V_8V_8P_8$ coupling estimated from the current hypothesis of $\phi\omega$ mixing. The $\phi\omega$-mixing theory tells us that the observed ϕ or ω is not the pure state of $SU(3)$, but a mixture of the singlet and the octuplet states,

$$\begin{aligned}\omega&=\omega^0\cos\theta+\phi^0\sin\theta\,,\\ \phi&=-\omega^0\sin\theta+\phi^0\cos\theta\,,\end{aligned}\tag{4.11}$$

where θ is the so-called mixing angle. The estimates of θ by several authors are in disagreement, but not far away from each other. Since we know that $g_{\phi\rho\pi}$ is practically zero as compared with $g_{\omega\rho\pi}$, we obtain

$$-g(\omega^0\rho\pi)\sin\theta+g(\phi^0\rho\pi)\cos\theta=0\,, \tag{4.12}$$

from which

$$g(\omega^0\rho\pi)/g(\phi^0\rho\pi)=\cot\theta\,. \tag{4.13}$$

In terms of the $SU(3)$-symmetric coupling constants previously defined,

$$(G/K)=(8/5)^{1/2}\cot\theta\,. \tag{4.14}$$

If one takes $\theta=30°$, one has

$$(G/K)=2.2\,. \tag{4.15}$$

Then the $V_8V_8V_8$ coupling is no longer neglected.

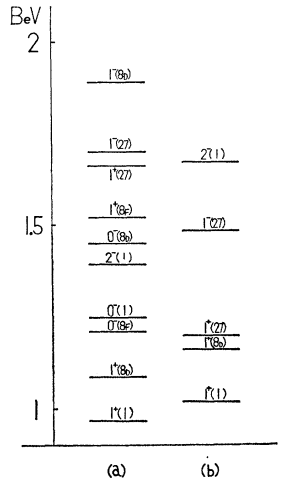

FIG. 3. The level schemes of the VP_8 resonances in Model B ($s_{\max}=8m^2$). V_8 is assumed to be the unitary gauge mesons in (a), while the $V_8V_8V_8$ coupling is zero in (b).

In the absence of the A-conservation rule, the 8_D channel of V_8P_8 is coupled with V_1P_8. The 8_F channel is not coupled owing to G-conjugation invariance and its analogs in $SU(3)$. The predicted level scheme has been drawn in Fig. 3. Just as in Model A, the low-lying levels are the $1^+(1)$ and $1^+(8_D)$ mesons. In contrast to Model A, $1^-(8_D)$ together with $1^-(V_1P_8)$ corresponds to the channel producing V_8.

The bootstrapping effects of the predicted resonances on the relevant channels are almost the same as given in Table II. We have only to regard $1^+(8_D)$ as $1^+(8_D,V_1P_8)$, keeping in mind that the effect of the $1^+(8_D,V_1P_8)$ on the $1^+(8_D,V_1P_8)$ row must be changed as follows: minus sign when V_8 is the gauge meson family, and plus sign when the $V_8V_8V_8$ coupling is weak.

In this way we have obtained the level scheme of the VP_8 resonances. The general features depend rather sensitively on the $V_8V_8V_8$ coupling constant. The absolute values of the mass levels are largely affected by the subtraction point of the D function. The scales of the ordinates in Figs. 2 and 3 shrink if one chooses a smaller subtraction point, and expand if one chooses a larger. We should not take literally the scales of the ordinates in the level schemes. However, it is the prominent feature independent of the involved coupling constants and the validity of the A-conservation rule that there exist a low-lying $1^+(1)$ and a $1^+(8_D)$ supermultiplet. The charge parities of these mesons are uniquely determined. Indeed, since in a tensorial form

$$P_i{}^j \to P_j{}^i \quad \text{and} \quad V_i{}^j \to -V_j{}^i \tag{4.16}$$

under charge conjugation, $1^+(1)$, $1^+(8_D)$, and $1^+(27)$ have negative charge parity, while $1^+(8_F)$ has positive charge parity. On the other hand, the axial-vector baryonic currents transforming like 1-, 8_D-, 8_F-, and 27-dimensional tensors under $SU(3)$ have positive charge parity. Therefore the $1^+(1)$ and $1^+(8_D)$ mesons are never coupled with the axial-vector baryonic currents. Moreover, the singlet meson cannot be coupled with the axial-vector currents composed of the octuplet baryons and the decuplet baryonic isobars, owing to the selection rule of $SU(3)$. It is the bilinear currents composed of the octuplet baryons and the octuplet baryonic isobars, if they exist, that $1^+(1)$ is coupled with. We feel that the production of $1^+(1)$ is through peripheral boson-nucleon collisions.

We add here a comment on the possibility of the boson icosuplet. Lee, Okubo, and Schecter[33] proposed the possible existence of a boson icosuplet, namely the 10 and 10* mesons which are connected with each other through charge conjugation. The present investigations, however, show that no icosuplet appears in any of the partial waves. The reason is quite simple in Model A. The process contributing to the 10 and 10* channels of V_8P_8 is only the V_1 exchange in the u channel. It produces a repulsive force for 1^-, 1^+, and 2^-, and an attractive force for 0^-, but not so strong as to produce a resonance. In Model B, the V_8 exchange contributes to the 10 and 10* channels in addition to the V_1 exchange in the u channel. Consulting the unitary crossing matrix elements, we find that these processes largely cancel each other in the 10 and 10* channels. Thus we do not have a force strong enough to produce a resonance in the 10 and 10* channels.

Another comment concerns the 2^+ channels. In the models which do not take account of the coupling-in of the P_8P_8 and VV channels, the force is in general too weak in 2^+. Actually the branching to P_8P_8 was reported for some of the VP_8 resonances. Therefore we can correctly estimate the 2^+ levels as well as the 1^- levels only after properly taking account of the P_8P_8 and the VV channels. Then the problem becomes too complicated to estimate numerically. If one considers V_8P_8 and V_1P_8, one has repulsive forces in $2^+(1)$, $2^+(8_D)$, and $2^+(27)$, and weakly attractive forces in $2^+(8_F)$ and $2^+(10,10^*)$. Owing to G-conjugation invariance and its analogs in $SU(3)$, only the $2^+(8_F)$ of V_8P_8 can be coupled with the 2^+ of P_8P_8 and V_8V_8. In the $2^+(8_D)$ channels of P_8P_8 and V_8V_8, the V_8 exchanges in the t and u channels produce no force, cancelling each other. We can expect the strongest force in the P_8-exchange process of the V_1V_8 channel. We might obtain a low-lying 2^+ resonance level, since 2^+ can be constructed from the s wave of V_1V_8.

5. GELL-MANN–OKUBO FORMULAS FOR THE VP_8 RESONANCES

Up to now all investigations have been limited to perfect symmetry. In the actual world, however, we find a small and sometimes considerable departure from unitary symmetry. This departure is always characterized by the eighth component of unitary spin. It results in the Gell-Mann–Okubo mass formula[1,34] and

[33] B. W. Lee, S. Okubo, and J. Schecter, Phys. Rev. **135**, B219 (1964).

[34] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

its extension to the relations among the coupling constants.[35,36] The mass formula is, as is well known, given for a boson supermultiplet by

$$m_i^2 = m_{0,i}^2 + \alpha_i[T(T+1) - \tfrac{1}{4}Y^2], \tag{5.1}$$

with α_i being a physical parameter dependent on an irreducible representation i under consideration. We shall discuss the physical parameters for the VP_8 resonances by means of simple dynamical models. When there exist nearly degenerate supermultiplets with the same dynamical properties (for example, the $\phi\omega$ mixing), the Gell-Mann–Okubo formula will be appreciably disturbed. Nevertheless we shall assume its validity here since it may help us in assigning the existing VP_8 resonances into supermultiplets and in predicting other members of the multiplets to be discovered.

Let us provisionally assume that the most violent breakdown is due to the mass splitting of P_8 which is of the same order of magnitude as the mean mass. On this assumption a simple-minded approximation is to count the constituent P_8 by means of the lowest order perturbation. Consulting the Clebsch-Gordan coefficients of $SU(3)$, we immediately find that the physical parameter α_i defined above obeys

$$\alpha_i = C_i K \alpha(P_8), \tag{5.2}$$

where $\alpha(P_8)$ is the α_i of P_8 and is given by

$$\alpha(P_8) = \tfrac{2}{3}(\mu_\pi^2 - \mu_K^2). \tag{5.3}$$

C_i is a numerical factor depending only on the representation, while K is a dynamical factor depending on the spin-parity and the other dynamical properties of the meson family under consideration. They can be easily calculated to lowest order of the perturbation expansion. We find that K turns out to be always positive independently of the spin-parity, if the Q value of the $V_8 \to P_8P_8$ decay is sufficiently small. C_i is calculated to be $\frac{1}{2}$, $-\frac{3}{10}$, $\frac{1}{6}$, and $\frac{3}{10}$ for 8_F, 8_D, $(10,10^*)$, and 27, respectively. Therefore this model predicts that a $T=0$, $Y=0$ member is heavier than a $T=1$, $Y=0$ member in 8_F and 27, and vice versa in 8_D. This model is easy to understand intuitively, but it has the defect of neglecting the symmetry-breaking effects coming from the renormalized coupling constants.

Another model follows faithfully the context of the bootstrap calculation and counts changes in Born amplitudes due to the P_8 mass differences. The mass differences of the constituent P_8 will also be operative. In Models A and B, there are three independent dynamical parameters for each Born amplitude, the changes caused by mass differences of the exchanged P_8, and the constituent P_8 in the intermediate states and external lines. The results are, in general, dependent on the dynamics involved. Fortunately, however, we have found by detailed numerical estimates that the same level pattern as in the former model is predicted for each lower supermultiplet, namely, for all the spin-parities

$$\alpha_i = k_i \alpha(P_8), \tag{5.4}$$

$$k_i > 0 \text{ for } 8_F \text{ and } 27, \quad k_i < 0 \text{ for } 8_D, \tag{5.5}$$

for V_8P_8, and

$$k_8 > 0 \tag{5.6}$$

for V_1P_8, in Model A. In Model B,

$$k_i > 0 \text{ for } 8_F \text{ and } 27. \tag{5.7}$$

We find that k_8 for $(8_D, V_1P_8)$ is negative when V_8 are the gauge mesons, and positive for vanishing $V_8V_8V_8$ coupling. We assume these level patterns within the respective supermultiplets and proceed to assignment of the existing meson resonances to the predicted supermultiplets.

6. ASSIGNMENT OF THE EXISTING RESONANCES

For many of the observed VP_8 resonances, quantum numbers have not yet been determined. First consider the $\pi\omega$ resonance (1220 MeV) or the B meson. Some authors have assigned B to the 2^- octuplet on the basis of a static approximation to the $\pi\omega$ scattering. In contrast to the case without recourse to any higher symmetry, the static approximation, $\mu/m=0$, is not good enough in the present unified investigation based on $SU(3)$. Even a low-energy approximation is not satisfactory. The B meson should be assigned to the 1^+ octuplet. If one takes Model A, the other members must lie above B. In Model B the $K\pi\pi$ resonance (1230 MeV) or the C meson, and its antiparticle, may be the $T=\frac{1}{2}$, $|Y|=1$ member, since they appear to be 1^+ from the observation of the decay angular distribution. The $K\pi\pi$ resonance around 1175 MeV is also a promising candidate for the $T=\frac{1}{2}$, $|Y|=1$ member of this octuplet.

Next, consider the $\pi\rho$ resonances. It has been reported that the A_2 meson has branchings to $K\bar{K}$ and $\pi\eta$. According to purely group-theoretical arguments, A_2 must belong without ambiguity to the 8_F family of V_8P_8 with $J^P=2^+$. This resonance is outside of the scope of the present investigation. For the A_1 meson (1090 MeV), there are two experiments which disagree on the spin-parity assignment. One of them[18] asserts 0^-, 1^+, 2^-, and so on, among which 0^- seems to be most favored, on the basis of the absence of the $K\bar{K}$ decay mode; while the other[19] claims to observe the $\pi\eta$ decay mode, hence 1^-, 2^+, and so on. If the former turns out to be true, A_1 will belong to $0^-(8_F)$ or $1^+(27)$. The possibility that it belongs to $1^+(8_F)$ cannot be completely excluded. Consulting the calculated level scheme, we conclude that A_1 belongs to $0^-(8_F)$ if the V_8 is the gauge meson family, and to $1^+(27)$ if the $V_8V_8V_8$ coupling is sufficiently weak. In the case of $0^-(8_F)$, the other members lie above A_1 since $T=1$ for A_1. In the

[35] M. Muraskin and S. L. Glashow, Phys. Rev. **132**, 482 (1963).
[36] K. Kikkawa, Progr. Theoret. Phys. (Kyoto) **30**, 636 (1963).

case of $1^+(27)$, A_1 is necessarily the $T=2$ member and the remaining members are above A_1.

We can easily see that the X meson ($\eta\pi\pi$) is not $0^-(1)$ as in the vector-pseudoscalar system. It will not be interpreted as a VP_8 resonance.

Another $K\pi\pi$ resonance which is not firmly established was reported to be around 1270 MeV.[37] If $T=\frac{1}{2}$, it will constitute $2^+(8_F)$ together with f^0 and A_2. The Gell-Mann–Okubo mass formula holds almost exactly for them. If $T=\frac{3}{2}$, it may be a member of $1^+(27)$.

Finally, some evidence has been reported for the isoscalar $\pi\rho$ resonance around 975 MeV.[38] From the Dalitz plot of the decay products, the spin-parity seems to be 1^+ or 2^-. This is the lowest VP_8 resonance that has ever been observed. It is quite welcome for the present level schemes. Indeed, a low-lying $1^+(1)$ has been predicted in both of the models. It is one of the most striking characteristics of the present work.

[37] For not firmly established resonances which were recently observed, see Proceedings of the 1964 International Conference on High-Energy Physics at Dubna, USSR, 1964 (to be published).

ACKNOWLEDGMENT

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[38] J. Bartsch, L. Bondar, W. Brauneck, M. Deutschmann, K. Eikel *et al.*, Phys. Letters **11**, 167 (1964).

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High-Energy Limit of Pion-Nucleon Total Cross Section*

Masao Sugawara and Arnold Tubis
Department of Physics, Purdue University, Lafayette, Indiana
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A simple estimate of the high-energy limit of the pion-nucleon total cross section is given in terms of the mass and the width of the 33 resonance and the S-wave pion-nucleon scattering lengths. The underlying assumptions are that the forward elastic (charge-nonexchange) scattering amplitude satisfies the usual (once-subtracted) dispersion relation, and that the scattering becomes dominantly absorptive in the high-energy region. This estimate gives 23 mb as the limit of the total cross section, which is in close agreement with what one obtains from a simple extrapolation of the available high-energy cross sections. The present analysis strongly suggests that a simple correlation exists between a pronounced low-energy resonance and the high-energy limit of the scattering amplitude.

IN dispersion theory, the high-energy behavior of the scattering amplitude is uniquely correlated to the low-energy behavior. Therefore, a detailed knowledge of low-energy scattering should yield some information about high-energy scattering as a consequence of the underlying analyticity assumption. If there is a pronounced low-energy resonance, it is possible that this resonance plays a dominant role in the discussion of high-energy scattering. An explicit assumption of such a correlation is made in the Regge-pole theory of strongly interacting particles.[1]

The purpose of this note is to show that one can actually obtain an estimate of the high-energy limit of the pion-nucleon total cross section in terms of the mass and the width of the 33 resonance and also the S-wave scattering lengths. The basic assumptions to be made are that the forward (charge-nonexchange) pion-nucleon scattering amplitude satisfies the usual dispersion relation and that this amplitude rapidly approaches a pure imaginary limit in the high-energy region. The latter of these may be regarded as the assumption that pion-nucleon scattering becomes dominantly absorptive in the high-energy limit. This estimate is then substantiated by a careful numerical analysis. This analysis also enables one to compute the high-energy limit of the total cross section if the high-energy phase of the scattering amplitude is given. This computed limit becomes 23 mb if one assumes the high-energy phase predicted by a reasonable optical potential model[2] of high-energy scattering.

Major conclusions are summarized at the end of this note, together with important remarks, the last of which concerns the question whether the same analysis can be applied to pion-pion scattering.

Let $T(\omega)$ be the forward (charge-nonexchange) pion-nucleon scattering amplitude as a function of the laboratory pion energy ω. We normalize $T(\omega)$ as $\mathrm{Im}T(\omega) = q\sigma(\omega)$, where q is the laboratory pion momentum and

* Work supported by the National Science Foundation and the U. S. Air Force.

[1] According to G. F. Chew and V. L. Teplitz, Phys. Rev. **136**, B1154 (1964), one expects in the Regge-pole theory a reasonable value of the high-energy limit of the pion-pion total cross section from a knowledge of the low-energy pion-pion resonances.

[2] Y. Nambu and M. Sugawara, Phys. Rev. Letters **10**, 304 (1963), and Phys. Rev. **132**, 2724 (1963). The high-energy phase is derived in the second of these references.