Electromagnetic Interactions of High-Energy Muons*

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The production of knock-on electrons and of electron pairs by muons of mean energy >50 BeV has been measured from cloud-chamber photographs of soft showers produced in lead plates. The cloud chamber was operated at a depth of 8.42×10^4 g/cm^2 underground. About half the data were taken while the cloud chamber was tilted 66° in order to favor observation of the high-average-energy muons that come in at large zenith angle. The energies of the electrons that initiated the showers was obtained from an experimental calibration (to be published). The shower energies that were studied extended from 85 MeV to about 10^4 MeV. The observed frequency is compared with the expected frequency calculated from the results of Bhabha for knock-on electrons and those of Zapolski and of Murota, Ueda, and Tanaka for electron pairs. The agreement is satisfactory except for knock-on electrons in the region of 10^9 eV, where the predicted frequency appears to be significantly lower than the observed frequency.

1. INTRODUCTION

THE results of two previous cloud-chamber experiments designed to measure the cross section for direct production of electron-positron pairs by high-energy muons are in disagreement with theoretical predictions for this process. Roe[1] and Gaebler *et al.*[2] found direct pair-production cross sections about half that predicted by the Murota-Ueda-Tanaka (MUT) theory.[3] Stoker *et al.*,[4] however, obtained agreement with the theory, but their conclusions were based upon only 24 direct pair-production events.

Measured cross sections for the knock-on process of production of electrons have been found to be in agreement with theory by some authors[1–5] although one author reports disagreement with the theory for large energy transfer.[6] The bremsstrahlung cross section is very small for muons with energy less than several hundred BeV.

2. EXPERIMENT

This experiment was designed to measure the rate of direct pair production by high-energy muons. For this purpose, a multiplate Wilson cloud chamber with a sensitive volume of $(25\times80\times60)$ cm^3 was operated at a depth of 1132 ft below the surface of the ground in a salt mine. The chamber contained 21 lead plates, $\frac{1}{8}$ in. thick, each with two 0.02-in. polished aluminum plates for better light reflection. Triggering of the chamber was accomplished by coincidence of light pulses from plastic scintillators mounted above and below the chamber.

During the first half of the experiment, the chamber was operated in a position that favored muons traversing the equipment in approximately a vertical path. During the second part of the experiment, the chamber and plastic scintillator assembly was tilted to an angle of 66° to observe muons arriving at steep angles. Direct pair-production theory predicts an increased cross section with the higher energy muons; and since the average energy of muons is greater at steeper angles in an underground location, the rate of direct pair production should increase with the angle of the muon arrival.

Most pictures show tracks of single, energetic particles traversing the chamber. The pictures were scanned for the electron showers produced by these particles. In order to be counted, a picture had to contain the track of a single penetrating particle visible in at least twelve consecutive intervals in both of the stereoscopic views. Pictures with tracks of more than one penetrating particle were discarded since it was assumed that these particles were produced in a local nuclear interaction and were therefore not muons. However, a particle entering the chamber accompanied by what appeared to be a low-energy knock-on electron produced in the roof of the chamber was allowed.

Tracks of particles that missed the defining scintillators were not counted. Tracks at angles greater than 75° from the zenith were not counted because of the uncertainties in the amount of material traversed before reaching the mine location.

Parallax measurements were made to determine front-to-rear location of the tracks. If a muon track was closer than two centimeters to the edge of the lighted region, it was rejected.

There were 1287 pictures taken at the vertical that met the scanning criteria. Showers of three or more tracks produced in any except the last four plates in any picture were recorded as events. The pictures taken at the vertical contained 236 such events. There were 1376 acceptable pictures taken with the chamber tilted to 66° among which were 317 shower events.

The number of track segments of each event in each interval was recorded. The few tracks at an angle greater than 60° with the penetrating particle were not

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[1] B. P. Roe, thesis, Cornell University, 1959 (unpublished).
[2] J. F. Gaebler, W. E. Hazen, and A. Z. Hendel, Nuovo Cimento **19**, 265 (1961).
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[6] R. F. Deery and S. H. Neddermeyer, Phys. Rev. **121**, 1803 (1961).

recorded, nor were the equally few "reflected" (backscattered) electrons. Sometimes the number of tracks had to be estimated from ionization, especially in dense intervals of an energetic shower.

3. ENERGIES OF THE SHOWERS

In order to compare the results with theory, it is necessary to find the number of events as a function of the energy of the showers. This is done in two stages: (1) An energy is associated with each event. (2) Energy intervals are chosen and the number of events in each energy interval is determined.

For showers completed in the chamber, the results of a recent experiment by Hazen and Hendel[7] were used to determine the shower energy. Preliminary results of Hazen and Hendel show that energies of showers as estimated by Wilson's shower curves[8] are too low by a factor of about 2, while the shower curves of Crawford and Messel[9] result in only a slight underestimate of the energy. Since the results of Hazen and Hendel are applicable only to showers completed in the chamber, the energies of showers not completed in the chamber were estimated by cutting off the shower curves of Crawford and Messel at a depth corresponding to the last visible interval.

The shower energy is directly proportional to the amount of material traversed. Since the core of a shower follows the muon direction very closely, each shower energy was increased by the secant of the angle between the muon axis and the lead plate. No correction need be made for the angular spread of the electron tracks about the shower core since the calibration showers of Hazen and Hendel were similarly "uncorrected."

With an energy assigned to each shower, we can construct a histogram of the number of events versus assigned energies. There is not, however, a unique correspondence between the number of track segments and energy. In their preliminary analysis, Hazen and Hendel estimated that the average standard deviation of energies corresponding to a given number of track segments amounted to about 25%. Logarithmic energy intervals of $E_0 \pm 21\%$ were chosen for display of the data. This interval size was chosen for better visualization of possible uncertainties and for direct comparison with the results of Gaebler *et al.*[2] Figure 1 gives the histogram of the number of showers versus the energy of the shower. The theoretical curves will be discussed in the next section.

4. THEORY

Equations for both bremsstrahlung and knock-on probabilities are given by Rossi.[5] The bremsstrahlung cross section is very small at most of the primary energies of concern in this experiment. The knock-on probabilities are nearly independent of the energy of

[7] W. E. Hazen and A. Z. Hendel (private communication).
[8] R. R. Wilson, Phys. Rev. **79**, 204 (1950).
[9] D. F. Crawford and H. Messel, Phys. Rev. **128**, 2352 (1962).

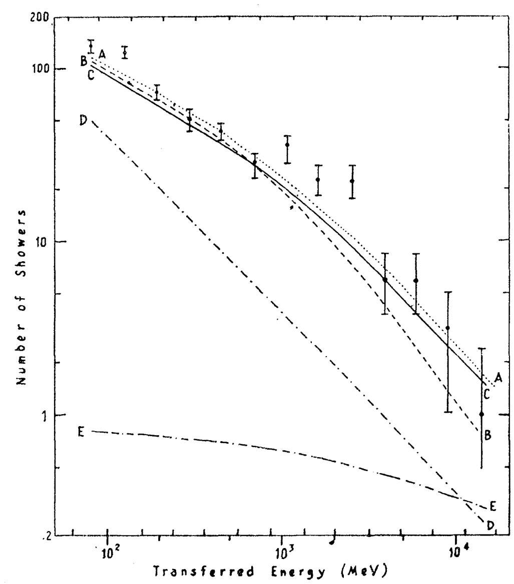

FIG. 1. Total number of showers versus shower energy (entire mine-muon distribution). Observed and predicted numbers of showers are compared for logarithmic energy intervals indicated by the vertical lines at the bottom. A—Total predicted number of events using the Murota cross section with $\alpha=2$. B—Total predicted number of events using the Zapolsky cross section. C—Total predicted number of events using the Murota cross section with $\alpha=1$. D—Predicted number of knock-on events. E—Predicted number of bremsstrahlung events. The error flags on the experimental points are statistical standard deviations.

the primary muon. Murota *et al.*[3] derived an equation for the direct pair production cross section. Roe[1] and Kearney[10] evaluated and integrated this equation. Recently, Zapolsky[11] derived another expression for direct pair production and this was evaluated by Kearney. A comparison of the two theories may be seen in Fig. 1.

Since the above cross sections are functions of the energy of the muons, it is necessary to take into account the energy spectrum of the muons reaching the mine location. For this purpose, the energy spectra of muons arriving at the surface were caluclated for zenith angles of 9°, 30°, 45°, 60°, and 70°. These calculations were based on zenith-angle measurements of muon spectra made by Pine *et al.*[12] and Pak *et al.*,[13] who found that their results could be fitted very well by an equation derived by Barrett *et al.*[14]

To find the underground spectra, the amount of energy each muon was expected to lose during its traversal of the rock above the equipment location was subtracted from its energy at the surface. For this, the energy-loss

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equation presented by Barrett *et al.* was used, namely,

$$dE/dx = 1.88 + 0.0766 \ln(E_m'/\mu C^2) + bE \text{ MeVcm}^2/\text{g}, \quad (1)$$

where

$$E_m' = E^2[E + \mu^2c^4/(2m_ec^2)]^{-1} \quad (2)$$

is the maximum transferable energy of a muon of rest energy μc^2 to an electron. A value of 3.0×10^{-6} was used for b. This value of b differs slightly from the value given by Barrett *et al.*, but it results in a better fit to the intensity measurements at the mine location.[15] The resulting integral spectra for the various zenith angles are presented in Fig. 2. With these spectra folded into the direct-pair functions, the expected number of pairs in the logarithmic energy intervals was calculated.

Figure 1 shows the calculated number of pairs, knock-on and bremsstrahlung events, as a function of the shower energy. Curves A and C give the calculated total number of events using the Murota calculation for direct pair production with the arbitrary parameter α equal to 2 and 1, respectively. Curve B is the same calculation using the Zapolsky theory. Curves D and E give predicted knock-on and bremsstrahlung events, respectively. This figure includes results of the entire experiment, i.e., for all muons at all angles. Figure 3 shows a similar calculation for muons arriving at steep angles (38–75°). Here, however, only the total predicted number of events, as calculated using the Murota calculation with $\alpha=2$, is shown.

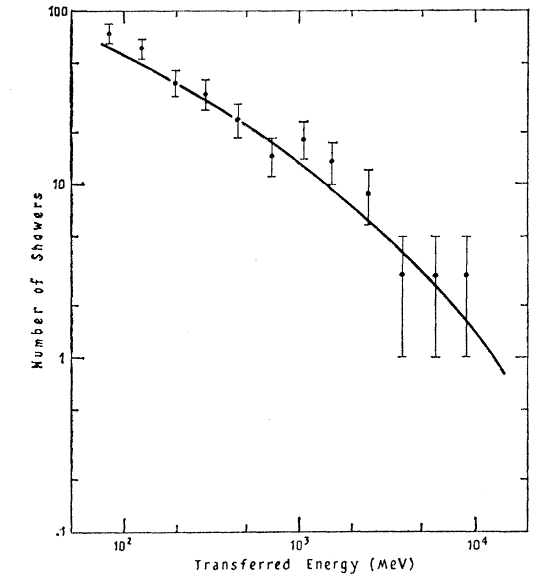

FIG. 3. Total showers produced versus shower energy (all muons with zenith angles 38 to 75°). The curve is based on knock-on and bremsstrahlung cross sections and the direct pair production theory of MUT for the case $\alpha=2$. The error flags on the experimental points are statistical standard deviations.

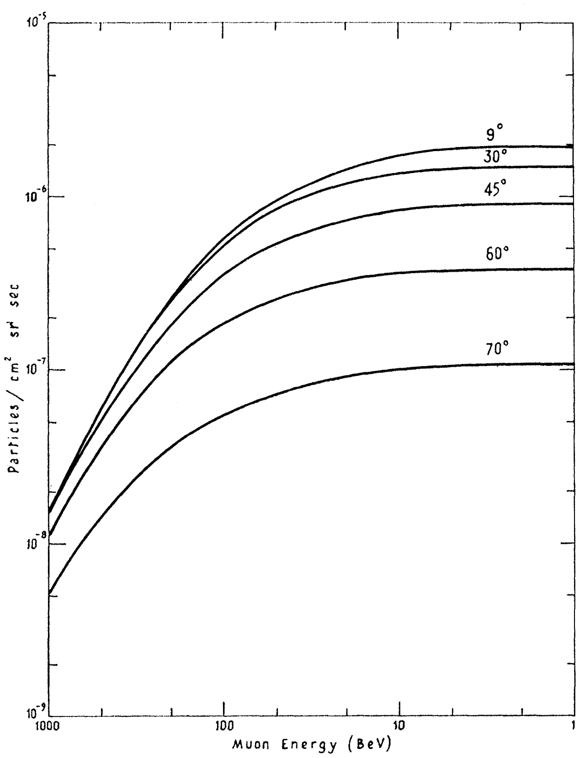

FIG. 2. Integral muon spectra for various zenith angles at the underground location.

5. RESULTS

Figures 1 and 3 show that the experimental results give generally good agreement with the expected values. However, possible systematic errors were large enough to prevent distinguishing between the two direct-pair calculations.

In order to check the direct-pair and knock-on theories separately, we separate the events by observation of the number of track segments below the plate in which a shower originates. For most direct-pair events there should be two tracks in addition to the muon, while knock-on events should have but one. A correction was made for single electrons that multiply and for direct pairs that lose one member before leaving the first plate. The correction made here was taken from a calculation of Gaebler *et al.* (See Table I in Ref. 2.)

Figures 4 and 5 show the calculated and experimental results for the entire muon spectrum in this experiment. Figure 4 compares experiment and theory for showers with two or more track segments in the first interval of the shower, and Fig. 5 gives the results for showers with one track below the first plate. Figures 6 and 7 show the same comparison for the steep-angle muons.

Figures 4 and 6 depend almost entirely on the number of direct-pair events. For these there seems to be a good agreement with predicted values. Figures 5 and 7 show that the number of showers with one track segment

[15] W. E. Hazen and C. A. Randall, Nuovo Cimento 8, 878 (1958).

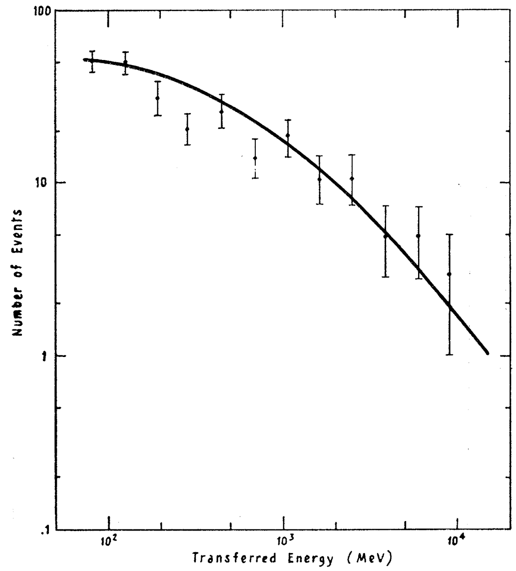

FIG. 4. Number of showers with two or more tracks below the first plate versus shower energy (entire mine muon distribution). The theoretical curve was calculated from pair production (MUT; $\alpha=2$) and knock-on theories, and from Wilson's charts.

below the first plate, primarily the knock-on events, is considerably larger than predicted, particularly in the region of transferred energy from about 1 to 3 BeV.

It is interesting to examine, briefly, the results of Gaebler *et al.* in view of the shower calibration of Hazen and Hendel. If the experimental points in Figs. 1, 3(a), and 3(b) (in Ref. 2) are assigned new locations for the energies (about twice the energies given), the results show good agreement with those obtained here.

6. DISCUSSION OF RESULTS

Before drawing any conclusions, it is necessary to examine and evaluate any possible systematic errors. In the scanning process there were several possible sources of error. Some pictures should contain two or more events. Since the number of observed multiple events was approximately the number expected, no correction was made for unresolved multiple showers. The relative number of events might be overestimated if a shower produced in the chamber could activate the lower scintillator when the primary would not. However, six inches of lead between the chamber and the lower scintillator virtually eliminated this possibility. Since it is not always possible to identify, unambiguously, the origin of a shower, some showers that originate above the first plate might have been included. The opposite is also possible. For example, a shower that originates four plates above the last visible interval, and hence should not be counted, might be accepted owing to a coincident knock-on electron from the fifth plate above the last visible interval. However, analysis shows that the net effect of these two processes should be negligible.

The number of events might also be influenced by the criterion for accepting a picture. To evaluate possible bias in picture selection and track-counting methods, one roll of film was rescanned about six months after the initial scan. The total number of pictures accepted for both scans agreed within 1%, but not all of the

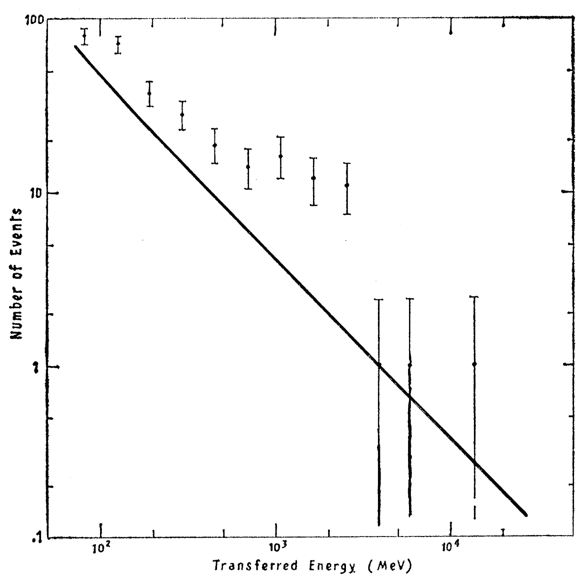

FIG. 5. Number of showers with one track below the first plate versus shower energy (entire mine muon distribution). The curve was calculated from pair production and knock-on theories, and from Wilson's charts. The error flags on the experimental points are statistical standard deviations.

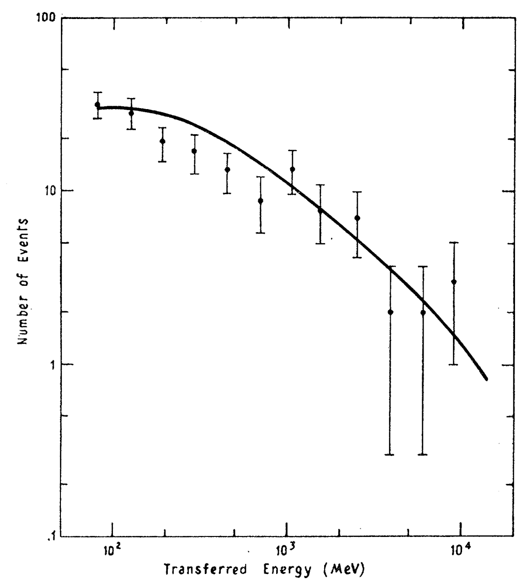

FIG. 6. Number of showers with two or more tracks below the first plate versus shower energy (all muons with zenith angles 38 to 75°). The theoretical curve was calculated from pair production (MUT; $\alpha=2$) and knock-on theories, and from Wilson's charts.

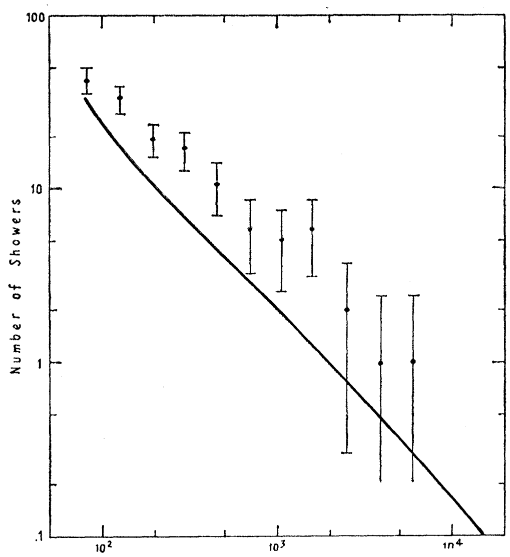

FIG. 7. Number of showers with one track below first plate versus shower energy (all muons with zenith angles 38 to 75°). Predicted curves are based on knock-on and direct pair theory (MUT; $\alpha=2$) and Wilson's charts. Error flags are statistical standard deviations.

pictures selected were the same in both cases. It was estimated from the rescan that bias in picture selection could be as much as 10%. However, there was no indication of bias for or against showers in the selection of pictures. Shower energies for both scans agreed within 7% in all cases.

Shower energies are determined from the number of track segments associated with an interaction. Some tracks might be outside of the illuminated region of the chamber. However, since muon tracks were required to be 2 cm inside of the edge of the lighted region, only a few tracks of the largest showers would be outside the lighted region. Other tracks might be far from the core of the shower and therefore be considered as background. Conversely, background tracks might be mistaken for shower tracks. However, owing to the depth underground and the low radioactivity of the salt, background tracks contributed a negligible amount to the entire count in the chamber and no corrections were made. For small showers, errors in the observed number of track segments are believed to be small. Frequently, the large showers (>1 BeV) contained track densities that were too great to count and required an estimate of the number of tracks from the amount of ionization. It was estimated that there might be a systematic bias of as much as 10% in track counting for the larger showers. Hazen and Hendel estimated that there could be possible systematic errors amounting to no more than 5% in their calibration of shower energy versus the number of track segments.

The standard deviation of the distribution of energies that corresponds to a given number of track segments was estimated to be about 25% for a single event. The resulting uncertainty in the energy of the experimental points depends on the number of events per interval. The resulting statistical uncertainty in the abscissa is about 4% in Fig. 1, 6% in Figs. 3, 4, and 6, and 8% in Figs. 5 and 7. It would seem safe to assume that the possible errors in the energies of the showers would be less than 15%.

The theoretical curves depend on the muon spectrum. The mine spectrum was calculated using known and extrapolated spectrum values at the surface. For muon energies appropriate to this experiment (>220 BeV), the surface spectrum is known only to an accuracy of about ±20%. For muons reaching the mine, there could be a systematic error in the spectrum as great as 25% due to uncertainties of the surface spectrum and the amount of material traversed by the muon reaching the mine. Since the knock-on cross section used here is virtually independent of the spectrum, little error would be introduced in the predicted number of knock-on events. For energies of the order of 60 BeV (the median energy of the mine muons), the direct-pair cross section varies quite slowly, i.e., logarithmically. For the lowest energy pairs, we should expect a difference of only 8% if there were a systematic error of 25% in the average energy of the spectrum. However, a systematic error in the spectrum of this size would result in a change of about 25% in the predicted number of pairs with energies above about 1 BeV.

It is possible that a number of direct pairs have been mistakenly classified as knock-on events. This could occur if, in the first interval of the shower, the tracks were superimposed upon the muon track. The scanner must then estimate whether one or two extra tracks exist. For low-energy events, the angle of emission of the pairs is relatively large, and the number of superimposed tracks should be small. For high-energy events, superposition is a serious problem. In the region of transferred energy from 1 to 3 BeV, where the greatest discrepancy occurs, 70% of all showers required the scanner's estimate as to the number of secondary tracks superimposed on the muon track in the first interval. In order that the knock-on results agree with theory, all of the knock-on showers that required an estimate would have to have been incorrectly judged. While the possibility exists that this could be the case, it is highly improbable that the percentage of misjudged events could be that large. If resolution of superimposed tracks is responsible for the disagreement, Gaebler *et al.* should have less disagreement, for the following reason. The space between the plates for the experiment of Gaebler *et al.* was about twice the space used here, allowing better separation of superimposed tracks and more track length for ionization estimates. However, their results, when corrected for the more recent energy estimates, are in agreement with those found here.

If it is assumed that perhaps 30% of the cases where estimates were required as to the number of tracks in the first interval could be in error, agreement could be obtained between theory and experiment for both knock-on and direct-pair interaction below 1 BeV of transferred energy.

The disagreement with knock-on theory is not unique to this experiment. As mentioned, the results of Gaebler *et al.*, when corrected for more recent energy estimates, show the same deviation. In addition, similar results have been found by Derry and Neddermeyer[6] for approximately the same region of energy transfer.

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Proton-Antiproton Annihilations into Two Mesons

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Proton-antiproton annihilation cross sections into two mesons are calculated assuming that contributions from the diagrams with a single intermediate meson dominate. Unitary symmetry and ω-ϕ mixing together with the total rates and cross sections for annihilation into two pseudoscalar mesons are used to obtain estimates of the derivative coupling constants for the ρ and Y mesons. (The Y meson is the member of the vector octet which is coupled to the hypercharge current.) The model is thus able to account for the annihilation into two pions and into two kaons, but yields results which are generally too large by an order of magnitude for the other two-meson final states of $p\bar{p}$ annihilations. It is pointed out that if the $\omega\rho\pi$ coupling constant, which is estimated from the 3π width of the ω, were smaller, and if the pseudoscalar meson intermediate states were neglected, then the model would yield a reasonably good description of the experiments.

I. INTRODUCTION

THE purpose of this paper is to discuss the consequences of a simple model for proton-antiproton annihilation into two mesons. The model assumes that the annihilation proceeds through a single intermediate vector meson or pseudoscalar meson state.

Berman and Oakes[1] have discussed nucleon-antinucleon annihilation from the point of view of a vector theory of strong interactions in which vector mesons are the dominant intermediate states. They have listed selection rules and several experimental consequences of these selection rules.

In this paper we will calculate explicitly cross sections and relative rates for proton-antiproton annihilations using both intermediate vector mesons and intermediate pseudoscalar mesons.

We will assume that the ρ meson is coupled universally to the isovector current.[2] From this assumption we obtain the ρNN vector coupling constant. The ρNN derivative coupling constant is obtained by fitting the the experimental $p\bar{p}$ annihilation cross section into two pions. Other coupling constants are obtained from considerations of unitary symmetry[3] and ω-ϕ mixing.[4,5] The results of the calculation for $K\bar{K}$ final states are, to a good approximation, independent of the amount of mixing as shown in Appendix B. We use experimental information at rest[6–8] and at 1.61 BeV/c.[9–12]

Our simple model for $p\bar{p}$ annihilations has been motivated by the following considerations. It has been observed[6] that the $p\bar{p}$ annihilations at rest occur predominantly from the S states of protonium. There are four distinct S states of protonium with quantum numbers J, I, G, C, P which exactly correspond to the

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