The nuclear temperature is defined by

$$E_f^* = \tfrac{1}{8}T^2 - T\,,$$

where

$$E_f^* = E^* - E_f - \epsilon\Delta_f\,.$$

The gap parameter Δ was arbitrarily taken to be 1.0 MeV and $\epsilon=1$ for *e-e*; 0 for *e-o* and *o-e*; and -1 for *o-o* nuclei. The reduction of the rigid-body moment of inertia due to pairing effects at low energies was calculated according to the pairing model of Lang.[25] In general, for these excitation $\mathcal{I}\sim\frac{1}{2}\mathcal{I}_0$.

[25] D. W. Lang, Nucl. Phys. 42, 353 (1963).

The results of these calculations are compared with the data[26,27] in Fig. 7. It is observed that the experimental data are well bracketed by the theoretical curves representing the two assumptions regarding fissionability as a function of angular momentum. Hence, within the limitations of this model, the so-called "anomaly" in the Bohr theory of fission anisotropies is not apparent if one includes the experimentally derived differences in $\mathcal{I}_{\rm eff}/\mathcal{I}_0$ with nuclear species and permits *l*-dependent fission.

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PHYSICAL REVIEW VOLUME 138, NUMBER 6B 21 JUNE 1965

Elastic Scattering of 320-MeV K^+ Mesons from Emulsion Nuclei Compared with the Predictions of the Known K^+-Meson–Nucleon Phase Shifts

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Exact diffuse-surface optical-model calculations were carried out to analyze the reaction and differential elastic-scattering cross sections for K^+ mesons with a mean energy of 320 MeV in nuclear emulsion. The real and imaginary nuclear-potential volume integrals per nucleon have also been obtained via the forward scattering amplitude from published K^+-nucleon phase shifts, and compared with those obtained from the optical-model analysis. The agreement is satisfactory and well within the error limits.

1. INTRODUCTION

THE early experiments[1–5] on K^+-neutron scattering were all of an indirect character; K^+ mesons were scattered from complex nuclei and a number of parameters were measured: the K^+-p elastic angular distribution and its cross section, the charge-exchange cross section σ_{ce} for the reaction $K^+ + n_{\rm bound} \to K^0 + p$; the small-angle elastic scattering from complex nuclei which when analyzed on the basis of the optical model yielded the real part of the K^+-meson–nucleus potential; and the total cross section for inelastic interaction. These parameters were then related to the basic K^+-neutron phase shifts through evaluation of the forward scattering amplitude.

In view of the substantial improvement in the accuracy of the direct measurements of the K^+-nucleon phase shifts it is worthwhile to reverse the procedure and test whether or not they predict the correct optical-model potentials via the forward scattering amplitudes. A new experiment has been performed at 320 MeV in which the optical-model potentials have thus been determined with a view to comparing them with the predictions of the $T=0$ and $T=1$ phase shifts which have been directly determined.

2. EXPERIMENTAL DETAILS

An emulsion stack was exposed to a separated K^+-meson beam of about 649-MeV/c average momentum. A total of 300 m of K^+-meson track was followed. All elastic scattering events $\geq 2°$ projected angle were

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† Work partially supported by the National Science Foundation.
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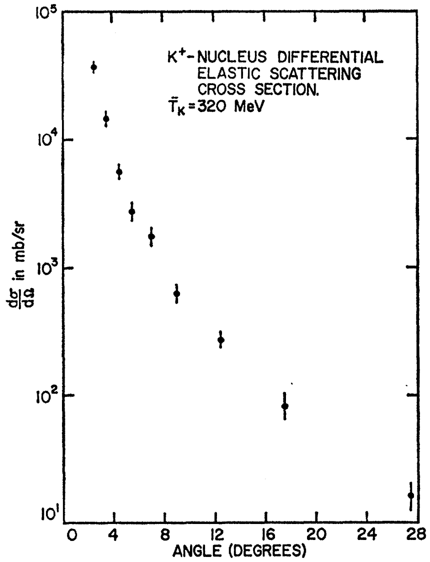

FIG. 1. Differential cross section for the scattering of 320-MeV K^+ mesons from emulsion nuclei.

measured, and the angular distribution was obtained in the usual way after due correction for geometrical bias. The reaction cross section was also determined and is 355±16 mb. The mean energy of the K^+ mesons at the point of scatter was 320 MeV (649-MeV/c momentum). The differential cross sections for the elastic scattering are listed in Table I and plotted in Fig. 1. Optical-model

TABLE I. Differential scattering cross sections for 320-MeV K^+ mesons from emulsion nuclei.

Angular interval in degrees	Cross section in mb/sr
2–3	37 000±3800
3–4	14 500±1900
4–5	5630± 800
5–6	2760± 440
6–8	1850± 220
8–10	628± 114
10–15	275± 41
15–20	82± 19
20–35	16± 4

fits to this angular distribution were then obtained using methods similar to those described by Helmy *et al.*[6] in their analysis of data at 260 MeV. The volume integrals of the potentials per nucleon were determined, which are for the Saxon well:

$$I_v+iI_w=\tfrac{4}{3}\pi R_0^3(V_0+iW)(1+9.88(a^2/R^2)),$$

where R_0 is the radius parameter, R is $R_0A^{1/3}$, and a is the rounding parameter in the Saxon well.

The value of I_w was determined from the reaction cross section. As always I_w, I_v, a, and R_0 are to an extent interdependent, and we have explored a range of 0.20 to 0.85 F for a and 1.07 to 1.35 for R_0. The values

[6] E. Helmy, M. A. Melkanoff, D. J. Prowse, and D. H. Stork, Phys. Rev. 127, 254 (1962).

TABLE II. Real and imaginary nuclear-potential-volume integrals per nucleon in MeV-F³ at 300 MeV as a function of a and R_0 (in F).

	a=0.20		a=0.57		a=0.85	
R_0	I_v	I_w[a]	I_v	I_w	I_v	I_w
1.07	30±25	136±10	57±17	98±9	58±16	91±7
1.20	51±14	104±8	60±15	89±6	60±16	86±6
1.35	65±15	88±6	65±16	82±5	65±18	82±5

[a] The errors in I_w arise mainly from the error in the reaction cross section. There is a small additional uncertainty which arises from the errors in I_v; this is included.

of χ^2 are shown plotted in Fig. 2 as a function of I_v for various combinations of a and R_0. In each case, the value of I_w is adjusted so that with the particular combination of I_v, a, and R_0 the reaction cross section of 355±16 mb is reproduced. For all these parameters a minimum is apparent in the repulsive region at about 65 MeV-F³. The exact position is only weakly dependent on the form-factor parameters. In Fig. 3 we show the variation of I_w required to hold the reaction cross section constant at the correct value for various a and R_0 combinations. We notice, as have previous workers at 125 and 260 MeV, that the best fits are obtained (at least insofar as the χ^2 fits are concerned) at low values of the radius parameter and at small rounding parameters. As can be seen from Fig. 3, however, such form-factor parameters require much larger values of I_w to match the reaction cross section for a particular I_v. The values of I_v and I_w as determined from Fig. 2 are tabulated in Table II as a function of a and R_0.

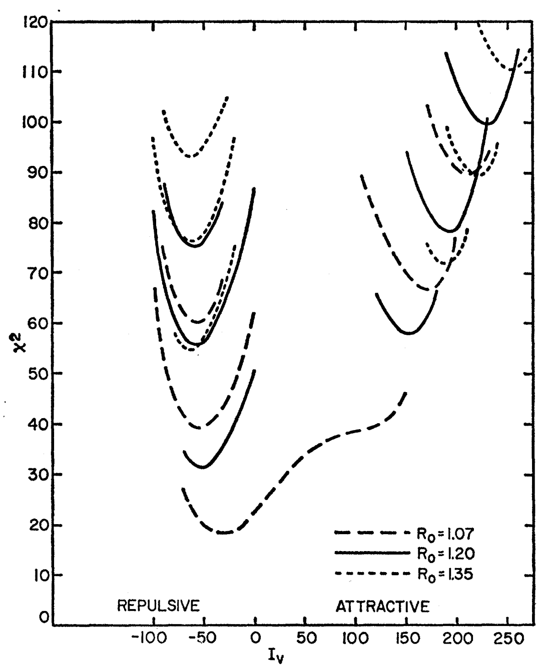

FIG. 2. χ^2 plotted as a function of V for various a and R_0 parameters. The R_0 values are labeled. For a particular R_0, decreasing χ^2 results from decreasing the a value. The a values used were 0.85, 0.57, and 0.20 F.

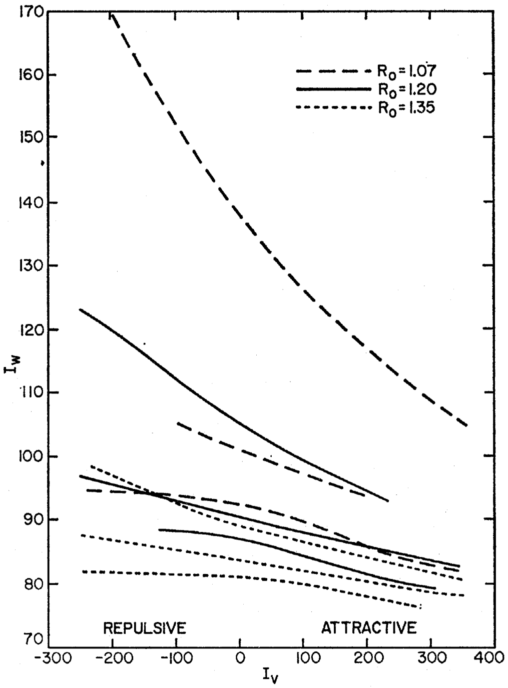

FIG. 3. The value of I_w required to reproduce the reaction cross section as a function of I_v and various a and R_0 parameters. The R_0 parameters are listed. For a particular I_v and R_0, decreasing I_w results from increasing a. The values used were 0.20, 0.57, and 0.85 F.

3. CALCULATION OF I_v AND I_w FROM THE PUBLISHED PHASE SHIFTS

It is assumed that the combined effect of the nuclear and Coulomb potential is to reduce the energy of the K^+ meson by 20 MeV; the effective K^+-meson energy as far as the interactions with nucleons are concerned is thus taken as 300 MeV or 623 MeV/c in momentum. The phase shifts used were interpolated from those given by Stenger *et al.*[7] for the $T=0$ state and by Goldhaber *et al.*[8] for the $T=1$ state with the following restrictions: that the s-wave $T=1$ phase shift be constrained by the effective range approximation with the scattering length and effective range given as -0.29 ± 0.015 and 0.5 ± 0.15 F, respectively; that the s-wave $T=0$ phase shift agree with the zero-range approximation, i.e., that $\tan\delta_{00}=a_{00}k$;[9] that the $T=1$ p-wave phase shifts be zero; that the $T=0$ p-wave phase shifts be proportional to the cube of the relative c.m. momentum, $\tan\delta_{01}=a_{01}k^3$ and $\tan\delta_{03}=a_{03}k^3$. Owing to the Fermi-Yang ambiguity there are two solutions to the $T=0$ p-wave phase shift, and both naturally give the same optical-model potentials. As a check on the calculations both solutions, labeled A and B by Stenger *et al.*, were taken independently, and identical potentials were in fact obtained. The exact values used are given in Table III.

TABLE III. Phase shifts (degrees). Minus sign indicates repulsion. K^+-meson energy = 300 MeV.

δ_{00}	$6.8_{-2.8}^{+2.8}$
δ_{10}	$-36.7_{-2.1}^{+2.1}$
$\delta_{01}(A)$	$-14.6_{-3.9}^{+4.1}$
$\delta_{03}(A)$	$15.9_{-5.1}^{+5.4}$
$\delta_{01}(B)$	$29.2_{-7.9}^{+9.1}$
$\delta_{03}(B)$	$-4.0_{-4.4}^{+3.4}$

Saxon and Lipperheide[10] have shown that

$$I_v+iI_w=(-2\pi\hbar^2/m)\langle f(0)\rangle_{\rm av},$$

where m is the reduced mass of the K^+-nucleon system, and $\langle f(0)\rangle_{\rm av}$ the forward scattering amplitude in the K^+-nucleon c.m. system is averaged over the isotopic spin:

$$\langle f(0)\rangle_{\rm av}=[(A+Z)/2A]f_1(0)+[(A-Z)/2A]f_0(0).$$

Here $(A+Z)/2A$ and $(A-Z)/2A$ are averaged over emulsion nuclei,

$$f_1(0)=(1/k_{\rm c.m.})e^{i\delta_{01}}\sin\delta_{01},$$

and

$$f_0(0)=(1/k_{\rm c.m.})(e^{i\delta_{00}}\sin\delta_{00}+e^{i\delta_{01}}\sin\delta_{01}+2e^{i\delta_{03}}\sin\delta_{03}).$$

As the atomic number does not cancel out of the expression for I_v and I_w in terms of V and W, V and W for a given I_v and I_w vary with A, and the appropriate V and W were therefore taken for the optical-model averaging procedure over emulsion nuclei.

4. DISCUSSION OF RESULTS

The calculated values of I_v and I_w are shown as a function of energy with their related errors in Fig. 4. The energy dependence taken was that given by the dependence of the phase shifts on k.

In order that the Saxon-Lipperheide result apply at these energies, corrections must be applied to the observed values of I_v and I_w. The correlation correction has been estimated by Melkanoff *et al.*[1] to be less than 10% at 93 MeV and even less at 230 MeV, so we neglect it. The effect of the Pauli exclusion principle, however, is not negligible. Sternheimer[11] and Bhowmik *et al.*[12] have calculated the effect to be some 20% at 93 MeV and 10% at 230 MeV. Extension of their calculation gives 7% at 300 MeV.

The value thus expected at 300 MeV for I_v is 72_{-24}^{+22} MeV-F^3 (repulsive) and for I_w is 94_{-12}^{+19} MeV-F^3. Those obtained by the optical-model analysis shown in Table II and Fig. 3 (the I_w values given should be raised by the 7% given above before comparison) are thus in good agreement—well within the error limits—

[7] V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, UCLA Report 1002 (to be published).

[8] S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halleran, T. F. Stubbs, G. M. Pjerrou, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **9**, 135 (1962).

[9] The first subscript denotes the isospin state and the second $2J$.

[10] R. Lipperheide and D. S. Saxon, Phys. Rev. **120**, 1458 (1960).

[11] R. M. Sternheimer, Phys. Rev. **106**, 1027 (1956).

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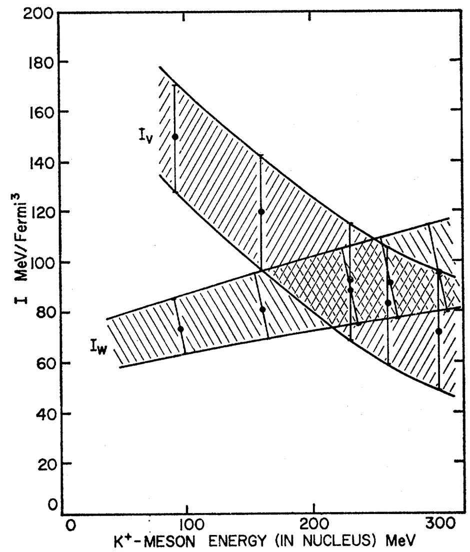

FIG. 4. The values of I_v and I_w predicted by the Stenger-Goldhaber phase shifts, as a function of energy. The points are energies that were explicitly calculated. The two bands contain all potential values that are allowed by the phase shifts.

for most radius parameters. The agreement with I_w is especially good—all radius parameters giving the value of I_w within the limits of error except for the extreme values of $R_0=1.07$ F and $a=0.20$ F taken together. If $a=0.20$ F, R_0 apparently should be more than 1.20 F; if R_0 is 1.07 F, a must be 0.5 F or more.

The analysis probably should not be extended much beyond 300 MeV as the d-wave phase shifts in the $T=0$ state and the p-wave phase shifts in the $T=1$ state will probably appear with significant strength at higher energies.

5. COMPARISON WITH PREVIOUS WORK

At lower energies, there have been two determinations of optical-model potentials for K^+ mesons from complex nuclei which can be directly compared with our analysis as they were obtained in an identical manner. At 93 MeV Melkanoff *et al.*[13] have obtained 123 ± 26 MeV-F^3 and 56 ± 8 MeV-F^3 for I_v and I_w, respectively, which agree reasonably well with the predictions from the directly determined phase shifts of 149 ± 22 and 73_{-11}^{+12} MeV-F^3. The agreement at 230 MeV between the potentials obtained by Melkanoff *et al.*[1] and Helmy *et al.*[6] and those predicted is also reasonable. These authors obtained $I_v=123\pm44$ MeV-F^3 and $I_w=103\pm13$ MeV-F^3, whereas the Stenger-Goldhaber phase shifts predict 93 ± 23 and 91_{-15}^{+18} MeV-F^3, respectively.

In conclusion, we may say that the agreement between the direct and indirect methods of obtaining the optical-model potentials is satisfactory and well within the error limits. It will be useful to repeat the calculation when the errors on the phase shifts have been reduced significantly.

ACKNOWLEDGMENTS

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Photodisintegration of the Deuteron by Polarized Photons*

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The asymmetry in the photodisintegration of the deuteron by polarized photons has been measured between photon energies of 75 and 230 MeV. Measurements were made mostly at 90° in the center-of-mass system, but limited data at 45° and 135° were also obtained. The data below 140 MeV are compared with current theories. At 90° our results are generally smaller than theoretical calculations. The measured asymmetry changes sign at about 130 MeV and shows a backward peaking at the higher energies.

I. INTRODUCTION

PHOTODISINTEGRATION of the deuteron has been extensively studied from threshold energy to several hundred MeV and a comprehensive bibliography is now available.[1] The photodisintegration process may be divided roughly into two energy regions, one below the pion-production threshold and one above this threshold. At energies below pion threshold, viz.,

* Work supported in part by the U. S. Office of Naval Research, the U. S. Atomic Energy Commission and the U. S. Air Force Office of Scientific Research.

[1] Bibliography of photonuclear and electronuclear disintegrations compiled by M. E. Toms, July 1963, U. S. Naval Research Laboratory.