ments is concerned. They were obtained from the nonrelativistic Schrödinger theory in which spin effects have been ignored and the inelastic processes were simply accounted for by making the potential complex. Nevertheless, in spite of these oversimplifications, the physical idea, that strong absorption at short distances determines, in a specific way, the high-energy large-angle scattering, may become a useful guide for further investigation toward a successful theory.

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Hypernuclear Spectroscopy in the Unitary Symmetry Model. II*

YUKIO TOMOZAWA
The Institute for Advanced Study, Princeton, New Jersey
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The unitary-symmetry classification of hypernuclear systems is discussed taking into consideration (a) the mixing of representations due to the symmetry-breaking interactions and (b) the Pauli principle. There is also suggested a possible relevance of the group $SU(16)\supset SU(8)\times[SU(2)]_{\rm spin}\supset SU(3)\times[SU(2)]_{\rm spin}$ for light hypernuclei.

AN application of $SU(3)$ symmetry to nuclear systems has been discussed by several authors.[1–5] The observed hyperfragments have been classified as members of the unitary multiplets which contain the ground states of ordinary nuclei. However, the situation is rather complicated because of the effects due to (a) symmetry-breaking interactions and (b) the Pauli principle. Taking into consideration these effects, we show that none of the observed hyperfragments, except $_\Lambda He^5$ and $_\Lambda C^{13}$, is a member of a pure unitary multiplet and that the ground state of a hyperfragment having baryon number $A\geqslant 5$ is a mixture of unitary multiplets which do not contain ordinary nuclei.[6,7] Also a possible relevance of the group $SU(16)\supset SU(8)\times[SU(2)]_{\rm spin}\supset SU(3)\times[SU(2)]_{\rm spin}$ for light hypernuclei is suggested.

Hypernuclei are bound states or resonant states composed of the lowest octuplet baryons. It is convenient, therefore, to consider the group $SU(8)$ which contains the group $SU(3)$ as a subgroup, for classifying hypernuclear systems. Furthermore, hypernuclei consisting of baryons in the s orbital state are the completely antisymmetric states of the group $SU(16)$ which is the Wigner extension[8] of the group $SU(8)\times[SU(2)]_{\rm spin}$. In Table I, we list such states and their reduction due to the subgroups for the case $A\leqslant 5$. Table II gives the dimensionality $N(\lambda_1,\lambda_2)$ of the representation $D(\lambda_1,\lambda_2)$ of the group $SU(3)$ and its isospin and hypercharge $[I,Y_{\rm max}]\equiv[(\lambda_1/2),(\lambda_1+2\lambda_2)/3]$ for the isospin multiplet which has the maximum hypercharge. We put a bar over the dimensionality when $\lambda_1<\lambda_2$, as usual.

Let us consider the lightest stable hyperfragment $_\Lambda H^3$ the spin of which is identified[9,10] to be $\frac{1}{2}$. Assuming that[11] it is an isotopic singlet, we may classify $_\Lambda H^3$ to be the $[I=0,\ Y=2]$ member of either $\overline{35}$ or $\overline{10}$ or their mixture according to[12] Tables I and II. The relevant wave functions are

$$|\overline{35},I=0,\ Y=2\rangle=\tfrac{1}{2}\sqrt{3}\,|pn\Lambda,\ I=0\rangle+\tfrac{1}{2}\,|pn\Sigma,\ I=0\rangle \quad (1)$$

and

$$|\overline{10},I=0,\ Y=2\rangle=-\tfrac{1}{2}\,|pn\Lambda,\ I=0\rangle+\tfrac{1}{2}\sqrt{3}\,|pn\Sigma,\ I=0\rangle, \quad (2)$$

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[12] For the $[I,Y]$ contents of $\overline{35}$ etc., see, e.g., Table I of Ref. 5.

TABLE I. The classification of hypernuclei with $A \leqslant 5$ by the groups $SU(16) \supset SU(8) \times [SU(2)]_{\text{spin}} \supset SU(3) \times [SU(2)]_{\text{spin}}$.

Baryon number	$SU(16)$	$SU(8) \times [SU(2)]_{\text{spin}}$	$SU(3)$ $Y_{\max}=4$	3	2	1	0	Spin
2	120	(36,1)			27	8	1	0
		(28,3)			$\overline{10}$	8, 10		1
3	560	(168,2)		$\overline{35}$	$\overline{10}(27)^2 35$	$(8)^3 10$		$\frac{1}{2}$
		(56,4)			$\overline{10}$, 27	8, 10	1	$\frac{3}{2}$
4	1820	(336,1)	$\overline{28}$	$\overline{35}$, 64	$\overline{10}(27)^4 35$, 28	$(8)^2 10$	$(1)^2$	0
		(378,3)		$(\overline{35})^2 64$	$(\overline{10})^3(27)^3(35)^2$	$(8)^4(10)^3$	1	1
		(70,5)			$(27)^2$	$(8)^2$		2
5	4368	(1008,2)	$\overline{28}$, $\overline{81}$	$(\overline{35})^4(64)^3 81$	$(\overline{10})^4(27)^7(35)^4 28$	$(8)^6(10)^4$	1	$\frac{1}{2}$
		(504,4)		$(\overline{35})^2(64)^2$	$(\overline{10})^3(27)^5(35)^2$	$(8)^5(10)^3$	1	$\frac{3}{2}$
		(56,6)			$\overline{10}$, 27	8, 10	1	$\frac{5}{2}$

where

$$|pn\Lambda, I=0\rangle\uparrow = \tfrac{1}{6} \sum_{\text{antisym}} \{2(pn\Lambda - np\Lambda) + p\Lambda n - \Lambda pn + \Lambda np - n\Lambda p\}\uparrow\uparrow\downarrow, \quad (3)$$

and

$$|pn\Sigma, I=0\rangle\uparrow = \tfrac{1}{6} \sum_{\text{antisym}} \{\sqrt{2}(p\Sigma^- p - \Sigma^- pp + n\Sigma^+ n - \Sigma^+ nn) + \Sigma^0 pn - p\Sigma^0 n + \Sigma^0 np - n\Sigma^0 p\}\uparrow\uparrow\downarrow, \quad (4)$$

respectively. Here, the arrow stands for the spin wave function and $\sum_{\text{antisym}}$ implies the antisymmetric sum, antisymmetric in the total wave functions.[13] Equation (1) can be obtained from the He^3 wave function by applying the lowering operators of isospin and of U spin, and Eq. (2) is determined by orthogonality with Eq. (1). (For details, see a forthcoming article of the author.)

From Eqs. (1) and (2), it would follow that the state $|\overline{35}, I=0, Y=2\rangle$ has lower energy than the state $|\overline{10}, I=0, Y=0\rangle$, since the former contains more of the $|pn\Lambda\rangle$ component than does the latter. If, however, the binding energy between the nuclear system and the Λ or Σ particle is small compared with the mass difference $\Sigma-\Lambda$, the eigenstate of the Hamiltonian is neither $\overline{35}$ nor $\overline{10}$, but Eqs. (3) and (4). Experimental evidence for this can be obtained from consideration of the wave function and the binding energy of ${}_\Lambda He^5$, the most abundant hyperfragment so far observed (or of ${}_\Lambda C^{13}$). In fact, the wave function of ${}_\Lambda He^5$ which is an isotopic singlet of $\overline{28}$ (see Table I) is given by

$$|{}_\Lambda He^5\rangle\uparrow = (1/\sqrt{5}) \sum_{\text{antisym}} He^4\Lambda\uparrow, \quad (5)$$

which does not contain the Σ component at all. (In general, hyperfragments of which the core nucleus is a closed shell have such a property.) Therefore, the binding energy of ${}_\Lambda He^5$, 3.1 MeV, is due purely to the force between the He^4 and Λ, and is quite small compared to the value $\Sigma-\Lambda=77$ MeV. This is, of course, consistent with the usual observation that the Λ-N force has a short range and thus is expected to give small binding energy.[11]

We conclude, then, that the observed hyperfragment ${}_\Lambda H^3$ is a mixture of $\overline{35}$ and $\overline{10}$:

$$|{}_\Lambda H^3\rangle \approx |pn\Lambda, I=0\rangle = \tfrac{1}{2}\sqrt{3}\,|\overline{35}, I=0, Y=2\rangle - \tfrac{1}{2}|\overline{10}, I=0, Y=2\rangle. \quad (3')$$

The deviation from Eq. (3′) for the wave function of ${}_\Lambda H^3$ would be of the order

$$\frac{\frac{1}{2}(\text{Binding Energy of } {}_\Lambda He^5)}{\Sigma-\Lambda} = \frac{1.6}{77} = 0.02.$$

We expect the existence of the state orthogonal to (3′),

$$|{}_\Lambda H^{3*}\rangle \approx |pn\Sigma, I=0\rangle = \tfrac{1}{2}|\overline{35}, I=0, Y=2\rangle + \tfrac{1}{2}\sqrt{3}\,|\overline{10}, I=0, Y=2\rangle, \quad (4')$$

around the $\Sigma+H^2$ threshold.[14] This could be found as a

TABLE II. The $SU(3)$ multiplet.

Representation $D(\lambda_1,\lambda_2)$	Dimensionality $N(\lambda_1,\lambda_2)$	Isospin, hypercharge component with the maximum hypercharge $[I, Y_{\max}] \equiv [\lambda_1/2, (\lambda_1+2\lambda_2)/3]$
$D(0,0)$	1	$[0,0]$
$D(1,1)$	8	$[\frac{1}{2},1]$
$D(3,0)$	10	$[\frac{3}{2},1]$
$D(0,3)$	$\overline{10}$	$[0,2]$
$D(2,2)$	27	$[1,2]$
$D(4,1)$	35	$[2,2]$
$D(6,0)$	28	$[3,2]$
$D(1,4)$	$\overline{35}$	$[\frac{1}{2},3]$
$D(3,3)$	64	$[\frac{3}{2},3]$
$D(5,2)$	81	$[\frac{5}{2},3]$
$D(0,6)$	$\overline{28}$	$[0,4]$
$D(2,5)$	$\overline{81}$	$[1,4]$

[13] Antisymmetrization should be performed only between the first two baryons and the third one, since that between the first two is already made.

[14] Such a state is expected to exist, of course, independent of the consideration of unitary symmetry.

resonance peak in a reaction such as

$$p+\mathrm{H}^2 \to {}_\Lambda\mathrm{H}^{3*}+K^+ \searrow \Lambda+\mathrm{H}^2, \text{ etc.} \qquad (6)$$

The energies which we should compare with the Gell-Mann–Okubo mass formula are, therefore,

$$\langle \overline{35}, I=0, Y=2|H|\overline{35}, I=0, Y=2\rangle = {}_\Lambda\mathrm{H}^3+\tfrac{1}{4}({}_\Lambda\mathrm{H}^{3*}-{}_\Lambda\mathrm{H}^3)$$

and

$$\langle \overline{10}, I=0, Y=2|H|\overline{10}, I=0, Y=2\rangle = {}_\Lambda\mathrm{H}^3+\tfrac{3}{4}({}_\Lambda\mathrm{H}^{3*}-{}_\Lambda\mathrm{H}^3).$$

For the hyperfragment ${}_\Lambda\mathrm{He}^4$, the wave functions turn out to be

$$|{}_\Lambda\mathrm{He}^4\rangle \approx |ppn\Lambda, I=\tfrac{1}{2}\rangle = (1/\sqrt{2})(|\overline{28}, I=\tfrac{1}{2}, Y=3\rangle + |\overline{35}, I=\tfrac{1}{2}, Y=3\rangle) \qquad (7)$$

and

$$|{}_\Lambda\mathrm{He}^{4*}\rangle \approx |ppn\Sigma, I=\tfrac{1}{2}\rangle = (1/\sqrt{2})(-|\overline{28}, I=\tfrac{1}{2}, Y=3\rangle + |\overline{35}, I=\tfrac{1}{2}, Y=3\rangle), \qquad (8)$$

which lead to

$$\langle \overline{28}, I=\tfrac{1}{2}, Y=3|H|\overline{28}, I=\tfrac{1}{2}, Y=3\rangle = \langle \overline{35}, I=\tfrac{1}{2}, Y=3|H|\overline{35}, I=\tfrac{1}{2}, Y=3\rangle \approx \tfrac{1}{2}({}_\Lambda\mathrm{He}^4+{}_\Lambda\mathrm{He}^{4*}). \qquad (9)$$

The wave function of the yet-to-be-found unstable hyperfragment ${}_\Lambda\mathrm{H}^2$ is $(1/\sqrt{2})(|\overline{10}\rangle+|8_a\rangle)$, which follows from the wave functions for $\overline{10}$ given in Ref. 1. This leads to the relation similar to Eq. (9).

A similar consideration can be applied to all the other members of the unitary multiplets. From Table I, it follows that the smaller the magnitude of the hypercharge, the greater the number of representations to be mixed. Also a complication comes from the facts that the systems with $|\text{strangeness}| \geqslant 2$ could have a mixing of states, the constituents of which have a smaller mass difference, since, e.g., $N+\Xi-2\Lambda\approx 25$ MeV and $3\Lambda+\Sigma-2(N+\Xi)\approx 20$ MeV, and that the binding energy between hyperons is not known. If the ratio (binding energy)/(mass difference) were not small, we would have difficulty in finding the eigenstate of the Hamiltonian unless we knew the details of the interactions.

Hypernuclei belonging to the unitary multiplet which contains ordinary nuclei have four baryons in the s orbital state. Since the Pauli principle does not apply between Λ or Σ and nucleons, or, equivalently, the generalized Pauli principle should apply between them, the ground states of hyperfragments with $A\geqslant 5$ belong to the unitary multiplet which does not contain ordinary nuclei. To find the unitary multiplet of ordinary nuclei for $A\geqslant 5$, we have to look for excited states of hypernuclei, which may be strongly unstable and could be found as resonances.[15] Also they are subject to the mixing of representations.

For heavier hyperfragments the mixing of the Σ component becomes important, the ratio (binding energy)/$(\Sigma-\Lambda)$ being non-negligible. The determination of the masses of the unitary multiplets is, therefore, difficult for such cases, too.

Finally, we mention the possibility of examining the unitary-spin independence or the spin and unitary-spin independence in hypernuclear systems. The former leads to the group $SU(8)$ and correlates the mass formulas of the $SU(3)$ multiplets which have the same baryon number and spin, and are on the same row in Table I. In other words, their masses are determined by those of one $SU(3)$ multiplet. Similarly, the spin and unitary-spin independence leads to the group $SU(16)$ and the correlates the mass splitting of the unitary multiplets which have the same baryon number in Table I. The group $SU(16)\supset SU(8)\times[SU(2)]_{\text{spin}}$ is an extension of the group $SU(4)\supset SU(2)\times[SU(2)]_{\text{spin}}$ of Wigner[8], and is a more straightforward extension than the group $SU(6)\supset SU(3)\times[SU(2)]_{\text{spin}}$ of Gürsey and Radicati and Sakita[16,17] the constituents of hypernuclei being the octuplet baryons instead of the triplet quarks.

Since experimental evidence for $SU(3)$ symmetry itself has yet to be found in nuclear systems, it would be premature to discuss such higher symmetries. However, they might afford a guide in experiments for finding various hypernuclear levels.

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[15] We have to amend Table I and the relevant discussions of Ref. 5, accordingly.

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