Sense-Nonsense Channels in an Approximate N/D Model*

Y. N. SRIVASTAVA AND P. NATH
Department of Physics, University of California, Riverside, California
(Received 15 January 1965)

The decoupling of sense-nonsense Regge amplitudes at a physical angular momentum when external particles have spin is investigated. As discussed by Abers and Zachariasen, various approximation schemes to the N/D matrix equations that are available do not seem to decouple the sense and nonsense channels in an appropriate manner. Here we show, through the specific example of vector-scalar scattering, that an approximate amplitude suggested earlier by the present authors seems to decouple properly. The amplitude, being unitary and symmetric (time-reversal invariant), and having the same degree of simplicity as (for example) the determinantal approximation, may be of use in rudimentary spinological bootstraps.

RECENTLY, considerable interest has been generated about bootstrapping beyond scalar and pseudoscalar external particles. Gell-Mann and Zachariasen[1] have considered a vector-spinor bootstrap on the basis of Regge trajectories and sense-nonsense channels. A rather detailed analysis of spinological structure of trajectories has been made by Abers and Zachariasen (AZ).[2] They show that when external particles have spin, the solution obtained from first-order determinantal approximations to the N/D matrix equations[3] *does not* decouple the sense-nonsense amplitude at a physical j. For the particular case of a vector-scalar (or vector-pseudoscalar) elastic scattering, it was found that a singularity at $j=0$ was present only in *one* of the nondiagonal (sense-nonsense) amplitudes. Thus, the symmetry of the above solution was assumed to be responsible for this anomalous behavior, and hence they considered the $j=0$ limit of some of the approximate solutions available, which are time-reversal invariant (and hence symmetric).[4,5] However, it was found that at the said limit these approximate amplitudes vanish identically.

Now let us focus our attention upon the vector-scalar problem and see if this amplitude decouples at $j=0$ to lowest order in g^2, the vector-scalar-scalar coupling constant. AZ have obtained the "input" Born term for all j to order g^2 by projecting out the Feynman diagram (with scalar exchange) at high j. Near $j=0$, for the parity $(-)^j$ helicity amplitude, with index 1 sense and index 2 nonsense, we have

$$B^j(s)\approx\frac{1}{qW}\begin{pmatrix} b_1(s) & b_x(s)/W(j)^{1/2} \\ b_x(s)/W(j)^{1/2} & b_2(s)/sj \end{pmatrix}, \quad (1)$$

where

$$b_1(s)=-(g^2/4\pi)(q/m)^2(E_s+XE_v)^2Q_0(x)$$
$$b_x(s)=-(g^2/4\pi)(q^2s/mW(E_s+XE_v)$$
$$b_2(s)=-(g^2/4\pi)q^2sX$$
$$X=1+((\mu^2+2m^2-s)/2q^2).$$

E_s (E_v) and μ (m) are the c.m. energy and mass of the scalar (vector) particle. As usual, q denoted the c.m. momentum and s the total c.m. energy squared.

With the input as given in Eq. (1), the first-order matrix determinantal approximation to the amplitude, viz.,

$$T_{\det}{}^j(s)\sim B^j(s)D^{-1}, \quad (2)$$

near $j=0$, looks like

$$T_{\det}{}^j(s)\sim\begin{pmatrix} t_{11}(s) & t_{12}(s)(j)^{1/2} \\ t_{21}(s)/(j)^{1/2} & t_{22}(s) \end{pmatrix}, \quad (3)$$

where $t_{\mu\nu}(s)$ are some functions of s, independent of j. Thus, in violation of unitarity, the determinantal approximation does not decouple sense and nonsense states at $j=0$.

As mentioned in the introduction, since only one off-diagonal element in Eq. (3) blows up (at $j=0$), it becomes obviously pertinent to inquire whether some approximate scheme which is capable of producing symmetric output may avoid the anomaly.

With this end in view, AZ investigated various first-order symmetrized versions of the determinantal approximation. An amplitude, suggested by Bjorken,

$$T_B{}^j(s)=\tfrac{1}{2}[B^{-1}D+D^TB^{-1}]^{-1} \quad (4)$$

seems to make both T_{12} and $T_{21}\sim1/(j)^{1/2}$ and hence is even less acceptable.

More complicated symmetrizations due to Martin and Wali[4] and to Shaw[5] seem to restore unitarity at $j=0$ by obtaining T_{12} and $T_{21}\sim(j)^{1/2}$. However, for these cases, $T_{11}\sim j$, so that the sense-sense amplitude vanishes identically at $j=0$: again, a rather unsatisfactory situation.

Now we would like to show that there indeed exists an approximate solution to the N/D equations suggested recently,[6] which has the same degree of simplicity as the determinantal method, is subtraction-point-independent and symmetric, and also seems to provide a lowest order amplitude which at a physical j neither blows up nor vanishes identically but remains finite.

* Supported in part by U. S. Atomic Energy Commission.
[1] M. Gell-Mann and F. Zachariasen, Phys. Letters **10**, 129 (1964).
[2] E. Abers and F. Zachariasen, Phys. Rev. **136**, B749 (1964).
[3] M. Baker, Ann. Phys. (N. Y.) **4**, 271 (1958).
[4] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).
[5] G. Shaw, Phys. Rev. Letters **12**, 345 (1964).
[6] Y. N. Srivastava and P. Nath (to be published).

The coupled N equations are given by

$$N(s)=B(s)\,\mathrm{Re}D(s)+\frac{1}{\pi}\int_0^\infty B(s')\theta\rho(s')N(s')\frac{ds'}{s'-s}. \quad (5)$$

To the first-order approximation we replace $N(s')$ by $B(s')\,\mathrm{Re}D(s')$ and approximate $\mathrm{Re}D(s')$ by $\mathrm{Re}D(s)$ inside the integral to obtain the scattering amplitude $\tilde{T}(s)$:

$$\tilde{T}(s)=\left[\left(B(s)+\frac{1}{\pi}\int\frac{B(s')\theta\rho(s')B(s')ds'}{s'-s}\right)^{-1}-i\theta\rho(s)\right]^{-1}. \quad (6)$$

Now putting Eq. (1) into Eq. (6), after some algebra, one obtains, to lowest order in g^2:

$$\tilde{T}(s)\approx\frac{1}{[j(1-iA)+B^2-iC(1-iA)]}\begin{pmatrix} Aj-iAC+iB^2, & iA'B(j)^{1/2}+(1-iA)B(j)^{1/2} \\ iA'B(j)^{1/2}+(1-iA)B(j)^{1/2}, & (1-iA)C+iB^2\end{pmatrix}, \quad (7)$$

where A, A', B, and C are functions of s and are $\sim g^2$. Now the continuation to $j=0$ can easily be made:

$$\tilde{T}(s)\sim\begin{pmatrix}-iA-B, & 0 \\ 0, & 1-iA-B'\end{pmatrix}. \quad (8)$$

Hence, we see that the amplitude does indeed decouple and also that the amplitude is, in general, finite at $j=0$.

Thus, the suggested approximation to the N/D equations[6] seems to possess the basic necessary ingredients. It is symmetric, unitary, and subtraction-point-independent, and decouples sense from nonsense in an appropriate manner (thus preserving unitarity). Since the amplitude generated by our approximation has about the same range of validity as any first-order amplitude, is equally simple, and lacks any obvious faults, there seems good reason to expect that it may be of some use in approximate spinological bootstraps.

ACKNOWLEDGMENTS

We would like to thank Professor P. Kaus and Professor F. Zachariasen for an interesting discussion.

PHYSICAL REVIEW VOLUME 138, NUMBER 5B 7 JUNE 1965

Broken Symmetry in Quantum Electrodynamics and Zero-Mass Bosons

R. ACHARYA AND P. NARAYANASWAMY
Tata Institute of Fundamental Research, Bombay, India
(Received 7 December 1964)

The recent attempt of Baker, Johnson and Lee at proving the existence or otherwise of zero-mass bosons in quantum electrodynamics with vanishing bare-electron mass is reexamined. It is argued that the renormalized vertex operator $\Gamma_c{}^5$ exists provided that the electron–pseudoscalar-boson vertex renormalization constant vanishes. No conclusion about the existence or otherwise of zero-mass bosons can be reached within the framework of their high-energy approximation.

1. INTRODUCTION

THE existence or otherwise of zero-mass particles in theories with broken symmetry has been widely discussed in the literature.[1] Essentially, there have been two main approaches to this problem: (a) formal proofs which are based on a study of the matrix elements of products of field operators[2]; (b) detailed investigation of a given theory (or model). Specifically, one studies scattering amplitudes (fermion-antifermion), vertex functions, and propagators and examines the analytic structure of these objects to see whether zero-mass bosons exist. Such is the case in the recent paper of Baker, Johnson, and Lee, who have studied quantum electrodynamics with vanishing bare fermion mass.[3] By virtue of the zero bare-electron mass, the Lagrangian of the theory is invariant under γ_5 transformation. Then, one assumes that the vacuum state is not invariant under γ_5 (thus breaking γ_5 symmetry) and one asks the question whether zero-mass bosons exist in the theory.

Baker *et al.* could not reach any definite conclusion because they argued that within the framework of

[1] J. S. Goldstone, Nuovo Cimento **19**, 154 (1961); J. S. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962); J. C. Taylor, *Proceedings of the 1962 Annual International Conference of High Energy Physics at CERN* (CERN, Geneva, 1962), p. 670; S. A. Bludman and A. Klein, Phys. Rev. **131**, 2364 (1963); Y. Nambu and G. Jona-Lasinio, *ibid.* **122**, 345 (1961); **124**, 246 (1961).

[2] See, for example, A. Klein and B. W. Lee, Phys. Rev. Letters **12**, 266 (1964).

[3] M. Baker, K. Johnson, and B. W. Lee, Phys. Rev. **133**, B209, 1964.