$O^{16}(\alpha,Be^8)C^{12}$ Reaction*

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Measurements have been made of the differential cross sections for the $O^{16}(\alpha,Be^8)C^{12}$ reactions to the ground state and to the 4.43-MeV first excited state of C^{12} at nine bombarding energies between 35.5 and 41.9 MeV. The Be^8 nuclei (in their ground state) were identified by detecting the two breakup alpha particles in coincidence, and by imposing the energy and angular requirements for breakup into a narrow cone. The measurements extend, for the most part, from 18° to 100° (c.m.). The angular distributions display oscillatory behavior with typical separations between peaks of 15° to 20° (c.m.). Strong fluctuations in differential cross sections were observed with changes in incident energy. The ground-state and 4.43-MeV-state yields are uncorrelated, except perhaps at 35.5 MeV where both yields are particularly large. With the exclusion of the 35.5-MeV data, the cross sections integrated from 18° to 100° (c.m.) have mean values of 0.6 mb for the ground-state reaction, and 1.0 mb for the 4.43-MeV-state reaction. The existence of the observed fluctuations refutes the original expectation that simple direct interaction processes would account for the $O^{16}(\alpha,Be^8)C^{12}$ reaction. The results can be qualitatively understood in terms of the statistical theory of compound nuclear reactions, although a substantial nonstatistical component cannot be excluded. The average width of the compound nuclear states, given by the application of the statistical viewpoint to the experimental results, is 1.1 MeV.

I. INTRODUCTION

THE present investigation was undertaken with the intent of studying direct alpha-particle pickup processes via the (α,Be^8) reaction. It has long been conjectured[1] that groups of nucleons within a nucleus are associated in clusters resembling alpha particles. Support for the alpha-particle model in light nuclei is provided by examination of shell-model wave functions in the L-S limit[2] and by explicit variational calculations which assume alpha-particle clustering.[3] The possible existence of these clusters at the nuclear surface of heavier nuclei has been recently discussed by Wilkinson.[4] Substantial "alpha-particle" clustering in C^{12} and at the surface of several heavy nuclei has been reported by Igo, Hansen, and Gooding[5] in a study of $(\alpha,2\alpha)$ processes at 915 MeV. Alpha-particle transfer in (d,Li^6) reactions on light nuclei also has been investigated by several groups,[6] and results obtained to date in these studies have been interpreted as being consistent with a description of the (d,Li^6) process as a direct cluster transfer.

Alpha-particle transfer in the (α,Be^8) reaction has not previously been investigated. There are several reasons to expect that this reaction may be a direct process and may be particularly suitable for the study of alpha-particle clustering. The generalized optical model[7] and simple approximations thereto[8] have provided a satisfactory description of the elastic and inelastic scattering of medium-energy alpha particles as well as other heavy ions.[9] The fact that both the alpha particle and the ground-state (g.s.)Be^8 nucleus have zero spin leads to a simple angular-momentum decomposition of the reaction amplitudes proceeding to final states in an even-even nucleus (in contrast to the (d,Li^6) reaction). Further, there is considerable evidence that the alpha-particle model is quite appropriate for the Be^8 nucleus in its ground-state rotational band.[3,10] It was decided to begin the investigation with an observation of the (α,Be^8) reaction on some light nuclei which themselves might be interpreted in terms of the alpha-particle model, as it was anticipated that the Be^8 yields might be relatively high here.

Because the Be^8(g.s.) nucleus has a short lifetime for breakup into two alpha particles, one must resort to indirect means for its detection. One way of determining that a Be^8(g.s.) nucleus has been produced is to use the conventional technique of observing the energy spectrum of the associated particle. Another means of detecting Be^8 production is the observation of the two breakup alpha particles in coincidence. This was the method used in the present experiment. Weinman and

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[1] J. A. Wheeler, Phys. Rev. **52**, 1083 and 1107 (1937); L. R. Hafstad and E. Teller, *ibid.* **54**, 681 (1938); D. M. Dennison, *ibid.* **57**, 454 (1940).
[2] J. K. Perring and T. H. R. Skyrme, Proc. Phys. Soc. (London) **A69**, 600 (1956).
[3] A. Herzenberg, Nucl. Phys. **3**, 1 (1957); L. D. Pearlstein, Y. C. Tang, and K. Wildermuth, *ibid.* **18**, 23 (1960).
[4] D. H. Wilkinson, in *Proceedings of the Rutherford Jubilee International Conference*, edited by J. E. Birks (Academic Press Inc., New York, 1961), p. 339.
[5] G. Igo, L. F. Hansen, and T. J. Gooding, Phys. Rev. **131**, 337 (1963).
[6] M. N. Huberman, M. Kamegai, and G. C. Morrison, Phys. Rev. **129**, 791 (1963); D. S. Gemmell, J. R. Erskine, and J. P. Schiffer, *ibid.* **134**, B110 (1964); W. W. Daehnick and L. J. Denes, *ibid.* **136**, B1325 (1964); J. B. Gerhart, P. Mizera, and F. W. Slee, Cyclotron Research, University of Washington, 1964, p. 21 (unpublished).
[7] R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).
[8] J. S. Blair, Phys. Rev. **115**, 928 (1959).
[9] R. H. Bassel, G. R. Satchler, and R. M. Drisko, in *Proceedings of the Third Conference on Reactions between Complex Nuclei*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, California, 1963), p. 45.
[10] R. R. Haefner, Rev. Mod. Phys. **23**, 228 (1951); D. R. Inglis, *ibid.* **25**, 390 (1952).

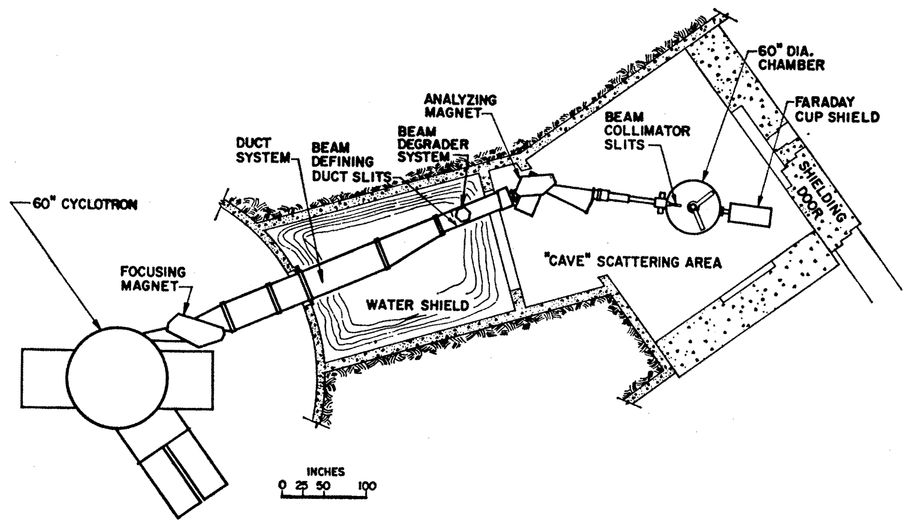

FIG. 1. General arrangement for scattering experiments at the University of Washington cyclotron.

Smither[11] previously reported on the use of this technique in a study of the reaction $Be^9(He^3,\alpha)Be^8$, although the bulk of their results were analyzed using the energy spectrum of the associated alpha particles alone. Moazed *et al.*[12] have observed alpha-particle coincidences from this same reaction; however, in their geometry only one of the observed alpha particles was from the Be^8(g.s.) breakup. Donovan *et al.*[13] have investigated the $O^{16}(\alpha,2\alpha)C^{12}$ reaction by observing coincidences between the two alpha particles. For almost all of their data, the detectors were too far apart in angle to observe alpha-particle coincidences from Be^8(g.s.) breakup. They do report, however, a brief unsuccessful attempt to detect (α,Be^8(g.s.)) events.

In the present experiment[14] the Be^8 nuclei from the (α,Be^8(g.s.)) reaction on O^{16} and Mg^{24} were detected by an observation of the two breakup alpha particles in close-angle coincidence. The differential cross sections were found to be rather strongly dependent on incident energy and, therefore, the initial hope for an explanation in terms of a simple direct alpha-particle pickup mechanism was not borne out. In consequence, the energy dependence has been investigated in some detail. The present paper is primarily concerned with a description of this investigation for the $O^{16}(\alpha,Be^8)C^{12}$ reactions leading to the ground state and 4.43-MeV state of C^{12}.

General experimental facilities are described in Sec. II. The next two sections describe aspects of the experimental arrangement which were peculiar to the present study, namely, the detailed method of Be^8 detection and the technique used in the establishment of stable incident alpha-particle energies. Section V contains a description of the procedures for data accumulation and a statement of the results. These results are then discussed in Sec. VI.

II. GENERAL EXPERIMENTAL FACILITIES

The experiment was performed using the 42-MeV alpha-particle beam of the University of Washington 60-in. cyclotron. The beam is defined and analyzed by an array of slits and magnets, and directed through beam collimator slits into a 60-in. diameter scattering chamber (see Fig. 1). Detectors can be placed on two independently movable arms. A target holder at the center of the chamber permits rapid interchange of several targets, and allows continuous adjustment of the target angle. Details of the Be^8 detector system, including the associated electronics, are described in Sec. III.

The incident beam was stopped and monitored in a Faraday cup attached to the scattering chamber. The Faraday cup was split by a vertical division through its center.[15] By monitoring the current into each side of the cup, as well as the total current, the proper alignment of the beam direction could be verified. (This was not strictly necessary in the present experiment because, with the collimators used, the beam which struck the target could not change appreciably in direction or location.) The currents from the Faraday cup were fed to an electronic integrating system,[16] which gives a number of counts proportional to the integrated charge. As a supplement to the Faraday cup, one or two detec-

[11] J. A. Weinman and R. K. Smither, Nucl. Phys. **45**, 260 (1963).
[12] C. Moazed, J. E. Etter, H. D. Holmgren, and M. A. Waggoner, Phys. Letters **12**, 45 (1964).
[13] P. F. Donovan, J. V. Kane, Č. Zupančič, C. P. Baker, and J. F. Mollenauer, Phys. Rev. **135**, B61 (1964).
[14] Initial data have been reported by R. E. Brown, J. S. Blair, D. Bodansky, N. Cue, and C. D. Kavaloski, Bull. Am. Phys. Soc. **9**, 545 (1964).
[15] C. Williamson and H. Fauska, Cyclotron Research, University of Washington, 1964, p. 58 (unpublished).
[16] The main unit is a Voltage-to-Frequency Converter, Dymec Model 2211 BR (Dymec, Palo Alto, California).

tors, held at fixed angles, were used to monitor scattered alpha particles.

NiO foils were used to provide an oxygen target. The Be^8 yield from Ni was determined to be negligible. Two such NiO targets were used. The first, which was made by passing current through a Ni foil (1 mg/cm^2) in an oxygen atomosphere, was found to be only about 60% oxidized. A completely oxidized NiO target was later made by baking a Ni foil (1 mg/cm^2) in air at 800°C. The oxygen contents of the NiO targets were determined by comparing the $O^{16}(\alpha,\alpha)$ elastic scattering yield from these targets to the yield from a Mylar target of known oxygen content. Target uniformity was investigated by rotating the target and observing the scattered alpha-particle yield at a fixed detector angle. The uncertainty in the determination of the NiO target thickness, including the effects of the nonuniformity, was about 5%. An isotopically enriched Mg^{24} target[17] was used for a brief investigation of the $Mg^{24}(\alpha,Be^8)Ne^{20}$ reaction.

Control of the incident-beam energy, which was desired for reasons discussed in Sec. I, was achieved by means of a water-cooled degrader system[18] installed between the beam-defining duct slits and the analyzing magnet (see Fig. 1). The degrader system consists of beryllium foils and a mechanism designed to provide a continuous variation of the total foil thickness from 0.001 to 0.022 in. This corresponds to a continuous energy variation from 41.3 to 25.6 MeV for a 42-MeV incident alpha-particle beam. With a $\frac{3}{32}$-in. wide by $\frac{3}{8}$-in. high beam collimator slit, the beam intensity was typically reduced by a factor of 20 when the energy was degraded to 35.5 MeV, which was the lowest energy used in the present experiment.

The total beam energy spread was investigated by the observation of the width of the elastically scattered alpha-particle peak from Au using a *p-n* junction solid-state detector. After allowing for other factors contributing to this width, the total spread in energy of the beam incident on the target was estimated to be less than 160 keV for all incident energies used. The energy loss in the target ranged from about 175 to 300 keV depending upon the angle of target rotation (which never exceeded 55°). For a typical loss of 250 keV, the spread in energy of alpha particles interacting with target nuclei was about 300 keV (full width at half-maximum).

III. DETECTION OF Be^8

A. Basic Kinematics

The Be^8 nucleus in its ground state is unstable with respect to breakup into two alpha particles. It has a breakup energy of 94 keV and a mean life of a few times 10^{-16} sec.[19] This lifetime is sufficiently long for the Be^8(g.s.) nucleus to be treated as a stable particle in its participation in nuclear reactions. Thus the Be^8(g.s.) nuclei emitted in (α,Be^8) reactions have a unique energy at a given laboratory angle, as for any two-body reaction product. On the other hand, the Be^8 lifetime is short enough for the nucleus to break up while still in the target, and thus the detection of Be^8 can be accomplished by the coincident detection of the two breakup alpha particles.

The breakup alpha particles from a rapidly moving Be^8 nucleus will be confined within a cone whose axis is the velocity vector of the original Be^8 nucleus and whose half-angle α_{max} is given by

$$\sin\alpha_{max} = [B/E_8]^{1/2}, \qquad (1)$$

where B is the breakup energy [94 keV for Be^8(g.s.)] and E_8 is the Be^8 kinetic energy (lab). For the Be^8(g.s.) reactions of interest in this work, E_8 ranges from approximately 10 to 35 MeV, and α_{max} ranges from 5.5° to 3°. In contrast, at the same energy E_8, α_{max} for the breakup of a Be^8 nucleus in its 2.9-MeV first excited state is about six times as large.

Even though the individual alpha-particle energies from the breakup of Be^8 nuclei with energy E_8 can vary over a wide range, the sum of the energies of an alpha particle and its breakup companion must equal E_8+B, which is a constant for a given observation angle in an (α,Be^8) reaction. Because the spin of the Be^8 nucleus in its ground state is zero, the alpha-particle breakup is isotropic in the Be^8 rest system. The probability distribution of alpha particles with respect to lab angle α is then given by a standard solid-angle transformation. This breakup while in motion yields an increasing density of alpha particles as the angle α increases from 0° toward α_{max}, and one therefore gains in detection efficiency by placing the two alpha-particle detectors near the cone edge. The specific formulas for the alpha-particle energies, the solid-angle transformation, and other kinematic quantities are given in the Appendix.

B. Effective Solid Angle for Be^8 Detection

The geometric efficiency, or effective solid angle Ω, for the detection of Be^8 is a function not only of the area of the detectors but also of their shape and orientation with respect to the breakup cone. For example, it has already been noted that relatively high detection efficiencies are obtained when the detectors are placed near the cone edge. Thus, if one starts with two detectors close together near the cone center and then gradually separates them, the solid angle increases at first, reaches a maximum as the detectors pass the cone edge, and then rapidly drops to zero as the detectors pass outside the breakup cone. This approach to zero detection probability as the detectors are moved

[17] Obtained from Fodor Accelerator Targets, Pittsburgh, Pennsylvania.

[18] T. J. Morgan, Cyclotron Research, University of Washington, 1963, p. 30 (unpublished).

[19] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 1 (1959).

outside the breakup cone is one of the key criteria for the identification of alpha particles from Be^8 breakup.

The dependence of detection efficiency on detector orientation implies that, with a fixed detector geometry, the solid angle Ω depends on α_{max}, which in turn depends on the Be^8 energy. Hence, for the method used here to detect Be^8 nuclei from an $(\alpha,Be^8(g.s.))$ reaction, Ω is a function, through kinematic relations, of the Be^8 lab angle θ_L, the beam energy, and the state reached in the residual nucleus.

A rough estimate of the effective solid angle can be made for small detectors close together, neglecting the higher density of alpha particles near the edge of the breakup cone and assuming instead a uniform distribution over this cone. Let Ω_0 be the solid angle subtended by one detector and Ω_c be the solid angle formed by the breakup cone. The solid angle for detection of one breakup alpha particle in one detector is Ω_0. The possible Be^8 directions will be distributed over a cone of size Ω_c, and the possible directions of the companion alpha particle will be distributed over a cone of size $4\Omega_c$. Thus the probability that the companion alpha particle will be detected in the other detector is $\Omega_0/(4\Omega_c)$. Multiplying by 2 because the "first" alpha particle could have gone into either detector, one obtains

$$\Omega \approx \Omega_0^2/(2\Omega_c). \quad (2)$$

This expression is only approximate but can be made more accurate in some cases by replacing the cone solid angle Ω_c by an appropriately normalized quantity Ω_c'. An application of such a modified version of Eq. (2) is discussed in the Appendix. For the actual data analysis, however, a more accurate calculation was made of the effective solid angle Ω in which careful attention was paid to the actual kinematics of the Be^8 breakup and to the details of the detector geometry.

In this calculation an expression of the form $\int G(\omega)d\omega$ is evaluated, where $G(\omega)$ is the joint probability that one alpha particle is detected in one detector and the companion alpha particle is detected in the other detector. The integral is carried out over all Be^8 directions ω. The probability $G(\omega)$ involves an integral over the detector areas. A program for an IBM-709 computer was written to evaluate the integrals. The details of the calculation are given in the Appendix. Plots of the effective solid angle as a function of the Be^8 kinetic energy are shown in Fig. 2 for the normal experimental geometry.

C. Preliminary Two-Detector System

In the first phase of the experiment a detection system was used which was particularly convenient for verifying that the observed events corresponded to Be^8 emissions. Solid-state detectors were mounted on each of the two movable arms in the scattering chamber, permitting the independent adjustment of the angular position of each detector. One detector was $16\frac{1}{16}$ in. from the

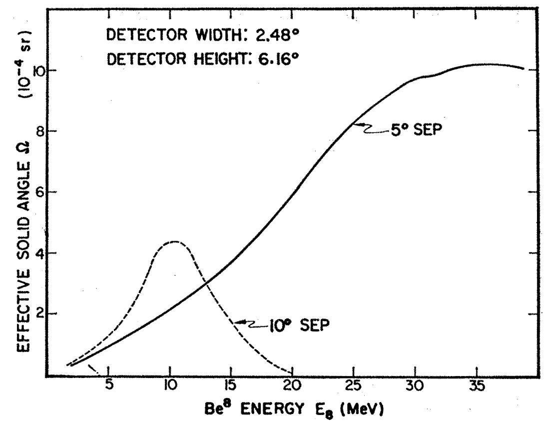

FIG. 2. Effective solid angle for the detection of Be^8(g.s.) nuclei. The indicated detector separations and detector dimensions correspond to the running conditions described in Sec. III D.

center of the chamber; the other was $14\frac{3}{16}$ in. from the center. The detectors were each covered with circular collimators $\frac{11}{16}$ in. in diameter and were encased in holders with outer diameters of $1\frac{1}{8}$ in. With this geometry, the detectors could be brought to within 3.5° of each other before the front detector shielded the back detector. The detectors each subtended a maximum angle of about 2.6°.

The detector biases were adjusted to give a depletion layer exceeding the range of the alpha particles from Be^8 breakup. Pulses from the detectors were amplified and placed in coincidence in a fast-slow coincidence system (Cosmic Model 801), in which the fast coincidence was established by the zero crossover time of a double delay line clipped signal. The resolving time was chosen to be somewhat shorter than the 87-nsec period of the cyclotron beam. Amplified signals from the two detectors were presented to the two axes of a 32×32 channel two-dimensional pulse-height analyzer (model ND-150), which was gated by the fast-slow coincidence system.

An energy calibration for each axis of the analyzer was obtained by scattering 42-MeV alpha particles from carbon. Coincidences between alpha particles and recoiling carbon ions were observed at several pairs of angles. The resulting monoenergetic alpha-particle groups provided convenient calibration points.

The detector system described above was initially used to make comparisons between spectra taken with the detectors at several different angular separations. The results of such a measurement are shown in Fig. 3. In this measurement the detectors were centered at 30° and the incident-beam energy was 42 MeV, leading to a full cone angle of 6.4° for the (α,Be^8) reaction to the ground state of C^{12}. A favorable detector separation was 5.6°, because the detectors then included the edges of the breakup cone as well as much of the central region. At this separation angle a relatively intense diagonal band resulting from events of constant total energy is

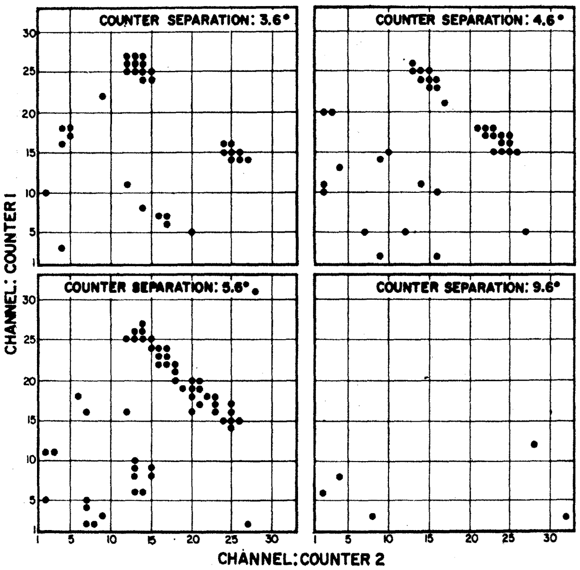

FIG. 3. Two-dimensional pulse-height distributions at several detector separations, for coincidence events in the bombardment of O^{16} by 42-MeV alpha particles. Each dot represents a channel containing two or more counts. Each axis spans an alpha-particle energy interval from about 11 to 17 MeV. The detectors were centered about 30°.

seen in the two-dimensional spectrum of Fig. 3. From the energy calibration it was determined that the band energy corresponds to (α,Be^8) events leading to the ground state of C^{12}. The energies at the sharply defined endpoints of the band correspond to the calculated maximum and minimum alpha-particle energies for the breakup of Be^8 from its ground state. The 5.6° data thus indicate that the prominent band arises from $O^{16}(\alpha,Be^8)C^{12}$ reactions to the ground states of both the Be^8 and C^{12} nuclei.

This identification is further confirmed by the data at other angles. When the detector separation is increased to 9.6° the band disappears because the separation at the inner edges of the detectors exceeds the Be^8 breakup cone full-angle. When the detectors are separated by only 4.6° or 3.6°, the band divides into two "islands." At these smaller separations the observed alpha particles are more likely to be close to the original Be^8 direction. In this case the forward alpha particle will have a high laboratory kinetic energy, while its backward companion will have a low kinetic energy, thus producing the observed "islands" near the band endpoints.

There is an indication in Fig. 3 of a second band lying below the conspicuous highest band. The second band is attributed to transitions to the 4.43-MeV first excited state of C^{12}. At some angles other than 30° these events become more conspicuous, and the identification of the reaction can then readily be confirmed by examining the total energy and the endpoint energies.

One might expect that the observed bands could also be attributed to events leading to the 2.9-MeV first excited state of Be^8 rather than to the Be^8 ground state, because the total energy of the two alpha particles is almost the same in the two cases. (There is a difference of a few hundred keV because the recoiling C^{12} nuclei carry different energies.) However, the breakup energy for a Be^{8*} exceeds the breakup energy for the ground state by a factor of more than 30, implying six times as large a cone angle and more extreme values for the maximum and minimum alpha-particle energies. Thus Be^{8*} events would appear, if at all, at energies considerably beyond the ground-state-band endpoints, and could not contribute to the band actually observed.

The absence of observed Be^{8*} events does not mean that the cross section for the formation of Be^8 in its 2.9-MeV state is necessarily small. The observation of such Be^{8*} events was normally precluded by the limited region of pulse heights scanned by the 32×32 channel analyzer in the two-detector arrangement. Even more restrictively, in both the two-detector arrangement and in the later three-detector arrangement, electronic energy thresholds were normally high enough to exclude the very low-energy alpha particles from Be^{8*} breakup. However, in one unusual geometry, discussed below, there were indications of a large Be^{8*} yield. With this minor exception, the present experiment provides no information on Be^{8*} processes and, in fact, was arranged to prevent their detection.

Another possible source of coincident alpha-particle events comes from $O^{16}(\alpha,2\alpha)C^{12}$ processes in which no Be^8 intermediate states are formed, such as scattering to alpha-particle emitting states of O^{16}. These events might be mistaken for Be^8 events. The work of Donovan *et al.*[13] demonstrates that the yield for $(\alpha,2\alpha)$ events to the ground state of C^{12} is appreciable. However, as the data they present is predominantly for detectors placed on opposite sides of the incident-beam direction, rather than close together, no direct prediction can be made from their data of the $(\alpha,2\alpha)$ rate which will be encountered in the present experiment.

It might be expected that $(\alpha,2\alpha)$ events would appear only as a continuous background because, in principle, the sum of the two alpha-particle energies is not constant for fixed detector angle. However, the magnitude of the sum extends over only a narrow interval. This interval is made still more narrow by the exclusion of the lowest energy alpha particles by electronic thresholds. Thus, in the present arrangement, $(\alpha,2\alpha)$ events will have essentially the same total alpha-particle energy as Be^8 events (for the same residual state in C^{12}), but the individual alpha-particle energies will extend beyond the limits characteristic of Be^8 breakup.

It is seen that the $(\alpha,2\alpha)$ contribution is negligible for the data presented in Fig. 3. This is proven by the sharpness of the band endpoints, by the appearance of the "islands" at 3.6° and 4.6°, and by the great reduction in the number of events at 9.6°. More quantitatively, the number of events of total energy corresponding to the C^{12} g.s. band, is reduced from 118 at 5.6°

separation to 3 at 9.6° separation. It is virtually inconceivable that there would be such a sharp angular dependence in the resultant of the many different contributing $(\alpha,2\alpha)$ reactions.

The conclusion that $(\alpha,2\alpha)$ events play a small part in the present experiment is supported by the two-detector pulse-height spectra found at other angles, and by results, discussed below, obtained with the three-detector system. It should be noted that this conclusion does not contradict the findings of Donovan *et al.*[13] The effective solid angle for the detection of Be^8(g.s.) events is here typically more than a factor of 100 greater than the effective solid angle for the detection of coincidence events for two alpha particles that are uncorrelated in direction. Actually one does expect correlations in laboratory angle for $(\alpha,2\alpha)$ reactions, but these are not of a nature likely to enhance significantly the yield for two detectors placed close together. Thus the present observed low yield of $(\alpha,2\alpha)$ events does not necessarily imply that the total $(\alpha,2\alpha)$ cross section is small.

Preliminary studies of the $O^{16}(\alpha,Be^8)C^{12}$ reactions with the two-detector system showed that there was an observable yield of Be^8(g.s.) events, that the cross section was small, and that the cross section varied strongly with incident alpha-particle energy. This made measurements at a succession of different incident energies desirable. To avoid the long running times which would have been required to obtain data with the two-detector system, a three-detector system with improved geometry was adopted.

D. Three-Detector System

All final data in this experiment were taken using a three-detector system, with an array of three rectangular solid-state detectors placed side by side on a line perpendicular to the direction of scattered particles. In order to take advantage of the full sensitive area of the detectors, and to permit them to be positioned close to one another, no collimators were used in front of the detectors. The sensitive area of each of the three detectors was measured by comparing the counting rate produced in each by an alpha-particle source to the rate produced in a fourth detector which was covered by a collimator of accurately known area. The sensitive areas of the three rectangular detectors were found to be equal to one another to within about 2%, with an average value of 0.153 in.^{2}. Because the effective solid angle depends upon the detector shape, the width W and height H of the detectors were measured with a traveling microscope. The values of W and H obtained for each of the three detectors agreed to within about 4%, and the products WH agreed to within 3%. The average of these products differed by 8% from the area determined by the alpha-source technique, presumably because of the difficulty in estimating the sensitive region in the microscope measurement. The dimensions were scaled to yield the correct area, and the values adopted were $W=0.248$ in., $H=0.619$ in. The detectors were mounted so that their centers were $\frac{1}{2}$ in. apart, with the middle detector $5\frac{3}{4}$ in. from the target. The three detectors were placed in an aluminum box covered in front by a 0.0001-in. aluminum foil.

With this geometry each detector subtended an angle of 2.5°, and adjacent detectors were separated in angle by 5°. For the Be^8(g.s.) breakup at the kinetic energies encountered in the present experiment, the detection efficiency is greatest when the Be^8 nucleus is directed midway between the detectors, and falls to zero as the Be^8 direction moves 1.25° away from this central direction. The efficiency falls to one-half its central value for a displacement of about 0.6°. Thus the "effective" angular resolution for the detection of Be^8 events was approximately $\pm 0.6°$.

Part of the improvement in counting rate with this system stemmed from the use of rectangular detectors, which can provide larger solid angles than circular detectors with no loss in angular resolution. The increase in effective solid angle cannot be estimated from the detector dimensions alone, because it involves a detailed consideration of the kinematics of Be^8 breakup (see Sec. III B and the Appendix). At typical Be^8 energies, in the geometries described above, the effective solid angle was about nine times larger for a pair of rectangular detectors than for the circular detectors. The angle subtended by each detector in the scattering plane was about the same for the rectangular and circular detectors.

A typical full-cone angle for Be^8(g.s.) breakup was in the neighborhood of 7°. For cone angles less than 7.5°, Be^8 events could lead to coincidences between adjacent detectors, but not between the two outer detectors. With the three detectors, one can therefore obtain data simultaneously at two angles (1-2 and 2-3 events) as well as a measure of background rates (1-3 events). (The numbers here refer to the detectors, with increasing numbers corresponding to increasing angles from the incident beam.)

To implement the over-all scheme, separate fast-slow coincidences were established for 1-2, 2-3, and 1-3 events, using circuitry similar to that described above for the two-detector system. The three coincidence signals were simultaneously used in two ways. First, the individual coincidence signals were used as routing signals to a 512-channel pulse-height analyzer (model ND-120). Second, the coincidence signals were mixed, so that a coincidence in any pair would provide a gating signal for the 512-channel analyzer. Pulses proportional to energy from the three detectors were added, and the sum presented to the analyzer as the pulse height to be stored. Aside from accidental triple coincidences, the sum contained contributions, for any one event, from only two of the detectors. With this arrangement, the gating signal (denoting a concidence in some pair) permitted the pulse height to be analyzed. The routing

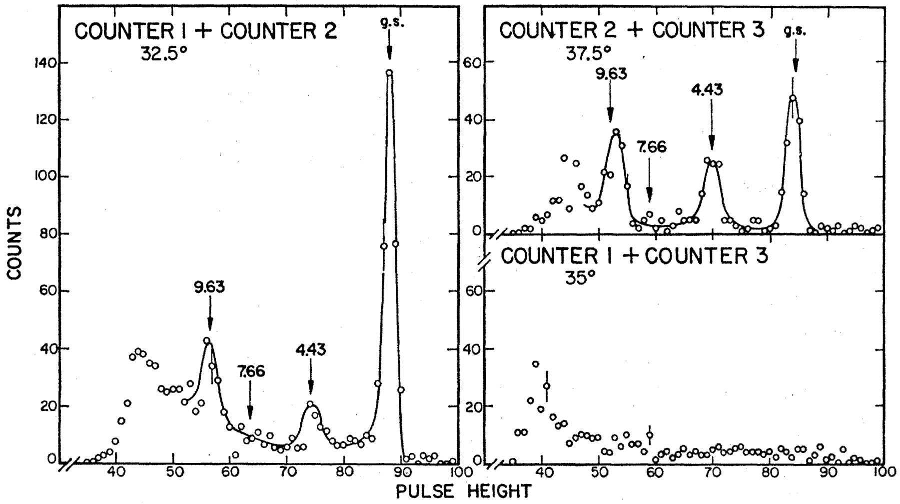

FIG. 4. Pulse-height distributions obtained with the three-detector system for coincidence events in the bombardment of O^{16} by 39.6-MeV alpha particles. The three plots correspond to coincidence-gated spectra taken simultaneously for the three possible detector pairs. The arrows show the pulse heights expected for (α,Be^8) reactions to the indicated states of C^{12}.

signal (denoting which two detectors were in coincidence) determined the 128-channel quadrant of the 512-channel analyzer in which the pulse height was stored. A 1-2 sum spectrum appeared in the first quadrant, a 2-3 sum spectrum appeared in the second quadrant, and a 1-3 sum spectrum appeared in the third quadrant. The fourth quadrant was not used.

Typical spectra are shown in Fig. 4. Peaks in the 1-2 and 2-3 spectra were identified as Be^8 events, involving monoenergetic sums of the breakup alpha particles. The identification was made from the energies of the peaks, employing an energy calibration based on alpha-particle sources and alpha-particle scattering from carbon. The sum energies were followed as a function of detector angle, and found to vary as expected for Be^8 events. Further confirmation of the identification is provided by the 1-3 spectrum. The absence of peaks in the 1-3 spectrum of Fig. 4 indicates that the peaks seen in the 1-2 and 2-3 spectra involve particles with small angular separation, as in Be^8(g.s.) breakup.

Some aspects of the over-all detection system could best be examined by changing the angular separation of the detectors. This could not here be conveniently done by changing the linear separation between the detectors because this would have necessitated opening the scattering chamber. However, it was possible to change remotely the distance of the counters from the target by mounting them on a tray driven in the radial direction by an externally controlled motor. A sequence of measurements was made in which the detector distance D was changed in order to obtain experimental confirmation of the solid-angle calculations discussed in Sec. II B and in the Appendix. The results are shown in Fig. 5, where the observed counting rate is plotted as a function of the radial position of the detectors. At large distances the counting rate decreases at a rate which asymptotically approaches D^{-4}. At small distances the rate decreases because the detector angular separation exceeds the Be^8 cone angle. Calculated magnitudes of the effective solid angle are also shown in this figure. The experimental results are normalized to the calculated solid angles at large distances.

The correspondence between the calculated and measured shapes is reasonably close, although there are discrepancies at small distances. A part, at least, of the discrepancy is due to multiple Coulomb scattering in the target. At small distances, where the full angle of the breakup cone is barely equal to or is just less than the angular separation of the inside edges of the detectors, multiple scattering can serve only to increase the observed yield. In the present experiment a typical rms angle for multiple scattering of an alpha particle in the target is 0.8°. Although no detailed calculation has been made of the resulting increase in counting rate, one can infer that it will be substantial by noting that the widely different calculated solid-angle curves at 19.6 and 29.9 MeV correspond to breakup cone angles differing by only 1.5°. The observed greater discrepancy at the lower Be^8 energy is qualitatively consistent with expectations for multiple scattering. A second possible contribution to the discrepancy comes from uncertainties in the detector width. As an example, if one were to increase the detector width by 5% (which is comparable to experimental uncertainties), then the discrepancy would be essentially removed from the 29.9-MeV data. This sensitivity suggests that collimators should be used to define the detectors if a more accurate absolute measurement is desired. A third possible contribution to the discrepancy comes from (α,2α) events. These will increase in rate proportional to D^{-4} as the distance is decreased.

Because the discrepancy between the observed points and the calculated curves in Fig. 5 is small, a comparison of the counting rates at normal distances and at small distances can be used to establish an upper limit for the (α,2α) yield. In this calculation the discrepancy is attributed entirely to (α,2α) events and the effect of multiple scattering is ignored. It is concluded from the D^{-4} dependence of the $(\alpha,2\alpha)$ rate, that at $5\frac{3}{4}$ in. the 29.9-MeV group contains less than a 1.5% contribution from $(\alpha,2\alpha)$ events, and that the 19-MeV group has less than a 6% contribution from $(\alpha,2\alpha)$ events.

These results are not necessarily representative of the situation as a whole. On the one hand, they provide an excessively high upper limit by neglecting multiple scattering. On the other hand, they refer to data taken at angles where the Be^8 yield was particularly large. More comprehensive information on the $(\alpha,2\alpha)$ yield can be obtained from examination of the 1-3 spectra. As discussed above, $(\alpha,2\alpha)$ events will produce peaks in the spectra at the same pulse heights as peaks from Be^8 breakup. Because the $(\alpha,2\alpha)$ yield is expected to be essentially the same in the 1-3 detector pair as in the 1-2 and 2-3 pairs, the magnitudes of any 1-3 peaks provide information concerning the $(\alpha,2\alpha)$ contribution to the 1-2 and 2-3 peaks. For the most part, peaks do not appear in the 1-3 spectra at forward angles, as is illustrated in Fig. 4. When such peaks do appear the yield is small.

[Peaks in the 1-3 spectra do become prominent when the detector angle is large and the Be^8 energy is correspondingly low. The peaks in such cases are attributed to Be^8(g.s.) events which can reach detectors 1 and 3 at reduced Be^8 energies owing to the increased breakup cone angle. The identification of these 1-3 peaks with the Be^8(g.s.) events is supported by data in which the yield was found to decrease markedly when the distance of the detector array from the target was reduced from $5\frac{3}{4}$ in. to about 4 in. while maintaining a constant linear separation between the individual detectors. It is further supported by detailed pulse-height information obtained from an auxiliary electronic system used during some of the runs. In this system the sum energy and the individual pulse heights from detectors 1 and 3 were recorded separately on paper tape whenever a 1-3 coincidence occurred. It was found that the two coincident alpha particles tended to share the available energy equally in a typical 1-3 peak in the region where the edges of the breakup cone just begin to reach both detectors 1 and 3. This is consistent with their being companions in Be^8(g.s.) breakup events in which the alpha particles were emitted with maximum possible angular separation.]

From the combined results of the examinations of two-dimensional spectra, of coincidence rates as a function of detector distance, and of 1-3 spectra, it was concluded that probably less than 10 μb/sr of the quoted Be^8 yields are due to $(\alpha,2\alpha)$ reactions. The magnitude of this estimated upper limit is smaller than typical uncertainties in the subtraction of the continuum background. Therefore $(\alpha,2\alpha)$ events do not appear to represent a significant source of error in the present experiment.

In the sequence of studies in which the detector distance was changed, another 1-3 peak was noted at very small detector distances. This peak is tentatively attributed to events from the breakup of Be^{8*} in its 2.9-MeV first excited state. The total peak energy is consistent with this hypothesis. The peak presumably appears only at small distances because at larger distances the angular divergence of the detected alpha particles is small compared to the breakup cone angle for Be^{8*}, and hence the energy of one of the alpha particles will lie below the threshold for particle detection. At shorter distances, where the angular divergence of alpha particles that can enter the detectors increases sufficiently, the low-energy member of the pair from the Be^{8*} breakup has enough energy to be detected. This explanation accounts for the appearance of the peak only at small distances, and also predicts that the peak, when it first appears, will have one alpha particle just above the energy threshold and the other at a correspondingly high energy. This was supported by separate examination of the two alpha-particle energies, again using the auxiliary electronic system mentioned above.

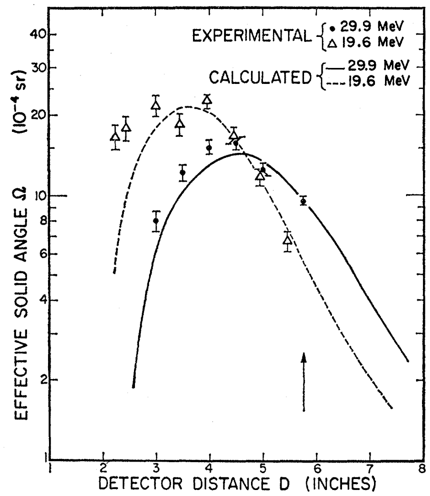

FIG. 5. Dependence of the effective solid angle for Be^8 detection on the target-detector distance. Results are shown for two different Be^8 energies. The curves are based on calculations outlined in the Appendix. The points correspond to experimental data normalized to the calculated curves at large distances. The arrow indicates the detector distance during data collection.

IV. BEAM-ENERGY MONITORING SYSTEM

In preliminary work it was found that the $O^{16}(\alpha,Be^8)$-C^{12} cross section was sensitive to the alpha-particle bombarding energy. A method for checking the beam energy was therefore desirable in order that one might be assured that no significant changes in the beam energy occurred during the course of a run, and to verify, when necessary, that one has returned to a previously studied energy. A device to accomplish this was constructed,[20] and was used in the present experi-

[20] C. D. Kavaloski, Cyclotron Research, University of Washington, 1964, p. 49 (unpublished).

ment. The method employed is identical to that used by Benveniste *et al.*,[21] in which the energy of elastically scattered alpha particles is degraded to energies convenient for comparison to those from a radioactive alpha-particle source. Such a method is independent of parameters which normally change with time and, in particular, makes virtually no demands on electronic stability.

The unit was designed to be placed into a side port on the scattering chamber at 30° to the incident beam. The three principal components of the unit are: (1) a solid-state detector, (2) a degrader holder permitting the insertion of aluminum foil to degrade elastically scattered beam alpha particles to an energy between 4 and 12 MeV, and (3) a source holder, located between the degrader and the detector, permitting the insertion and removal of the source without disrupting the vacuum system.

The signals from the detector are amplified and fed into a 512-channel analyzer. In one-half of the analyzer an alpha-particle spectrum from a Th*C*-Th*C*′ source is recorded. The most intense groups from this source are at 8.78, 6.05, and 6.09 MeV, the last two groups being unresolved in the spectrum. In the other half of the analyzer the degraded spectrum of the beam scattered from a gold target (about 1.2 mg/cm^2 thick) is recorded. Figure 6 shows the results with the two halves overlapped.

Because the degraded peak is relatively broad, a reference "center point" is defined to be midway between the two half-maximum positions. This point can be located to within one channel, corresponding to an uncertainty in the degraded peak energy of less than 50 keV. If the degraded energy is reproduced to within 50 keV then, typically, the incident-beam energy is reproduced to within about 15 keV. Actually, during the taking of an angular distribution, drifts of up to about 50 keV in incident energy were noticed. This is considered acceptable, because 50 keV is considerably less than the incident-beam energy spread of about 160 keV (see Sec. II).

This system does not provide an accurate absolute beam-energy determination because of uncertainties in the range-energy relations and in the determination of degrader thickness. However, an absolute beam-energy measurement was made by magnetic analysis of scattered particles with a broad-range spectrometer.[22] The measurements, accurate to within 100 keV,[23] were made at the full beam energy of 41.9 MeV and at a reduced beam energy of 40.8 MeV. Spectra were recorded with the beam-energy monitoring system simultaneously with these two beam-energy measurements. This allowed us to reproduce these two energies without the necessity of repeating the analysis with the magnetic spectrometer. New incident energies were initially calibrated by comparing the analyzing magnet field (see Fig. 1) with the field for the 40.8-MeV point, and return to the new energy in subsequent runs was provided for by use of the beam-energy monitor. Thus, in essence, all absolute energies are based on magnetic analysis (either of the scattered beam or the incident beam), and the reproduciblity of energies is based on the beam-energy monitoring system.

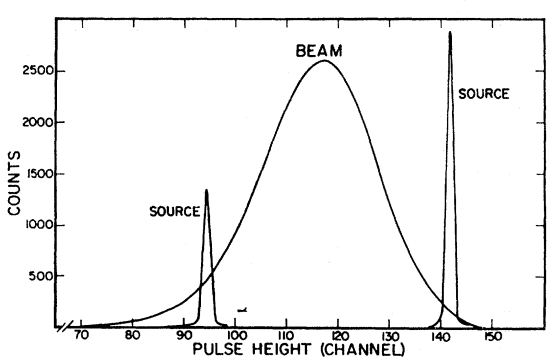

FIG. 6. Superposed pulse-height distributions obtained with the beam-energy monitor system. The broad curve is the spectrum of beam alpha particles after scattering and degradation. The sharp peaks correspond to alpha-particle groups at 6.05 and 6.09 MeV (unresolved) and at 8.78 MeV, from a Th*C*-Th*C*′ radioactive source.

V. DATA-TAKING AND RESULTS

At the beginning of each run, a standard set of alignment procedures was carried out. The gains of the three alpha-particle detecting channels were set equal and were calibrated with an alpha-particle source. Proper timing of the coincidence system was established by matching the individual channel delays in a sequence of observations of α-C^{12} coincidences from the $C^{12}(\alpha,\alpha')$-$C^{12*}(4.43)$ reaction. The C^{12} recoil nuclei were detected in a separate (fourth) detector, and the inelastically scattered alpha particles were detected in turn in each of the three detectors being matched. The energy of the beam entering the scattering chamber was set to the desired value using the procedure outlined in Sec. IV. Normally, the beam energy was initially set at a value previously used and (α,Be^8) data were taken again at several angles where the Be^8 yield was large. These repeat measurements served as a check on the over-all system reproducibility, including the beam energy, the detector geometry, and the electronic efficiency.

At the new beam energy, data were taken so as to obtain Be^8 differential cross sections at 2.5° intervals, usually over a lab angle range from 12.5° to about 75°. It was established that the 1-2 and 2-3 detector systems were essentially identical by repeating data points with each pair successively at the same angle. The acquisi-

[21] J. Benveniste, A. Mitchell, and R. Thomas, Nucl. Instr. Methods **23**, 349 (1963).

[22] D. K. McDaniels, W. Brandenberg, G. W. Farwell, and D. L. Hendrie, Nucl. Instr. Methods **14**, 263 (1961).

[23] D. L. Hendrie (private communication).

tion of data at lab angles greater than about 75° was often precluded by low laboratory yield, the broadening of peaks due to target thickness effects, and the higher background at the lower energy of the breakup alpha particles. At some incident energies data were obtained only over the angular region of the prominent forward peaks.

The basic results of this study of the $O^{16}(\alpha,Be^8)C^{12}$ reaction consist of differential cross sections at 11 energies from 35.5 to 41.9 MeV. The cross sections for the reaction leading to C^{12}(g.s.) are displayed in Fig. 7, and those leading to C^{12*}(4.43) are displayed in Fig. 8. It is seen that the differential cross sections all show strong oscillatory patterns, with a typical angular separation between successive peaks of 15° to 20° (c.m.). This rapid variation with angle implies that partial waves corresponding to high l values (about 10 or 11) make a significant contribution to the differential cross section.

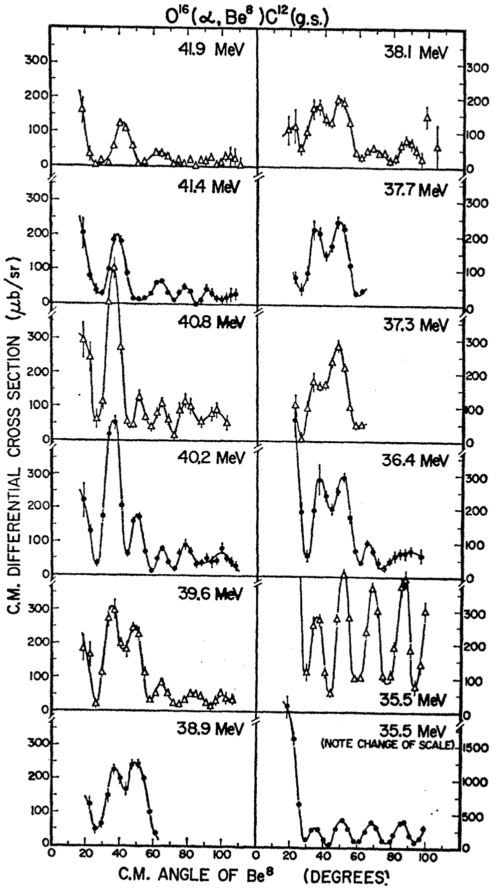

FIG. 7. Differential cross sections for the $O^{16}(\alpha,Be^8)C^{12}$(g.s.) reaction at several alpha-particle bombarding energies. The indicated errors correspond to statistical errors and to systematic uncertainties in background subtraction. There is an additional uncertainty in absolute normalization of about 10%. Curves have been drawn through the experimental points to guide the eye.

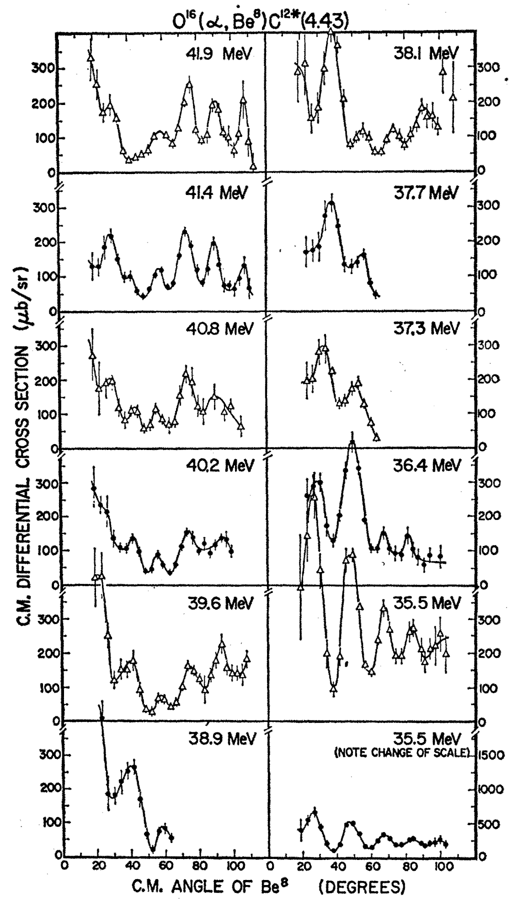

FIG. 8. Differential cross sections for the $O^{16}(\alpha,Be^8)C^{12*}$(4.43 MeV) reaction at several alpha-particle bombarding energies. The indicated errors correspond to statistical errors and to systematic uncertainties in background subtraction. There is an additional uncertainty in absolute normalization of about 10%. Curves have been drawn through the experimental points to guide the eye.

Another significant aspect of Figs. 7 and 8 is the strong dependence on bombarding energy of the shape and magnitudes of the differential cross sections. In some cases, it is not even possible to single out a prominent peak in the angular distribution and then follow it as the bombarding energy is changed. The energy dependence seems to be somewhat more pronounced in the case of the reaction leading to the ground state of C^{12}. This effect is emphasized in Fig. 9, where the integral of the differential cross section from 18° to 100° (c.m.) is plotted as a function of bombarding energy for the two reactions in question. The yield of the (α,Be^8) reaction to the 4.43-MeV state in C^{12} is rather flat except near 35.5 MeV, whereas the ground-state yield shows considerably more variation with energy. There is a sharp increase in yield at 35.5 MeV for reactions to both the 4.43-MeV state and the ground state. With the exclusion of the 35.5-MeV data, the cross sections integrated from 18° to 100° (c.m.) have mean values of 0.6 mb for the ground-state reac-

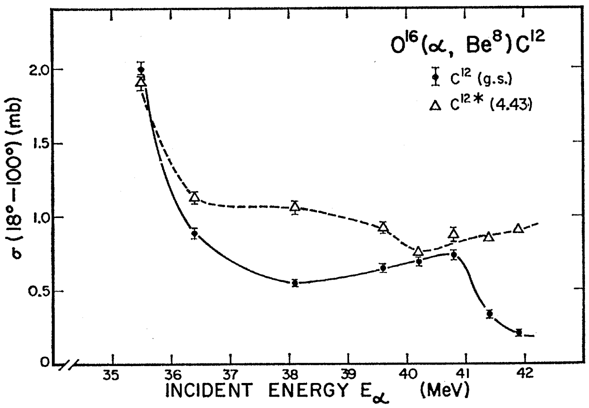

FIG. 9. Dependence of the $O^{16}(\alpha,Be^8)C^{12}$ cross sections on incident alpha-particle energy. The indicated cross sections are obtained by integrating the differential cross sections from 18° to 100° (c.m.). Curves have been drawn through the experimental points to guide the eye.

tion, and 1.0 mb for the 4.43-MeV-state reaction. Assuming no unusually rapid rise in yield in the backward direction, the results indicate that the total angle-integrated reaction cross section over most of the energy range studied is about 1 to 2 mb for the 4.43-MeV reaction and is somewhat smaller for the ground-state reaction.

Some data over a rather restricted angular range were also obtained for (α,Be^8) reactions leading to the 7.66- and 9.64-MeV states in C^{12}. The cross sections again showed marked variations with angle and indications of fluctuations with incident energy. The uncertainties in these data are considerably greater than those for the 4.43-MeV and ground states, due mainly to the higher background which one generally encounters in the detection of low-energy coincident alpha particles. Where comparisons were made (e.g., at an incident energy of 41.9 MeV), the average 7.66-MeV yield appeared to be about a factor of 3 smaller than the average 9.64-MeV yield, and the average 9.64-MeV yield was roughly equal to the average ground-state yield. Detailed angular distributions are not presented because of the relatively poor quality of the data. The cross sections at 35.5- and 36.4-MeV incident energies were particularly hard to determine due to the low Be^8 energies, and it was not possible to conclude from the present experimental data if the rise in yield which was seen at 35.5 MeV for the two lowest states also occurs for reactions to the 7.66- and 9.64-MeV states.

In addition to the O^{16} data, cross sections at 40.8- and 39.6-MeV incident energies were obtained for the (α,Be^8) reaction on Mg^{24}, leading to Ne^{20}(g.s.) and to $Ne^{20*}(1.63)$. Partial distributions (not shown here) were taken from 17.5° (lab) to about 50° (lab), with gaps due to an oxygen contaminant in the Mg target. Cross sections are typically about a factor of 4 smaller than those obtained in the O^{16} reaction. They also change rapidly with angle and with incident energy.

VI. DISCUSSION

The characteristics of the $O^{16}(\alpha,Be^8)C^{12}$ reaction described in the previous section do not appear to be immediately accounted for by any simple model for the reaction mechanism. The applicability of several of the more popular reaction models will therefore be considered in some detail in an attempt to understand the nature of the observed (α,Be^8) processes.

The most general expression for the differential cross section describing the $O^{16}(\alpha,Be^8(\text{g.s.}))C^{12}$ reaction is[24]

$$\frac{d\sigma}{d\Omega}(0\rightarrow I')=\sum_{M'=-I'}^{I'}|f_{I',M'}|^2, \tag{3}$$

where the reaction amplitude $f_{I',M'}$ is given by

$$f_{I',M'}=(-i\pi^{1/2}/k)\sum_{l,l'}(2l+1)^{1/2}\times(I'l'M',-M'|l0)U_{l;I'l'}{}^{l}Y_{l'}{}^{-M'}(\theta,\phi). \tag{4}$$

Here, k is the initial relative wave number, $(I'l'M',-M'|l0)$ is a Clebsch-Gordan coefficient linking the initial orbital angular momentum l, the final orbital angular momentum l', and the final nuclear angular momentum I', while $Y_{l'}{}^{-M'}(\theta,\phi)$ is a spherical harmonic. The quantum number M' is the projection of the final nuclear angular momentum along the direction of the incident beam. The dynamics of the reaction are expressed through the elements of the collision matrix $U_{l;I'l'}{}^{l}$. For a transition to a spin-zero nuclear state, as is the case when C^{12} is left in its ground state, the differential cross section has the simple form

$$\frac{d\sigma}{d\Omega}(0\rightarrow 0)=\frac{\pi}{k^2}\Big|\sum_{l}(2l+1)^{1/2}U_{l;0l}{}^{l}Y_{l}{}^{0}(\theta,\phi)\Big|^2. \tag{5}$$

In direct interaction theories (such as the distorted-wave Born-approximation theory) the coefficients of the spherical harmonics $Y_{l'}{}^{-M'}$ in the reaction amplitudes tend to be smooth functions of l' for a given value of incident energy, and slowly varying functions of energy for given l'. It is this functional dependence of the collision matrix which leads to the regularity in oscillations and the slow energy dependence characteristic of direct angular distributions.

When both incident and final projectiles are strongly absorbed and have nearly equal wave numbers, which is the case for the present experiment, the most important matrix elements in typical direct interaction calculations[25] are found to be those for which $l\approx l'\approx L$, where L is that orbital angular momentum at which the real part of the collision matrix for elastic scattering in the incident channel has the value $\frac{1}{2}$; in other words, the dominant contributions to the reaction amplitudes

[24] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958), Eq. (VIII-2.3).
[25] See, for example, E. Rost, Phys. Rev. **128**, 2708 (1962).

correspond to classically grazing collisions. [To emphasize the classical picture, this angular momentum is customarily related to a "sharp-cutoff radius" R through $kR\equiv(L+\frac{1}{2})$.] If only the single final orbital angular momentum L made an appreciable contribution, the angular dependence of any reaction amplitude $f_{I',M'}$ would be that of the spherical harmonic $Y_L^{-M'}(\theta,\phi)$, which for $L\gg 1$ is well approximated in the forward hemisphere by $[(2L+1)/4\pi]^{1/2}J_{M'}((L+\frac{1}{2})\theta)e^{-iM'\phi}$. In these circumstances, then, the direct cross section for a spin-zero transition is expected to be proportional to $J_0^2(kR\theta)$. However, neighboring partial waves also make contributions to the reaction amplitude which interfere constructively due to the smooth variation of the elements of the reaction matrix with l. The envelopes of the resulting angular distributions[26,27] are found to decrease more rapidly with angle than the envelopes for the prediction J_0^2. (It will be observed that the same approximate angular distributions are given by Fraunhofer diffraction theories,[28] which thus provide alternative and essentially equivalent descriptions of a direct reaction when $k\approx k'$ and when strong absorption is present.)

The observed angular distributions display strong oscillations. This is consistent with predictions of direct interaction theories, although any other reaction models in which high partial waves are important also imply rapid oscillations. From the number of oscillations and their spacing we estimate that partial waves with l in the neighborhood of 10 and 11 make large contributions to the reaction amplitudes; these values of l are nearly the same as the values (11 or 12) of the sharp-cutoff angular momentum L deduced from analysis of the elastic scattering of alpha particles by O^{16} in the same interval of energy.[29]

However, except for the case of 35.5-MeV incident energy, the angular distributions do not have the monotonically decreasing envelopes typical of direct interaction calculations, and often the maxima and minima of the patterns are spaced irregularly. More important, the character of the angular distributions and the magnitudes of the cross sections frequently change dramatically with relatively small variations in the incident energy. These qualitative observations are in striking disagreement with the predictions of any standard direct interaction theory. The large increase in cross section particularly precludes a dominant direct interaction contribution at 35.5 MeV, where the angular distribution alone might suggest it.

In view of the disagreement with direct interaction predictions, we next considered the possibility that the amplitude at a given incident energy is dominated by a strongly excited individual level in the compound system (perhaps not isolated) so that an amplitude of the typical Breit-Wigner form is superimposed upon a reaction amplitude which otherwise varies smoothly with energy. Such a procedure has had some limited success in describing elastic scattering of heavy ions[30] and of alpha particles[31] of somewhat lower incident energy than that used in the present experiment.

However, several observations cause us to discount the model of the preceding paragraph as a general description over the entire energy range investigated:

(a) If the variations of cross section are caused by individual dominant resonant states, then correlations between the cross sections to two different final states are expected since a resonant state could decay via either channel. The measurements, in general, show no positive correlation between the observed cross sections to the ground state and first excited state of C^{12}, although there is a sharp rise in both cross sections at an incident energy of 35.5 MeV. A quantitative test of a possible correlation between the two reactions was made by calculating the experimental cross-correlation ratio.[32] The value obtained, upon exclusion of the 35.5-MeV data, was 1.02, which is indicative of essentially no correlation between the two transitions. Were the 35.5-MeV data included, the cross-correlation ratio would of course be increased somewhat.

(b) When there is a single dominating resonance, the angular distribution for the ground-state transition should tend to conform to the pattern appropriate for that resonance, namely $[P_J(\cos\theta)]^2$, where J is the angular momentum of the resonant state. However with the exclusion of the data at 35.5 MeV, an increase in the observed ground-state cross section is not associated with greater regularity in the angular distribution.

At 35.5 MeV both the cross sections do increase markedly and both angular distributions show regularly spaced oscillations. A fair fit to the locations of the maxima and minima in the ground-state pattern is given by the function $[P_{11}(\cos\theta)]^2$. This suggests that at this energy there may be a state in the compound system which has a particularly large width for formation under alpha-particle bombardment. Because it was not possible at 35.5 MeV to make reliable measurements of the (α,Be^8) cross sections to the 7.66- and 9.64-MeV levels of C^{12}, we cannot test this suggestion by noting whether or not the enhancement is common to more than two final channels of the (α,Be^8) reaction.

It is interesting that at 35.5 MeV there is a phase relation between the Be^8 angular distribution to the

[26] W. E. Frahn and R. H. Venter, Ann. Phys. (N. Y.) **24**, 243 (1963).
[27] J. S. Blair, D. Sharp, and L. Wilets, Phys. Rev. **125**, 1625 (1962).
[28] J. S. Blair, Phys. Rev. **115**, 928 (1959); A. Dar, Phys. Letters **7**, 339 (1963); E. M. Henley and D. U. L. Yu, Phys. Rev. **133**, B1445 (1964).
[29] N. Cue and D. Shreve (private communication).
[30] D. A. Bromley, J. A. Kuehner, and E. Almqvist, Phys. Rev. **123**, 878 (1961).
[31] E. B. Carter, G. B. Mitchell, and R. H. Davis, Phys. Rev. **133**, B1421 (1964).
[32] See, for example, Ref. 35 for a quantitative discussion of cross-correlation ratios.

ground state of C^{12} and the angular distribution[29] of alpha particles elastically scattered from O^{16}: Specifically, the maxima of the Be^8 angular distribution coincide with the minima in the elastic angular distribution. This indicates that the partial wave dominating the reaction amplitude has an orbital angular momentum L corresponding to a grazing alpha-particle collision. (The location of maxima and minima of the elastic cross section, in situations of strong absorption, is well approximated by a sharp-cutoff model: $\eta_l=0$ for $l<L$, $\eta_l=\frac{1}{2}$ for $l=L$, and $\eta_l=1$ for $l>L$. The resulting elastic cross section is[27]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{el}}=\frac{1}{4k^2}\frac{1}{\sin^2\theta}\left[\frac{d}{d\theta}\left\{P_L+\tfrac{1}{2}(P_{L+1}+P_{L-1})\right\}\right]^2. \quad (6)$$

The phase relations can be seen by contrasting this expression with the Be^8 reaction cross section, which is proportional to $P_L{}^2$.)

The characteristics of the (α,Be^8) cross sections discussed above have led us to examine the statistical treatment of Ericson,[33,34] a theory which is pertinent to situations in which many overlapping levels are excited in the compound system. The phases of the reaction amplitudes corresponding to the individual compound levels are assumed to be randomly distributed, with the consequence that both total and differential cross sections display statistical fluctuations around the familiar predictions of the energy-averaged statistical theory. The "period" of these fluctuations is closely related to the average level width Γ of the compound nuclear levels. Recent papers by the Chalk River group[35,36] and by Bondorf and Leachman[37] consider the statistical theory in detail for heavy ion reactions which are similar to the reactions studied in the present paper. For the remainder of this discussion we lean heavily on the concepts and results developed in these papers.

For the statistical theory of fluctuations to be applicable it is necessary that the average level spacing D of the relevant compound nuclear levels be small compared to the average width Γ of these levels. Standard theoretical calculations have been carried out to ascertain whether the condition $\Gamma\gg D$ is fulfilled. (The question is not a trivial one since it is known that level spacings are rapidly increasing functions of angular momentum.) Let the average partial width for decay of compound nuclear levels with total angular momentum J to a particular final state α with channel spin s be denoted by $\langle\Gamma_\alpha{}^J\rangle$. This width is related to the average level spacing D^J and transmission function $T_l(\alpha)$ in a familiar approximation

$$\sum_{l,s} T_l(\alpha)\cong 2\pi\langle\Gamma_\alpha{}^J\rangle/D^J, \quad (7)$$

where l, s, and J must satisfy the requirements of angular momentum and parity conservation. It should be mentioned that this relation is strictly applicable only for $T_l\ll 1$, which is not the case for the present study. The reader is referred to the discussion of Vogt *et al.*[36] concerning the applicability of this expression to situations where $T_l\approx 1$.

Estimates indicate that nucleon decay does not make a large contribution to the width; thus, to determine the approximate ratio of the average total width $\langle\Gamma^J\rangle$ to level spacing, only the alpha-particle channels need be considered. The summation indicated above cannot be accomplished by an explicit enumeration of states because there is not sufficient information available on the states of O^{16} at the high excitation energies involved here. Instead, a code written for the IBM-7094 by Vandenbosch[38] was employed to carry out a summation based on the level density expression[39]

$$\Omega(U,j)=\tfrac{1}{12}a^{-1/4}(U+t)^{-5/4}[(2j+1)/(2ct)^{3/2}] \times\exp[2(aU)^{1/2}-(j+\tfrac{1}{2})^2/2ct], \quad (8)$$

where U and j are the excitation energy and spin of the residual nucleus, t is the thermodynamic temperature, and a and c are the conventional level density parameters. The transmission coefficients were computed using a Saxon-Woods optical potential which gives a fair fit to the elastic scattering of alpha particles from O^{16} ($V=38$ MeV, $W=21$ MeV, $a=0.41$ F, $r_0=1.88$ F).

This calculation of $\langle\Gamma^J\rangle/D^J$ was repeated for all relevant values of the compound nuclear spin J. It was found through a separate calculation, using the optical parameters cited above for the outgoing channel calculation, that the transmission functions in the incoming channel at 41 MeV (lab) rapidly become small when l exceeds 12. Therefore, because both particles in the incoming channel are spinless, compound nuclear states with spins of 12 or less dominate the reaction. This is consistent with the observations made above concerning the periodicity of the angular distributions. As expected, the ratio $\langle\Gamma^J\rangle/D^J$ was found to be smallest at high J. For example, using $c=1.4$ (corresponding to a spherical rigid body nucleus with $r_0=1.2$ F and $a=\frac{1}{8}A$ [40]) one finds that $\langle\Gamma^J\rangle/D^J$ is 600, 1350, and 40 at $J=0$, 6, and 12, respectively, for a compound nuclear excitation energy of 37.5 MeV (corresponding to 41-MeV incident energy). In general, $\langle\Gamma^J\rangle/D^J$ decreases rapidly if a is decreased. Reducing a to $\frac{1}{12}A$ was found to reduce this ratio at $J=12$ to about 7. Assuming that this represents a low choice for a, it is concluded that the condition $\langle\Gamma^J\rangle/D^J\gg 1$ is probably satisfied for $J\leq 12$ and that the

[33] T. Ericson, Phys. Rev. Letters **5**, 430 (1960); Ann. Phys. (N. Y.) **23**, 390 (1963).
[34] D. M. Brink and R. O. Stephen, Phys. Letters **5**, 77 (1963).
[35] E. Almqvist, J. A. Kuehner, D. McPherson, and E. W. Vogt, Phys. Rev. **136**, B84 (1964).
[36] E. W. Vogt, D. McPherson, J. A. Kuehner, and E. Almqvist, Phys. Rev. **136**, B99 (1964).
[37] J. P. Bondorf and R. B. Leachman (unpublished).
[38] R. Vandenbosch (private communication).
[39] D. W. Lang, Nucl. Phys. **26**, 434 (1961).
[40] D. Bodansky, Ann. Rev. Nucl. Sci. **12**, 79 (1962).

occurrence of isolated resonances in the compound system is unlikely. It is therefore reasonable to consider the experimental results in the light of the fluctuation theory.[40a]

The fluctuation pattern in excitation functions is closely related to the average width of the compound nuclear states. A prescription has been given by Brink and Stephen[34] for deducing the average level width from the spacing of excitation function peaks. Following Almqvist *et al.*[35] in applying this prescription to differential cross sections, we note that $\Gamma = 0.5/M$ for transitions to the zero-spin ground state and $\Gamma \cong 0.37/M$ for transitions to the spin-2 4.43-MeV state, where M is the number of peaks per unit energy interval. These expressions assume that the average width is independent of the compound nuclear angular momentum J, and, because this assumption is not necessarily fulfilled in the present experiment, the experimental "width" is to be viewed as an effective average width. Differential cross sections at different angles were examined as a function of the incident energy, over the experimental energy span of 5.1 MeV (c.m.). The average number of peaks was 2.1 for the ground-state reaction and was 1.9 for the 4.43-MeV state reaction, giving an average experimental width of about 1.1 MeV.

It would be of interest to compare this estimate of experimental width with a theoretical width deduced from $\langle \Gamma^J \rangle / D^J$, discussed above, and D^J, the reciprocal of the compound nuclear level density. However, serious difficulties stand in the way of obtaining a reliable theoretical estimate of Γ. A discussion of these difficulties will be presented, couched mainly in terms of difficulties in finding the average level density of the compound nucleus. However, many of the objections are also relevant for the determination of $\langle \Gamma^J \rangle / D^J$. These difficulties include:

(a) The compound nuclear lifetime corresponding to the 1.1-MeV experimental width is 6×10^{-22} sec. The transit time of a nucleon through the Ne^{20} compound nucleus is about 10^{-22} sec. Thus the nuclear lifetime exceeds the nucleon transit time by less than a factor of 10, and one is led to question the underlying assumption of the statistical theory that the partial decay widths are randomly distributed.

(b) The evidence supporting the description of level densities in terms of the statistical model has been primarily accumulated for nuclei with mass number A of 40 or greater. The application of the same expressions to light nuclei, near $A = 20$, is not supported by a compelling body of experimental evidence.

(c) The dependence of level density on excitation energy demands, at least, a knowledge of the level density parameter a. Uncertainties in this parameter lead to large uncertainties in the level density at high excitation energy. For example, at an excitation energy of 32 MeV (typical of the present experiment), changing a from $\frac{1}{8}A$ to $\frac{1}{12}A$ changes the level density by a factor of 25.

(d) The dependence of level density on spin is understood in only a very preliminary fashion. The failure of different experiments to give consistent values for the spin cutoff factor $\sigma^2 = ct$ illustrates this difficulty.[41] The use of common functional forms for the spin dependence, while suggested by simple theoretical models, has not yet received substantial experimental confirmation. Tentatively accepting the usual dependence, one is still left with great uncertainty in choosing σ^2. This has little effect on the level density at low spin, but has greater effect at high spin. For example, at $J = 11$, changing σ^2 from 8 to 12 changes the level density by a factor of about 7.

(e) Compound nuclear states with different values of spin contribute to the observed distributions. The relative importance of these states depends both on the cross section for compound nuclear formation and on the branching ratio for Be^8 emission to the observed states of C^{12}. In view of the other difficulties, and of uncertainties in this calculation itself, a calculation of of the Be^8 branching ratio has not been attempted for the present experiment. For qualitative orientation we refer to the calculations of Vogt *et al.*,[36] who conclude that for Mg^{24}, at 25-MeV excitation energy, the branching ratio for Be^8 emission lies between 1.6% and 4.9% for compound states with spin less than or equal to 10. This suggests that Be^8 evaporation should not be expected to be trivial here, but of course provides no quantitative basis for assigning weights to the various spin states.

(f) If it is assumed that the individual widths for different spin states are known, and that the relative contributions to the observed yield of these states are also known, it remains necessary to deduce a predicted over-all width to be compared to the experimental width. Existing theoretical formulations of the statistical theory of fluctuations provide no obvious guide as to how this is to be accomplished. The problem was not severe in previous work[35,37] on the reactions $C^{12} + C^{12} \rightarrow Ne^{20} + He^4$ because of the dominance of only a few spin states. However, the branching ratios calculated by

[40a] *Note added in proof.* The relation between Γ/D and the transmission functions has been investigated in several papers by Moldauer. [P. A. Moldauer, Phys. Letters **8**, 70 (1964); Phys. Rev. **135**, B642 (1964); **136**, B947 (1964); Rev. Mod. Phys. **36**, 1079 (1964).] It is pointed out in these papers that it is difficult to estimate Γ/D in the region of overlapping levels. Moldauer finds that the right-hand side of Eq. (7) should include a normalization factor N^2. In general, $N^2 > 1$. This implies that the procedure followed above overestimates the magnitude of Γ/D. However, it appears that N^2 only slightly exceeds unity when Γ/D is relatively small (and for boundary conditions such that the resonance energies of the R matrix and the collision matrix are the same). To the extent that this is true, the over-all conclusion that Γ is probably large compared to D is not qualitatively altered. The same considerations would lower the theoretical values of Γ calculated below, and thus bring them into somewhat closer agreement with the value found experimentally.

[41] N. MacDonald, At. Energy Res. Estab. (Gt. Brit.), Rept. AWRE NR/P-2/61 (1961).

Vogt *et al.*[36] indicate that the contributions of low-spin states are relatively more important in Be^8 emission than in the alpha-particle emission which they considered. Therefore the problem of superimposing the different spin states to obtain an over-all width may not be trivial in the present experiment. No solution of this problem in the statistical theory of fluctuations has been attempted here. It is likely that its resolution will depend upon the particular prescription used for extracting the experimental "width" from the observed data.

Despite these major difficulties, a theoretical estimate was made of the expected widths of Ne^{20} formed in different spin states with 40-MeV incident alpha particles. Values of the ratio $\langle\Gamma^J\rangle/D^J$ were taken from the analysis described above, for $c=1.4$ and $a=\frac{1}{8}A$. The level spacing D^J was also calculated from Eq. (8), again assuming $a=\frac{1}{8}A$ and, at the fixed energy of the compound nucleus, $\sigma^2=ct=10$. For $J=0$, 6, and 12, the spacings were found to be 1.7, 1.1, and 170 keV, respectively. This implies approximate theoretical widths of 1.0 MeV for $J=0$, 1.5 MeV for $J=6$, and 7 MeV for $J=12$. It is concluded, from the order-of-magnitude agreement between these theoretical results at different spins and the estimated experimental width of 1.1 MeV, that the statistical model can very possibly provide an explanation of the observed width, subject to modifications in parameters and a solution of the difficulties cited above.

More conclusive tests of specific predictions of the statictical theory are unfortunately difficult to make because of the limited extent of the data. One such test is in principle possible through the study of the correlation ratios. Consider, for example, the self-correlation ratio

$$R=\langle[\sigma(\theta)]^2\rangle/\langle\sigma(\theta)\rangle^2, \tag{9}$$

where $\sigma(\theta)$ is the differential cross section, and the brackets indicate an average over incident energy. For a purely statistical reaction the expectation value of this ratio depends on the number of independent channels and on the number of independent measurements (i.e., "sample size"). If direct processes play a role, the ratio further depends upon the relative magnitudes of the direct and compound nuclear contributions. For precise theoretical predictions, the sample size must be large. In the present experiment the sample size (given by the ratio of the experimental energy span of 5.1 MeV to the average level width of 1.1 MeV) is only about 5, and consequently there is a large statistical uncertainty in the correlation ratios. It is to be noted that the desired goal of a large sample size cannot be achieved in a simple fashion by an indefinite increase in the experimental energy span, because with large changes in incident energy the average cross sections are expected to change.

Experimental self-correlation ratios were found by averaging $\sigma(\theta)$ and $[\sigma(\theta)]^2$ over energy, and $\langle\sigma^2\rangle/\langle\sigma\rangle^2$ over angle. The values obtained were 1.40 and 1.30 for the g.s. and 4.43-MeV state reactions, respectively. These results were obtained using the data at all experimental incident energies, with a weighting based on the spacing between successive energies. If it is assumed that the high yields at 35.5 MeV represent a fluctuation to be understood within the framework of the statistical theory, then it is appropriate, and in fact essential, to retain the 35.5-MeV data in calculating R. However, if the 35.5-MeV yield is taken to represent a case where the reaction amplitude is not part of a population of amplitudes described statistically by a Gaussian distribution, then the 35.5-MeV data should not be used to calculate R. With the omission of the 35.5-MeV points, the self-correlation ratios are reduced to 1.21 and 1.24 for the g.s. and the 4.43-MeV states, respectively. Of course, the omission of the highest cross section will always serve to depress R, and in the absence of compelling arguments is not a justifiable step.

Since the g.s. differential cross section involves two degrees of freedom, the expectation value of the self-correlation ratio depends only on the sample size.[35] For a sample size of 5 the predicted value is 1.80. This is considerably in excess of the experimental result of 1.40. On omitting the 35.5-MeV points the reduction in sample size (and hence the reduction in the expectation value of R) is small, and the discrepancy between the calculated and measured self-correlation ratios is further increased. It is not clear which comparison is the more appropriate, nor is it clear how much significance can be attached to results obtained with such small samples sizes. Nevertheless the relatively low values of the experimental self-correlation ratio suggest that there may be a significant nonstatistical contribution to the ground-state reaction.

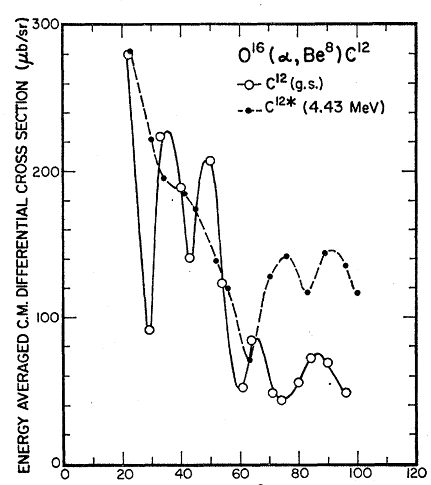

FIG. 10. Differential cross sections averaged over the incident-energy interval from 35.5 to 41.9 MeV for the $O^{16}(\alpha,Be^8)C^{12}$ reactions leading to the ground state and 4.43-MeV state of C^{12}.

The expectation value of R cannot be as reliably calculated for the 4.43-MeV state. If there are equal contributions to the cross section for the three possible absolute values of the magnetic quantum number of this state, then the effective number of degrees of freedom reaches an upper limit of 6. This corresponds, for infinite sample size, to a self-correlation ratio of 1.33.[37] For smaller sample size the expectation value of the ratio is less, and hence the experimental results appear reasonably consistent with the predictions of fluctuation theory.

Another specific prediction of the statistical theory concerns angular distributions. On averaging the differential cross sections over a large sample size, random fluctuations should disappear, and the resultant average distribution should be symmetric about 90°. The experimental energy-averaged distributions are presented in Fig. 10 for both the g.s. and 4.43-MeV state reactions. (The 35.5-MeV data are retained; their exclusion has little effect on the shape of the distributions.) In the absence of data in the backward hemisphere, the test of symmetry about 90° cannot be made. It is to be noted that the distribution for the g.s. reaction continues to show a strongly oscillating pattern even after averaging. The magnitude of the oscillations, particularly at small angles, appears to exceed the random fluctuations of about $\pm 30\%$ which are expected for a sample size of 5.[36] This suggests that states in a limited range of high angular momenta contribute predominantly to the g.s. reaction, but in itself does not indicate whether the process is mainly compound nuclear or direct. There is somewhat less structure in the 4.43-MeV angular distribution, which may be due to the superposition of cross sections to different magnetic substates in the residual nucleus.

In summary, we note that the observation of fluctuations in cross section is in qualitative agreement with the predictions of the statistical theory of compound nuclear reactions. However, the incomplete character of both the data and the statistical model precluded precise quantitative tests of this explanation of the experimental results. A larger experimental sample size (via a smaller average level width), more complete angular distributions, and a better knowledge of the level density for states of high spin and excitation energy in light nuclei are minimal requirements for more adequate comparisons. In the absence of such comparisons, a substantial direct interaction contribution cannot be excluded, and in fact there are suggestions in the results that nonstatistical processes are important for the ground-state transition. With our relatively limited data, it appears premature to consider processes intermediate between the conceptual extremes of compound nuclear and direct interactions.

The following conclusions can be drawn from the present experiment:

(1) It is practical and convenient to detect Be^8(g.s.) nuclei of moderate energy by coincident observation of the breakup alpha particles.

(2) There is an appreciable cross section for $O^{16}(\alpha,Be^8(g.s.))C^{12}$ reactions to both the ground state and the 4.43-MeV first excited state of C^{12}. The 4.43-MeV cross section is generally the higher of the two. Assuming no rapid rise in yield in the unobserved portions of the backward hemisphere, the angle-integrated cross sections are of the order of 1 or 2 mb, for incident alpha-particle energies between about 36 and 42 MeV (lab).

(3) The yields to both levels at 35.5-MeV incident energy are at least twice as great as the yields over most of the immediately higher energy range, suggesting the possibility of a strongly excited (probably not isolated) level in the compound system near 35.5 MeV.

(4) Throughout the remainder of the observed energy region, changes of the order of 1 MeV in incident energy usually produce large, and apparently random, changes in the differential cross sections. Therefore the reactions cannot be explained solely in terms of a conventional direct interaction process, contrary to initial expectations, nor by individual dominating compound nuclear resonances. The energy dependence can, however, be understood in the context of the statistical model.

ACKNOWLEDGMENTS

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APPENDIX

In this Appendix the kinematics of Be^8 breakup and the detailed calculation of the effective solid angle Ω are discussed. Figure 11 describes the Be^8 breakup by means of a velocity diagram. At a fixed lab angle $\alpha<\alpha_{max}$

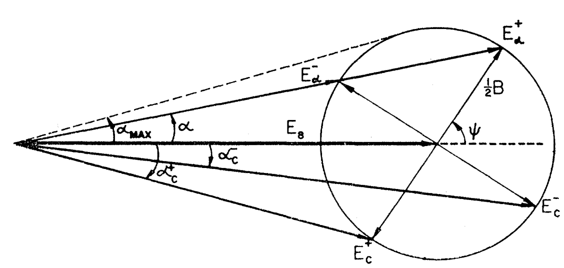

FIG. 11. Velocity diagram for the breakup of Be^8 into two alpha particles. The velocity vectors are labeled by the energies to which they correspond. The lab velocities (corresponding to energies E_α and E_c) are obtained by adding the alpha-particle velocity in the Be^8 rest system (energy $\frac{1}{2}B$) to the Be^8 velocity in the lab (energy E_8).

there are two possible alpha-particle energies E_α^+ and E_α^- corresponding to two different breakup angles ψ in the Be^8 rest system. To each of the two alpha particles corresponds a breakup companion moving at an angle α_c^+ or α_c^- with energy E_c^+ or E_c^-, respectively. The kinematic relations connecting these quantities are

$$E_\alpha^\pm=\tfrac{1}{2}(E_8\cos 2\alpha+B)\pm\cos\alpha[E_8(B-E_8\sin^2\alpha)]^{1/2}, \quad \text{(A1)}$$

$$\sin\alpha_c^\pm=(E_\alpha^\pm/E_c^\pm)^{1/2}\sin\alpha, \quad \text{(A2)}$$

and

$$E_c^\pm=E_8+B-E_\alpha^\pm. \quad \text{(A3)}$$

The number of alpha particles per unit solid angle at an angle α is proportional to the solid-angle transformation factor $|(\sin\psi d\psi)/(\sin\alpha d\alpha)|$ (see Sec. III A). For $\alpha<\alpha_{\max}$ this factor is double valued, and one obtains different transformation factors for the two rest system angles ψ which contribute to the yield at lab angle α. The total probability that a pair of alpha particles is emitted, with one member going into the solid angle $d\Omega_\alpha$ at α, will be designated as $2P(\alpha)d\Omega_\alpha$. The factor of 2 is included because either of the two alpha particles may be detected. With this notation $P(\alpha)$ refers to the probability of detection of the "first" alpha particle, although of course there can be no real distinction between it and its so-called "companion." Because $P(\alpha)$ contains contributions from two different rest system angles ψ, it is found through the summation

$$P(\alpha)=P^+(\alpha)+P^-(\alpha), \quad \text{(A4)}$$

where $P^+(\alpha)$ and $P^-(\alpha)$ are

$$4\pi P^\pm(\alpha)=\pm 2\beta\cos\alpha+(1+\beta^2\cos 2\alpha)(1-\beta^2\sin^2\alpha)^{-1/2}, \quad \text{(A5)}$$

and where

$$\beta=(E_8/B)^{1/2}=1/\sin\alpha_{\max}. \quad \text{(A6)}$$

The expression for Ω, the effective solid angle for Be^8 detection, is

$$\Omega=\Omega^++\Omega^-. \quad \text{(A7)}$$

Here

$$\Omega^\pm=\int d\omega\int \sin\alpha d\alpha d\phi\, P^\pm(\alpha)S^\pm(\alpha,\phi;\omega), \quad \text{(A8)}$$

where the integrations are over all Be^8 directions and over the faces of detectors 1 and 2. The azimuthal angle ϕ is indicated in Fig. 12, and the function $S^\pm(\alpha,\phi;\omega)$ is defined by

$$S^\pm(\alpha,\phi;\omega)=1 \quad \text{if the companion alpha is detected in the other detector}$$
$$=0 \quad \text{otherwise.} \quad \text{(A9)}$$

One must distinguish S^+ from S^- because, at a fixed angle α, the angles $\alpha_c^\pm$ of the companion are not equal to each other.

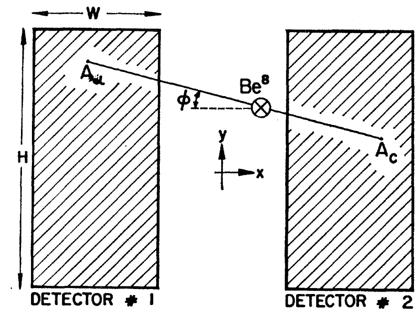

FIG. 12. Relative orientation of two detectors and the directions of the Be^8 nucleus and its breakup alpha particles (directed toward points A_α and A_c).

The application of this analysis to two equal-area rectangular detectors is illustrated in Fig. 12. By symmetry, one need integrate with respect to ω only over the first quadrant, and then multiply the result by four. In addition, one need integrate only over detector 1, and then multiply the result by two. The approximation will be used that all points on the faces of the detectors are at the same distance D from the target. Therefore, $d\omega=dxdy/D^2$, and Eqs. (A7) and (A8) give

$$\Omega=(8/D^2)\int_0^{x_{\max}}dx\int_0^{y_{\max}}dy[F^+(\omega)+F^-(\omega)], \quad \text{(A10)}$$

where the upper limits correspond to the maximum excursion of Be^8 directions which can result in coincidence events. Here

$$F^\pm(\omega)=\int_1 P^\pm(\alpha)S^\pm(\alpha,\phi;\omega)\sin\alpha d\alpha d\phi, \quad \text{(A11)}$$

where the integration is now only over detector 1. The functions S^+ and S^- are normally unity over a certain range in α and zero outside of this range, although, owing to a double-valued character of α as a function of α_c^+, there may be more than one range of α over which S^+ can have the value 1. In essence, this double-valuedness occurs because, as α increases from 0 to $\alpha_{\max}$, the angle α_c^+ reaches $\alpha_{\max}$ and starts to decrease again before α reaches $\alpha_{\max}$. For the Be^8 energies involved here, this effect is small. Nevertheless, it was taken into account in the final computer calculation of Ω. Equation (A11) is then integrated over all ranges $\alpha_1^\pm$ to $\alpha_2^\pm$ for which $S^\pm=1$. The result is

$$4\pi F^\pm(\omega)=\int_1 d\phi[\pm\beta\sin^2\alpha-\cos\alpha(1-\beta^2\sin^2\alpha)^{1/2}]_{\alpha_1^\pm}^{\alpha_2^\pm}. \quad \text{(A12)}$$

The limits $\alpha_1^\pm$ and $\alpha_2^\pm$ are functions of both ϕ and ω (i.e., of ϕ, x, and y).

The integrals in Eqs. (A10) and (A12) were performed on an IBM-709 computer. The geometry discussed in Sec. III D with both 5° and 10° separation was used, and Ω was calculated as a function of E_8. The result is shown in Fig. 2.

An approximation based on Eq. (2) can be used to check the computer calculation for the case of very small detectors near the cone center. One introduces an effective cone solid angle Ω_c' as if the breakup were uniform over a cone of size Ω_c' with breakup probability

$$P(\alpha=0)=(1+\beta^2)/2\pi. \tag{A13}$$

Thus

$$\Omega_c'=1/P(0)=2\pi/(1+\beta^2), \tag{A14}$$

and

$$\Omega\approx\Omega_0^2(E_8+B)/(4\pi B). \tag{A15}$$

Equation (A15) was found to agree with the more exact computer calculation under the conditions described above.

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Low-Energy Elastic K^+-d Scattering with Separable Potentials*

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Low-energy K^+-d elastic-scattering cross sections are calculated using a Fadeev type of multiple-scattering formalism. The two-body interactions are taken to be S-wave nonlocal separable potentials of the Yamaguchi form. Coulomb forces, and the K^+-K^0 and n-p mass differences are neglected. The elastic angular distribution and cross section, as well as the total cross section, are calculated for incident kaon laboratory momenta in the range 110–230 MeV/c. In this momentum range it is found that for the elastic scattering cross sections the impulse-approximation result is within 10–25% and the double-scattering-approximation result is within 10% of the correct value. It is also found that in order for the optical theorem to give a good result for the total cross section, triple-scattering terms must be included. A detailed examination of the multiple-scattering series is made in order to illuminate these results.

I. INTRODUCTION

IN a previous paper[1] a multiple-scattering formalism of the Fadeev type[2,3] for scattering from deuterons was derived and applied to low-energy K^--d scattering.[4] That particular application was chosen because the low-energy $\bar{K}$-N interactions were S-wave interactions and because some data on K^--d scattering was available. In fact the model for the $\bar{K}$-N interactions used in A did not completely justify using the results of the calculation to draw hard conclusions concerning experiment. As an investigation of the contribution of the single-scattering and the double-scattering terms to the exact solution of the multiple-scattering equations, however, these results were of interest. Even from this point of view the particular process studied was rather a special case in that the $\bar{K}$-N interactions are absorptive. The role played by unitarity was obscured by this absorption so that the scope of the interpretations which could be placed on the results was not clear.

In the present work we have applied the formalism developed in A to low-energy K^+-d scattering, a process in which the two-particle interactions are real. Partly because of our crude models for the two-particle interactions and partly because of the lack of data, our primary aim is not a comparison of our calculated results with experiment. Instead we shall concentrate on investigating the contributions of the low-order terms of the multiple-scattering series to the exact solution. Our aim is to see if there exist convenient approximations which may then be used in conjunction with more realistic two-particle interactions.

The model used here incorporates the same broad features as that used in A. Calculations are carried out for incident kaon laboratory momenta in the range 110–230 MeV/c. (For momenta $\gtrsim$300 MeV/c relativistic effects are large, while for momenta $\lesssim$100 MeV/c effects due to the K^+-K^0 and n-p mass differences become large. Neither relativistic nor mass-splitting effects are taken into account, although some calculations are done using relativistic kinematics.) Only S-wave interactions between pairs of particles are included. Each of the three interactions (K-N isospin

* Work supported by the U. S. Atomic Energy Commission.
[1] J. H. Hetherington and L. H. Schick, Phys. Rev. **137**, B935 (1965); hereafter referred to as A.
[2] L. D. Fadeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)]; Dokl. Akad. Nauk **138**, 561 (1961); **145**, 301 (1962) [English transls.: Soviet Phys.—Doklady **6**, 384 (1961); **7**, 600 (1963)].
[3] See also C. Lovelace, in *Strong Interactions in High Energy Physics*, edited by R. G. Moorhouse (Plenum Press, New York, 1964), p. 437; Phys. Rev. **136**, B1225 (1964).
[4] For similar developments of the n-d problem see A. N. Mitra and V. S. Bhasin, Phys. Rev. **131**, 1265 (1963); R. Aaron, R. D. Amado, and Y. Yam, *ibid.* **136**, B650 (1964); Phys. Rev. Letters **13**, 579 (1964).