Pion Production and the One-Particle-Exchange Mechanism in $\bar{p}$-p Interactions at 3–4 BeV/c*

T. FERBEL, A. FIRESTONE, J. SANDWEISS,† AND H. D. TAFT
Yale University, New Haven, Connecticut

AND

M. GAILLOUD,‡ T. W. MORRIS, AND W. J. WILLIS
Brookhaven National Laboratory, Upton, New York

AND

A. H. BACHMAN, P. BAUMEL, AND R. M. LEA
The City College, New York, New York

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A study of antiproton-proton collisions at 3–4 BeV/c indicates that about 25% (19 mb) of the total interaction cross section can be attributed to pion production without annihilation. The two outstanding characteristics of the pion-production channels are the strong forward peaking of the antibaryons in the center-of-mass system, and the prominence of the well-known $T=J=3/2$ pion-nucleon resonance $N^*(1238)$ and of its antiparticle in the final states. The cross section for the reaction $\bar{p}+p\rightarrow\bar{p}+p+\pi^++\pi^-$ is 3.43 ± 0.23 mb at 3.28 BeV/c and 3.67 ± 0.30 mb at 3.66 BeV/c. This reaction, which is the major channel contributing to the pion-production cross section, is found to be consistent with a one-pion-exchange mechanism. In particular, the theory of Selleri and Ferrari gives a good fit to many aspects of the data. However, a detailed analysis indicates that pion production in proton-antiproton collisions is not totally consistent with a simple one-pion-exchange mechanism.

I. INTRODUCTION

WE report the results of a bubble-chamber investigation of multiple-pion production without annihilation in antiproton-proton collisions at high energies. Annihilation of the $\bar{p}$-p system into pions will be dealt with in a forthcoming publication. The exposures were made at antiproton momenta of 3.28 BeV/c and 3.66 BeV/c utilizing the separated beam facility and the BNL 20-in. liquid-hydrogen bubble chamber at the AGS.[1] A total of 2000 two-pronged and 8000 four-pronged stars were measured and processed using the GUTS program.[2] Approximately 2000 of these measurements were fitted successfully to one of the pion-production final states listed in Table I; all the fitted events were visually checked for consistency of the interpretation with the ionization on film.

In this paper we shall briefly describe the properties of the pion-production channels enumerated in Table I, devoting most of our attention to a discussion of the largest pion-production state, namely $\bar{p}p\rightarrow\bar{p}p\pi^+\pi^-$. The methods of analysis and a summary of the cross-section data will be given in Sec. II. In Sec. III we shall consider some of the general characteristics of these final states. The production of the familiar $T=\frac{3}{2}$, $J=\frac{3}{2}$ pion-nucleon isobar and its charge conjugate will be discussed in Sec. IV. All the states listed in Table I have analogs in proton-proton collisions. A comparison of the two systems will be made in the Discussion in an attempt to understand the mechanisms involved in the production process.

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† Alfred P. Sloan Foundation Fellow.
‡ Present address: Laboratory of Nuclear Research, Ecole Polytechnique, Lausanne, Switzerland.
[1] T. Ferbel, A. Firestone, J. Sandweiss, H. D. Taft, M. Gailloud, T. W. Morris, A. H. Bachman, P. Braumel, and R. M. Lea, Phys. Rev. (to be published).
[2] J. P. Berge, F. T. Solmitz, and H. D. Taft, Rev. Sci. Instr. **32**, 538 (1961).

II. ANALYSIS AND CROSS SECTIONS

The presence of the annihilation channels in the interaction of antiprotons with protons contributes greatly to the complexity of the analysis procedures used in the study of the pion-production states. In the two-pronged final states for example, every interaction is kinematically consistent with at least one annihilation channel—not necessarily of a variety that can be fitted. Also approximately 40% of the two-pronged and four-pronged events which fit one-constraint pion production interpretations (i.e., one missing neutral) are inconsistent with the ionization estimates; hence it is necessary for a physicist to examine separately each candidate event. A study of a subsample of 1360 two

TABLE I. Final states with no more than one missing neutral.

Reaction	$\bar{p}$-p final state	Cross section (mb) 3.28 BeV/c	Cross section (mb) 3.66 BeV/c	p-p analog	Cross section (mb) 3.67 BeV/c[a]
1	$\bar{p}\pi^0$	2.3 ± 0.5		$pp\pi^0$	2.9 ± 0.3
2	$\bar{n}p\pi^-$	2.0 ± 0.4		$pn\pi^+$ (2, 3)	11.4 ± 0.7 (2, 3)
3	$\bar{p}n\pi^{+b}$	2.0 ± 0.4			
4	$\bar{p}p\pi^+\pi^-$	3.43 ± 0.23	3.67 ± 0.30	$pp\pi^+\pi^-$	2.67 ± 0.13
5	$\bar{p}p\pi^+\pi^-\pi^0$	0.3 ± 0.1	0.5 ± 0.1	$pp\pi^+\pi^-\pi^0$	0.74 ± 0.07
6	$\bar{p}n\pi^+\pi^+\pi^-$	0.11 ± 0.04	0.23 ± 0.08	$pn\pi^+\pi^+\pi^-$ (6, 7)	1.15 ± 0.09 (6, 7)
7	$\bar{n}p\pi^+\pi^-\pi^-$	0.18 ± 0.06	0.16 ± 0.07		

[a] G. A. Smith, H. Courant, E. C. Fowler, H. Craybill, J. Sandweiss, and H. Taft, Phys. Rev. **123**, 2160 (1961); E. L. Hart, R. I. Louttit, D. Luers, T. W. Morris, W. J. Willis and S. S. Yamamoto, Phys. Rev. **126**, 742 (1962).
[b] This cross section was assumed to be the same as that for reaction (2).

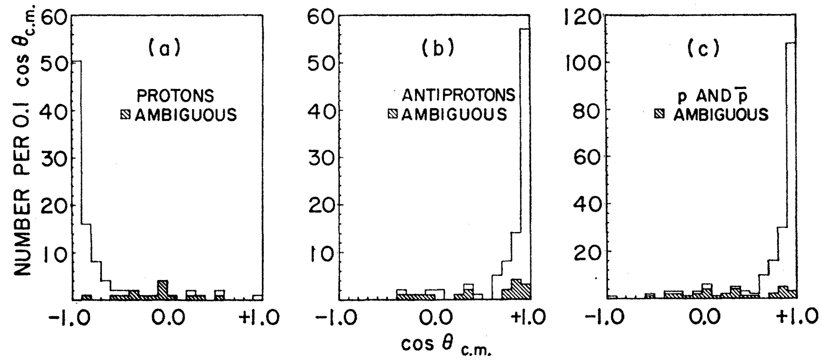

FIG. 1. Angular distribution of protons (a) and antiprotons (b) in the center-of-mass system for the reaction $\bar{p}p \rightarrow \bar{p}p\pi^0$. The cross-hatched area refers to events which are ambiguous with other single-pion production or annihilation final states. The graph at the right (c) contains the sum of (a) and (b) with the proton distribution (a) reflected about $\theta_{cm}=90°$. Based on 97 events.

prongs[3] indicates that approximately 70% of the events which account for reactions (1) and (2) can be unambiguously identified through kinematic fitting with the aid of ionization estimates. About 10% of the events contributing to these channels can be either reaction (1) or reaction (2) due to kinematic ambiguities between the two reactions. The remaining 20%, while indistinguishable from annihilation channels, can be attributed to reactions (1) and (2); this is determined through a comparison of various experimental distributions, such as the distribution in the missing mass for the ambiguous events, with the shapes predicted by phase space. Reactions (1) and (2) can generally be separated from the annihilation channels due to the presence of a relatively slow and heavily ionizing proton; this is not the case with reaction (3). For this reason the cross section for reaction (3) is assumed to be the same as that for reaction (2); this is required because of the invariance of the $\bar{p}p$ strong interaction under the *CP* operation.[4]

The background in reactions (4)–(7) is believed to be small. Reaction (4) is subject to four kinematic constraints, and it is therefore very unlikely for other reactions to simulate this final state. This is borne out by the fact that only 5% of the events which fit reaction (4) are inconsistent with the ionization estimates. Although the analysis of reaction (6) suffers somewhat from the same ambiguities encountered in reaction (3), the fact that reaction (6) is nearer threshold makes it relatively easy to separate from competing interpretations. Table I summarizes the cross sections for the reactions which can be fitted (i.e., those containing one or no neutral particles in the final state) as obtained in this experiment.[5] The analogous states for the p-p system are shown in the fifth column. It is interesting to note at this point that except for the $\bar{p}p\pi^+\pi^-$ state the cross sections for the analogous processes in the p-p system are generally larger. This point and a correction of the total pion-production cross section to account for the unobservable final states (such as $n+\bar{n}+\cdots$) will be discussed more fully in Sec. V.

III. GENERAL CHARACTERISTICS OF PION-PRODUCTION STATES

The pion-production reactions can be characterized by two dominant features. First is the apparently peripheral nature of the interaction exemplified by the fact that the antinucleon is peaked strongly forward while the nucleon is peaked backward in the center of mass system.[6] Second is the dominance of resonance production, especially the production of the $T=\frac{3}{2}$, $J=\frac{3}{2}$ pion-nucleon state $N^*(1238)$ and its antiparticle.[5,7]

Figures 1, 2, and 3 show the angular distributions in the center of mass system for nucleons, pions, and

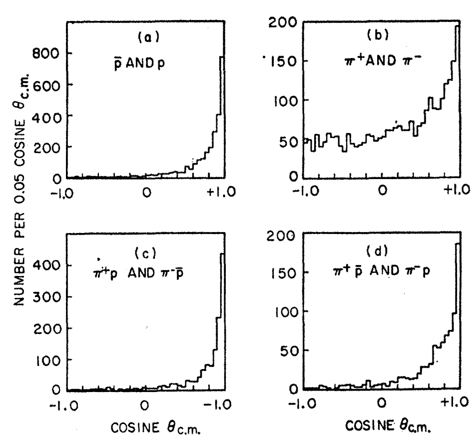

FIG. 2. Angular distribution of the pions, nucleons, and pion-nucleon systems in the center of mass for the reaction $\bar{p}p \rightarrow \bar{p}p\pi^+\pi^-$. The distributions for $T_z>0$ have been reflected about 90° and added to the distributions for $T_z<0$. Based on 1331 events.

[3] T. Ferbel, doctoral dissertation, Yale University, 1963 (unpublished).

[4] A. Pais, Phys. Rev. Letters **3**, 242 (1959).

[5] Similar results have been obtained by O. Czyzewski, B. Escoubes, Y. Goldschmidt-Clermont, M. Guinea-Moorhead, T. Hofmokl, R. Lewisch, D. R. O. Morrison, M. Schneeberger, and S. deUnamuno, *Proceedings of the Siena International Conference on Elementary Particles*, edited by G. Bernadini and G. P. Puppi (Societá Italiana di Fisica, Bologna, 1963) Vol. 1, p. 271. H. C. Dehne, E. Lohrmann, E. Ranbold, P. Soding, M. W. Teucher, and G. Wolf, *ibid*, p. 282; Phys. Rev. **136**, B843 (1964).

[6] The presence of a peripheral interaction in $\bar{p}$-p scattering was observed at Berkeley in the 1.61 BeV/c antiproton experiment. See, for example, G. R. Lynch, Rev. Mod. Phys. **33**, 395 (1961).

[7] T. Ferbel, J. Sandweiss, H. D. Taft, M. Gailloud, T. W. Morris, R. M. Lea, T. E. Kalogeropoulos, Phys. Rev. Letters **9**, 351 (1962).

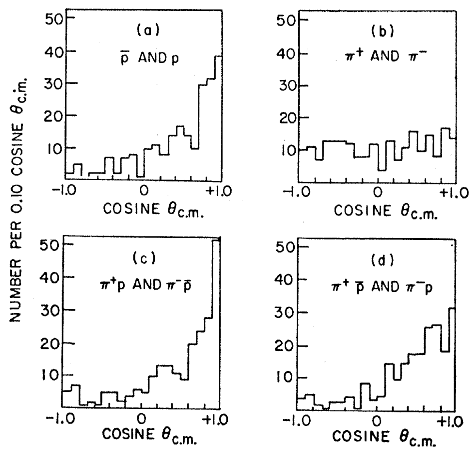

FIG. 3. Angular distribution of the pions, nucleons, and pion-nucleon systems in the center of mass for the reaction $\bar{p}p \to \bar{p}p\pi^+\pi^-\pi^0$. The distributions for $T_z>0$ have been reflected about 90° and added to the distributions for $T_z<0$. Based on 111 events.

various pi-nucleon combinations involved in some typical pion-production reactions. For reaction (1) we show the angular distribution of the proton and of the antiproton separately. We find that the distributions of particles with respect to the proton line of flight are identical to the distributions of the antiparticles with respect to the antiproton line of flight in the center of mass. This is again a consequence of the *CP* invariance of the unpolarized $\bar{p}p$ system.[4] In what follows, we assume that both *C* and *CP* invariance hold and we combine data which are predicted to be the same because of these invariance principles.

The strongly peripheral characteristics of these pion-production reactions are readily visible in Figs. 1, 2, and 3. It is of interest to note that whereas the angular distributions of the pions and protons decrease markedly in their peaking as the pion multiplicity rises, the effect does not appear to be as pronounced in the angular distributions of the doubly charged ($T_z=\pm\frac{3}{2}$) pion-nucleon systems.

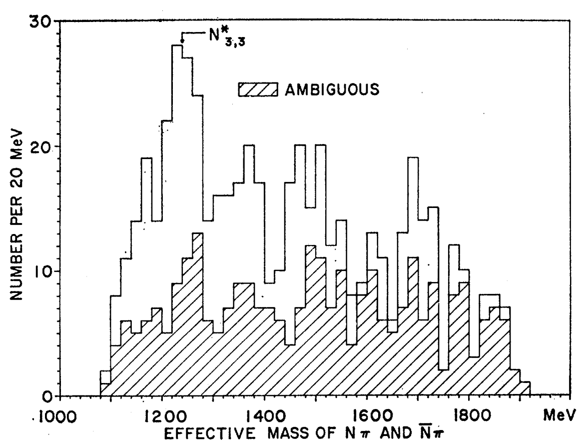

FIG. 4. Invariant-mass distribution for all the pion-nucleon systems in reactions (1), (2), and (3). The cross-hatched region refers to events which were either kinematically ambiguous between the single-pion production reactions, or could not be distinguished through ionization from pionic annihilations due to the presence of a charged nucleon of large momentum [as in reaction (3)]. Based on 276 events.

The reactions in Table I which are not represented in Figs. 1–3, have particle distributions similar to those shown. The angular distributions for the π^0 in reactions (1) and (5) are consistent with isotropy. The nucleons in reaction (2) are distributed as those in reaction (1) while the nucleons in the final states (6) and (7) do not exhibit as much peaking as do the nucleons in reaction (5).

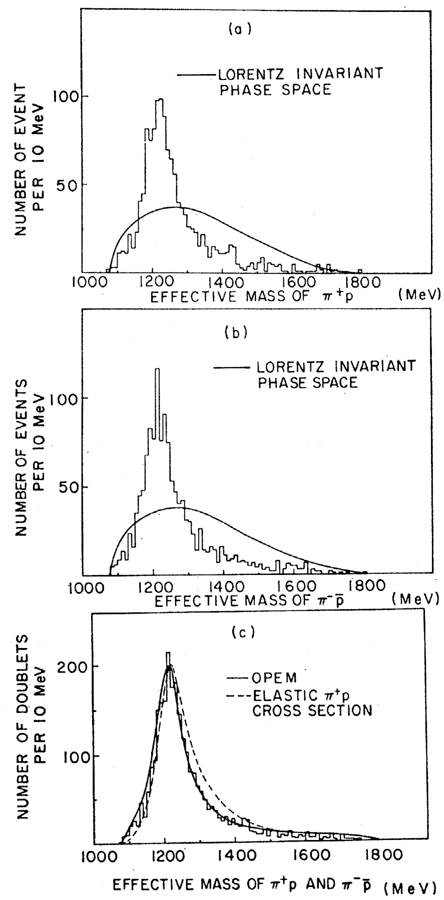

FIG. 5. Invariant-mass spectra for the π^+p (a), $\pi^-\bar{p}$ (b), and the sum of the two (c) for the reaction $\bar{p}p \to \bar{p}p\pi^+\pi^-$. In (a) and (b) we show the four-particle phase space. In (c) the comparison curves are the shape of the π^+p elastic cross section (broken line) and the prediction of the theory of Ferrari and Selleri (unbroken line). Based on 1331 events.

The importance of isobar production in high-energy interactions has long been recognized by Sternheimer and Lindenbaum.[8] Figures 4 through 6 clearly indicate the dominance of the $T=J=\frac{3}{2}$ isobar. It appears that this is the only resonance produced in significant amounts in these final states. The dipion invariant-

[8] R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. **105** 1874 (1957), Phys. Rev. Letters **5**, 24 (1960), Phys. Rev. **123** 333 (1961).

mass spectra[9] in these events show no evidence of ρ production, and only a very weak signal in the three-pion mass spectrum of zero charge is observed in the η and ω regions in Fig. 6. In Fig. 4 we show the effective-mass distribution of all the pion-nucleon systems available in the $N\bar{N}\pi$ final states. (Note that the combinations have $T_z=\pm\frac{1}{2}$). Figure 5(a) shows the invariant-mass spectrum of the π^+p combinations in reaction (4). The charge conjugate ($T_z=-\frac{3}{2}$) distribution is shown in 5(b). Four-body Lorentz-invariant phase space curves clearly give inadequate fits to the data. The sum of 5(a) and 5(b) is shown in 5(c); for comparison we give the shape of the π^+p elastic scattering cross section.[10] Also we show the prediction of a peripheral

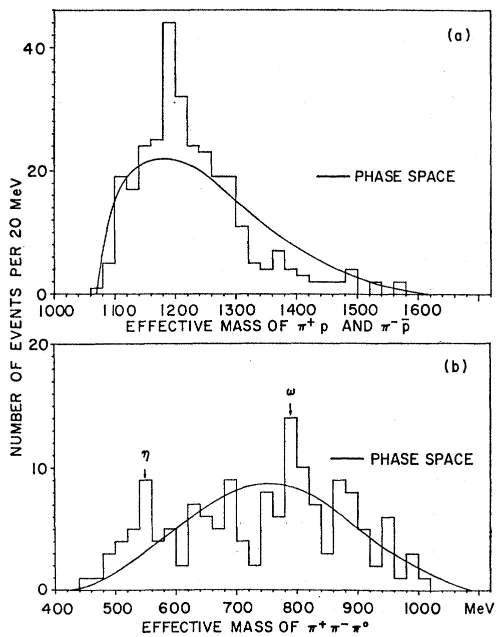

FIG. 6 Effective-mass distributions of π^+p and $\pi^-\bar{p}$ (a) and $\pi^+\pi^-\pi^0$ (b) in the reaction $\bar{p}p\rightarrow\bar{p}p\pi^+\pi^-\pi^0$. For comparison we show the spectra according to five-particle phase space. Based on 150 events.

model of Selleri and Ferrari. This will be described in the next section.

IV. DOUBLE-PION PRODUCTION

The largest single channel contributing to the pion-production process in $\bar{p}$-p interactions at these energies is the reaction:

$$\bar{p}+p\rightarrow\bar{p}+p+\pi^+\pi^-. \qquad (4)$$

[9] The invariant mass M of an N-particle combination is defined in the usual manner:

$$M=\left[\left(\sum_{i=1}^{N} E_i\right)^2-\left(\sum_{i=1}^{N} \mathbf{P}_i\right)^2\right]^{\frac{1}{2}},$$

where E_j and P_j are the energy and vector momentum of particle j.

[10] This shape is essentially the prediction of the isobar model of Ref. 8.

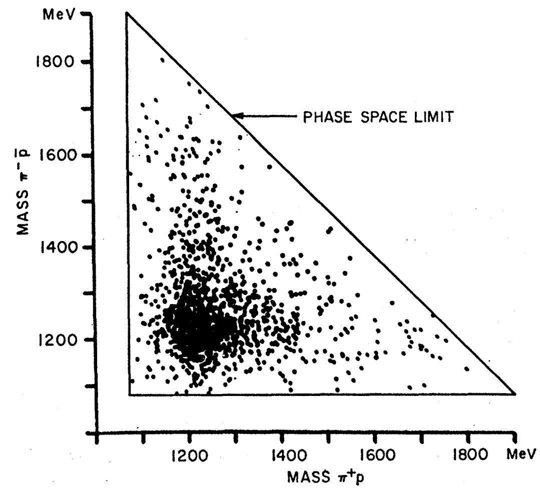

FIG. 7 Scatter plot of the effective mass of the π^+p versus the mass of $\pi^-\bar{p}$ for the reaction $\bar{p}p\rightarrow\bar{p}p\pi^+\pi^-$. Based on 1331 events.

Figure 7, which is a scattergraph of the invariant mass of π^+p versus the mass of $\pi^-\bar{p}$ system, clearly indicates that this reaction proceeds a major portion of the time through the production of a pair of $T=J=\frac{3}{2}$ (1238 MeV) isobars.[11] The formation and decay of the

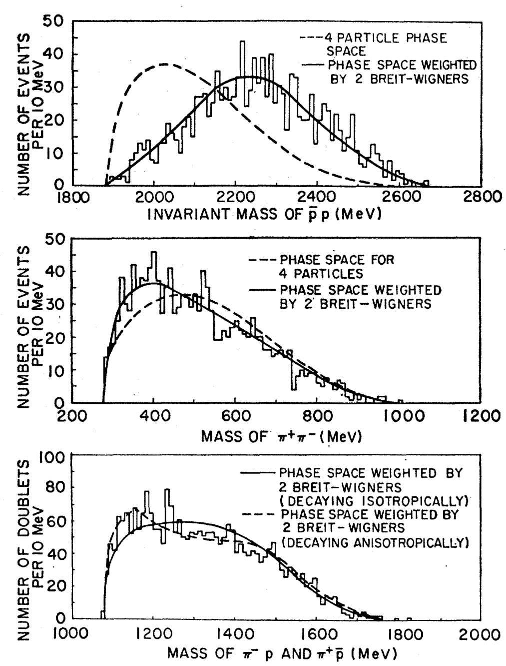

FIG. 8 Invariant-mass spectra in the reaction $\bar{p}p\rightarrow\bar{p}p\pi^+\pi^-$. The comparison curves for the double-isobar reflection were generated in a Monte-Carlo phase-space program by appropriately weighting the four-particle phase space. Based on 1331 events.

[11] For a summary of the effect of the $T=J=\frac{3}{2}$ pion-nucleon resonance on pion-nucleon scattering and pion photoproduction see M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4 (1954).

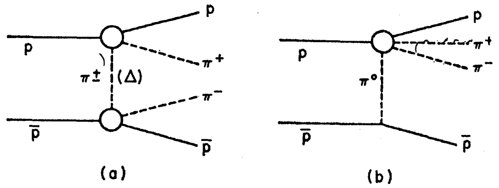

FIG. 9. The two dominant one-pion-exchange diagrams expected to contribute to reaction (4). The "double-isobar" diagram is shown in (a) and the "Drell" diagram in (b).

$T_z=\pm\frac{3}{2}$ isobars will strongly affect the shapes of the other spectra in this final state. In Fig. 8 we show the effects of the isobars on the $\pi^+\pi^-$, $\bar{p}p$, and π^-p and $\pi^+\bar{p}$ invariant-mass distributions. The four-particle Lorentz-invariant phase space does not represent the data very well. This is expected because of the dominance of the 1238-MeV isobars. The isobar reflection curves are obtained by weighting the four-particle phase space with two Breit-Wigner S-wave resonances in a Monte Carlo phase-space program. It is clear that the shapes of the experimental spectra of Fig. 8, as well as the other mass spectra in this reaction, can be understood on the basis of the dominant $T=J=\frac{3}{2}$ double-isobar production.[12]

The angular distributions of the participants in reaction (4) strongly suggest that a peripheral mechanism may be involved in this production process. The two diagrams which are here expected to dominate are shown in Fig. 9. These diagrams have been considered by various authors.[13–17] We shall follow the nomenclature of Ferrari[13] in referring to the two Feynman diagrams as the "double-isobar" and the "Drell" diagram. In what follows we shall quantitatively compare our results with the predictions of the one-pion-exchange model (OPEM) in two ways. One way will be with the theory of the type given by Salzman and Salzman (SS)[14] wherein the virtual exchanged pion can be treated as being "almost real" and no empirical form factors are used at the vertices. We shall refer to this type of theory as a pole approximation. The other comparison will be to the semiempirical theory proposed by Ferrari and Selleri (FS)[13,16] who have extended the OPEM to include off-shell pion-nucleon scattering effects. This extension is especially valid near $T=J=\frac{3}{2}$ isobar region.

The double-isobar diagram has the fortunate feature that if one neglects interferences between diagrams, the exchange diagram which leads to $T_z=\pm\frac{1}{2}$ vertices will be negligible in comparison with the diagram which leads to $T_z=\pm\frac{3}{2}$ pion-nucleon vertices. Hence if reaction (4) is interpreted as due to the double-isobar diagram, one can calculate essentially a unique four-momentum transfer for each event. This is not the situation in the case of the Drell diagram process. Here there are two equally reasonable and experimentally indistinguishable final states: pion production at either the proton or the antiproton vertex. Furthermore in the double-isobar situation the OPEM stands on rather strong footing in that the coupling constant for the πpN^* vertex is known, as is the nature of the (3,3) off-shell pion-nucleon scattering amplitude.[18]

On the other hand, nothing is known about the reaction necessary for a complete description of the Drell diagram, namely $\pi^0+p\rightarrow\pi^++\pi^-+p$. Ferrari[13] has however been able to derive this cross section after making certain simplifying assumptions as to the nature of pion-nucleon scattering. We shall use his results and his formulation of the Drell diagram in determining its contribution to reaction (4).[19] Also because of the simplifying feature in the double-isobar diagram (only one four-momentum transfer) we shall be interested in the "reflections" of a Drell process as seen in an event misinterpreted as double-isobar production.

Figure 10 shows a scattergram of the square of the four-momentum transfer (Δ^2) versus the invariant masses for the π^+p and $\pi^-\bar{p}$ systems, interpreting every event as a double-isobar process. Each event appears

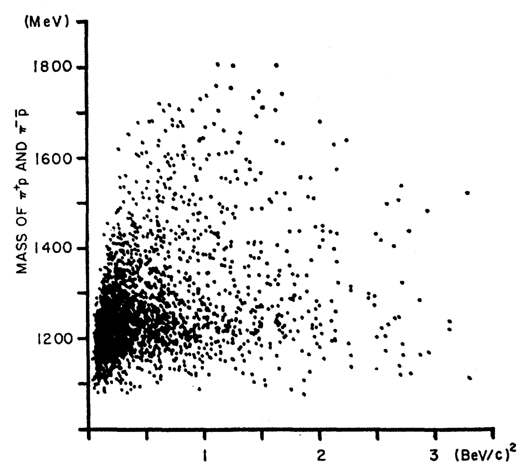

FIG. 10. Scatter plot of the invariant mass of the π^+p and $\pi^-\bar{p}$ systems versus the square of the four-momentum transfer (Δ^2) for the main double-isobar diagram. For each Δ^2 two points are plotted (π^+p and $\pi^-\bar{p}$ masses). The plot represents 1331 events in the reaction $\bar{p}p\rightarrow\bar{p}p\pi^+\pi^-$.

[12] The $\pi^+\bar{p}$ and π^-p mass spectra include the effect of introducing an anistropic decay in the π^+p and $\pi^-\bar{p}$ system. That is, to obtain the distribution given by the unbroken curve, we weighted the double-isobar-production phase space with an angular distribution in the rest frames of the isobars (A $\sin^2\theta$ dependence was used for the angle of the decay pion with respect to the "production" plane normal). The introduction of an anisotropy in the isobar decay did not significantly affect the $\pi^+\pi^-$ or $\bar{p}p$ distributions.

[13] E. Ferrari, Nuovo Cimento **30**, 290 (1963).

[14] F. Salzman and G. Salzman, Phys. Rev. **121** 1541 (1961).

[15] S. D. Drell, Rev. Mod. Phys. **33**, 458 (1961).

[16] E. Ferrari and F. Selleri, Nuovo Cimento **27**, 1450 (1963).

[17] For a review and further references on peripheral models see E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962).

[18] E. Ferrari and F. Selleri, Nuovo Cimento **21**, 1028 (1961).

[19] Dr. Selleri and Dr. Ferrari have kindly supplied us with their formulas for calculating the quantities of interest. We have used the parameters $A=0.28$, $\alpha=4.73\,\mu^2$ in the 3, 3-resonance region; also $\gamma=90\,\mu^2$ in the off-resonance region as well as in the Drell diagram calculation. See Refs. 13 and 16.

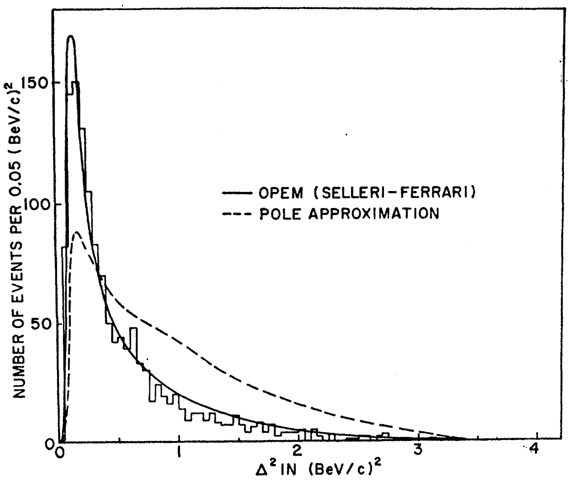

FIG. 11 Projection of points in Fig. 10 along the Δ^2 axis. The distribution in Δ^2 is compared with the prediction of the theory of Ferrari and Selleri and to the pole approximation.

twice on the plot; i.e., for a given Δ^2 there are the two masses. As expected a very strong concentration of events is seen at low Δ^2 as well as in the region of the 1238-MeV isobar.

The mass projection of this scattergram was given in Fig. 5(c). In calculating the comparison FS-theory curve for Fig. 5(c) we have included contributions from both diagrams. The calculations of the double-isobar contribution is straightforward. The Drell-diagram contribution, which accounts for 45% of the curve marked OPEM, was calculated by a Monte Carlo technique. A number of events was generated according to the distributions predicted by Ferrari for the Drell diagram.[13] Since the Drell diagram does not fully specify the nature of the three-body state at the pion-production vertex, but only the total mass, it was thought sensible to consider the off-shell process $\pi^0 p \to \pi^+\pi^- p$ as being dominated by $N^*(1238)+\pi^-$ production (just as is the case of real πp scattering at comparable center of mass energies). To introduce the effect of the $N^*(1238)$ for the Drell diagram we further weighted the generated Monte Carlo distributions with a Breit-Wigner S-wave denominator for the $\pi^+ p$ system. The effective mass of $\pi^-\bar{p}$ was then calculated, added to the resonating $\pi^+ p$ distribution, and the result normalized to the cross section predicted by Ferrari. It should be noted that the fit to the shape of the data is good, but it is essentially forced by our choice of $N^*(1238)$ dominance at the pion-production vertex of the Drell diagram. (A somewhat better fit can be obtained by assuming more double-isobar production.[20]) The fact that the normalization, i.e., the cross section for reaction (4) as predicted by Ferrari and Selleri, is correct marks the success of the FS theory.[21]

Figure 11 shows the projection of the scattergram given in Fig. 10 along the Δ^2 axis. Again, the theoretical contributions from both diagrams under consideration have been included in this plot. The curve marked "pole approximation" contains the double-isobar contribution due to the SS theory and the contribution from the Drell diagram (calculated in a manner similar to that described for the invariant-mass plot) without the empirical form factors of FS. It is clear that the pole-approximation distribution gives too small a contribution for low Δ^2 where the double-isobar diagram should dominate, and much too large a contribution for high Δ^2 where the misinterpreted Drell diagram contributes. The FS theory fits the data remarkably well.

As noted in the previous section the double-isobar diagram dominates the low Δ^2(to $\pi^+ p$) region, and therefore for low Δ^2, where the Drell diagram can be neglected, the theory should be on relatively firm ground. In Fig. 12 we show the invariant-mass spectrum for the $\pi^+ p$ and $\pi^-\bar{p}$ for the events with $\Delta^2 < 0.2 (\text{BeV}/c)^2$. For comparison we give the predictions of the SS and the FS theories where they are supposed to be most reliable. Both the SS and FS theories exhibit similar shapes, but the FS theory again gives the proper normalization.

A necessary condition for a pion-exchange mechanism is the independence of the cross section of spatial

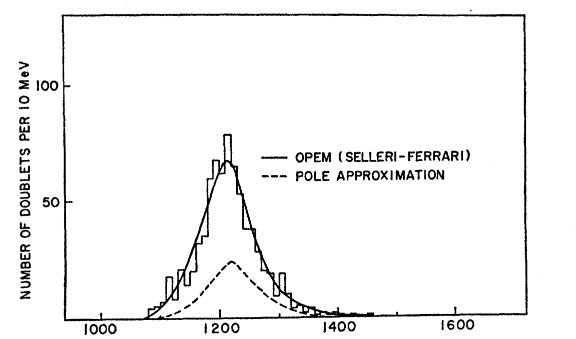

FIG. 12 Invariant mass of the $\pi^+ p$ and $\pi^-\bar{p}$ systems for Δ^2 (to $\pi^+ p$) <0.2 (BeV/c)^2 (calculated according to the double-isobar diagram) from the $\bar{p}p \to \bar{p}p\pi^+\pi^-$ reaction. Comparison curves are due to the theories of Ferrari and Selleri, and Salzman and Salzman (pole approximation). Based on 378 events.

[20] We believe that 80% is a more reasonable estimate for double-isobar production. Although this number is strongly model-dependent, it seems certain that 50% is low. The 80% figure was obtained by comparing the data with various admixtures of phase space and s-wave Breit-Wigner resonant shapes (resonance peak at 1215 and $\Gamma = 90$ MeV).

[21] It should perhaps be mentioned that the empirical corrections to the pole approximation were extracted by Selleri and Ferrari through a detailed comparison of single-pion production in pp scattering with their theory. Specifically, their theory predicted a modification of the pole term in off-shell pion-nucleon scattering (near the 3,3 region) by a multiplicative "form factor" which is a function of Δ^2 alone (having previously calculated the off-shell 3,3 amplitude). This function is only appropriate near the 3,3 region. A similar function was suggested for the off-resonance region and a good over-all fit to the pp data was obtained. We have used these same functions in our analysis of the $\bar{p}p$ data and the success of the fit suggests a degree of universality to the "form factors" of Selleri and Ferrari.

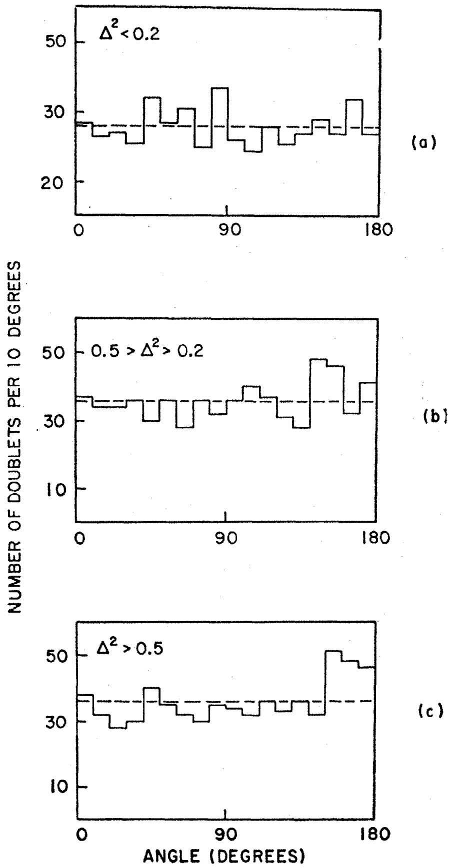

FIG. 13. Distributions in the Treiman-Yang angle for various regions of Δ^2 (to π^+p) in $(\mathrm{BeV}/c)^2$: (a) contains 234 events; (b), 321 events; (c) 324 events.

rotations about the exchange direction. This is the Treiman-Yang[22] test. The distribution in the Treiman-Yang (TY) angle for any spinless-particle-exchange mechanism is expected to be isotropic when one diagram dominates and interferences in the final states can be neglected.[23] In Fig. 13 we show the TY angle distribution interpreting the events according to the double-isobar diagram. It is seen that for all regions of Δ^2 the distributions are consistent with isotropy and therefore with an OPEM. The TY test is not appropriate to the Drell-process diagram because of the inherent interference problem.[23]

If for low Δ^2 (to π^+p) the virtual pion can be considered real then the distribution in the scattering angle of the pion in the π-N rest frame along the exchange direction should follow the scattering distribution observed in real pion-nucleon scattering.[24] Figure 14 shows a comparison of our results in terms of forward (F) to backward (B) and polar (P) to equatorial (E) pion ratios with the experimental π^+p scattering data[25] as a function of the invariant mass of the $\pi^+p(\pi^-\bar{p})$ system. The Coulomb terms in the real scattering (solid curves) have been omitted due to the fact that even for the minimum Δ^2(to π^+p) the virtual scattering occurs far from the photon pole. The agreement is poor, especially for the scattering before the resonance value of 1238 MeV. What is more, at resonance the virtual scattering does not appear to show a change in the sign of the $(\mathrm{F}-\mathrm{B})/(\mathrm{F}+\mathrm{B})$ ratio expected from s- and p-wave interference in the scattering. The distribution for $\Delta^2 < 0.2$ $(\mathrm{BeV}/c)^2$ is quite similar to those for $\Delta^2 > 0.2$ $(\mathrm{BeV}/c)^2$. All this indicates that either the off-shell πN scattering amplitude has to be further corrected or that there are strong interference effects in the final states.

In Fig. 15 we show the distribution in the scattering angles for $\Delta^2 < 0.2(\mathrm{BeV}/c)^2$ integrated over the masses of the isobars. The experimental spectrum is again peaked forward and does not follow the expected $1+3\cos^2\theta$ decay distribution of the $J=\frac{3}{2}$, $J_z=\frac{1}{2}$ isobar; a χ^2 test gives a discrepancy of 5 standard deviations. In the same figure we show the decay distribution of the isobars in their rest frames taken along the normal to the production plane (as defined by the two $T_z=\frac{3}{2}$ isobars). This distribution is, as it should be, symmetric about 90°. The comparison spectrum, $1+\frac{3}{2}\sin^2\theta$, is the transformed $1+3\cos^2\theta$ dependence along the exchange direction, assuming the observed azimuthal isotropy about the exchange direction. The fit to the data is adequate.

V. DISCUSSION

A comparison of the cross sections for pion production in $\bar{p}p$ and pp collisions indicates general agreement of the production process with a single-pion-exchange mechanism in the 3-4 BeV/c momentum region. For example, the cross section for the final state $\bar{p}p\pi^0$ is about the same as the cross section for its analog in pp scattering, namely the $pp\pi^\circ$ state. The usual OPEM requires these cross sections to be the same if interferences or final state interactions are dis-

[22] S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962). We define the Treiman-Yang angle in the usual manner. For the π^+p combination we have

$$\cos(T-Y)=\frac{(\mathbf{k}_{\mathrm{inc}}\times\mathbf{k}_{\bar{M}})\cdot(\mathbf{k}_p\times\mathbf{k}_{\pi^+})}{|\mathbf{k}_{\mathrm{inc}}\times\mathbf{k}_{\bar{M}}|\cdot|\mathbf{k}_p\times\mathbf{k}_{\pi^+}|},$$

where $\mathbf{k}_{\mathrm{inc}}$, $\mathbf{k}_{\bar{M}}$, $\mathbf{k}_p$, and $\mathbf{k}_{\pi^+}$ are the unit vectors in the directions of the incident $\bar{p}$, the outgoing $\pi^-\bar{p}$ resultant momentum, the outgoing proton, and π^+ meson, respectively. All these vectors are defined in the laboratory system. An analogous expression holds for the T-Y angle for the $\pi^-\bar{p}$ combination, where the analogous vectors are defined in the rest frame of the incident antiproton. The distributions in the T-Y angle for the π^+p and the $\pi^-\bar{p}$ systems must be identical due to the invariance of reaction (4) under C; the two distributions are, however, statistically independent and may therefore be added.

[23] See, for example, E. Ferrari, Phys. Letters 2, 66 (1962).

[24] The scattering angle of the virtual pion is defined for the π^+p system (considering double-isobar production) in terms of the unit vectors $\mathbf{k}^*$ and $\mathbf{k}_M$ as $\cos\theta=\mathbf{k}^*.\ \mathbf{k}_M$. Here $\mathbf{k}^*$ is the direction of the π^+ and the π^+p rest frame and $\mathbf{k}_M$ is the direction of the π^+p system in the laboratory (this is the exchange "direction"). Again the analogous expression for the $\pi^-\bar{p}$ scattering is independent and the distributions in the two scattering angles may be added.

[25] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. II, p. 63.

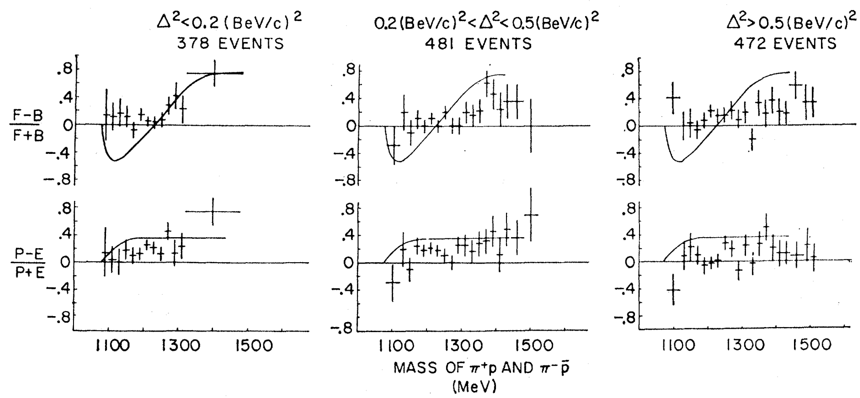

FIG. 14. Scattering angle of the virtual pion as a function of the effective mass of the $\pi^+p(\pi^-\bar{p})$ system for various regions of Δ^2 (to π^+p). The forward (F) to backward (B) and polar (P) to equatorial (E) ratios are compared with experimental (real) π^+p scattering data without the inclusion of Coulomb terms.

regarded. Also the cross section for reaction (5) is about the same size as its pp analog. In both of these and the other pion-production states we find that the cross sections in $\bar{p}p$ collisions are relatively smaller than their counterpart cross sections in pp interactions when they are compared with the predictions of the OPEM. This is especially interesting with respect to reaction (5) where any simple OPEM would predict $\sigma(\bar{p}p\pi^+\pi^-\pi^0)$ to be greater than $\sigma(pp\pi^+\pi^-\pi^0)$ (when all single-pion-exchange diagrams but not interferences or absorption effects, are considered in the final states). The opposite appears to be the case experimentally.

Several authors have recently observed similar effects in the size of the single-pion-production cross sections in $\bar{p}p$ collisions.[5] They note that although the Selleri-Ferrari theory gives a far better fit to the data than does the pole approximation, the cross-section predictions are nevertheless too high by about 50%.

All these results are not entirely surprising since the empirical correction factors of the FS theory were obtained from data in which other absorptive processes were not as strong as in the case of the $\bar{p}p$ system. That is, more damping of the low partial waves is required in the OPEM treatment of the $\bar{p}p$ system than is afforded by the FS factors because of the presence of the large annihilation channels and strong interactions in the $\bar{p}p$ final states.

The double-pion-production reaction (4) was discussed earlier in the paper where it was shown that the FS theory gives a very reasonable fit to the data. The predicted cross section at 3.66 BeV/c for the dominant double-isobar diagram is 2.1 mb, for the Drell diagrams it is 1.8 mb, and for the "weak" double-isobar graph ($T_z=\pm\frac{1}{2}$ at the πN vertices) it is approximately 0.2 mb.[26] The sum of these, 4.1 mb, is to be compared with the experimental result of 3.7 mb.

Although the FS theory successfully predicts the experimental cross section, it is suspect for two reasons. First, reaction (4) appears to proceed a good deal more than the predicted 50% of the time through double-isobar production ($T_z=\pm\frac{3}{2}$).[20,27] Second, Ferrari in a recent comparison of double-pion production in pp collisions with OPEM predictions noted sizable discrepancies between theory and experiment,[13] especially in the energy dependence of the cross sections, this probably being due to interference in the final states. These discrepancies make the seeming agreement of our data with the FS theory for all Δ^2 somewhat surprising and less satisfying. The excellent agreement of the FS double-isobar contribution with our data for

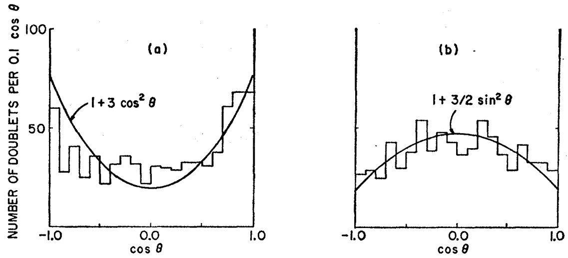

FIG. 15. The distribution in the scattering angle of the virtual pion for low Δ^2 is compared with the expected decay distribution of a $J=\frac{3}{2}$, $J_z=\frac{1}{2}$ resonance in (a). The distribution in the angle between the $\pi^+(\pi^-)$ in the $\pi^+p(\pi^-\bar{p})$ rest frame and the normal to the double-isobar production plane is shown in (b). The comparison curve is the transformed distribution as given in (a) assuming azimuthal isotropy of the decay about the exchange "direction." Based on 378 events.

[26] The cross sections for these processes according to the pole approximation, that is, without the use of off-shell correction factors, are 1.3, 4.0, and 0.1 mb in the above order.

[27] During the preparation of the present work our attention was brought to a paper presented by Selleri at the Boulder Conference on Particle Physics (July 1964). In this paper Selleri discusses some objections which other authors have to the FS theoretical approach to the OPEM (see, for example, the paper by Durand and Chiu which was presented at the same conference). He also presented his most recent off-shell corrections to the virtual πN scattering amplitude. These corrections are important for $\Delta^2>10\,\mu^2$ and essentially have no effect on the theory of the double-isobar contribution for $\Delta^2<10\,\mu^2$. The effect of these corrections is to increase the size of the double isobar contribution for $\Delta^2>0.2$ (BeV/c)^2 by about 50% and so raise the total double-isobar production cross section to approximately 80% of the reaction $\bar{p}p\rightarrow\bar{p}p\pi^+\pi^-$. This brings the double-isobar production cross section prediction of FS in agreement with the experimental results (see Footnote 20). The theory is unchanged for $\Delta^2<0.2$ (BeV/c)^2 and therefore remains in good agreement with the data. The FS contribution due to the Drell diagram remains unchanged thereby making the predicted total cross section according to the OPEM to large.

TABLE II. Breakdown of the total pion-production cross section.

Final states	Cross sections at 3.28 BeV/c (mb)
Reactions of Table I	10.32 ± 0.8
Other observable states	
$\bar{p}+p+>1\pi^0$ and $p+\bar{n}+\pi^-+\geq1\pi^0(\bar{p}+n+\pi^++\geq1\pi^0)$[a]	2.6 ± 0.9
$\bar{p}+p+2\pi^++2\pi^-$[b]	≥0.01
Inferred states	
$n\bar{n}\pi^0$[c]	2.3
$n\bar{n}+>1\pi^0$[c]	1.5
$n\bar{n}\pi^+\pi^-$(and $+\pi^0$)	2.5
All pion production	19.2 ± 2.0 mb

[a] $\sigma(\bar{p}+n+\pi^++\geq1\pi^0)$ assumed to be the same as $\sigma(p+\bar{n}+\pi^-+\geq1\pi^0)$.
[b] Based on the observation of one event.
[c] When these corrections are applied to the 0-prong cross section it is seen that the cross section for charge exchange ($\bar{p}p\rightarrow\bar{n}n$) becomes $\sim0.0_{-0.0}^{+1.3}$ mb (See Ref. 1.)

low Δ^2 does however appear to be genuine since at low Δ^2 interferences and absorptive effects are less important and may be neglected.

Recently several authors have introduced into the calculation of one-particle-exchange diagrams the effects of absorptive processes in the incoming and outgoing channels.[28–30] Specifically, Durand and Chiu have been able to show that one-pion exchange must be important in the pion production reaction (4), especially in the double-isobar process.[31]

These results and the partial success of the theory of Selleri and Ferrari[24] in predicting the cross sections for the pion production reactions in $\bar{p}p$ and pp collisions indicate consistency of the pion-production process with the one-pion-exchange mechanism.

Since the reactions listed in Table I appear to have strongly peripheral characteristics and reaction (4) is apparently dominated by a one-pion-exchange mechanism, it is therefore appropriate to estimate the correction to the total pion-production cross section for the states that are not directly measurable, such as $\bar{n}n\pi^0$, $\bar{n}n\pi^+\pi^-$, and the like, on the basis of a one-pion-exchange model.

Table II lists the measured and inferred cross sections in this experiment. An estimate of the states that are not directly observable was obtained in the following manner: Reaction (4) was assumed to proceed approximately 80% of the time via the dominant double-isobar process, 15% of the time via the Drell process, and about 5% of the time via the "weak" double-isobar process.[20] Then Clebsch-Gordan coefficients and single-pion-production cross section estimates in π-N scattering, obtained in a manner similar to that used by Ferrari,[13] determined the ratios of the processes not directly observable in $\bar{p}p$ collisions.

The total pion-production cross section for $\bar{p}p$ interactions at 3.28 BeV/c is 19 ± 2 mb which, when compared to the pion-production cross section of 26 mb in pp collisions, would seem to indicate on the basis of an OPEM interpretation the presence of strong absorptive processes in the $\bar{p}p$ initial and final states.

[28] L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964). Also *ibid.* **13**, 45 (E) (1964).
[29] A. Dar, and W. Tobocman, Phys. Rev. Letters **12**, 511 (1964).
[30] N. Sopkovitch, Nuovo Cimento **26**, 186 (1962).
[31] L. Durand and V. T. Chiu (private communication).

VI. CONCLUSION

Our study of pion production in antiproton-proton collisions at 3-4 BeV/c has shown that the reactions proceed through a peripheral mechanism. Specifically, the reaction $\bar{p}+p\rightarrow\bar{p}+p+\pi^++\pi^-$ was found to be consistent with a one-pion-exchange process. This was seen in the isotropy of the Treiman-Yang angle for various regions of four-momentum transfer, and in the general agreement of the data with theoretical models.

Through the introduction of certain "universal" correction factors to the off-shell π-N scattering amplitude the theory of Selleri and Ferrari was shown to give a far better fit to the experimental data than that given by the pole approximation. The cross sections predicted by Selleri and Ferrari are about the right size despite the fact that no attempt has been made explicitly to introduce interferences or absorptive effects into their theory. However, the fact that strong interferences occur in these final states is borne out by the distributions in the scattering angles for reaction (4). The importance of absorptive effects in the antiproton-proton system has already been suggested in the previous section. Hence, it is clear that a theory of pion-production in antiproton-proton collisions should consider these effects.

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