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Calculation of the K^* Vector Meson Parameters from One- and Two-Particle Exchange Forces

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Previous calculations of the K^* parameters by Diu, Gervais, and Rubinstein, based on the approximation of Zachariasen and Zemach, are improved by taking into account two-particle exchange forces using a method already proposed by the author for the ρ bootstrap. The forces due to $K\pi$ and $\pi\pi$ exchange are added to the ρ and K^* forces considered by Diu, Gervais, and Rubinstein. Self-consistent solutions are obtained both in the one-channel problem ($K\pi$ scattering with elastic unitarity) and in a two-channel proble m involving the $K\pi$ and $K\eta$ states. In contrast to the one-particle exchange forces, the forces obtained are r eally strong enough to generate the K^*. Consequently, the numerical results show a much better agreement with experiment. Assuming that Sakurai's universality holds for the ρ coupling, a one-parameter family of solutions is obtained. The coupling constants $(K^*K\pi)$, $(K^*K\eta)$, and $(\rho\pi\pi)$ are calculated for several values of $m_{K^*}{}^2$. The first two have the right order of magnitude while the third one comes out in agreement with experiment. The qualitative features of the one-particle exchange model are shown to be maintained when two-particle exchange forces are added. This raises the possibility that they are not linked to the approximation of the forces. Thus one might understand why they are in agreement with experiment even though the one-particle exchange forces do not really give a good approximation of the unphysical jumps considered in the problem.

INTRODUCTION

RECENTLY, the equations obtained by expressing self-consistency conditions in very simplified models have been shown to possess many qualitative features in agreement with experiment, particularly when combined with SU_3 symmetry.[1–4]

However, attempts to derive numerical results from those models have not yet been very successful. In particular, Diu, Rubinstein, and I tried to calculate the K^* vector meson parameters from one-particle exchange (OPE) forces.[3] ρ and K^* forces were taken into account. A one-channel model was considered where the K^* pole was generated in the $K\pi$ scattering amplitude satisfying elastic unitarity. In a two-channel model, we improved the unitarity approximation by adding the $K\eta$ channel to the $K\pi$ one. We used a method already proposed by Zachariasen and Zemach for the ρ bootstrap.[2] The conclusion of that paper was that no quantitative agreement could be reached, though the qualitative features of the model appeared to be very interesting. Moreover, this seems to be also true for other mesonic amplitudes.[4]

As we pointed out in Ref. 5, that failure casts a doubt on the validity of the whole approximation scheme, since one is led to believe that the forces considered are not sufficient to generate bound states or resonances. In particular, the qualitative success of the bootstrap in its simplified version, as it has been developed till now, becomes questionable since, clearly, the terms which one neglects do in fact give important contributions. Consequently one is faced with the problem of improving the dynamical models which are used to express the self-consistency requirements.

In Ref. 5 we studied that question in $\pi\pi$ scattering with elastic unitarity. We tried to improve the left-hand cut approximation by adding the $\pi\pi$-exchange forces to the ρ forces on which Refs. 2 and 4 were based. It seems natural to consider the forces due to the exchange of more than one particle since the deficiency of the OPE model shows that they are important. The difficulty in doing this is that one gets infinite contributions, the corresponding diagrams being divergent. However, the absorptive parts of the fourth-order square diagrams are finite. In Ref. 5 we used that fact to get finite answers from a model including $\pi\pi$-exchange forces by considering only the exchange of two pions with an angular momentum equal to 0 or to 1 in the crossed channels. Results were encouraging; in particular the $\pi\pi$ forces considered are attractive for $l=1$, $I=1$ in the direct channel so that they facilitate the generation of the ρ. In fact the forces considered in Ref. 5 seemed to be really strong enough to produce a resonance.

Our aim in this work is to see whether the K^* generation studied in Ref. 3 is improved when two-particle exchange (TPE) forces are added approximately in the

[1] See, for instance: R. H. Capps, Phys. Rev. Letters **10**, 312 (1963); Nuovo Cimento **30**, 341 (1963); **27**, 1268 (1963); Phys. Rev. **131**, 1307 (1963); **134**, B461 (1964); H. M. Chan, P. C. De Celles, and J. E. Paton, Phys. Rev. Letters **11**, 521 (1963); R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963); Ann. Phys. (N. Y.) **23**, 415 (1963); R. E. Cutkosky, J. Kalckar, and P. Tarjanne, Phys. Letters **1**, 93 (1962); R. E. Cutkosky and M. Leon (unpublished); R. E. Cutkosky and K. Y. Lin (unpublished); A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); Nuovo Cimento **31**, 1324 (1964); R. E. Cutkosky, M. Leon, and K. Y. Lin (unpublished); B. Diu, H. R. Rubinstein, and J. L. Basdevant, Preprint. Orsay Report No. Th. 62 1964 (unpublished); R. E. Cutkosky and P. Tarjanne, Phys. Rev. **133**, B1292 (1964).

[2] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).

[3] B. Diu, J. L. Gervais, and H. R. Rubinstein, Nuovo Cimento **31**, 27 (1964).

[4] B. Diu, J. L. Gervais, and H. R. Rubinstein, Nuovo Cimento **31**, 341 (1964).

[5] J. L. Gervais, Nuovo Cimento **34**, 1347 (1964).

same way as in Ref. 5. Among the TPE contributions we take into account only the $K\pi$ forces and the $\pi\pi$ forces. In fact it will appear that, for the other TPE forces, the rest mass of the two particles which are exchanged is at least of the order of 1 BeV, and we shall show that those forces are likely to give small contributions by considering explicitly the influence of the $K\eta$ exchange.

Section 1 is devoted to the calculation of the TPE forces. The method used is the same as in Ref. 5. However, it has been modified because the mass differences among the external particles introduce some new difficulties. In that section we also try to justify the approximation made to include the TPE forces using a qualitative argument based on the nearby singularity approximation.

In Sec. 2, the K^* generation with TPE forces is studied in $K\pi$ scattering with elastic unitarity. Numerical results are discussed. Similarly, the two-channel model including TPE forces is studied in Sec. 3. Finally, the main results of the TPE model are presented in the conclusion.

The qualitative features of our model are studied very carefully in this paper. In fact it is very important to see whether the qualitative features of the OPE models are preserved when TPE forces are considered. Effectively, if this is true, the qualitative results of the OPE approximation will appear to be model-independent. Then we will understand why they seem to agree with experiment despite the failure of the OPE forces on which they are based. In the conclusion it will appear that this is indeed the case in our problem and that one really improves the numerical results by including TPE forces.

1. CALCULATION OF THE FORCES

As in Ref. 3, we shall study the K^* generation both with elastic unitarity in the $K\pi$ scattering amplitude (one-channel problem) and with the unitarity improved by adding the $K\eta$ channel to the $K\pi$ one (two-channel problem).

We then consider the reactions

$$\pi(p_1)+K(p_2)\to\pi(p_3)+K(p_4),$$
$$\pi(p_1)+K(p_2)\to\eta(p_3)+K(p_4),$$
$$\eta(p_1)+K(p_2)\to\pi(p_3)+K(p_4),$$
$$\eta(p_1)+K(p_2)\to\eta(p_3)+K(p_4),$$

where p_1, p_2, p_3, p_4 are the four-momenta of the reacting particles. The usual Mandelstam variables are

$$s=-(p_1+p_2)^2,\quad t=-(p_1+p_3)^2,\quad u=-(p_1+p_4)^2;$$

and the T matrix is defined, in the s channel, according to

$$\langle p_3,p_4|S_{ij}|p_1,p_2\rangle=\delta_{ij}\langle p_3,p_4|p_1,p_2\rangle +(2i/(\omega_1\omega_2\omega_3\omega_4)^{1/2})\delta_4(p_1+p_2+p_3+p_4)T_{ij}, \quad (1.1)$$

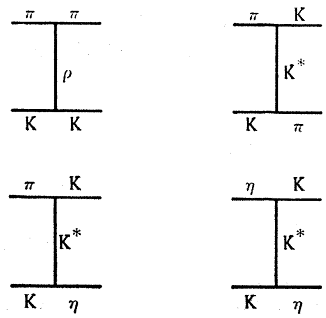

FIG. 1. The OPE diagrams.

the $K\pi$ and the $K\eta$ channels being labeled by subscripts 1 and 2, respectively.

We shall use a matrix N/D method to calculate the partial-wave amplitude $t_{ij}(s)$ corresponding to the K^* quantum numbers in the s channel ($l=1$, $I=\frac{1}{2}$). It is deduced from T_{ij} in such a way that the two-channel unitarity takes the form[6]:

$$\mathrm{Im}t_{ij}=\sum_k(q_k{}^3/\sqrt{s})t_{ik}t_{kj}{}^*\Theta(s-s_k), \quad (1.2)$$

where $\Theta(x)$ is the unit step function, and s_k and q_k are, respectively, the threshold and the center-of-mass momentum of channel k:

$$s_1=(m_k+1)^2\sim21.3,\quad s_2=(m_k+m_\eta)^2\sim57.5,$$
$$q_1{}^2(s)=(1/4s)[s-(m_k+1)^2][s-(m_k-1)^2], \quad (1.3)$$
$$q_2{}^2(s)=(1/4s)[s-(m_k+m_\eta)^2][s-(m_K-m_\eta)^2].$$

In (1.3), as in the following, we take $m_\pi{}^2$ to be 1. The properties of the OPE diagrams have been discussed in Ref. 3. In $\pi+K\to\pi+K$, the ρ and the K^* can both be exchanged. For $l=1$, $I=\frac{1}{2}$ in the s channel, the K^* force is repulsive and no self-generation is possible. In that channel, the sign of the ρ forces depends upon the relative sign of the ρ-$\pi\pi$ and ρ-$K\bar{K}$ coupling constants. In the OPE model, one is led to assume that the ρ forces are attractive ($f_\pi f_K<0$) and stronger than the K^* forces. In $\pi+K\to\eta+K$ and in $\eta+K\to\eta+K$, only the K^* exchange is allowed. The corresponding diagrams are shown in Fig. 1.

The following Hamiltonian indicates our coupling-constant definition:

$$\mathcal{H}=\{f_\pi\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}+if_K[(\partial_\mu K^\dagger)\tfrac{1}{2}\boldsymbol{\tau}K-K^\dagger(\tfrac{1}{2}\boldsymbol{\tau})\partial_\mu K]\}\boldsymbol{\rho}_\mu +\gamma_{K^*}[iK_\mu{}^{*\dagger}\boldsymbol{\tau}(K\partial_\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial_\mu K)+\mathrm{h.c.}] +\gamma_\eta[iK_\mu{}^{*\dagger}(K\partial_\mu\eta-\eta\partial_\mu K)+\mathrm{h.c.}]. \quad (1.4)$$

We calculate the graphs of Fig. 1, and project them on the angular momentum l and on the isospin I in the

[6] t_{ij} is deduced from T_{ij} by projecting on the partial wave $l=1$, $I=\frac{1}{2}$ and dividing the result by $q_i(s)q_j(s)$. This division does not introduce any new singularity, since the partial-wave projection of T_{ij} vanishes at thresholds; moreover, by performing it, one avoids the kinematical singularities which otherwise would appear in t_{12} and t_{21}, and one obtains convergent integrals into the N/D equations.

s channel, writing the result in the convenient form:

$$
\begin{aligned}
B_{11}{}^{\rho lI}(s,m_\rho{}^2) &= -(f_\pi f_K/4\pi)C_{st}{}^{I1}F_{11}{}^{\rho l1}(s,m_\rho{}^2)\,,\\
B_{11}{}^{K^*lI}(s,m_{K^*}{}^2) &= (-1)^{l+1}(\gamma_{K^*}{}^2/4\pi)\\
&\quad\times 3C_{su}{}^{I\frac{1}{2}}F_{11}{}^{K^*l1}(s,m_{K^*}{}^2)\,,\\
B_{12}{}^{l}(s,m_{K^*}{}^2) &= (-1)^{l+1}(\gamma_{K^*}\gamma_\eta/4\pi)\\
&\quad\times\sqrt{3}F_{12}{}^{l1}(s,m_{K^*}{}^2)\,,\\
B_{22}{}^{l}(s,m_{K^*}{}^2) &= (-1)^{l+1}(\gamma_\eta{}^2/4\pi)F_{22}{}^{l1}(s,m_{K^*}{}^2)\,.
\end{aligned}
\tag{1.5}
$$

(In B_{11}, a superscript ρ or K^* indicates what particle is exchanged.) In (1.5) $C^{II'}$ represents the isospin crossing matrix elements of the $K\pi$ scattering, the subscripts being such that, for instance, $C_{st}{}^{II'}$ describes the crossing from the t channel to the s channel. Explicitly,

$$
C_{su}{}^{II''}=\begin{pmatrix}-\frac{1}{3} & \frac{4}{3}\\ \frac{2}{3} & \frac{1}{3}\end{pmatrix},\qquad
C_{st}{}^{II'}=\begin{pmatrix}1/\sqrt{6} & 1\\ 1/\sqrt{6} & -\frac{1}{2}\end{pmatrix}.
\tag{1.6}
$$

Since the η is an isoscalar particle, only one value of the isospin is allowed in $\pi+K\to\eta+K$ and $\eta+K\to\eta+K$; therefore, B_{12} and B_{22} have no isospin indices.

We want to introduce the TPE forces in the same way as in Ref. 5, so we need the functions $F^{ll'}(x,y)$ which appear in (1.5) for ll' equal to zero or to one. They can be written in the form

$$
\begin{aligned}
F_{ij}{}^{l1}(x,y) &= \frac{1}{4}\int_{-1}^{+1}P_l(v)\frac{a_{ij}+v}{b_{ij}-v}dv\,,\\
F_{ij}{}^{l0}(x,y) &= \frac{1}{4}\int_{-1}^{+1}P_l(v)\frac{1}{b_{ij}-v}dv\,,
\end{aligned}
\tag{1.7}
$$

where

$$
\begin{aligned}
b_{11}{}^{\rho} &= 1+\frac{y}{2q_1{}^2(x)}\,,\\
a_{11}{}^{\rho} &= 1+\frac{1}{2q_1{}^2(x)}\left[x-\frac{(m_K{}^2-1)^2}{x}\right],\\
b_{11}{}^{K^*} &= 1+\frac{1}{2q_1{}^2(x)}\left[y-\frac{(m_K{}^2-1)^2}{x}\right],\\
a_{11}{}^{K^*} &= 1+\frac{1}{2q_1{}^2(x)}\left[x-\frac{(m_K{}^2-1)^2}{y}\right],
\end{aligned}
\tag{1.8}
$$

$$
b_{12}=\frac{1}{2q_1(x)q_2(x)}\left[q_1{}^2(x)+q_2{}^2(x)-\frac{1}{4x}(2m_K{}^2-m_\eta{}^2-1)^2+y\right],
$$

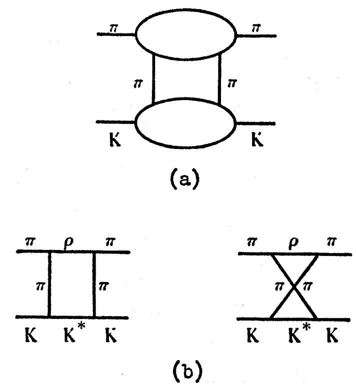

FIG. 2(a). Two-pion exchange in $\pi+K\to\pi+K$. (b) Two-pion exchange in the fourth order.

$$
a_{12}=\frac{1}{2q_1(x)q_2(x)}\left[q_1{}^2(x)+q_2{}^2(x)+x-\frac{(m_\eta{}^2-1)^2}{4x}-\frac{(m_K{}^2-1)(m_K{}^2-m_\eta{}^2)}{y}\right],
$$

$$
b_{22}=1+\frac{1}{2q_2{}^2(x)}\left[y-\frac{(m_K{}^2-m_\eta{}^2)^2}{x}\right],
$$

$$
a_{22}=1+\frac{1}{2q_2{}^2(x)}\left[x-\frac{(m_K{}^2-m_\eta{}^2)^2}{y}\right].
$$

In the OPE model, we introduce the functions

$$
\begin{aligned}
g_{11}{}^{(1)}(s) &= \frac{1}{q_1{}^2(s)}[B_{11}{}^{\rho 1\frac{1}{2}}(s,m_\rho{}^2)+B_{11}{}^{K^*1\frac{1}{2}}(s,m_{K^*}{}^2)]\,,\\
g_{12}{}^{(1)}(s) &= g_{21}{}^{(1)}(s)=\frac{1}{q_1(s)q_2(s)}B_{12}{}^{1}(s,m_{K^*}{}^2)\,,\\
g_{22}{}^{(1)}(s) &= (1/q_2{}^2(s))B_{22}{}^{1}(s,m_{K^*}{}^2)\,,
\end{aligned}
\tag{1.9}
$$

and assume that the unphysical jump of $t_{ij}(s)$ is equal to the jump of $g_{ij}{}^{(1)}(s)$:

$$
2\delta t_{ij}(s)=2\delta g_{ij}(s)\,.
$$

As explained in the introduction, we add to the OPE forces given by (1.9) the TPE forces due to the exchange of the lightest two-particle states.

A. Two-Pion Exchange in $\pi+K\to\pi+K$

The t channel corresponds to the reaction $\pi+\pi\to K+\bar{K}$ and the first intermediate state is $\pi\pi$ [Fig. 2(a)]. The diagram which provides the forces involves the $\pi\pi$ scattering and again the $\pi+\pi\to K+\bar{K}$ reaction. In the same way as in Ref. 5, we take those forces into account in fourth order by replacing the black boxes of Fig. 2(a) by the one-particle exchange terms. Thus, in the graph 2(a), the $\pi\pi$ scattering will be approximated by the ρ exchange amplitude, and the $\pi\pi\to K\bar{K}$ reaction by the K^* exchange contribution. This leads to the graphs of Fig. 2(b).

In order to compute the corresponding jump of t_{11} one has to evaluate the absorptive part A_{11t} of the graphs 2(b) for $t\geqslant 4$. This can be done by writing the unitarity in the t channel. According to (1.1) one gets

$$A_{11t}{}^{I'}(t,s)=\left(\frac{t-4}{t}\right)^{1/2}\Theta(t-4)\sum_{l'=0}^{\infty}\frac{2l'+1}{4\pi}\times B_{33}{}^{l'I'}(t,m_\rho{}^2)B_{34}{}^{l'I'}(t,m_{K^*}{}^2)P_{l'}(\cos\theta'). \quad (1.10)$$

In this formula, l' and I' are, respectively, the angular momentum and the total isospin in the t channel; θ', the scattering angle in that channel, is defined by

$$\cos\theta'=2/((t-4)(t-4m_K{}^2))^{1/2}(\tfrac{1}{2}t-m_K{}^2-1+s). \quad (1.11)$$

Only even or odd values of l' appear within the expansion (1.10) according to whether I' is even or odd. The $\pi\pi$ state has been labeled by 3 and the $K\bar{K}$ state by 4. Therefore, in (1.10), $B_{33}{}^{l'I'}(t,m_\rho{}^2)$ and $B_{34}{}^{l'I'}(t,m_{K^*}{}^2)$ are associated, respectively, with $\pi+\pi\rightarrow\pi+\pi$ and to $\pi+\pi\rightarrow K+\bar{K}$. They are the partial-wave projections in the t channel of the exchange of one particle in the s channel.

Those Born terms are calculated according to (1.4). The result can be written in the form

$$\begin{aligned}B_{33}{}^{l'I'}(t,m_\rho{}^2)&=(f_\pi{}^2/4\pi)2A^{I'1}F_{33}{}^{l'1}(t,m_\rho{}^2)\,,\\ B_{34}{}^{l'I'}(t,m_{K^*}{}^2)&=(\gamma_{K^*}{}^2/4\pi)3C_{ts}{}^{I'\frac{1}{2}}F_{34}{}^{l'1}(t,m_{K^*}{}^2)\,.\end{aligned} \quad (1.12)$$

$A^{II'}$ is the isospin crossing matrix of the $\pi\pi$ problem

$$A^{II'}=\begin{bmatrix}1/3 & 1 & 5/3\\ 1/3 & 1/2 & -5/6\\ 1/3 & -1/2 & 1/6\end{bmatrix},$$

and $C_{ts}{}^{I'I}$ is easily deduced from (1.6). The functions F_{33} and F_{34} are also given by (1.7) if one defines the a_{ij}'s and the b_{ij}'s according to

$$\begin{aligned}b_{33}&=1+m_\rho{}^2/2q_3{}^2(t)\,,\\ a_{33}&=1+t/2q_3{}^2(t)\,,\\ b_{34}&=(1/2q_3(t)q_4(t))[\tfrac{1}{2}t-m_K{}^2-1+m_{K^*}{}^2]\,,\\ a_{34}&=(1/2q_3(t)q_4(t))[\tfrac{3}{2}t-m_K{}^2-1+(m_K{}^2-1)^2/m_{K^*}{}^2]\,.\end{aligned} \quad (1.13)$$

The center-of-mass momenta q_3 and q_4 may be written

$$q_3(t)=\tfrac{1}{2}(t-4)^{1/2},\quad q_4(t)=\tfrac{1}{2}(t-4m_K{}^2)^{1/2}. \quad (1.14)$$

The half-jump $(\delta t_{11})_t$ of t_{11} associated with A_{11t} can be calculated from (1.10) by projecting on $l=1$, $I=\frac{1}{2}$. In the same way as in Ref. 5, we approximate the TPE forces by keeping only the first two terms of (1.10), i.e., $l'=0$ and $l'=1$. By doing so we avoid the difficulties due to the divergence of the square diagrams considered. In fact, the terms corresponding to $l'\geqslant 2$ would lead to divergent integrals in the N/D equations.

In Ref. 5 we studied an expansion similar to (1.10) in the $\pi\pi$ problem; we showed that it converges on a fairly large part of the left-hand cut.

Moreover one deduces from (1.7), (1.11), and (1.17) that, for fixed s,

$$B_{33}(t,m_\rho{}^2)B_{34}(t,m_{K^*}{}^2)P_{l'}(\cos\theta')=O[(t-4)^{l'}],\quad\text{as}\quad t\rightarrow 4.$$

From this it follows that, in (1.10), the terms which we neglect do not contribute much to the longest range TPE forces, since they are very small at the beginning of the t cut. We believe, on physical grounds, that the longest range forces dominate. Thus one would expect that, in a more satisfactory theory where no divergence would appear, those terms would in fact lead to really small contributions. This makes it plausible that useful results will be obtained if the divergences are removed by discarding the terms corresponding to $l'\geqslant 2$ as we do here.

It is convenient, in order to solve the N/D equations approximately, to introduce a function $g_{11t}{}^{(2)}(s)$ analytic everywhere in the s complex plane except on the left-hand cuts associated with A_{11t}, its half-jump being given by

$$\delta g_{11t}{}^{(2)}(s)=(\delta t_{11})_t(s). \quad (1.15)$$

We shall show that $g_{11t}{}^{(2)}(s)$ can be calculated from

$$g_{11t}{}^{(2)}(s)=\frac{f_\pi{}^2}{4\pi}\frac{\gamma_{K^*}{}^2}{4\pi}\frac{2}{\pi q_1{}^2(s)}\int_{-1}^{+1}\cos\theta\, d(\cos\theta)\left\{(t-t_1)\int_4^\infty dt'\frac{q_3(t')}{\sqrt{t'}}\frac{1}{(t'-t)(t'-t_1)}\times F_{33}{}^{01}(t',m_\rho{}^2)F_{34}{}^{01}(t',m_{K^*}{}^2)+3q_3(t)q_4(t)\cos\theta'\int_4^\infty\frac{dt'}{q_4(t')\sqrt{t'}}\frac{1}{(t'-t)}F_{33}{}^{11}(t',m_\rho{}^2)F_{34}{}^{11}(t',m_{K^*}{}^2)\right\}. \quad (1.16)$$

θ is the scattering angle in the s channel,

$$\cos\theta=1+t/2q_1{}^2(s). \quad (1.17)$$

In (1.16) the integration over $\cos\theta$ has to be performed after expressing $\cos\theta'$ and t as functions of s and $\cos\theta$. The integrals over t' are convergent, since the functions F_{ij} behave like $\ln(t')$ as t' goes to infinity. Equation (1.16) is obtained by writing dispersion relations for the two values of l' separately:

For $l'=0$, one has to perform a subtraction at an arbitrary point t_1. However, since we project on $l=1$, $g_{11t}{}^{(2)}(s)$ does not depend on t_1.

For $l'=1$, we have obtained a convergent integral by writing a dispersion relation for the partial-wave amplitude divided by the product $q_3(t)q_4(t)$. This does not in-

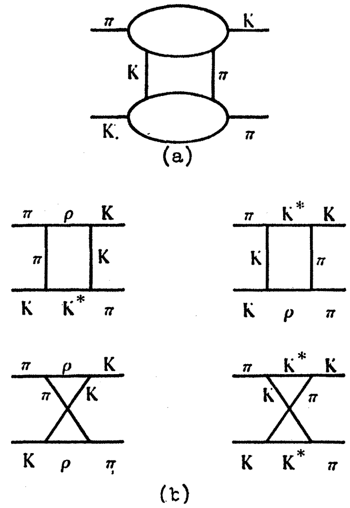

FIG. 3(a). $K\pi$ exchange diagram in $K+\pi\to K+\pi$. (b) $K\pi$ exchange in the fourth order.

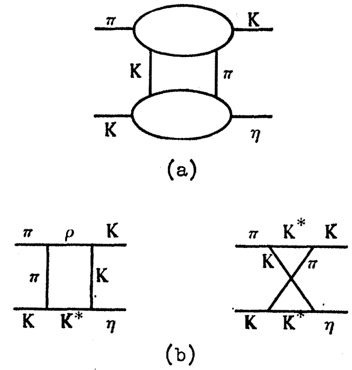

FIG. 4(a). $K\pi$ exchange in $\pi+K\to\eta+K$. (b) $K\pi$ exchange in the fourth order.

troduce any new singularity, since the P-wave amplitude in the t channel has to vanish both at the $\pi\pi$ and at the $K\bar{K}$ thresholds. Besides, in that way, one gets rid of the kinematical singularities, since for $4\leqslant t'\leqslant 4m_K^2$, the quantity $F_{34}^{11}(t'm_{K^*}^2)/q_4(t')$ remains real, though F_{34}^{11} and q_4 become complex according to (1.14) and (1.7).

We immediately see from (1.11) and (1.14) that the product $q_3(t)q_4(t)\cos\theta'$ is a polynomial in s and t. Therefore the singularities of $g_{11t}^{(2)}$ come only from the denominators $(t'-t)$, so that the right-hand side of (1.12) has only the two-pion exchange cuts. Moreover, by replacing $1/(t'-t)$ by $i\pi\delta(t'-t)$, we can verify that its half-jump is given by (1.15) and by (1.10) restricted to its first two terms. We see then that the $g_{11t}^{(2)}(s)$ defined by (1.16) do have the desired properties. Integrating over $\cos\theta$ by means of (1.7), we get finally

$$g_{11t}^{(2)}(s)=\frac{f_\pi^2}{4\pi}\frac{\gamma_{K^*}^2}{4\pi}\frac{2}{\pi q_1^2(s)}\left\{\frac{2}{q_1^2(s)}\int_4^\infty dt'\frac{q_3(t')}{\sqrt{t'}}F_{33}^{01}(t',m_\rho^2)\times F_{34}^{01}(t',m_{K^*}^2)F_{11}^{\rho 10}(s,t')+3\int_4^\infty\frac{dt'}{q_4(t')\sqrt{t'}}F_{33}^{11}(t',m_\rho^2)F_{34}^{11}(t',m_{K^*}^2)F_{11}^{\rho 11}(s,t')\right\}. \quad (1.18)$$

In this calculation, we have not taken into account the other two-particle exchange forces. In fact the first intermediate state neglected is $\pi\omega$, which has a rest mass much larger than the $\pi\pi$ rest mass ($m_\omega+m_\pi\sim 920$ MeV). This will be discussed again in Sec. 3.

B. $K\pi$ Exchange Forces in $\pi+K\to\pi+K$

The u channel corresponds again to the $K\pi$ scattering and the first intermediate state is $\pi+K$ [see Fig. 3(a)]. These forces will be taken into account in fourth order of the coupling constants by replacing the black boxes of Fig. 4(a) by the ρ and K^* exchange contributions. This leads to the graphs of Fig. 3(b).

Writing the unitarity in the u channel gives the corresponding absorptive part $A_{11u}^{I''}(u,s)$:

$$A_{11u}^{I''}(u,s)=\frac{q_1(u)}{\sqrt{u}}\Theta(u-s_1)\times\sum_{l''=0}^{\infty}\frac{2l''+1}{4\pi}[B_{11}^{\rho l''I''}(u,m_\rho^2)+B_{11}^{K^*l''I''}(u,m_{K^*}^2)]^2P_{l''}(\cos\theta''), \quad (1.19)$$

where l'', I'', and θ'' are respectively the angular momentum, the total isospin and the scattering angle in the u channel:

$$\cos\theta''=1+t/2q_1^2(u). \quad (1.20)$$

The half-jump of t_{11} corresponding to the graphs 3(b) is obtained by projecting (1.19) on $l=1$, $I=\frac{1}{2}$. As in Subsec. A, we keep only the first two terms of (1.19) and introduce an analytic function $g_{11u}^2(s)$. However, since the external masses are not equal, the method of Subsec. A has to be modified. For $l''=1$, the partial-wave amplitude vanishes at the $K\pi$ threshold, so that one's first idea is to write a dispersion relation for the partial-wave amplitude $l''=1$ divided by $q_1^2(u)$. This does not work because, according to (1.3) and (1.20), the product $q_1^2(u)\cos\theta''$ which would appear in $g_{11u}^{(2)}(s)$ is

$$q_1^2(u)\cos\theta''=\tfrac{1}{4}[t-s+(m_K^2-1)^2/u], \quad (1.21)$$

so that a kinematical pole at $u=0$ would be introduced.

To avoid that difficulty, we define $g_{11u}^{(2)}(s)$ by

$$g_{11u}^{(2)}(s)=\frac{1}{2\pi q_1^2(s)}\int_{-1}^{+1} d(\cos\theta)\,\cos\theta(u-u_1)\int_{s_1}^{\infty} du'\frac{q_1(u')}{\sqrt{u'}}\frac{1}{(u'-u)(u'-u_1)}\bigg\{5\left(\frac{\gamma_{K^*}^2}{4\pi}\right)^2[F_{11}^{K^*01}(u',m_{K^*}^2)]^2$$
$$-2\frac{f_\pi f_K}{4\pi}\frac{\gamma_{K^*}^2}{4\pi}F_{11}^{\rho01}(u',m_\rho^2)F_{11}^{K^*01}(u',m_{K^*}^2)\bigg\}+\frac{1}{2\pi q_1^2(s)}\int_{-1}^{+1} d(\cos\theta)\,\cos\theta\, 3\int_{s_1}^{\infty}\frac{du'}{q_1(u')(u'-u)\sqrt{u'}}$$
$$\times\frac{1}{4}\left[t-s+\frac{(m_K^2-1)^2}{u'}\right]\bigg\{5\left(\frac{\gamma_{K^*}^2}{4\pi}\right)^2[F_{11}^{K^*11}(u',m_{K^*}^2)]^2+2\frac{f_\pi f_K}{4\pi}\frac{\gamma_{K^*}^2}{4\pi}F_{11}^{\rho11}(u',m_\rho^2)F_{11}^{K^*11}(u',m_{K^*}^2)\bigg\}\,. \quad (1.22)$$

$g_{11u}^{(2)}(s)$ does not contain any term in $(f_\pi f_K/4\pi)^2$ since the isospin factor vanishes.

In (1.22) the singularities of the right-hand side come only from the denominators $(u'-u)$ since, for $l''=1$, we have replaced the term $(m_K^2-1)^2/u$ which would appear according to (1.21) by $(m_K^2-1)^2/u'$.

Consequently, $g_{11u}^{(2)}(s)$ is analytic except on the $K\pi$ exchange cuts. Moreover, its half-jump, calculated by replacing $1/(u'-u)$ by $i\pi\delta(u'-u)$, is equal to the projection of the first two terms of (1.19) on $l=1$, $I=\frac{1}{2}$, as it should be.

Integrating over $\cos\theta$ by means of (1.7), we get finally

$$g_{11u}^{(2)}(s)=\left(\frac{\gamma_{K^*}^2}{4\pi}\right)^2\frac{5}{\pi q_1^2(s)}\bigg\{-\frac{1}{q_1^2(s)}\int_{s_1}^{\infty} du'\frac{q_1(u')}{\sqrt{u'}}[F_{11}^{K^*01}(u',m_{K^*}^2)]^2F_{11}^{K^*10}(s,u')$$
$$+\frac{3}{2}\int_{s_1}^{\infty}\frac{du'}{q_1(u')\sqrt{u'}}[F_{11}^{K^*11}(u',m_{K^*}^2])^2F_{11}^{K^*11}(s,u')\bigg\}+\frac{\gamma_{K^*}^2}{4\pi}\frac{f_\pi f_K}{4\pi}\frac{2}{\pi}\bigg\{\frac{1}{q_1^2(s)}\int_{s_1}^{\infty} du'\frac{q_1(u')}{\sqrt{u'}}F_{11}^{\rho01}(u',m_\rho^2)$$
$$F_{11}^{K^*01}(u',m_{K^*}^2)F_{11}^{K^*10}(s,u')+\frac{3}{2}\int_{s_1}^{\infty}\frac{du'}{q_1(u')\sqrt{u'}}F_{11}^{\rho11}(u',m_\rho^2)F_{11}^{K^*11}(u',m_{K^*}^2)F_{11}^{K^*11}(s,u')\bigg\}\,. \quad (1.23)$$

In the u channel, the first intermediate state which we neglect is $K\eta$. It leads to forces of shorter range than the $K\pi$ exchange since $m_K+m_\eta\sim 1040$ MeV while $m_K+m_\pi\sim 630$ MeV. In fact we shall show, in Sec. 3 that the $K\eta$ forces are negligible.

C. Two-Particle Exchange in $\pi+K\to\eta+K$

In the t channel the reaction is $\eta+\pi\to K+\bar{K}$. Among the two nonstrange-particle intermediate states only $\pi+\eta$, $\pi+\rho$, and $\omega+\rho$ are allowed. Moreover one can see easily that in $\pi+\eta\to\pi+\eta$ and $\pi+\eta\to\pi+\rho$, there exist no OPE terms. Thus the corresponding forces in $\pi+K\to\eta+K$ vanish in fourth order; and in that approximation the longest TPE forces are given by the $K\bar{K}$ exchange. We shall neglect those forces since they have a shorter range than the $K\pi$ and $\pi\pi$ forces which we take into account ($2m_K\sim 1$ BeV). This will be discussed again in Sec. 3.

In the u channel, the reaction is also $\pi+K\to\eta+K$ and the lowest intermediate state is $\pi+K$ (as in Subsec. B, the next intermediate state is $\eta+K$). The $K\pi$-exchange diagram is shown on Fig. 4(a). It involves the $K\pi$ scattering and the $\pi+K\to\eta+K$ reaction, which we approximate by their OPE terms [see Fig. 4(b)].

The corresponding absorptive part $A_{12}(u,s)$ is

$$A_{12}(u,s)=\frac{q_1(u)}{\sqrt{u}}\Theta(u-s_1)\sum_{l''}\frac{2l''+1}{4\pi}[B_{11}^{\rho l''1/2}(u,m_\rho^2)+B_{11}^{K^*l''1/2}(u,m_{K^*}^2)]B_{12}^{l''}(u,m_{K^*}^2)P_{l''}(\cos\theta'')\,. \quad (1.24)$$

Accordingly, $g_{12}^{(2)}(s)$ is defined by

$$g_{12}^{(2)}(s)=\frac{-1}{q_1(s)q_2(s)}\frac{\gamma_{K^*}\gamma_\eta}{4\pi}\frac{f_\pi f_K}{4\pi}\sqrt{3}\bigg\{\frac{1}{q_1(s)q_2(s)}\int_{s_1}^{\infty} du'\frac{q_1(u')}{\sqrt{u'}}F_{11}^{\rho01}(u',m_\rho^2)F_{12}^{01}(u',m_{K^*}^2)F_{12}^{10}(s,u')+\frac{3}{2}\int_{s_1}^{\infty}\frac{du'}{q_2(u')\sqrt{u'}}$$
$$\times F_{11}^{\rho11}(u',m_\rho^2)F_{12}^{11}(u',m_{K^*}^2)F_{12}^{11}(s,u')\bigg\}+\frac{1}{q_1(s)q_2(s)}\frac{\gamma_{K^*}\gamma_\eta}{4\pi}\frac{\gamma_{K^*}^2}{4\pi}\sqrt{3}\bigg\{\frac{1}{q_1(s)q_2(s)}\int_{s_1}^{\infty} du'\frac{q_1(u')}{\sqrt{u'}}F_{11}^{K^*01}(u',m_{K^*}^2)$$
$$\times F_{12}^{01}(u',m_{K^*}^2)F_{12}^{10}(s,u')-\frac{3}{2}\int_{s_1}^{\infty}\frac{du'}{q_2(u')\sqrt{u'}}F_{11}^{K^*11}(u',m_{K^*}^2)F_{12}^{11}(u',m_{K^*}^2)\cdot F_{12}^{11}(s,u')\bigg\}\,. \quad (1.25)$$

In that relation, the kinematical singularities have been removed using the same method as in (1.16), i.e., by dividing the $l''=1$ contribution by the product $q_1(u)q_2(u)$. This introduces the factor

$$q_1(u)q_2(u)\cos\theta''=\tfrac{1}{4}[t-s+(m_K{}^2-1)(m_K{}^2-m_\eta{}^2)/u],$$

which has a pole at $u=0$. Consequently, in the same way as in Subsec. B, we have replaced $(m_K{}^2-1)(m_K{}^2-m_\eta{}^2)/u$ by $(m_K{}^2-1)(m_K{}^2-m_\eta{}^2)/u'$ in the definition of $g_{12}{}^{(2)}(s)$.

D. Two-Particle Exchange in $\eta+K\rightarrow\eta+K$

Here also we shall not consider the exchange of two particles in the t channel since, in the same way as in $\pi+K\rightarrow\eta+K$, the lowest intermediate state in that channel which gives a nonvanishing contribution in the fourth order is $K+\bar{K}$.

In the u channel, the reaction is again $\eta+K\rightarrow\eta+K$ and the first intermediate state is $\pi+K$ [Fig. 5(a)]. We approximate those forces by the square diagram of Fig. 5(b) which leads to the absorptive part.

$$A_{22}(u,s)=\frac{q_1(u)}{\sqrt{u}}\Theta(u-s_1)\sum_{l''}^{\infty}\frac{2l''+1}{4\pi}[B_{12}{}^{l''}(u,m_{K^*}{}^2)]^2P_{l''}(\cos\theta''). \tag{1.26}$$

A reasoning similar to the previous ones shows that $g_{22}{}^{(2)}(s)$ is

$$g_{22}{}^{(2)}(s)=\frac{\gamma_{K^*}{}^2}{4\pi}\frac{\gamma_\eta{}^2}{4\pi}\frac{3}{\pi q_2{}^2(s)}\left\{\frac{-1}{q_2{}^2(s)}\int_{s_1}^{\infty}du'\frac{q_1(u')}{\sqrt{u'}}[F_{12}{}^{01}(u',m_{K^*}{}^2)]^2F_{22}{}^{10}(s,u')\right.$$
$$\left.+\frac{3}{2}\int_{s_1}^{\infty}\frac{du'}{\sqrt{u'}}\frac{q_1(u')}{q_2{}^2(u')}[F_{12}{}^{11}(u',m_{K^*}{}^2)]^2F_{22}{}^{11}(s,u')\right\}. \tag{1.27}$$

E. Conclusion

We are now ready to introduce our new approximation of the forces into the N/D equations, namely, we shall assume that $t_{ij}(s)$ has the same left-hand cut as the function $(t_L)_{ij}(s)$ defined by

$$(t_L)_{11}=g_{11}{}^{(1)}(s)+g_{11t}{}^{(2)}(s)+g_{11u}{}^{(2)}(s),$$
$$(t_L)_{12}=(t_L)_{21}=g_{12}{}^{(1)}(s)+g_{12}{}^{(2)}(s), \tag{1.28}$$
$$(t_L)_{22}=g_{22}{}^{(1)}(s)+g_{22}{}^{(2)}(s),$$

and that its unphysical jump is given by

$$2\delta t_{ij}=2\delta(t_L)_{ij},\quad \text{for } i,\, j=1,\,2. \tag{1.29}$$

A last point is worth emphasizing about the question of forces: There exists another πK resonance called κ which has $l=0^+$ and $I=\frac{1}{2}$.[7] As in Ref. 3 we do not consider the κ-exchange forces. That approximation can be justified as follows:

We define the $\kappa\pi K$ interaction by the Hamiltonian

$$\mathcal{H}=gK^\dagger\boldsymbol{\tau}K\boldsymbol{\pi}+\text{h.c.} \tag{1.30}$$

so that g is deduced from the full width Γ_κ according to

$$\Gamma_\kappa=\tfrac{3}{2}(q_1(m_\kappa{}^2)/m_\kappa{}^2)(g^2/4\pi).$$

Taking[7] $\Gamma_\kappa\sim10$ MeV this gives $g^2/4\pi\sim0.3m_\pi{}^2$. The κ-exchange diagram in $K\pi$ scattering is calculated from (1.30). If $B_{11}{}^{\kappa11}$ is its partial-wave projection on $l=1$, $I=\frac{1}{2}$, it is easy to deduce from (1.5) that

$$\frac{B_{11}{}^{\kappa11}}{B_{11}{}^{K^*11}}\simeq\frac{g^2}{\gamma_{K^*}{}^2}\frac{1}{2q_1{}^2(a_{11}{}^{K^*}+b_{11}{}^{K^*})}. \tag{1.31}$$

Evaluating (1.31) at the $K\pi$ threshold, one gets

$$\frac{B_{11}{}^{\kappa11}}{B_{11}{}^{K^*11}}\simeq\frac{1}{50}\frac{g^2}{\gamma_{K^*}{}^2}\frac{1}{m_\pi{}^2};$$

since $\gamma_{K^*}{}^2/4\pi\sim0.9$, Eq. (1.31) is very small at threshold. On the other hand, at high-energy Eq. (1.31) goes

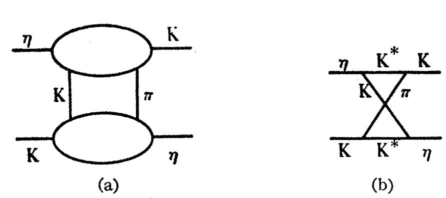

FIG. 5(a). $K\pi$ exchange in $K+\eta\rightarrow K+\eta$. (b) $K\pi$ exchange in the fourth order.

[7] Evidences of the existence of the κ meson have been reported by G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962); D. H. Miller, G. Alexander, O. I. Dahl, L. Jacobs, G. R. Kalbfleisch, and G. A. Smith, Phys. Letters **5**, 279 (1963); S. G. Wojcicki, G. R. Kalbfleisch, and M. H. Alston, *ibid.* **5**, 283 (1962); M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, G. R. Lynch, F. Muller, and J. M. Perreau, Dubna International Conference 1964 (unpublished).

The spin-parity of the κ meson is not yet definitely known; however, as pointed out by Wojciki *et al.*, if it were 0^+, the decay $K^*\rightarrow\kappa+\pi$ via strong interactions would be forbidden. Ferro-Luzzi *et al.* have observed that the ratio

$$\frac{K^*\rightarrow\kappa+\pi}{K^*\rightarrow K+\pi}<0.01,$$

which strongly favors the 0^+ assignment.

to zero. Thus the κ-exchange forces are really negligible compared to the K^*-exchange contribution.

2. ONE-CHANNEL SOLUTION

In this section, we shall determine the $K\pi$ scattering amplitude assuming that this elastic unitarity holds everywhere on the right-hand cut. Consequently, we write

$$t_{11}(s)=N/D. \tag{2.1}$$

According to (1.28), N and D satisfy the coupled integral equations

$$N(s)=\frac{1}{\pi}\int_{\text{left-hand cuts}} ds' \frac{\delta(t_L)_{11}(s')}{(s'-s)} D(s'), \tag{2.2}$$

$$D(s)=1-\frac{s-s_0}{\pi}\int_{s_1}^{\infty} \frac{q_1^3(s')}{\sqrt{s'}} \frac{N(s')ds'}{(s'-s)(s'-s_0)}. \tag{2.3}$$

One has to perform at least one subtraction in the dispersion relation of $D(s)$ since $\mathrm{Im}D(s)$ does not go to zero at infinity.

By eliminating D between (2.2) and (2.3), N can be shown to satisfy the integral equation

$$N(s)=(t_L)_{11}(s)+\frac{1}{\pi}\int_{s_1}^{\infty} ds' \frac{q_1^3(s')}{\sqrt{s'}} [(s'-s_0)(t_L)_{11}(s') - (s-s_0)(t_L)_{11}(s)] \frac{N(s')}{(s'-s)(s'-s_0)}. \tag{2.4}$$

According to (1.9), (1.18), (1.23), and (1.28), $(t_L)_{11}$ is a polynomial in the coupling constants. If one solves (2.4) by calculating successive approximations, one gets a power series. Since the forces have been determined up to fourth order, we approximate $N(s)$ by keeping only the terms of second and fourth degrees. Those terms appear in the first two iterations of (2.4) and we get

$$N(s)=(t_L)_{11}(s)+\frac{1}{\pi}\int_{s_1}^{\infty} ds' \frac{q_1^3(s')}{\sqrt{s'}} [(s'-s_0)g_{11}^{(1)}(s') - (s-s_0)g_{11}^{(1)}(s)] \frac{g_{11}^{(1)}(s')}{(s'-s)(s'-s_0)}, \tag{2.5}$$

$D(s)$ will be deduced from $N(s)$ according to (2.3). By doing so we preserve elastic unitarity on the physical cut.

Let us look now at the self-consistency conditions. They can be stated as follows:

(a) $t_{11}(s)$ given by (2.1), (2.3), and (2.5) must have a pole which corresponds to the K^* intermediate state in the direct channel, i.e.,

$$\mathrm{Re}D(m_{K^*}^2)=0. \tag{2.6}$$

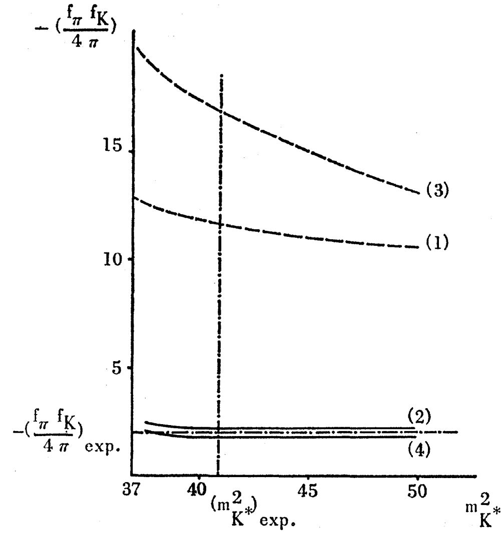

FIG. 6. Curve (1): one-channel results in the OPE model; curve (2): one-channel results in our model; curve (3): two-channel results in the OPE model; curve (4): two-channel results in our model.

(b) The width of that resonance,

$$\Gamma=\frac{1}{m_{K^*}} \frac{\mathrm{Im}D(m_{K^*})}{[\partial\, \mathrm{Re}D/\partial s]_{s=m_{k^*}^2}},$$

has to be identified with the physical width of the K^*:

$$\Gamma_{\exp}=(2q_1^3(m_{K^*}^2)/m_{K^*}^2)(\gamma_{K^*}^2/4\pi). \tag{2.7}$$

This leads to the condition

$$N(m_{K^*}^2)+2\frac{\gamma_{K^*}^2}{4\pi}\left[\frac{\partial}{\partial_s}\mathrm{Re}D\right]_{s=m_{k^*}^2}=0. \tag{2.8}$$

Accordingly one gets two equations. Looking at our expressions for the forces one sees that, *a priori*, six unknown parameters are involved: m_K^2, m_ρ^2, $m_{K^*}^2$, $f_\pi f_K/4\pi$, $f\pi^2/4\pi$, $\gamma_{K^*}^2/4\pi$. The self-consistent solutions will be studied in the following case: We give their physical values to m_ρ^2 and m_K^2, and assume that the ρ coupling satisfies

$$f_\pi^2/4\pi=-f_\pi f_K/4\pi. \tag{2.9}$$

This relation can be deduced either from exact SU_3 symmetry or from Sakurai's universality.[8] Though f_K is not known experimentally, this last hypothesis is not in disagreement with the present experimental situation.[9]

Equation (2.9) enables us to eliminate $f_\pi^2/4\pi$ which appears in g_{11t} according to (1.18), so that only $m_{K^*}^2$, $f_\pi f_K/4\pi$, and $\gamma_{K^*}^2/4\pi$ remain to be determined. Further-

[8] J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960).

[9] Some time ago, that question was discussed by J. J. Sakurai, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 176.

more we must take into account only the solutions for which $f_\pi f_K<0$; otherwise, we get an inconsistency, since (2.9) would lead to $f_\pi^2<0$.

From the self-consistency conditions we shall get a one-parameter family of solutions. We choose this parameter to be $m_{K^*}^2$ as in the OPE approximation studied in Ref. 3.

In order to discuss the numerical results of the model, it is convenient to write N and D in the form of polynomials in the coupling constants:

$$N(s,m_{K^*}^2) = -\frac{f_\pi f_K}{4\pi}\left[M_{11}{}^{\rho}+\frac{\gamma_{K^*}{}^2}{4\pi}M_{11}{}^{\rho K^*}-\frac{f_\pi f_K}{4\pi}M_{11}{}^{\rho\rho}\right] +\frac{\gamma_{K^*}{}^2}{4\pi}\left[-M_{11}{}^{K^*}+\frac{\gamma_{K^*}{}^2}{4\pi}M_{11}{}^{K^*K^*}\right], \quad (2.10)$$

$$\mathrm{Re}D(s,m_{K^*}^2) = 1+\frac{f_\pi f_K}{4\pi}\left[G_{11}{}^{\rho}+\frac{\gamma_{K^*}{}^2}{4\pi}G_{11}{}^{\rho K^*}-\frac{f_\pi f_K}{4\pi}G_{11}{}^{\rho\rho}\right] -\frac{\gamma_{K^*}{}^2}{4\pi}\left[-G_{11}{}^{K^*}+\frac{\gamma_{K^*}{}^2}{4\pi}G_{11}{}^{K^*K^*}\right]. \quad (2.11)$$

The functions M_{11} are calculated from the results of Sec. 1 by adding the contribution of the second iteration given by (2.5) and extracting the coupling-constant factors. The G_{11}'s are deduced from the M_{11}'s according to (2.3) by letting

$$G_{11}(s,m_{K^*}^2)=\frac{s-s_0}{\pi}P\int_{s_1}^{\infty} ds'\frac{q_1{}^3(s')}{\sqrt{s'}}\frac{M_{11}(s',m_{K^*}^2)}{(s'-s)(s'-s_0)}. \quad (2.12)$$

We put the subtraction point s_0 at the same place as in the OPE model by setting

$$s_0=2(m_K^2+1)-m_{K^*}^2. \quad (2.13)$$

This can be justified by the same qualitative arguments as in Ref. 3, since the TPE cuts which we consider enclose the cuts due to ρ and K^* exchange forces.

According to (2.10) and (2.11) the self-consistency relations (2.6) and (2.8) lead, for a given value of $m_{K^*}^2$, to two equations of second degree in $f_\pi f_K/4\pi$, the coefficients of those equations being polynomials in $\gamma_{K^*}{}^2/4\pi$.

For a fixed value of $m_{K^*}^2$, the functions M_{11}, G_{11}, $\partial G_{11}/\partial s$ which are involved in (2.6) and (2.8) were computed on a UNIVAC 1107. Then we determined the relationship between the two coupling constants by solving the two second-order equations for several values of $\gamma_{K^*}{}^2/4\pi$. No ambiguity appeared since each of them had only one negative solution. The intersection of the two curves so obtained gave the self-consistent solution.

On Fig. 6, curve (2) shows the relation between $f_\pi f_K/4\pi$ and $m_{K^*}^2$ which follows from our model; for comparison we have also reproduced on that figure the results of the OPE model given in Ref. 3 [curve (1)]. Similarly, on Fig. 7, curves (1) and (2) represent $\gamma_{K^*}{}^2/4\pi$ as function of $m_{K^*}^2$ in both cases.

The OPE equations discussed in Ref. 3 can be determined from (2.6), (2.8), (2.10), and (2.11) by keeping only the second-order terms in N and D.

All the parameters which appear in our equations can be deduced from experiment if one assumes that (2.9) holds:

$$\begin{gathered} m_{K^*}{}^2\sim 41\,,\\ \gamma_{K^*}{}^2/4\pi\sim 0.9\,,\\ f_\pi{}^2/4\pi=-f_\pi f_K/4\pi\sim 2 \text{ to } 2.5\,. \end{gathered} \quad (2.14)$$

As pointed out in Ref. 3, the OPE model leads to values of the coupling constants which are much too large. For instance, one sees from Figs. 6 and 7 that, taking the physical value of the K^* mass, one gets, in that model,

$$f_\pi f_K/4\pi\sim -12\,,\quad \gamma_{K^*}{}^2/4\pi\sim 4.5\,. \quad (2.15)$$

This, clearly, reflects the fact that the OPE forces are not strong enough to generate the K^* pole in the direct channel.

On the contrary, our model leads to much lower values; for instance, setting $m_{K^*}{}^2=41$, one has (see Figs. 6 and 7):

$$f_\pi f_K/4\pi\sim -2.2\,,\quad \gamma_K{}^2/4\pi\sim 2.2\,. \quad (2.16)$$

As a matter of fact, comparing (2.16) with (2.14), we see that one obtains a much better agreement with experiment in our model. In particular the product $f_\pi f_K/4\pi$ given by (2.16) is equal to the physical value indicated in (2.14). Consequently, one is led to believe that the left-hand-cut approximation is really improved when our model of TPE forces is included.

It is interesting to compare the signs of the OPE forces and of the TPE forces. Looking at (1.18), we see that g_{11t} is given by a positive integral since the functions $F^{ll'}$ are positive. This shows that the $\pi\pi$-exchange forces are attractive and have, therefore, the same sign as the ρ-exchange forces.

The sign of the $K\pi$-exchange forces is not obvious, since $g_{11u}(t)$ contains contributions of both signs. To determine that sign we have also solved the K^* problem neglecting the $K\pi$-exchange forces, i.e., keeping only in N and D the OPE contributions and the term coming from g_{11t}. It appears that the absolute magnitude of $f_\pi f_K/4\pi$ is reduced when the $K\pi$-exchange forces are discarded.[10] This shows that these forces are repulsive, since the attraction due to $\pi\pi$ and to ρ exchanges needs to be larger when they are taken into account; they have, thus, the same sign as the K^*-exchange forces.

The OPE model predicts that no $K\pi$ resonance should appear with isospin $\frac{3}{2}$ and spin 1, since it leads to repulsive forces in that channel. The situation is not so clear

[10] This is also true in the OPE model; see Ref. 4.

cut in our model since, projecting our TPE forces on $l=1$, $I=\frac{3}{2}$, one gets contributions of both signs. However, we have determined the $l=1$, $I=\frac{3}{2}$ amplitude by the same method as for $l=1$, $I=\frac{1}{2}$, assuming that the same forces dominate. In that problem, $m_{K^*}^2$, $f_\pi f_K/4\pi$, and $\gamma_{K^*}^2/4\pi$ appear as unknown parameters. We have found that, if we take the physical value of $m_{K^*}^2$ and assume that the coupling constants are given by (2.16), no resonance appears, since we have verified (at least in the region where the model can be trusted, i.e., for $s\lesssim 150$) that ReD does not vanish.[11]

A last point is worth emphasizing. Using the same method as in Ref. 5 one can show that, in our model, the cuts associated with the $K\pi$ intermediate states in the s channel and in the u channel satisfy the crossing symmetry to fourth order in the coupling constant if one takes into account only the S and P waves in those channels. This was not of course true in the OPE model, since no cut appeared in the u channel, while the cut due to the $K\pi$ intermediate state was considered in the s channel. Consequently, our solution appears to be more crossing symmetric.

3. TWO-CHANNEL SOLUTION

In this section, we improve the unitarity approximation by taking into account also the $K\eta$ channel assuming that the jumps across the unphysical cuts are given by (1.28). As shown by Bjorken[12] the N/D method can still be used if one introduces 2×2 matrices.

If we write

$$t=ND^{-1}, \tag{3.1}$$

the matrices N and D are to be determined according to

$$N_{ij}(s)=\frac{1}{\pi}\int_{\text{left-hand cuts}} ds' \sum_k \frac{\delta(t_L)_{ik}(s')D_{kj}(s')}{(s'-s)}, \tag{3.2}$$

$$D_{ij}(s)=\delta_{ij}-\frac{(s-s_0)}{\pi}\int_{s_1}^{\infty}\frac{q_i^3(s')}{\sqrt{s'}}\frac{N_{ij}(s')ds'}{(s'-s)(s'-s_0)}. \tag{3.3}$$

In the same way as in Sec. 2, N will be approximated by

$$N_{ij}(s)=(t_L)_{ij}(s)+\frac{1}{\pi}\sum_{k=1,2}\int_{s_k}^{\infty}ds'\frac{q_k^3(s')}{\sqrt{s'}}[(s'-s_0)g_{ik}^{(1)}(s')-(s-s_0)g_{ik}^{(1)}(s)]\frac{g_{kj}^{(1)}(s')}{(s'-s)(s'-s_0)}, \tag{3.4}$$

$g_{ij}^{(1)}(s')$ being given by (1.9). $D(s)$ will be deduced from $N(s)$ according to (3.3). We put the subtraction point s_0 which appears in (3.3) and (3.4) in the same place as in the one-channel model [see formula (2.13)].

It is convenient to rewrite N_{ij} and Re$D_{ij}=R_{ij}$ showing the dependence upon the coupling constants:

$$\begin{aligned}
N_{11}&=N+\frac{\gamma_{K^*}^2}{4\pi}\frac{\gamma_\eta^2}{4\pi}M_{11}^{K\eta},\\
N_{12}&=\frac{\gamma_{K^*}\gamma_\eta}{4\pi}\left\{M_{12}^{K}-\frac{\gamma_{K^*}^2}{4\pi}M_{12}^{KK}-\frac{f_\pi f_K}{4\pi}M_{12}^{K\rho}+\frac{\gamma_\eta^2}{4\pi}M_{12}^{K\eta}\right\},\\
N_{21}&=\frac{\gamma_{K^*}\gamma_\eta}{4\pi}\left\{M_{12}^{K}-\frac{\gamma_{K^*}^2}{4\pi}M_{21}^{KK}-\frac{f_\pi f_K}{4\pi}M_{21}^{K\rho}+\frac{\gamma_\eta^2}{4\pi}M_{21}^{K\eta}\right\},\\
N_{22}&=\frac{\gamma_\eta^2}{4\pi}\left\{M_{22}^{\eta}+\frac{\gamma_{K^*}^2}{4\pi}M_{22}^{K\eta}+\frac{\gamma_\eta^2}{4\pi}M_{22}^{\eta\eta}\right\},
\end{aligned}\tag{3.5}$$

$$\begin{aligned}
R_{11}&=\text{Re}D-\frac{\gamma_{K^*}}{4\pi}\frac{\gamma_\eta}{4\pi}G_{11}^{K\eta},\\
R_{12}&=-\frac{\gamma_{K^*}\gamma_\eta}{4\pi}\left\{G_{12}^{K}-\frac{\gamma_{K^*}^2}{4\pi}G_{12}^{KK}-\frac{f_\pi f_K}{4\pi}G_{12}^{K\rho}+\frac{\gamma_\eta^2}{4\pi}G_{12}^{K\eta}\right\},\\
R_{21}&=-\frac{\gamma_{K^*}\gamma_\eta}{4\pi}\left\{G_{21}^{K}-\frac{\gamma_{K^*}^2}{4\pi}G_{21}^{KK}-\frac{f_\pi f_K}{4\pi}G_{21}^{K\rho}+\frac{\gamma_\eta^2}{4\pi}G_{21}^{K\eta}\right\},\\
R_{22}&=1-\frac{\gamma_\eta^2}{4\pi}\left\{G_{22}^{\eta}+\frac{\gamma_{K^*}^2}{4\pi}G_{22}^{K\eta}+\frac{\gamma_\eta^2}{4\pi}G_{22}^{\eta\eta}\right\},
\end{aligned}\tag{3.6}$$

where N and ReD are given by the one-channel approximation (2.10) and (2.11). The functions G_{ij} are calculated from the M_{ij}'s according to (3.3).

Owing to the π-η mass difference, a difficulty occurs in t_{22}. In fact, looking at the expression (1.27) of $g_{22}^{(2)}$, we see that it contains $F_{22}^{ll'}(s,u')$ integrated over u' for $u'\geqslant s_1$. According to (1.7) and (1.8), $F_{22}^{ll'}$ becomes complex if

$$s+u'\langle 2(m_K^2+m_\eta^2).$$

The corresponding singularity occurs on the right-hand cut if this inequality holds for $s>s_1$, i.e., for

$$u'\leqslant 2(m_K^2+m_\eta^2)-s_1\sim 37.$$

[11] The same result holds if one takes the physical values of the coupling constants. This is to be compared to a similar discussion in the $\pi\pi$ problem (Ref. 4): assuming that the ρ forces dominate, we found in Ref. 4 that the $l=2$, $I=0$ amplitude had a resonance behavior which was tentatively identified with the f^0.

[12] J. D. Bjorken, Phys. Rev. Letters, 4, 473 (1960).

Consequently, the part of the integral corresponding to $s_1<u'<37$ in $g_{22}^{(2)}$ leads to singularities on the right-hand cut. This reflects a general property of the $K\eta$-scattering amplitude: The cuts due to $K\pi$ exchange and the unphysical part of the right-hand cut overlap for $s_1\leqslant s\leqslant 37$, so that, strictly speaking, (3.2) and (3.3) do not hold. However, the integrands of (1.27) vanish for $u'=s_1$ and it appears that the undesired jump of $g_{22}^{(2)}$ is small compared to $\mathrm{Re}g_{22}^{(2)}$, since it comes only from the beginning of the integration in (1.27) where the integrand is close to zero. Therefore we shall neglect that cut, assuming that (3.2) and (3.3) hold so that N will have a small imaginary part for $s_1\leqslant s\leqslant 37$. However, since the physical value of the K^* mass ($m_{K^*}{}^2\sim 41$) is outside that interval, we shall avoid that difficulty by restricting ourselves to $m_{K^*}{}^2>37$. On the other hand, the K^* does not decay into $K+\eta$. Thus we shall look for solutions satisfying

$$37<m_{K^*}{}^2<s_2. \qquad (3.7)$$

By doing so, we also avoid the difficulties due to the location of the OPE cuts (see Appendix of Ref. 3).

We consider now the self-consistency equations. The first condition is that a pole should occur in the three amplitudes considered corresponding to the K^* intermediate state in the s channel. This leads to

$$\mathrm{Re}[\det D(m_{K^*}{}^2)]=0. \qquad (3.8)$$

Moreover, the residue matrix at that pole has to be identified with the one obtained from the graphs of Fig. 8 where the K^* appears in the s channel:

$$\mathcal{T}=\frac{1}{(s-m_{K^*}{}^2+i\Gamma m_{K^*})}\times\begin{pmatrix} -2\dfrac{\gamma_{K^*}{}^2}{4\pi} & -\dfrac{2}{\sqrt{3}}\dfrac{\gamma_{K^*}\gamma_\eta}{4\pi} \\ -\dfrac{2}{\sqrt{3}}\dfrac{\gamma_{K^*}\gamma_\eta}{4\pi} & -\dfrac{2}{3}\dfrac{\gamma_\eta{}^2}{4\pi}\end{pmatrix}. \qquad (3.9)$$

In principle, that condition provides only two more equations, since the residue matrix should have zero determinant and should be symmetric. In fact our solution automatically has zero determinant but is not symmetric since we have not exactly solved the N/D equations (see Refs. 2 and 3).

We look for self-consistent solutions under the same hypothesis as in the one-channel model, keeping only four parameters: $m_{K^*}{}^2$, $f_\pi f_K/4\pi$, $\gamma_{K^*}{}^2/4\pi$, and $\gamma_\eta{}^2/4\pi$. The method used is the same as in Refs. 2 and 3. We satisfy exactly the conditions

$$t_{11}=\mathcal{T}_{11},\quad t_{22}=\mathcal{T}_{22}. \qquad (3.10)$$

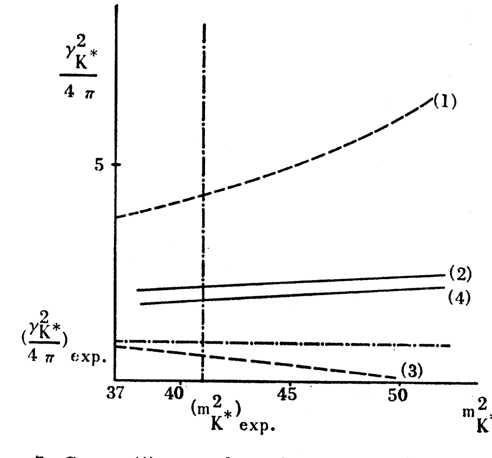

FIG. 7. Curve (1): one-channel results in the OPE model; curve (2): one-channel results in our model; curve (3): two-channel results in the OPE model; curve (4): two-channel results in our model.

According to (3.9) and (3.1) this leads to the equations (we neglect $\mathrm{Im}D$, which is small compared to $\mathrm{Re}D$)!

$$\begin{aligned}2(\gamma_{K^*}{}^2/4\pi)(\mathrm{Re}\,\det D)'+N_{11}R_{22}-N_{12}R_{21}=0,\\ \tfrac{2}{3}(\gamma_\eta{}^2/4\pi)(\mathrm{Re}\,\det D)'+N_{22}R_{11}-N_{21}R_{12}=0,\end{aligned} \qquad (3.11)$$

where

$$(\mathrm{Re}\,\det D)'=[\partial(\mathrm{Re}\,\det D)/\partial s]_{s=m_{K^*}{}^2}$$

and

$$\mathrm{Re}\,\det D=R_{11}R_{22}-R_{12}R_{21}.$$

The conditions (3.8) and (3.11) enable us to determine the solution. Then we calculate an asymmetry parameter (3.12)

$$\mathcal{Q}=\frac{t_{12}}{\mathcal{T}_{12}}=\frac{-\sqrt{3}}{2\gamma_{K^*}\gamma_\eta/4\pi}\,\frac{1}{(\mathrm{Re}\,\det D)'}\times\{N_{12}R_{11}-N_{11}R_{12}\}. \qquad (3.12)$$

$\mathcal{Q}$ would be equal to 1 for an exactly symmetric t matrix.

The numerical calculations have been performed using the same method as in the one-channel approximation (see Sec. 2). For a fixed value of $m_{K^*}{}^2$ satisfying (3.7), Eqs. (3.8) and (3.10) led to three equations of second degree in $f_\pi f_K/4\pi$. No ambiguity appeared, since each of them had only one negative solution and we got three relations expressing $f_\pi f_K/4\pi$ as functions of $\gamma_{K^*}{}^2/4\pi$ and $\gamma_\eta{}^2/4\pi$. That system of equations has been

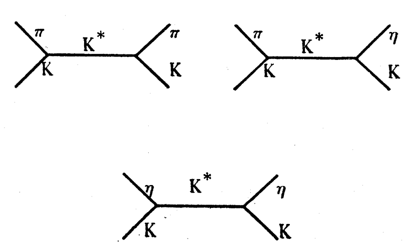

FIG. 8. The K^* pole in the s channel.

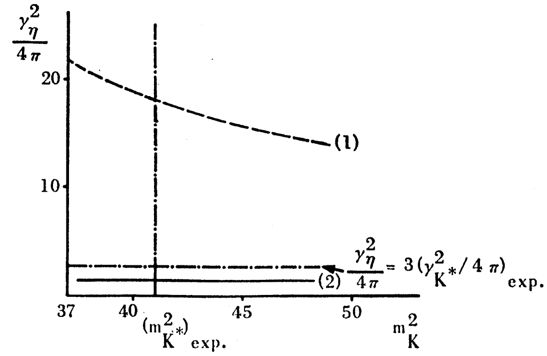

FIG. 9. Curve (1): result of the OPE model; curve (2): result of our model.

solved numerically by computing $f_\pi f_K/4\pi$ for several values of the two K^* coupling constants. For a given value of $\gamma_\eta^2/4\pi$ we then have three curves expressing the relation between $f_\pi f_K/4\pi$ and $\gamma_{K^*}^2/4\pi$. Self-consistency is achieved for the value of $\gamma_\eta^2/4\pi$ such that these curves coincide at a point. For comparison we have reproduced the calculated values of $\gamma_{K^*}^2/4\pi$ and $f_\pi f_K/4\pi$ on the same figure as the one-channel solution; i.e., on Figs. 6 and 7.

On Fig. 6, curve (3) shows the relation between $f_\pi f_K/4\pi$ and $m_{K^*}^2$ given by our model; on that figure, we have also reproduced the results of the two-channel OPE model [curve (4)]. Similarly, in Fig. 7, curves (3) and (4) represent $\gamma_{K^*}^2/4\pi$ as a function of $m_{K^*}^2$ in both cases. Finally, the relation between $\gamma_\eta^2/4\pi$ and $m_{K^*}^2$ is displayed in Fig. 9.

We see from those figures that, setting $m_{K^*}^2=41$, one gets

$$f_\pi f_K/4\pi=-1.8\,,\quad \gamma_{K^*}^2/4\pi=1.9\,,\quad \gamma_\eta^2/4\pi=1.4\,. \qquad (3.13)$$

For that solution, the asymmetry parameter $\mathcal{Q}$, given by (3.12) is

$$\mathcal{Q}\simeq 1.15\,.$$

Since $\mathcal{Q}$ is close to 1, the matrix t_{ij} is nearly symmetric and we may believe that our approximation of the N/D solution is reliable.

In the OPE model, setting $m_{K^*}^2=41$ gives the following results:

$$f_\pi f_K/4\pi=-17\,,\quad \gamma_{K^*}^2/4\pi=0.5\,,\quad \gamma_\eta^2/4\pi=18\,. \qquad (3.14)$$

The coupling constant $\gamma_\eta^2/4\pi$ which appears in the two-channel model is not known from experiment. However, arguments based on SU_3 lead to

$$\gamma_\eta^2/4\pi=3\gamma_{K^*}^2/4\pi\simeq 2.7\,. \qquad (3.15)$$

This can be deduced either from exact SU_3 symmetry or from the broken SU_3 symmetry proposed by Muraskin and Glashow,[13] combined with Sakurai's universality[8] (see Ref. 14).

[13] M. Muraskin and S. L. Glashow, Phys. Rev. **132**, 482 (1963).
[14] B. Diu, H. R. Rubinstein, and J. L. Basdevant, Nuovo Cimento **35**, 460 (1965).

Comparing (3.11) with (3.12) and (2.14), we see that, as pointed out in Ref. 3, adding a second channel does not improve the numerical results in the OPE model. Effectively, $f_\pi f_K/4\pi$ is increased, and the new coupling constant $\gamma_\eta^2/4\pi$ which is introduced comes out much too large. $\gamma_{K^*}^2/4\pi$ is reduced to a more correct value; however, this is due to the fact that $\gamma_\eta^2/4\pi$ is much larger than expected.

In the TPE model, on the contrary, $\gamma_\eta^2/4\pi$ has the right order of magnitude, so that the values of $f_\pi f_K/4\pi$ and $\gamma_{K^*}^2/4\pi$ are not much changed when the $K\eta$ channel is added. This raises the possibility that they do not depend much on the unitarity approximation, so that the numerical results can be trusted.

As in Sec. 2, we compare the signs of the OPE and of the TPE forces. In $\pi+K\to\eta+K$ one can verify numerically, by putting the calculated values of the coupling constants into (1.25), that the $K\pi$ forces are attractive as are the K^* ones. The same result is immediately seen to hold for $\eta+K\to\eta+K$, since $g_{22}^{(2)}$ is given by a positive integral [see formula (1.25)].

In our calculations, we considered only the longest range forces among the TPE contributions. The lightest intermediate states which we neglected were $K\bar{K}$, $K\eta$, and $\pi\omega$. They all have a rest mass of the order of 1 BeV. Consequently, in order to get an insight into the validity of our approximations, we study the effect of one of those forces, hoping that the other ones will give contributions of the same order. The $K\eta$ forces are easier to consider since they can be taken into account to fourth order in the coupling constant without introducing any new parameter. The corresponding self-consistency problem has been solved in the same way as the one we studied previously. It appears that the numerical results are almost unchanged when the $K\eta$ forces are added, so that these forces seem to be unimportant. Thus the TPE forces we neglected are likely to give small contributions. This justifies our approximation *a posteriori*.

The model including $K\eta$ exchange forces has an interesting property. Using the same method as in Ref. 5 one can show that, in that model, the two cuts of the s channel and of the u channel due to $K\pi$ and $K\eta$ intermediate states satisfy the crossing symmetry to fourth order if one takes into account only the S and P waves in those channels.

4. CONCLUSION

The method of approximately including TPE forces, already studied in Ref. 5 for the ρ bootstrap, has been shown to work also in the K^* problem where the external particles have unequal masses.

By adding fourth-order terms, one restores approximately the crossing symmetry between the cuts of the s channel and of the u channel which correspond to the lowest lying two-particle intermediate states.

The TPE forces which one is led to consider in our model have the same sign as the corresponding OPE forces:

(A) In $\pi+K\to\pi+K$ for $I=\frac{1}{2}$, the forces due to ρ exchange and to $\pi\pi$ exchange both are attractive for $f_\pi f_K<0$, while, on the contrary, the K^* forces and the $K\pi$ forces are repulsive. In our model, as in the OPE model, the total forces are attractive so that a pole can appear in the s channel.

(B) In $\pi+K\to\pi+K$ for $I=\frac{3}{2}$, the total forces are repulsive in the two approximations and we predict that no $K\pi$ resonance should exist with $l=1$, $I=\frac{3}{2}$. That prediction is, till now, in agreement with experiment.[15]

(C) In $\pi+K\to\eta+K$ and in $\eta+K\to\eta+K$, the $K\pi$ exchange forces are attractive, as are the K^* exchange forces.

In both models, one has to assume that $f_\pi f_K<0$ in order to generate the K^*. Though it is not yet known from experiment, that negative sign is also predicted by SU_3 symmetry or by Sakurai's universality.[8]

For a given value of $m_{K^*}^2$, our model, like the OPE model, leads to a well-defined self-consistent solution in the one-channel approximation as well as in the two-channel approximation. Moreover, these solutions are not pathological; i.e., $\gamma_{K^*}^2/4\pi$ and $\gamma_\eta^2/4\pi$ come out positive. This was not at all obvious *a priori*, since our system of equations is rather involved.

Assuming that

$$f_\pi^2/4\pi=-f_\pi f_K/4\pi\,,$$

three coupling constants have been determined:

$$f_\pi f_K/4\pi\simeq-1.8\,,$$
$$\gamma_{K^*}^2/4\pi\simeq1.9\,,$$
$$\gamma_\eta^2/4\pi\simeq1.4\,.$$

The calculated value of $f_\pi f_K/4\pi$ is in agreement with experiment, while for $\gamma_{K^*}^2/4\pi$ and $\gamma_\eta^2/4\pi$, the results have the right order of magnitude.

When TPE forces are considered, the numerical results show a much better agreement with experiment than in the OPE model. Moreover, the influence of the $K\eta$ channel becomes much more reasonable. Therefore one really improves the approximation of the forces by adding TPE forces in the way we studied in Ref. 5 and in this paper. (This was also our conclusion in Ref. 5.)

As a matter of fact, as pointed out previously,[5] the bootstrap mechanism is by no means connected with the OPE approximation. On the one hand, one uses self-consistency requirements which have to hold anyway since they express crossing symmetry properties; on the other hand, one introduces a very simplified dynamical model to express those conditions. We have shown that, by improving this dynamical model, one really gets better answers from the self-consistency equations. On the other hand, since the qualitative features of the OPE model have been shown to be maintained when TPE forces are added, we may believe that they do not depend on the model, so that they have some deep connection with the self-consistency of the exact S matrix. If that result holds in general, we will understand why the bootstrap theory in its simplified version, as it has been developed till now, has been qualitatively successful despite the failure of the OPE approximation on which it is based.[16]

Note added in proof. Since this article was submitted for publication, the author has studied the generation of the ρ in a two-channel $\pi\pi$ and $K\bar{K}$ problem, including $\pi\pi$ and $K\pi$ forces, by the same method. The results obtained, which will be described in a forthcoming paper, also tend to show the reliability of our approximation scheme.

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[15] This has been observed in particular, by S. G. Wojcicki *et al.* (see Ref. 7).

[16] The qualitative success of the OPE bootstrap model may not be complete, since some evidences of a K^+K^+ resonance have been reported [M. Ferro-Luzzi, R. George, Y. Goldsmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, G. R. Lynch, F. Muller, and J. M. Perreau, Dubna International Conference, 1964 (unpublished)]. As we pointed out in Ref. 4 with B. Diu and H. R. Rubinstein, the OPE bootstrap model predicts that no KK resonance should exist with $I=1$ since all the OPE forces are repulsive in that channel. I am indebted to Dr. H. R. Rubinstein who pointed out this discrepancy to me.