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Mass Difference of Ξ^- and Ξ^0

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As an application of the method for computing coupling constants proposed in a previous note, the $\Xi^- - \Xi^0$ mass difference is calculated from the consistency requirement on the various determinations of the same coupling constant. We also make some predictions concerning $NN\pi$ and $\Sigma\Sigma\pi$ couplings.

IN previous notes,[1–3] we have proposed a method of computing the residue of the pole of the amplitude in the first sheet, i.e., the coupling constants, in terms of masses without introducing any approximation in the context of strong interactions. In addition to the unitarity condition and analyticity, we assumed that one of the "residues" of the pole-like singularity[4] on the second sheet, which has no definite angular momentum, must vanish.[5]

In the previous papers it has been shown that[1] the "residues" of the pole-like singularity $R(a,\Delta)$ are given (apart from numerical proportionality constants) by

$$R(a,\Delta)=\frac{1}{2a}\left\{\int_{C_\Delta}\frac{\xi[K_0(\xi)]^2J_0(-i\xi)}{1+a^{-1}K_0(\xi)}d\xi-\int_{C_\Delta}\frac{\xi[K_0(\xi)]^2J_0(-i\xi)}{1-a^{-1}K_0(\xi)}d\xi\right\},\quad(1)$$

where $J_0(z)$ and $K_0(z)$ are Bessel functions of the first and third kind, respectively, and C_Δ is a contour in the ξ plane from 0 to ∞ in the sector Δ (see Fig. 1). In Fig. 1, $P_{j,\,j+1}$ are the points where the denominator of the integrand of Eq. (1) becomes zero. The numbering of the sectors is also shown in Fig. 1. The graphs of

$$R_\Delta(a)\equiv\tfrac{1}{2}[R(a,\Delta)+R(a^*,\Delta)]\quad(2)$$

are given in Fig. 2 for even integers Δ.

If there is a pole at $u=m_1^2$ on the first sheet, then there occur pole-like singularities at $s=s_-$ and $s=s_+$ on the second sheet, $A(s^{\rm II},t)$, where s_- and s_+ are defined by the solutions of $u-m_1^2=0$ for fixed $z=-1$ and $z=+1$, respectively. For the case of a t pole at $t=m_1^2$, the pole-like singularity appears at $p^2=-\frac{1}{4}m_1^2$. The connection between a, the solution of $R_\Delta(a)=0$, and the coupling constant G^2 is given in Table I (See Fig. 3), where G^2, for convenience of normalization, is defined by the Lagrangian density

$$\mathcal{L}=iG\bar{\psi}'(x)\gamma_5\psi(x)\varphi(x)+\cdots.$$

For the $\Xi\Xi\pi$ and $NN\pi$ coupling, $G=\sqrt{2}g_c$ or $G=g_0$, depending on whether the pion is charged or neutral, respectively. Charge independence requires $g_c\approx g_0$. On the other hand, the nonzero $\Sigma\Sigma\pi$ coupling constants are the same for all the charge states except for sign differences. These three cases are the only ones where the pion interacts with baryons belonging to the same charge multiplet. Up to this point, we have not made any approximation in the context of strong interactions.

We now introduce the approximation that $\Delta M/M$ and μ^2/M^2 can be neglected compared to unity, where M, μ, ΔM are the baryon mass, pion mass, and mass difference of baryons within the same charge multiplet, respectively (maximal error in this approximation is

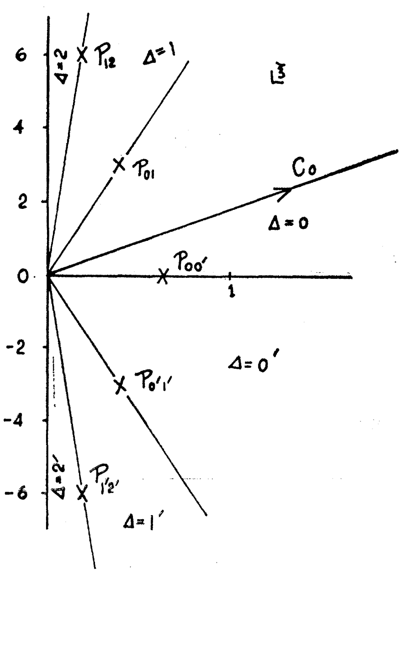

FIG. 1. Integration contour C_Δ of Eq. (1) in the sector Δ.

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[1] T. Sawada, Nuovo Cimento (to be published).
[2] A. O. Barut and T. Sawada, Nuovo Cimento (to be published).
[3] T. Sawada, Phys. Rev. Letters (to be published).
[4] A *pole-like singularity* is an essential singularity whose behavior is, however, the same as a pole, as long as the singular point is approached in a given sector Δ, that is,

$$\lim_{z\to a}(z-a)f(z)=R_\Delta \text{ for } b_\Delta<\arg(z-a)<b_{\Delta+1}.$$

Thus, a pole-like singularity has several "residues" R_Δ, each of which corresponds to a given sector (Δ).
[5] This rather complicated condition can be reduced to an assumption that a particular series of poles must vanish on the second sheet of both s and t surfaces. The equivalence of these two assumptions is shown in a separate paper (Ref. 3). The latter assumption is a special case of the usually accepted hypothesis that the scattering amplitude is as analytic as possible, as long as it is consistent with the unitarity condition for all channels.

TABLE I. Connection between a^{-1} and the coupling constant G^2. Here $K=\{[\mu^2-(M-m)^2]/[(M+m)^2-\mu^2]\}^{+1/2}$ and $L=\mu[\frac{1}{2}(2M^2+2m^2-\mu^2)+\frac{1}{2}\{(2M^2+2m^2-\mu^2)^2-4(M^2-m^2)^2\}^{1/2}]^{-1/2}$. The corresponding pole graphs are given in Fig. 3.

Amplitude	Position of pole in s or p^2	$1/a$	Corresponding pole graph number
	$s_+=2M^2+2\mu^2-m^2$	$(G^2/4\pi)K$	(a)
	$s_+=2M^2+2\mu^2-M^2$	$(G^2/4\pi)K$	(a')
Pion-baryon scattering amplitudes	$s_-=\dfrac{(M^2-\mu^2)^2}{m^2}$	$\dfrac{G^2}{4\pi}\dfrac{m^2}{M^2-\mu^2}K$	(a)
	$s_-=\dfrac{(m^2-\mu^2)^2}{M^2}$	$\dfrac{G^2}{4\pi}\dfrac{M^2}{m^2-\mu^2}K$	(a')
	t pole $p^2=-\mu^2/4$	$(G^2/4\pi)L$	(b)
Baryon-baryon scattering amplitudes	u pole $s_+=2M^2+2m^2-\mu^2$	$(G^2/4\pi)K$	(c)
	$s_-=\dfrac{(M^2-m^2)^2}{\mu^2}$	$\dfrac{G^2}{4\pi}\dfrac{\mu^2}{M^2-m^2}K$	(c)

about 2%). Then all the relations in Table I which connect the value of a^{-1} [where a is the solution of $R_\Delta(a)=0$], and the coupling constant G^2 reduce to a single relation of the form

$$a^{-1}=\frac{G^2}{4\pi}\frac{\mu}{2M}\left\{1-\frac{\mu^2}{4M^2}\right\}^{-1/2}, \tag{3}$$

except one case, namely, that of the pole-like singularity at $s=s_-$ generated by the u pole of the baryon-baryon scattering, in which case we get instead of (3)

$$a'^{-1}=\frac{G^2}{4\pi}\frac{\mu^2}{M_1^2-M_2^2}\frac{\mu}{2M}\left\{1-\frac{\mu^2}{4M^2}\right\}^{-1/2}. \tag{4}$$

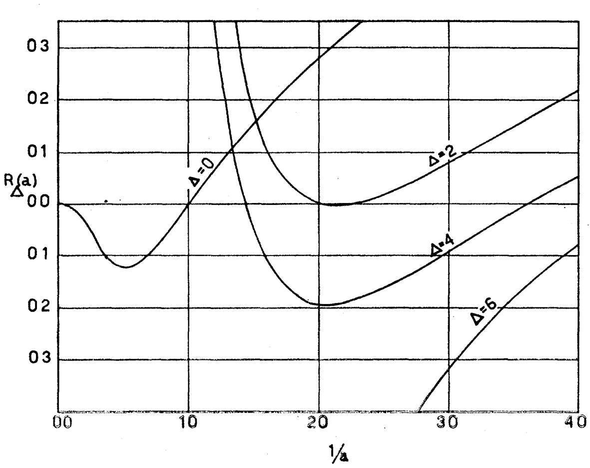

FIG. 2. Graph for $R_\Delta(a)$ versus a^{-1} for $\Delta=0,2,4,6$.

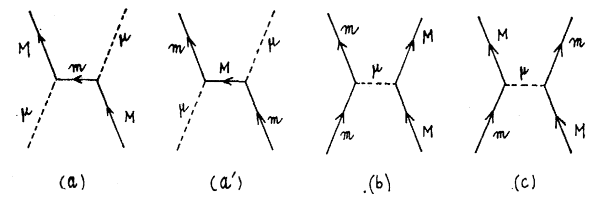

FIG. 3. Graphs for the corresponding pole term in Table I.

In Eq. (4), M_1 and M_2 are the masses of baryons which belong to the same charge multiplet but which have different charges. At this point, we use the consistency requirement that different determinations of the same coupling constant must give the same value. In the case of $\Xi\Xi\pi$ or $NN\pi$, the values of G^2 corresponding to the poles shown in Figs. 4(a), (b), and (c) are $2g_c^2$, $\sqrt{2}g_cg_0$, and g_0^2, respectively. Since the equation $R_\Delta(a)=0$ has

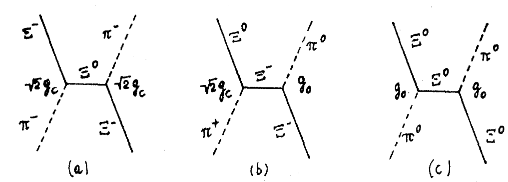

FIG. 4. Pole graph of pion-baryon scattering amplitude with various charge states of pion.

the following solutions (see Fig. 2)

$$\begin{aligned} a^{-1}&=0.0,\\ a^{-1}&=1.006,\\ a^{-1}&=1.445,\\ a^{-1}&=2,\\ a^{-1}&=3.7,\\ &\cdots, \end{aligned} \tag{5}$$

it is evident which solution should be used in each case of Figs. 4(a), (b), and (c), in order to restore the charge independence $g_c\approx g_0$. (About this consistency with charge independence, a detailed explanation is given in Ref. 2.) In this note, we examine the consistency of Eqs. (3) and (4). As the u pole of the baryon-baryon scattering comes from the exchange of a *charged* pion (see Fig. 5), G^2 is $2g_c^2$ for the case of NN or $\Xi\Xi$ scattering. Writing down Eq. (3) for the case of $G^2=2g_c^2$,

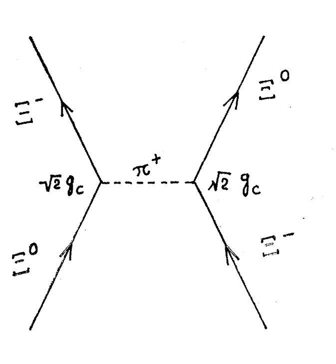

FIG. 5. u-pole diagram of baryon-baryon scattering.

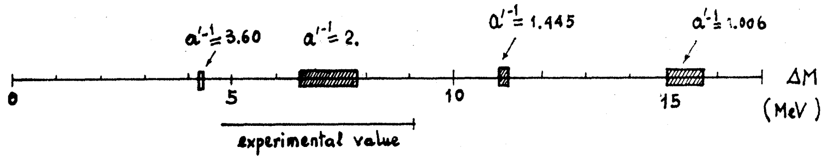

FIG. 6. Possible solution for the mass difference of a Ξ particle, and its experimental value.

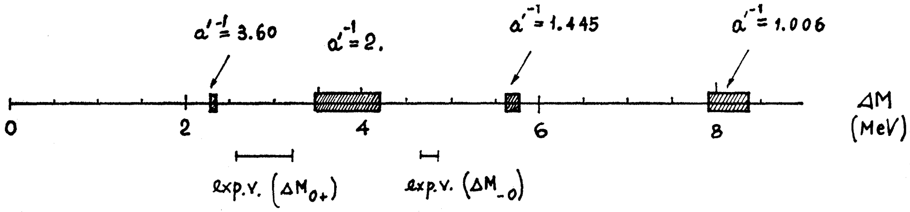

FIG. 7. Possible solution for the mass difference of Σ particles ($M_- - M_0$ and $M_0 - M_+$), and their experimental values (marked "exp.v.")

namely, for the pole corresponding to Fig. 4(a) or $s=s_+$ of Fig. 5, and taking the ratio of this equation and Eq. (4), we obtain

$$\mu_c^2/(M_1^2-M_2^2)=a'^{-1}/a^{-1}\,, \tag{6}$$

where μ_c is the mass of the *charged* pion, and a is the solution of $R_\Delta(a)=0$ corresponding to Fig. 4(a): a' is also a solution of $R_\Delta(a)=0$. For the case of $NN\pi$ or $\Xi\Xi\pi$ coupling, we get[6]

$$a^{-1}=2.076\,.$$

In the following we examine Eq. (6) in each case separately.

(*i*) *$\Xi\Xi\pi$ coupling*. The mass difference of the charged and neutral Ξ particle ΔM is

$$\Delta M=(\mu_c^2/2M)(2.076/a'^{-1})\,, \tag{7}$$

where M is the average mass of Ξ. In Fig. 6 possible values of ΔM are given which follow from Eq. (7) and the solutions a'^{-1} in Eq. (5). In order to take into account possible errors in the numerical solutions of $R_\Delta(a)=0$, we have used solutions which lie in the range

$$-0.01\leq R_\Delta(a)\leq 0.01 \tag{8}$$

from which we get the theoretical range for ΔM shown in Fig. 6.

[6] Since the root of $R_\Delta(a)=0$ at $a^{-1}\approx 2$ is not sharply determined numerically, we compute the right-hand side of Eq. (3) for $a^{-1}\approx 2$ in the following way, using the solutions $a^{-1}=1.006$ and $a^{-1}=1.445$:

$$1.006=a''^{-1}=\frac{g_0^2}{4\pi}\frac{\mu_0}{2M}\left\{1-\frac{\mu^2}{4M^2}\right\}^{-1/2},\quad \text{for Fig. 4(c)}$$

$$1.455=a'^{-1}=\frac{\sqrt{2}g_0g_c}{4\pi}\frac{(\mu_0\mu_c)^{1/2}}{2M}\left\{1-\frac{\mu^2}{4M^2}\right\}^{-1/2},\quad \text{for Fig. 4(b)}$$

$$a^{-1}=\frac{2g_c^2}{4\pi}\frac{\mu_c}{2M}\left\{1-\frac{\mu^2}{4M^2}\right\}^{-1/2},\quad \text{for Fig. 4(a)}.$$

Eliminating the coupling constants, we have

$$(1.455)^2=1.006\times a^{-1},$$
$$a^{-1}=2.076.$$

The experimental errors are at present larger. According to the measurement by London *et al.*[7]

$$(\Delta M)_{\rm exp}=6.9\pm 2.2 \text{ MeV},$$

and according to that by Carmony *et al.*,[7]

$$(\Delta M)_{\rm exp}=6.1\pm 1.6 \text{ MeV}.$$

Thus we see that the solution $a^{-1}=2$ should be used to get the more accurate value of the mass difference. In spite of the fact that the solution $a^{-1}=2$ is not very sharp (see Fig. 2), we obtain the following theoretical limits:

$$\Delta M=7.2\pm 0.7 \text{ MeV}. \tag{9}$$

The range given in Eq. (8) covers also the forward-backward discrepancy[2] in the pion-baryon case.

(*ii*) *$NN\pi$ coupling*. In this case, the mass difference of the proton and the neutron has already been measured very accurately. Putting the experimental value into the left-hand side of Eq. (6), we obtain

$$(\mu_c^2/(M_n^2-M_p^2))_{\rm exp}=8.01\pm 0.01\,,$$

which implies a solution of the equation $R_\Delta(a)=0$ at $a'^{-1}=16.66$. The higher roots of $R_\Delta(a)=0$ have not yet been determined, but we predict a solution around this value in some higher sector Δ.

(*iii*) *$\Sigma\Sigma\pi$ coupling*. In this case, since there is no restriction on account of charge independence consistency, we cannot determine the value a^{-1} in Eq. (3). However, if we assume that the coupling constant of $\Sigma\Sigma\pi$ is not much larger than that of $NN\pi$ (say $g_\Sigma^2<1.4g_N^2$), then there are only two possibilities for a^{-1}, namely,

$$a^{-1}=1.006$$

or

$$a^{-1}=0.00\,,$$

where the normalizations of g_Σ and g_N are defined by the Lagrangian density,

$$\mathcal{L}_I=ig_N\bar{\psi}_N\gamma_5\boldsymbol{\tau}\psi_N\cdot\boldsymbol{\varphi}_\pi+ig_\Sigma(\bar{\boldsymbol{\psi}}_\Sigma\gamma_5\times\boldsymbol{\psi}_\Sigma)\cdot\boldsymbol{\varphi}_\pi+\cdots. \tag{10}$$

[7] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastein, J. Kirz and M. Roos, Rev. Mod. Phys. **36**, 977 (1964).

Using Eq. (6) and the values of the roots given in Eq. (5), possible mass differences of Σ particles ($M_- - M_0$ and $M_0 - M_+$) can be calculated for $a^{-1} = 1.006$. The result is shown in Fig. 7. The experimental values,[7]

$$M_- - M_0 = 4.75 \pm 0.10 \text{ MeV},$$
$$M_0 - M_+ = 2.9 \pm 0.4 \text{ MeV}$$

do not agree with the calculated values. The only way to restore the consistency of Eqs. (3) and (4) under our assumption is to choose $a^{-1} = 0.0$. From Eq. (3), that choice means[8]

$$g_\Sigma^2/4\pi = 0.0.$$

[8] R. Capps, in *Theoretical Physics*, edited by A. Salam (International Atomic Energy Agency, Vienna 1963), p. 163.

A very small value for this coupling constant is also implied by the experimental branching ratio

$$Y_1^* \to \Sigma\pi / Y_1^* \to \Lambda\pi < 3\%.$$

We also remark that under a pure D-type coupling g_Σ^2 is identically zero.[9]

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[9] J. J. Sakurai, Ref. 8, p. 244.

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Pion Theory of Relativistic Nucleon-Nucleon Interaction*

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The one- and two-pion-exchange contributions to the nucleon-nucleon interaction, which seem to predominate outside the phenomenological core, are investigated in the exact relativistic form up to fourth order in the pion-nucleon coupling constant. It is shown that the fourth order radiative correction to the usual one-pion-exchange interaction is not negligible for larger scattering angles and higher energies. Further, the fourth-order two-pion-exchange interaction is found to contain appreciable relativistic effects, and in particular the relativistic spin-orbit interaction is found to be larger than the nonrelativistic estimates. These results can be expected to yield a more precise determination of the role of the one- and two-pion-exchange interactions in nucleon-nucleon scattering.

1. INTRODUCTION

It has become increasingly clear in recent years that the problem of nuclear forces can best be treated by a combination of field-theoretical nucleon-nucleon interaction and a phenomenological core.[1] It is of particular importance for this purpose to carry out an accurate determination of the one- and two-pion-exchange contributions to the nucleon-nucleon interaction.

Although the calculation of the one-pion-exchange interaction is quite simple, the two-pion-exchange interaction presents considerable difficulty. In an earlier paper,[2] an improved calculation of the two-pion-exchange interaction was given by taking into account the effect of the nucleon recoil, but this treatment involved the nonrelativistic approximations. Since it is hardly reasonable to expect that the nonrelativistic results can be accurately applied to nucleon-nucleon scattering up to incident energies of about 310 MeV in the laboratory system, we shall now present the relativistic results for nucleon-nucleon interaction up to fourth order in the pion-nucleon coupling constant.

It is convenient to divide the fourth-order pion-theoretical interaction into two parts. The first part represents the so-called radiative correction to the well-known relativistic one-pion-exchange interaction,[3] and it corresponds to processes in which one pion is exchanged between the nucleons while another pion is emitted and reabsorbed by the same nucleon. The second part represents the effect of the exchange of two pions between the nucleons. All these processes have been described earlier,[2] but we shall now express their exact relativistic contributions in a form suitable for numerical evaluation, and thus carry out a very accurate determination of the relativistic effects appearing there.

As far as possible we shall follow the notation of Ref. 2, and for numerical evaluations we shall take $g^2/4\pi c\hbar = 14$, $\mu c^2 = 138$ MeV, and $M/\mu = 6.8$. It should also be noted that $\lambda = \mu c/\hbar$ and $\kappa = Mc/\hbar$.

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[1] G. Breit, Rev. Mod. Phys. **34**, 766 (1962).
[2] S. N. Gupta, Phys. Rev. **117**, 1146 (1960).
[3] G. Breit and M. H. Hull, Nucl. Phys. **15**, 216 (1960).