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Decay of Decuplet Baryon Resonances and the $SU(6)$ Symmetry*

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The $SU(6)$ symmetry is applied to derive the sum rules between the decay amplitudes of decuplet baryon resonances, in the presence of symmetry-breaking interactions. We obtain two relations. The first is the same relation as in the complete $SU(3)$ symmetry and is known to be well satisfied experimentally. Experimental fit of the second shows 1.4 standard deviations, while that of the corresponding relation of complete $SU(3)$ symmetry shows 5 standard deviations.

THE $SU(6)$ symmetry, which couples spin and unitary spin as an extension of the supermultiplet theory of Wigner,[1] has been proposed[2] for classifying the hadrons. Mass formulas and electromagnetic properties of some multiplets have been discussed,[3,4] in a way similar to that used in the case of the $SU(3)$ symmetry.[5] In this note, we apply the $SU(6)$ symmetry to the decay of decuplet baryon resonances in the presence of symmetry-breaking interactions.

In the case of the $SU(3)$ symmetry, we have the following relations between the observed decay amplitudes:

$$(N^*\to N\pi)=-\sqrt{2}(Y^*\to\Lambda\pi)=\sqrt{3}(Y^*\to\Sigma\pi)=\sqrt{2}(\Xi^*\to\Xi\pi)\,,\quad(1)$$

for the case of complete symmetry,[6] and

$$(N^*\to N\pi)+\sqrt{2}(\Xi^*\to\Xi\pi)=-(3/\sqrt{2})(Y^*\to\Lambda\pi)+(\sqrt{3}/2)(Y^*\to\Sigma\pi)\,,\quad(2)$$

in the presence of the first-order perturbation of the symmetry breaking interactions.[7,8] Here, $(B^*\to B\pi)$ stands for the decay amplitude of a baryon resonance B^* into the eigenstate of isotopic spin composed of a baryon B and a pion. Being a higher symmetry group, the $SU(6)$ with symmetry-breaking interactions which transform as adjoint representation gives relations intermediary between Eqs. (1) and (2).

We assume that the $SU(6)$ symmetry-breaking interactions are the sum of $SU(2)\times SU(3)$ invariant term and a term coupled with spurions $S_3{}^3$ and $S_3{}'^3$ which form a member of a 35-plet (adjoint) representation,

$$T_{(b,3)}{}^{(a,3)}=\delta_b{}^aS_3{}^3+\boldsymbol{\sigma}_b{}^a\hat{p}S_3{}'^3.\quad(3)$$

Here, $\boldsymbol{\sigma}$ and $\hat{p}$ are the Pauli spin matrices and the unit vector in the direction of the decaying particle, respectively; the $SU(6)$ index which shall be denoted by a Greek letter is $\alpha=(a,A)$, where a and A stand for indices of $SU(2)$ and $SU(3)$ subgroups, respectively, ($a=1,2;A=1,2,3$). Equation (3) can be considered as an extension of the octuplet transformation property[9] of symmetry-breaking interactions in the $SU(3)$ symmetry. Since, however, the first-order perturbation of 35-plet symmetry-breaking interactions cannot split the Λ-Σ mass degeneracy,[3] our approximation is to neglect the quantity $\delta=(\Sigma-\Lambda)/\frac{1}{2}(\Sigma+\Lambda)=0.067$.

Let $\Psi_{\alpha\beta\gamma}$ and $\Phi_\beta{}^\alpha$ be field operators for the 56-plet baryon and the 35-plet meson, respectively.[10] The S matrix for the decuplet baryon decay into octuplet baryon and pion can be written as

$$S=a_1\bar{\Psi}^{\alpha\beta\gamma}(\boldsymbol{\sigma}\hat{p})_\gamma{}^\rho\Psi_{\alpha\rho\nu}\bar{\Phi}_\beta{}^\nu+a_2\bar{\Psi}^{\alpha\beta\gamma}(\boldsymbol{\sigma}\hat{p})_\gamma{}^\rho\Psi_{\alpha\beta\nu}\bar{\Phi}_\rho{}^\nu+a_3\bar{\Psi}^{\alpha\beta\gamma}(\boldsymbol{\sigma}\hat{p})_\nu{}^\rho\Psi_{\alpha\beta\rho}\bar{\Phi}_\gamma{}^\nu+b\bar{\Psi}^{\alpha\beta(c,3)}\boldsymbol{\sigma}_c{}^d\hat{p}\Psi_{\alpha\nu(d,3)}\bar{\Phi}_\beta{}^\nu\,,\quad(4)$$

where

$$(\boldsymbol{\sigma}\hat{p})_\beta{}^\alpha=\boldsymbol{\sigma}_b{}^a\hat{p}\delta_B{}^A.$$

No $SU(6)$ invariant term appears in (4), since it would not conserve parity. The first three terms are $SU(2)\times SU(3)$ invariant and the last is due to the spurion $S_3{}'^3$ of Eq. (3). The term due to the spurion $S_3{}^3$ does not contribute to the considered process, since only p wave is permissible. Incidentally, the second and the third terms of (4) are equal as far as the pseudo-

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10 Explicit forms are (see Ref. 4)

$$\Psi_{\alpha\beta\gamma}=\chi_{abc}D_{ABC}+(1/3\sqrt{2})[\epsilon_{ab}\chi_c\epsilon_{ABD}N_C{}^D+\epsilon_{bc}\chi_a\epsilon_{BCD}N_A{}^D+\epsilon_{ca}\chi_b\epsilon_{CAD}N_B{}^D]$$

and

$$\Phi_\beta{}^\alpha=\delta_b{}^aM_B{}^A+\boldsymbol{\sigma}_b{}^a\mathbf{V}_B{}^A.$$

D_{ABC}, $N_B{}^A$, $M_B{}^A$, and $\mathbf{V}_B{}^A$ are baryon decuplet, baryon octuplet, *ps* meson octuplet, and vector meson nonuplet, respectively. χ_{abc} and χ_a are spins $\frac{3}{2}$ and $\frac{1}{2}$ wave functions, respectively. [$F_{AB}{}^{CD}=\epsilon^{CDE}D_{ABE}$ is a mixed tensor for decuplet baryon given by Okubo (Ref. 6).]

TABLE I. Experimental data on the decay of decuplet baryon resonance into baryon and pion.

Process	Width Γ (MeV)	Branching ratio (%)	$\rho=\dfrac{m_B}{m_B^*}p_f^3$ $[(\text{MeV}/c)^3]$	$\gamma^{1/2}=(\Gamma/\rho)^{1/2}$	$\gamma^{1/2}$ Prediction of complete $SU(3)$
$(N^*\to N\pi)$	125	100	959	0.36	input
$(Y^*\to\Lambda\pi)$	53 ± 2	94 ± 4	695	0.27 ± 0.01	0.25
$(Y^*\to\Sigma\pi)$		6 ± 4[a]	165	0.14 ± 0.05	0.21
$(\Xi^*\to\Xi\pi)$	7.5 ± 1.7	100	280	0.16 ± 0.02	0.25

[a] The corresponding entry in p. 982 of Ref. 12 should be this value instead of the value 9 ± 4. (We thank Dr. A. Barbaro-Galtieri for this information.)

scalar meson components of $\Phi_\beta{}^\alpha$ are concerned. Note also that terms which contain $\Phi_\beta{}^{(c,3)}$, $\Phi_{(d,3)}{}^\alpha$, or $\Phi_{(d,3)}{}^{(c,3)}$ never occur in Eq. (4), since they do not contribute to pionic processes.

Calculation of matrix elements of Eq. (4) gives expression of decay amplitudes in terms of the invariant amplitudes a_i and b;

$$\begin{aligned}(N^{*-}\to n\pi^-)&=a',\\ (Y^{*-}\to\Lambda\pi^-)&=-(1/\sqrt{2})a',\\ (Y^{*-}\to\Sigma^0\pi^-)&=(1/\sqrt{6})a'+(\sqrt{2}/\sqrt{3})b',\\ (\Xi^{*-}\to\Xi^0\pi^-)&=-(1/\sqrt{3})a'-(1/\sqrt{3})b',\end{aligned}\tag{5a}$$

where[10]

$$a'=(1/3\sqrt{2})(a_1-2a_2-2a_3)(\bar{\chi}^a\epsilon^{bd}\boldsymbol{\sigma}_d{}^c\chi_{abc})\hat{p}$$

and

$$b'=(1/3\sqrt{2})b(\bar{\chi}^a\epsilon^{bd}\boldsymbol{\sigma}_d{}^c\chi_{abc})\hat{p}.\tag{5b}$$

Using Eq. (5a) and the relations due to charge independence,[11] we obtain the following two relations:

$$(N^*\to N\pi)=-\sqrt{2}(Y^*\to\Lambda\pi)\tag{6}$$

and

$$2\sqrt{2}(\Xi^*\to\Xi\pi)=(N^*\to N\pi)+\sqrt{3}(Y^*\to\Sigma\pi).\tag{7}$$

We see that Eq. (2) is split into Eqs. (6) and (7) [i.e., $2\times$Eq. (2)$=3\times$Eq. (6)+Eq. (7)], and Eq. (6) is contained in Eq. (1). This is due to the fact that while the first three terms of Eq. (4) correspond to those for complete symmetry of the $SU(3)$ group, the last one which corresponds to the symmetry breaking term $T_3{}^3$ does not contain the matrix elements for the processes $N^*\to N\pi$ and $Y^*\to\Lambda\pi$.

To compare Eqs. (6) and (7) with experiment, we give a list of observed values[12] for the reduced width $\gamma^{1/2}=(\Gamma/\rho)^{1/2}$ in Table I, where Γ is the decay width and $\rho=(m_B/m_B{}^*)p_f{}^3$, p_f being the final-state momentum. From Eq. (6), we have

$$[\gamma(N^*\to N\pi)]^{1/2}=[2\gamma(Y^*\to\Lambda\pi)]^{1/2},\tag{8}$$

while experimental values are 0.36 and 0.38 ± 0.01 for the left- and right-hand sides, respectively, which is rather satisfactory and is a prediction of complete $SU(3)$ symmetry as was noted already.

Equation (7) leads to

$$2[2\gamma(\Xi^*\to\Xi\pi)]^{1/2}=[\gamma(N^*\to N\pi)]^{1/2}+[3\gamma(Y^*\to\Sigma\pi)]^{1/2}.\tag{9}$$

Time-reversal invariance essentially fixes the phases of the three amplitudes to be equal, up to a $\pm$ sign, which we fix in order to obtain best agreement with experiment.[8] Note that we can express the decay amplitudes as linear combinations of baryon-baryon-meson coupling constants with real number coefficients, according to Eq. (4).

From the data of Table I we obtain the value 0.45 ± 0.06 for the left-hand side and the value $0.36+(0.24\pm0.09)=0.60\pm0.09$ for the right-hand side of Eq. (9). We obtain agreement with current experimental data within 1.4 standard deviations. In contrast with this, the experimental fit of the relation

$$-(Y^*\to\Lambda\pi)=(\Xi^*\to\Xi\pi)$$

of complete $SU(3)$ symmetry shows 5 standard deviations.

Finally, we should mention the relation between the amplitudes $(NN\pi)$ and $(N^*N\pi)$. These amplitudes are determined by the $SU(2)\times SU(3)$ invariant terms of Eq. (4), since the last term does not contribute to the amplitudes containing nucleons. In fact, we obtain

$$(n\to p\pi^-)=[1/(3\sqrt{2})^2](-2a_1+10a_2+10a_3)\times(\bar{\chi}^a\boldsymbol{\sigma}_a{}^b\chi_b)\hat{p}.\tag{10}$$

Comparing Eq. (10) with Eq. (5), we find that no relation can be deduced in this case between the amplitudes $(NN\pi)$ and $(N^*N\pi)$, contrary to the case of Ref. 13.

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[11] $(N^*\to N\pi)=(N^{*-}\to n\pi^-)$, $(Y^*\to\Lambda\pi)=(Y^{*-}\to\Lambda\pi^-)$, $(Y^*\to\Sigma\pi)=\sqrt{2}(Y^{*-}\to\Sigma^0\pi^-)$ and $(\Xi^*\to\Xi\pi)=-(\sqrt{\tfrac{3}{2}})(\Xi^{*-}\to\Xi^0\pi^-)$.
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