$K_2 \to \pi^+ + \pi^-$ and the Question of Bose Statistics for Pions*

S. A. BLUDMAN
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
(Received 21 October 1964)

K_2 decay into $\pi^+ + \pi^-$ is possible in a CP-invariant way if the $\pi^+\pi^-$ wave function contains a term antisymmetric in $\pi^+\pi^-$ exchange. This possibility requires a small violation of Bose symmetry for identical particles (extended to a particle-antiparticle pair) which, because the evidence for pion Bose statistics has consisted in the absence until recently of $K_2 \to 2\pi$, is consistent with present experimental data. Irrespective of any question of Bose statistics, if PC is conserved, $K_2 \to \pi^0 + \pi^0$ is forbidden. The absence or presence of this latter decay will thus discriminate between the theory discussed and theories in which PC is violated for the K-π system either in the dynamics or because of environmental effects. We consider the contribution of an antisymmetric $\pi^+\pi^0$ state to $K_2\pi^+$ decay, assuming the same statistical mixing $\mathcal{E}$ for the $\pi^+\pi^0$ final state in K^+ decay as for the $\pi^+\pi^-$ state in K^0 decay. If $\Delta T = \frac{1}{2}$ amplitudes dominate, this antisymmetric $\pi^+\pi^0$ state contributes negligibly to the $K^+{}_{2\pi}$ decay rate, which is therefore still large compared with what might be expected from electromagnetic corrections to an otherwise exact $\Delta T = \frac{1}{2}$ rule. If $\Delta T = \frac{1}{2}$ amplitudes dominate, then $\mathcal{E} = 1.5 \times 10^{-3}$, but if $\Delta T = \frac{1}{2}, \frac{3}{2}$, and $\frac{5}{2}$ amplitudes are comparable, then a value as large as $\mathcal{E} = 0.07$ is possible. The paper contains a short critical review of what is called for in order to test pion Bose statistics in the light of the recent experiment of Christensen *et al.*, and remarks on the theoretical consequences of impure Bose statistics.

I. INTRODUCTION

A RECENT experiment[1] shows that the long-lived component K_L of the K^0-$\bar{K}^0$ complex decays into the $\pi^+\pi^-$ mode at a rate 3.5×10^{-6} that with which the short-lived component K_S decays into the same mode. The particles in the 2π system are interchanged by the operator PC, so that if one assumes symmetry under exchange of two particles (Bose statistics), the $\pi^+\pi^-$ final state must be an eigenstate of CP ($CP = +1$). The observed nonorthogonality of the K_S and K_L states then demonstrates a violation of CP invariance. [If CP is violated in the dynamics, the theoretical problem is to explain the very small observed violation in terms of an interference between the dominant $T=0$ 2π decay amplitude and relatively complex amplitudes for the minor decay modes (a) $T=2$ 2π decay, (b) 3π decay, (c) leptonic decay.[2] One may also attempt to retain CP-invariant dynamics and to attribute the small effect observed to interactions of the K^0-$\bar{K}^0$ system with its environment: with the source and chamber walls, with the unsymmetric Fermi sea of neutrinos, with the galactic neighborhood of baryons, or with a CP-unsymmetric vacuum.]

Since the only experimental evidence for Bose statistics of pions has heretofore consisted[3] in the *absence* of $K_2 \to 2\pi$ (assuming CP invariance), in this note we shall consider the possibility of the pion system not being exactly symmetric under exchange of the two particles, i.e., we shall consider the possibility of $K_2{}^0 \to \pi^+ + \pi^-$ with PC conservation. We shall also find in $K_2 \to \pi^0 + \pi^0$ a test for pion statistics that is still applicable in the face of the results of Christensen *et al.*

This idea of mixed statistics is not to be identified with parafield quantization, a particular generalization of ordinary field quantization.[4] With mixed statistics a degeneracy occurs between conventional Bose and conventional Fermi particles of pion mass and spin. The concomitant departure from strong locality is discussed at the end of this paper.

II. IMPURE STATISTICS IN $K_2 \to 2\pi$ DECAY

The Bose nature of photons is established by macroscopic experiments. The Fermi nature of electrons and nucleons follows from the selection rules and stability of atoms, molecules, and nuclei. Mesons, on the other hand, are massive, relatively complex unstable particles whose statistics can be studied only through decay or production processes leading to two (or more) identical mesons. For pions, the evidence for Bose statistics has heretofore come[3] from the absence of $K_2 \to 2\pi$, and, to a lesser extent, from the slow rate of $K_{\pi 2}{}^+$ decay relative to $K_{\pi 2}{}^0$.

We assume an amplitude

$$C = (\tfrac{1}{2})^{1/2} \langle \phi_A | H_w | K^0 \rangle = -(\tfrac{1}{2})^{1/2} \langle \phi_A | H_w | \bar{K}^0 \rangle$$

for the decay of K^0 (or $\bar{K}^0$) into two pions in an antisymmetric ($PC = -1$) state ϕ_A; since the space configuration is that of an S state, this state is the anti-

* Supported in part by the U. S. Atomic Energy Commission.

[1] J. H. Christensen, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2] Possibility (a) and (c) are investigated by T. N. Truong, Phys. Rev. Letters 13, 358 (1964) and R. G. Sachs, *ibid.* 13, 286 (1964), respectively. Possibility (b) is interesting because the opposite G parities of the 2π and 3π final states will allow different roles for first- and second-class currents having opposite PC transformation character. See also the phenomenological analysis by T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

[3] H. C. von Baeyer, Phys. Rev. 135, B189 (1964); A. M. L. Messiah and O. W. Greenberg, *ibid.* 136, B248 (1964). In this reference, latter conditions are derived in ordinary quantum mechanics which imply symmetric statistics. I am grateful to Professor R. Prange for a discussion bringing this reference to my attention when it appeared in abstract form.

[4] For H. S. Green's trilinear commutation relations, O. W. Greenberg and A. M. L. Messiah (to be published) show that paraparticles cannot be singly produced from ordinary particles.

symmetric charge state

$$\phi_A^{+-}=(\tfrac{1}{2})^{1/2}[|\pi^+(1)\pi^-(2)\rangle-|\pi^-(1)\pi^+(2)\rangle].$$

The antisymmetric charge state ϕ_A^{00} of two π^0's vanishes. The total Hamiltonian is PC-invariant and has as eigenstates the conventional orthogonal states $K_{1,2}=(\frac{1}{2})^{1/2}(K^0\pm\bar{K}^0)$. We take for the final states in K^0 decay

$$|\pi^+\pi^-\rangle=c\phi_S^{+-}+s\phi_A^{+-},$$
$$|\pi^0\pi^0\rangle=\phi_S^{00},$$

where $CP\phi_S=\phi_S$, $CP\phi_A=-\phi_A$, and $|\pi^+\pi^-\rangle$ which is a linear combination of $|\pi^+(1)\pi^-(2)\rangle$ and $|\pi^-(1)\pi^+(2)\rangle$ of mixed symmetry, has been normalized so that $c=(1+\mathcal{E}^2)^{-1/2}$, $s=\epsilon/(1+\mathcal{E}^2)^{-1/2}$. The states of different symmetry character are noninterfering so that $\mathcal{E}$ can be chosen real. In terms of the remaining decay amplitudes,

$$A=(\tfrac{1}{2})^{1/2}\langle\phi_S^{+-}|H_w|K^0\rangle=(\tfrac{1}{2})^{1/2}\langle\phi_S^{+-}|H_w|\bar{K}^0\rangle,$$
$$B=(\tfrac{1}{2})^{1/2}\langle\phi_S^{00}|H_w|K^0\rangle=(\tfrac{1}{2})^{1/2}\langle\phi_S^{00}|H_w|\bar{K}^0\rangle,$$
$$\langle\pi^+\pi^-|H_w|K_1\rangle=2cA,\quad\langle\pi^0\pi^0|H_w|K_1\rangle=2cB,$$
$$\langle\pi^+\pi^-|H_w|K_2\rangle=2sC,\quad\langle\pi^0\pi^0|H_w|K_2\rangle=0,$$

and the measured branching ratios are

$$R^{+-}=\frac{R(K_2\to\pi^++\pi^-)}{R(K_1\to\text{all }2\pi)}=\frac{\mathcal{E}^2|C|^2}{|A|^2+|B|^2}=2.3\times10^{-6},$$

$$R_1=\frac{R(K_1\to\pi^0+\pi^0)}{R(K_1\to\text{all }2\pi)}=\frac{|B|^2}{|A|^2+|B|^2}=0.335\pm0.014.$$

$\mathcal{E}$ is thus expected to be small if $|C|^2$ and $|A|^2+|B|^2$ are comparable in magnitude.

Because of the identity of the two π^0's, *if PC is conserved* $K_2\to\pi^0+\pi^0$ *is forbidden*, irrespective of what statistics π^0 obeys. On the other hand, if pions obey strict Bose statistics and if PC is not conserved for dynamical or environmental reasons (or if K_2 decays into a new $PC=+1$ particle which then decays into 2π), the branching ratio

$$R_2=\frac{R(K_L\to\pi^0+\pi^0)}{R(K_L\to\pi^++\pi^-)}$$

would be expected to be of order unity unless a remarkable cancellation took place between $T=2$ and $T=0$ amplitudes A_2 and A_0.[5] Our starting point has been the realization that PC invariance together with pion Bose statistics forbade $K_2\to2\pi$. While the observation of any $K_L\to2\pi$ implies PC nonconservation and/or impure Bose statistics, only the presence of $K_L\to\pi^0+\pi^0$ can establish PC violation.

The detection of *any* $K_L\to\pi^0+\pi^0$ thus restores K_L decay as a test for pion Bose statistics. Except from K_L decay it is difficult to test pion statistics to the accuracy (0.1%) called for. (Most tests rely on strong-interaction selection rules derived from isospin invariance[6]: the effects of order $\alpha/2\pi$ induced by corrections to isospin symmetry cannot then be distinguished from the effects of a pion-statistics impurity of order $\mathcal{E}$.) The electromagnetic production of $\pi^+\pi^-$ pairs does depend on the symmetry of the $\pi^+\pi^-$ state.[7]

The detection of *any* $K_L\to\pi^0+\pi^0$ thus distinguishes between impure statistics and all other explanations of $K_L\to\pi^++\pi^-$. If PC is violated, a more quantitative measurement of R_2 is necessary in order to establish the role played by the $T=2$ amplitude.

III. IMPURE STATISTICS IN $K^+\to2\pi$ DECAY

In order to relate A, B, C, we decompose H_w into $\Delta T=\frac{1}{2},\frac{3}{2},\frac{5}{2}$ terms with corresponding amplitudes M_1, M_3, M_5. In isospin language, ϕ_A^{+-} is pure $T=1$ and

$$\phi_S^{+-}=(\tfrac{1}{3})^{1/2}|T=2\rangle+(\tfrac{2}{3})^{1/2}|T=0\rangle,$$
$$\phi_S^{00}=(\tfrac{2}{3})^{1/2}|T=2\rangle-(\tfrac{1}{3})^{1/2}|T=0\rangle,$$

so that

$$2A=(\tfrac{2}{3})^{1/2}M_1+(\tfrac{1}{3})^{1/2}M_3+(\tfrac{1}{3})^{1/2}M_5,$$
$$2B=-(\tfrac{1}{3})^{1/2}M_1+(\tfrac{2}{3})^{1/2}M_3+(\tfrac{2}{3})^{1/2}M_5,$$
$$2C=M_1+M_3.$$

The observed value for the K_1 branching ratio

$$R_1=\frac{1}{3}\frac{|M_1-\sqrt{2}(M_3+M_5)|^2}{|M_1|^2+|M_3+M_5|^2},$$

which is independent of the statistical impurity $\mathcal{E}$, determines two solutions: (i) $|M_3+M_5|/|M_1|\approx0$ or (ii) $|M_3+M_5|/|M_1|\approx2\sqrt{2}\cos(\delta_2-\delta_0)$. [Solution (i) is that preferred by the $\Delta T=\frac{1}{2}$ rule.] The branching ratio

$$R^{+-}=\mathcal{E}^2|M_1+M_3|^2/(|M_1|^2+|M_3+M_5|)^2$$

is then fitted by

$$\text{(i)}\ \mathcal{E}(1+M_3/M_1)=1.5\times10^{-3}$$

[5] Experimentally $R_2<50$. If $K_L=pK^0-q\bar{K}^0$ and $\epsilon=1-p/q\neq0$, then $R_2=\frac{1}{2}|\epsilon-2\sqrt{2}iF\ \mathrm{Im}A_2/A_0|^2/|\epsilon+\sqrt{2}iF\ \mathrm{Im}A_2/A_0|^2$, where $F=\exp[i(\delta_2-\delta_0)]$ is the factor for π-π S-wave final-state interaction. In the Sachs model, where the 2π amplitudes are PC conserving, A_2 is real and $R_2=\frac{1}{2}$. In the Truong model, where A_2 is maximally PC-violating, $R_2\approx0.85$.

[6] This is true of the test suggested by Messiah and Greenberg, Ref. 3: the absence of S waves in the $P=1$ 2π states issuing from $\pi^++p\to n+\pi^++\pi^+$, $\pi^\pm+d\to\pi^\pm+\pi^0+d$, $p+\mathrm{He}^3\to\mathrm{He}^4+\pi^++\pi^0$ and in the $T=0$ 3π states issuing from $d+d\to\mathrm{He}^4+\pi^++\pi^-+\pi^0$. Messiah and Greenberg also suggest that meson resonances (other than ω, η, ρ) may be found whose quantum numbers forbid a particular pion decay only on grounds of pion statistics. Their test for Bose statistics in $\bar{p}+p\to\pi^++\pi^-+\pi^++\pi^-$ actually tests the Bose nature of the dipion state $\pi^+\pi^-$ rather than that of the pions themselves.

[7] For orientation: the $\mu^+\mu^-$ pair-production cross section has been observed to equal the Bethe-Heitler cross section (based on Fermi statistics of muons) to about 5%. See A. Alberigi-Quarenta, M. De Pretis, G. Marini *et al.*, *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 469.

or

(ii) $\mathcal{E}(1+M_3/M_1)$
$$=[1+8\cos^2(\delta_2-\delta_0)](1.5\times10^{-3})\leq4.5\times10^{-3}.$$

In order to determine $\mathcal{E}$, it is now necessary to know M_3/M_1.

We now consider the contribution of the antisymmetric $T=1$ state to $K^+\rightarrow\pi^++\pi^0$ decay. Assuming the same statistical mixing $\mathcal{E}$ for $\pi^+\pi^0$ as for $\pi^+\pi^-$, we write for this final state

$$|\pi^+\pi^0\rangle=c\phi_S{}^{+0}+s\phi_A{}^{+0},$$

where $\phi_S{}^{+0}$ and $\phi_A{}^{+0}$ are pure $T=2$ and $T=1$, respectively. The amplitude

$$\langle\pi^+\pi^0|H_w|K^+\rangle =c((\tfrac{3}{4})^{1/2}M_3-(\tfrac{1}{3})^{1/2}M_5)+s(M_1-\tfrac{1}{2}M_3)$$

leads to the branching ratio

$$R^{+0}=\frac{R(K^+\rightarrow\pi^++\pi^0)}{R(K_1\rightarrow\text{all }2\pi)} =\frac{\frac{3}{4}|M_3-\frac{2}{3}M_5|^2+\mathcal{E}^2|M_1-\frac{1}{2}M_3|^2}{|M_1|^2+|M_3+M_5|^2}.$$

The observed value $R^{+0}=5\times10^{-3}$ requires

$$|M_3-\tfrac{2}{3}M_5|\ll|M_1|^2+|M_3+M_5|^2$$

and

$$\mathcal{E}^2|M_1-\tfrac{1}{2}M_3|^2\ll|M_1|^2+|M_3+M_5|^2.$$

The first inequality demands M_3, $M_5\ll1$ or $M_3\approx\frac{2}{3}M_5$; the second inequality demands $\mathcal{E}\ll1$ or $M_1\approx\frac{1}{2}M_3$. Thus we expect $\mathcal{E}\leq4.5\times10^{-3}$ unless M_1, M_3, M_5 are of comparable magnitude. In this latter case $\mathcal{E}\approx(R^{+0})^{1/2}=0.07$ is possible. If the statistical impurity were so large, then it would show up even in tests whose validity depends on isospin invariance.

Note that even when the $T=1$ final state is allowed to contribute the value observed for R^{+0} is still uncomfortably large compared with what might be expected from electromagnetic corrections to an otherwise exact $\Delta T=\frac{1}{2}$ rule.

IV. REMARKS

To the above phenomenological analysis we now append some remarks concerning the theoretical possibility of impure Bose statistics for pions. We have emphasized that admitting an antisymmetric component ϕ_A to the 2π state still forbids the $PC=-1$ K_2 state from decaying into $\pi^0+\pi^0$, while permitting decay into $\pi^++\pi^-$. In its original sense the symmetrization postulate applies to the states of two identical particles such as e^-e^- or $\pi^0\pi^0$. It is important to realize what is involved when this principle is extended to two different particles such as e^+e^-, $\pi^+\pi^-$ (or even $\pi^+\pi^0$) which are distinguished by some internal quantum number like charge (or isotopic spin). The extension to a particle-antiparticle pair is justified by the relativistic field concept wherein particle and antiparticle creation are effected by the negative frequency parts of canonically conjugate variables.[8] In allowing $PC=-1$ for the $\pi^+\pi^-$ S state we are making the negative frequency parts of $\phi(\mathbf{x})$ and $\phi^\dagger(\mathbf{x}')$ not exactly commute on a space-like surface. The departure from local commutativity that we are admitting permits an interference between π^+ creation at $\mathbf{x}$ and π^- creation at $\mathbf{x}'$, but involves no restriction on the measurability of $\phi(\mathbf{x})$ or $\phi^\dagger(\mathbf{x})$ (which are anyway not measurable).

The idea of impure statistics is, nevertheless, hard to even express in field theory which, through the commutation relations, incorporates the statistics "at the ground floor." Within the framework of parafield quantization, Greenberg and Messiah[4] have derived from local commutativity of the interaction Hamiltonian density the result that particles produced singly from ordinary particles cannot be paraparticles. In S-matrix theory, the crossing symmetry of scattering amplitudes, an even stronger condition that Bose statistics, plays a basic role. Although tied to particular theoretical formulations, these comments indicate how surprising would be any failure of pion Bose statistics. It is therefore fortunate that in $K_L\rightarrow\pi^0+\pi^0$ we can still test pion statistics.

Note added in proof. Because of the different symmetry character of the K_1 and K_2 final states, so long as the decay is PC invariant, there will be no K_1-K_2 interference. It has been pointed out to me that this absence of any K_1-K_2 interference may be easier to check experimentally. Then the absence of $K_2\rightarrow\pi^0+\pi^0$.

[8] This is particularly clear in contrasting the role of the anticommutation relations in the e^-e^- and in the e^-e^+ systems. The antisymmetry of the e^-e^- state, which is a consequence of the anticommutation of $\bar{\psi}(x)$ and $\psi(x')$, simply forbids certain states. The antisymmetry of the e^+e^- system, which is a consequence of the anticommutation of $\bar{\psi}(x)$ and $\psi(x')$, permits all configurations to exist, but correlates the space-spin configuration with the C parity. C conjugation applied to the state $a^+(1)b^+(2)|0\rangle$ interchanges particle and antiparticle, $Ca^+(1)b^+(2)|0\rangle=b^+(1)a^+(2)|0\rangle$, but in order to relate this to the state $a^+(2)b^+(1)|0\rangle$ with space and spin coordinates 1 and 2 interchanged, anticommutation of a^+ and b^+ is needed. Only then is the selection rule C parity$=(-)^{L+S}$ obtained.