Study of the Absorption Model in Pion-Nucleon Charge-Exchange Scattering*

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The predictions of a peripheral ρ-exchange model with absorption are studied for the $\pi^- p$ charge-exchange reaction. Comparisons with experiment of predicted differential cross sections are made at laboratory momenta of 5.9 and 10.0 BeV/c. It is found that absorption parameters determined from pion-nucleon elastic-scattering data fail to provide a fit to the angular distribution, absolute magnitude, or energy dependence of the observed cross sections. Even the assumption of essentially complete absorption of low partial waves still fails to fit absolute magnitudes and energy dependence, but it can give an acceptable angular dependence if the anomalous magnetic coupling of the ρ is neglected. It is concluded that the simple absorption model is not an adequate representation of the experimental situation. Implications and possible modifications of the theory are discussed.

I. INTRODUCTION

The salient features of accumulating experimental data on a considerable number of inelastic reactions show (i) differential cross sections that are sharply peaked toward the forward direction; and (ii) total reaction cross sections that decrease at high energies. Although the Born terms for peripheral meson exchanges fail to reproduce observed production angular distributions like (i), the remarkable success of the use of the vector-exchange model of Stodolsky and Sakurai[1] in predicting other correlations in isobar-production reactions indicates that peripheral exchanges should form the backbone of a complete theoretical description of inelastic processes.[2] When momentum-transfer-dependent form factors are introduced into the Born model to reproduce the qualitative features of (i), then the theoretical predictions for (ii) are in strong disagreement with the data.[3] Consequently, a slightly more sophisticated theoretical approach seems to be necessary. In this connection both Regge[4] and absorption models[5–13] are immediate candidates. Several authors have suggested that the absorption model could account for (i), and they have inferred that the magnitude of the inelastic cross section would be correctly predicted, i.e., (ii). However, the actual application of the absorption model thus far has been limited to processes for which the final-state interactions were not completely determined by experiment. Secondly, a careful comparison of the predictions of the theory for a given process at two different energies has not been made to test the correspondence with (ii). Thirdly, in the processes thus far examined, more than one peripheral meson exchange was allowed by the selection rules, making a complete analysis formidable. For example, in the process $\pi^\pm p \to \rho^\pm p$ which has received considerable attention,[5–7] both π^0 and ω^0 exchanges are allowed. For these reasons it is desirable that the absorption model be examined for an inelastic reaction in which (a) all final-state interactions are directly obtained from experiment; (b) data on the reaction at different energies are available; and (c) only a single peripheral exchange is "allowed." Only with such a test can one be reasonably certain that *ad hoc* adjustment of arbitrary parameters does not produce a fortuitous agreement with the experimental results.

In this connection we examine the predictions of the absorption model for the charge-exchange scattering $\pi^- + p \to \pi^0 + n$ and compare the results with the recent data at 5.9 BeV/c and 10 BeV/c.[14] For this reaction the ρ meson is the only known $I=1$ resonance with an

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[1] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963); L. Stodolsky, Phys. Rev. **134**, B1099 (1964).

[2] A general qualitative feature indicative of the peripheral nature of many inelastic reactions is a characteristic forward peaking in reactions for which only meson exchanges are expected and backward peaking in reactions that baryon exchanges may dominate.

[3] M. A. Abolins, D. D. Carmony, Duong-N-Hoa, R. L. Lander, C. Rindfleisch, and N. Xuong, University of California at San Diego (unpublished). Some other results of the form factor treatment are also unreasonable; see, for example, V. Barger and E. McCliment, Phys. Letters **9**, 191 (1964).

[4] K. Gottfried and J. D. Jackson, Phys. Letters **8**, 144 (1964); Nuovo Cimento **33**, 309 (1964); M. M. Islam, *ibid.* **30**, 579 (1963); M. M. Islam and R. Piñon, *ibid.* **30**, 837 (1963). A straightforward experimental test of the Regge exchange mechanism (unmodified by absorption) for inelastic reactions is given in the letter: V. Barger, Nuovo Cimento **35**, 700 (1965).

[5] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

[6] L. Durand and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964); **13**, 45 (1964); in Proceedings of the 1964 Boulder Conference on Particles and High Energy Physics (to be published).

[7] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964).

[8] N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962).

[9] M. Baker and R. Blankenbecler, Phys. Rev. **128**, 415 (1962).

[10] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin, B. W. Downs, and J. Downs (Interscience Publishers, Inc., New York, 1959), Vol. I.

[11] A. Dar, Phys. Rev. Letters **13**, 91 (1964); A. Dar, M. Kugler, Y. Dothan, and S. Nussinov, *ibid.* **12**, 82 (1964); A. Dar and W. Tobocman, *ibid.* **12**, 511 (1964).

[12] G. A. Ringland and R. J. N. Phillips, Phys. Letters **12**, 62 (1964).

[13] I. Derado, V. P. Kenney, and W. D. Shephard, Phys. Rev. Letters **13**, 505 (1964).

[14] S. J. Lindenbaum, in Proceedings of the 1964 International Conference on High Energy Physics at Dubna (to be published). (Charge exchange scattering data used are those of Falk-Vairant *et al.*)

appreciable $\pi\pi$ coupling to mediate the exchange. The final-state interactions are fairly well-known in this reaction from π^-p and π^+p elastic-scattering data. We follow the usual methods of applying the distorted-wave Born approximation (DWBA) to this reaction.

II. STATEMENT OF MODEL

For the extension of the distorted-wave Born approximation methods to the domain of high-energy inelastic reactions, the basic underlying assumptions that appear to be inherent in the practical formulation of the model are: (i) the inelastic channel in question has a cross section which is very small compared to the total inelastic cross section; (ii) the range of the peripheral Born exchange is much smaller than the range of the forces in the entrance and exit channels; (iii) the final-state interactions are, in principal at least, determined by elastic scattering; (iv) only the helicity-nonflip elastic amplitudes are essential for determining the small-angle absorption; and (v) the elastic amplitude is adequately represented by a pure imaginary Gaussian distribution in momentum transfer. The DWBA result, which is derived on the basis of assumptions (i) through (iv), is

$$T^J=[S_{ff}^J]^{1/2}B^J[S_{ii}^J]^{1/2}, \qquad (1)$$

where S_{ii}^J and S_{ff}^J are the helicity nonflip elastic-scattering matrix elements for the initial and final states, respectively. Helicity indices on the transition amplitude (T) and the Born amplitude (B) have been suppressed. Assumptions (iv) and (v) are made primarily because of pragmatic considerations. Thus, the helicity-changing elastic S-matrix elements, which are presumably quite small, could be included in an exact treatment. Similarly, the assumption (v) of a pure imaginary elastic amplitude with a particular angular distribution is a useful but nevertheless inessential feature of the absorption model. In later sections we consider the relaxation of assumption (iii) as a procedure to "unitarize" the model. It has been further suggested that assumption (ii) is not a limitation of the model provided that the transition is sufficiently weak.[6,15]

For the π^-p charge exchange process both the magnitude of the cross section and the range of the Born ρ-meson exchange interaction are consistent with the standard assumptions (i) and (ii) above. The appropriate absorption factors are determined from the experimental data on π^-p and π^+p elastic scattering. Since the real part of the elastic amplitude contributes $\lesssim\frac{1}{16}$ as much as the imaginary part to the differential cross section,[14] it is reasonable to treat the elastic amplitudes as pure imaginary. This approximation is further justified by the fact that the absolute value of Eq. (1) contains no interference term between real and imaginary parts of the elastic amplitude. The experimental angular distributions of elastic scattering are well fitted for small Δ^2 by a Gaussian form:

$$f_{\rm el}{}^{(\pm)}(\Delta,q)=i(\sigma_t{}^{(\pm)}q/4\pi)\exp(-A^{(\pm)}\Delta^2/2), \qquad (2)$$

where the $(\pm)$ superscripts refer to π^+p and π^-p elastic scattering, respectively. Using charge independence, the π^0n elastic amplitude is given by:

$$f_{\rm el}{}^{(0)}(\Delta,q)=\tfrac{1}{2}[f_{\rm el}{}^{(+)}(\Delta,q)+f_{\rm el}{}^{(-)}(\Delta,q)]. \qquad (3)$$

As previously discussed, the amplitude in Eq. (2) is the helicity nonflip amplitude. The usual angular-momentum decomposition of $f_{\rm el}(\Delta,q)$ is converted to an approximate integral form for $\Delta^2\ll q^2$:

$$f_{\rm el}(\Delta,q)=-\frac{i}{q}\int_0^\infty dx\, xJ_0(x\Delta/q)(e^{i\chi(x)}-1), \qquad (4)$$

where the identification $J+\frac{1}{2}\rightarrow x$ has been made. Taking the Fourier-Bessel transform and substituting Eq. (2), we find

$$e^{i\chi(x)}=1-Ce^{-\gamma x^2}, \qquad (5)$$

where

$$C=\sigma_t/4\pi A, \quad \gamma=(2Aq^2)^{-1}. \qquad (6)$$

For momenta in the range 6 to 10 BeV/c and for low momentum transfer, the A's of both $\pi^\pm p$ diffraction peaks have an approximately constant value of 7.8 (BeV/c)$^{-2}$.[16] The values of the total cross section can be found from the empirical fits[17]

$$\begin{aligned}\sigma_t(\pi^+p)&=22.26+25.10/P_{\rm lab}\ {\rm mb},\\ \sigma_t(\pi^-p)&=24.37+24.94/P_{\rm lab}\ {\rm mb},\end{aligned} \qquad (7)$$

where the units of $P_{\rm lab}$ are BeV/c. From this experimental information the absorption parameters are completely determined. We obtain the following results:

at $P_{\rm lab}=5.9$ BeV/c,

$$C^{(+)}=0.677, \quad C^{(-)}=0.730, \quad C^{(0)}=0.704,$$
$$\gamma^{(+)}=\gamma^{(-)}=\gamma^{(0)}=0.0250;$$

at $P_{\rm lab}=10.0$ BeV/c,

$$C^{(+)}=0.633, \quad C^{(-)}=0.686, \quad C^{(0)}=0.659,$$
$$\gamma^{(+)}=\gamma^{(-)}=\gamma^{(0)}=0.0143.$$

The resultant absorption factor for the charge-exchange process

$$\exp\{i[\chi^{(-)}(x)+\chi^{(0)}(x)]/2\}$$

is also quite adequately represented by the form

$$1-Ce^{-\gamma x^2},$$

[15] R. Omnes, Phys. Rev. **137**, B649 (1965).

[16] S. Brandt *et al.*, Phys. Rev. Letters **10**, 413 (1963); M. L. Perl, L. W. Jones, and C. C. Ting, Phys. Rev. **132**, 1252 (1963); K. J. Foley *et al.*, Phys. Rev. Letters **10**, 376 (1963); **10**, 543 (1963).
[17] S. J. Lindenbaum *et al.*, Phys. Rev. Letters **7**, 352 (1961).

where the parameters are

$$C=0.717, \quad \gamma=0.0250$$

for $P_{\rm lab}=5.9$ BeV/c and

$$C=0.673, \quad \gamma=0.0143$$

for $P_{\rm lab}=10.0$ BeV/c.

III. CALCULATION

In terms of the familiar variables

$$\begin{aligned} s&=-(q_1+p_1)^2=M^2+m^2+2M\omega_{\rm lab},\\ t&=-(q_1-q_2)^2=-\Delta^2=-2q^2(1-\cos\theta),\\ Q_\mu&=\tfrac{1}{2}(q_1+q_2)_\mu, \end{aligned} \tag{8}$$

the standard decomposition of the meson-nucleon transition matrix into invariant amplitudes is[18]

$$T(s,t)=-A(s,t)+iQ\cdot\gamma B(s,t), \tag{9}$$

where the T matrix bears the following relationship to the scattering matrix

$$S_{fi}=\delta_{fi}-(2\pi)^4 i\delta^{(4)}(p_2+q_2-p_1-q_1)\times\left[\frac{M^2}{4E_1E_2\omega_1\omega_2}\right]^{1/2}\bar{u}(p_2)T(s,t)u(p_1). \tag{10}$$

The Jacob-Wick helicity amplitudes,[19] which are convenient for this analysis, may be expressed in terms of the amplitudes A and B by

$$\begin{aligned} f_{++}(s,\theta)&=\cos(\tfrac{1}{2}\theta)[2MA(s,t)+(s-M^2-m^2)B(s,t)]/(8\pi\sqrt{s}),\\ f_{+-}(s,\theta)&=\sin(\tfrac{1}{2}\theta)[(s+M^2-m^2)A(s,t)+(s-M^2+m^2)MB(s,t)]/(8\pi s). \end{aligned} \tag{11}$$

Then the unpolarized differential cross section is

$$d\sigma/d\Delta^2=(\pi/q^2)[|f_{++}(s,\theta)|^2+|f_{+-}(s,\theta)|^2], \tag{12}$$

where

$$q^2=[s-(M+m)^2][s-(M-m)^2]/4s \tag{13}$$

is the center-of-mass momentum.

The ρ-meson pole contributions to the $\pi^-p\to\pi^0n$ helicity amplitudes are

$$f_{++}{}^B(s,\theta)=G(s)(\omega^2+\epsilon^2)^{-1}(1-\tfrac{1}{4}\omega^2)^{1/2}\times[s-M^2-m^2+\tfrac{1}{2}q^2\omega^2\mu_{\rho NN}]$$

and

$$f_{+-}{}^B(s,\theta)=G(s)Ms^{-1/2}\omega(\omega^2+\epsilon^2)^{-1}\{\tfrac{1}{2}(s-M^2+m^2)-(q^2/4M^2)\mu_{\rho NN}[4s-\tfrac{1}{2}\omega^2(s+M^2-m^2)]\}, \tag{14}$$

where

$$\begin{aligned} \omega&=2\sin(\tfrac{1}{2}\theta),\\ \epsilon^2&=m_\rho{}^2/q^2,\\ G(s)&=f_{\rho\pi\pi}g_{\rho NN}/[4\pi q^2(2s)^{1/2}]. \end{aligned} \tag{15}$$

The dimensionless coupling constants are defined through the effective interaction Lagrangian

$$\mathfrak{L}_I=-ig_{\rho NN}\bar{N}\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}N\boldsymbol{\varrho}_\mu+g_{\rho NN}(\mu_{\rho NN}/2M)\bar{N}\sigma_{\mu\nu}\tfrac{1}{2}\boldsymbol{\tau}N\partial_\nu\boldsymbol{\varrho}_\mu-f_{\rho\pi\pi}\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\varrho}_\mu, \tag{16}$$

with $\sigma_{\mu\nu}=-\tfrac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$. According to this definition, universal coupling of the ρ meson to the conserved isovector current[20] gives $f_{\rho\pi\pi}=g_{\rho NN}$. We take the ρ-meson width to be $\simeq$100 MeV which gives $f_{\rho\pi\pi}{}^2/4\pi=2$. Moreover, assumed dominance of the isovector nucleon form factor by the ρ-meson contribution[21] yields $\mu_{\rho NN}=3.7$.

The angular-momentum expansion of the helicity amplitudes

$$f_{\mu\lambda}(s,\theta)=(1/p)\textstyle\sum_J(J+\tfrac{1}{2})f_{\mu\lambda}{}^J(s)d_{\lambda\mu}{}^J(\theta) \tag{17}$$

may be converted in the small-angle approximation to the form

$$f_{\mu\lambda}(s,\theta)\simeq(1/p)\int_0^\infty dx\,xf_{\mu\lambda}(x,s)J_n(\omega x) \tag{18}$$

with the aid of the formula

$$d_{\lambda\mu}{}^J(\theta)\simeq J_n(\omega[J+\tfrac{1}{2}]),\quad n=\mu-\lambda \tag{19}$$

valid for $\omega^2\ll1$. When we incorporate the fundamental assumption of the absorption model

$$f_{\mu\lambda}{}^A(x,s)=(1-Ce^{-\gamma x^2})f_{\mu\lambda}{}^B(x,s), \tag{20}$$

as discussed in Sec. II above, the modified helicity amplitudes for $\omega^2\ll1$ are easily determined by Eqs. (18) and (20) and the useful identity

$$\omega^\nu/(\omega^2+\epsilon^2)=\epsilon^\nu\int_0^\infty dx\,xJ_\nu(\omega x)K_\nu(\epsilon x). \tag{21}$$

The result of this straightforward calculation is

$$f_{++}(s,\theta)=G(s)\{[(\omega^2+\epsilon^2)^{-1}-CI_0(\epsilon,\omega)]\times(s-M^2-m^2-\tfrac{1}{2}q^2\epsilon^2\mu_{\rho NN})+\tfrac{1}{2}q^2(1-C)\mu_{\rho NN}\}, \tag{22}$$

$$f_{+-}(s,\theta)=\tfrac{1}{2}\omega G(s)Ms^{-1/2}\{[(\omega^2+\epsilon^2)^{-1}-CI_1(\epsilon,\omega)]\times[s-M^2+m^2-(q^2/4M^2)\mu_{\rho NN}(2s+\tfrac{1}{4}\epsilon^2(s+M^2-m^2))]+(q^2/4M^2)(1-C)\mu_{\rho NN}(s+M^2-m^2)\}, \tag{23}$$

where

$$I_\nu(\epsilon,\omega)\equiv(\epsilon/\omega)^\nu\int_0^\infty dx\,xJ_\nu(\omega x)K_\nu(\epsilon x)e^{-\gamma x^2}. \tag{24}$$

Introducing an integral representation for K_ν (the one given by Laplace transform) into the expression for I_ν

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[18] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

[19] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

[20] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); in Proceedings of the International School of Physics at Varenna, Italy, 1963, p. 50 (unpublished).

[21] This is only an approximate estimate of the magnetic coupling since the ρ meson alone does not seem to explain entirely the structure of the isovector nucleon form factor; see, for example, V. Barger and R. Carhart, Phys. Rev. **136**, B281 (1964).

and performing the integration over x, we find

$$I_\nu(\epsilon,\omega)=\frac{1}{4}\int_0^\infty dy(1+\gamma y)^{-\nu-1} \times\exp\{-\tfrac{1}{4}y[\epsilon^2+\omega^2/(1+\gamma y)]\}. \quad (25)$$

Since $\gamma=(2Aq^2)^{-1}$ (and A is not dependent on energy), it is possible to make a change of variable such that the integrand is independent of energy at fixed Δ^2:

$$I_\nu(\epsilon,\Delta^2)=\tfrac{1}{4}(m_\rho{}^2/\epsilon^2)\int_0^\infty dz(1+z/2A)^{-\nu-1} \times\exp\{-\tfrac{1}{4}z[m_\rho{}^2+\Delta^2(1+z/2A)^{-1}]\}. \quad (26)$$

This integration is performed numerically for each value of Δ^2. In the forward direction, however, the result can be expressed in closed form:

$$I_\nu(\epsilon,0)=(\beta/\epsilon^2)e^\beta[-\mathrm{Ei}(-\beta)], \quad (27)$$

where $\beta=\frac{1}{2}Am_\rho{}^2$ and Ei is the exponential integral. It is interesting to note that for very small Δ^2, $I_\nu(\epsilon,0)$ has the same energy dependence as the Born term. Consequently the energy dependence of the $d\sigma/d\Delta^2$ at $\Delta^2=0$ will be the same for both Born and absorption models. As we shall see in a later comparison with the experimental data, this feature of the absorption model proves to be a grave difficulty.

The helicity flip amplitude f_{+-} vanishes as ω for forward angles [see Eq. (23)]. With zero magnetic coupling $\mu_{\rho NN}=0$ only the helicity nonflip amplitude f_{++} makes an appreciable contribution to the differential cross section for the high energies and small momentum transfers of interest. Consequently, for the case $\mu_{\rho NN}=0$ we note that the apparent "structure"[22] which the experimental results show for $\Delta^2<0.1$ (see Fig. 1) cannot be explained in terms of the increasing helicity flip amplitude. On the other hand, if $\mu_{\rho NN}$ is different from zero, the helicity flip amplitude is no longer negligible. This reduces the falloff of the differential cross section with increasing Δ^2, and, as will be seen, makes the disagreement between experiment and model more severe.[23] The magnetic coupling (and hence the helicity flip amplitude) may be relevant to the explanation of the experimental results at small Δ^2. In this connection it must be noted that the absorption constitutes a real modification of the real Born amplitude, and thus does not provide the imaginary part of the amplitude required by the optical theorem.

In the DWBA treatment of the inelastic reactions, there is no provision for characteristics of the production amplitude to be reflected back into the elastic scattering channel to make a self-consistent unitarity calculation.[24] However, a weak form of the unitarity restriction on the amplitude for a *single* inelastic channel follows from the requirement that the single-channel partial-wave cross section be less than the *total* inelastic partial-wave cross section as determined from elastic scattering. For the charge exchange process, this unitarity limitation becomes

$$q^2[|f_{++}(s,x)|^2+|f_{+-}(s,x)|^2]\leq U, \quad (28)$$

where

$$U=1-|e^{i\chi(x)}|^2 =2Ce^{-\gamma x^2}-C^2e^{-2\gamma x^2}. \quad (29)$$

The explicit form of this restriction (with $\mu_{\rho NN}=0$ and neglecting the helicity flip amplitude) is

$$qG(s)K_0(\epsilon x)(1-Ce^{-\gamma x^2})(s-M^2-m^2)\leq\sqrt{U}. \quad (30)$$

The absorption model as formulated does not guarantee even that this weak form of unitarity will be satisfied when the experimental values of C and γ are used [in this connection note that $K_0(\epsilon x)\to\ln x$ as $x\to 0$]. However, if the connection between C and the elastic scattering parameters is relaxed, then the absorption model can be theoretically unitarized by the prescription $C\equiv 1$.

IV. DISCUSSION

The results of the calculation of the differential charge-exchange cross section for π^-p scattering $d\sigma/d\Delta^2$, are plotted as a function of the invariant momentum transfer Δ^2 in Fig. 1, both for $\mu_{\rho NN}=0$ for $\mu_{\rho NN}=3.7$. Also displayed is the cross section calculated with the unmodified Born amplitude, and the experimental observations. As was discussed previously, the two parameters of the absorption, the degree of absorption of the lowest partial wave C and the width of the diffraction peak A, have been obtained solely from experimental data on π^-p and π^0n elastic scattering. The values so derived are $A=7.8$ (BeV/c)$^{-2}$ and $C=0.717$ at 5.9 BeV/c and $C=0.673$ at 10 BeV/c. We note that the range of Δ^2 plotted corresponds to a range of momentum transfer where the model is expected to be valid. Thus, for example, $\Delta^2=0.4$ corresponds to $\omega^2=0.156$ at 5.9 BeV/c and $\omega^2=0.089$ at 10 BeV/c, both satisfying $\omega^2\ll 1$.

It is apparent that the absorption model in this form fails to fit the observed angular dependence and magnitude of the cross section at either energy, and that the energy dependence of the cross section does not reproduce the measured decrease with energy. We discuss

[22] The pion-nucleon charge-exchange amplitude is $1/\sqrt{2}$ times the difference of the π^+p and π^-p elastic amplitudes. Therefore the imaginary part of the forward charge-exchange amplitude is determined by the total $\pi^\pm p$ cross sections. The $\Delta^2=0$ experimental point in Fig. 1 is determined in this manner by neglecting the real part of the amplitude. G. Höhler, G. Ebel, and J. Giesecke (preprint Karlsruhe, 1964) have calculated the real part of the amplitude from a dispersion relation and still find that the forward charge-exchange amplitude is considerably below the data points for $\Delta^2<0.1$. This suggests a possible structure at small Δ^2. [cf. the discussion in F. Bruyant *et al.*, Phys. Letters **12**, 278 (1964).]

[23] The authors are indebted to J. D. Jackson for calling the preceding point to their attention.

[24] R. C. Arnold, Phys. Rev. **136**, B1388 (1964).

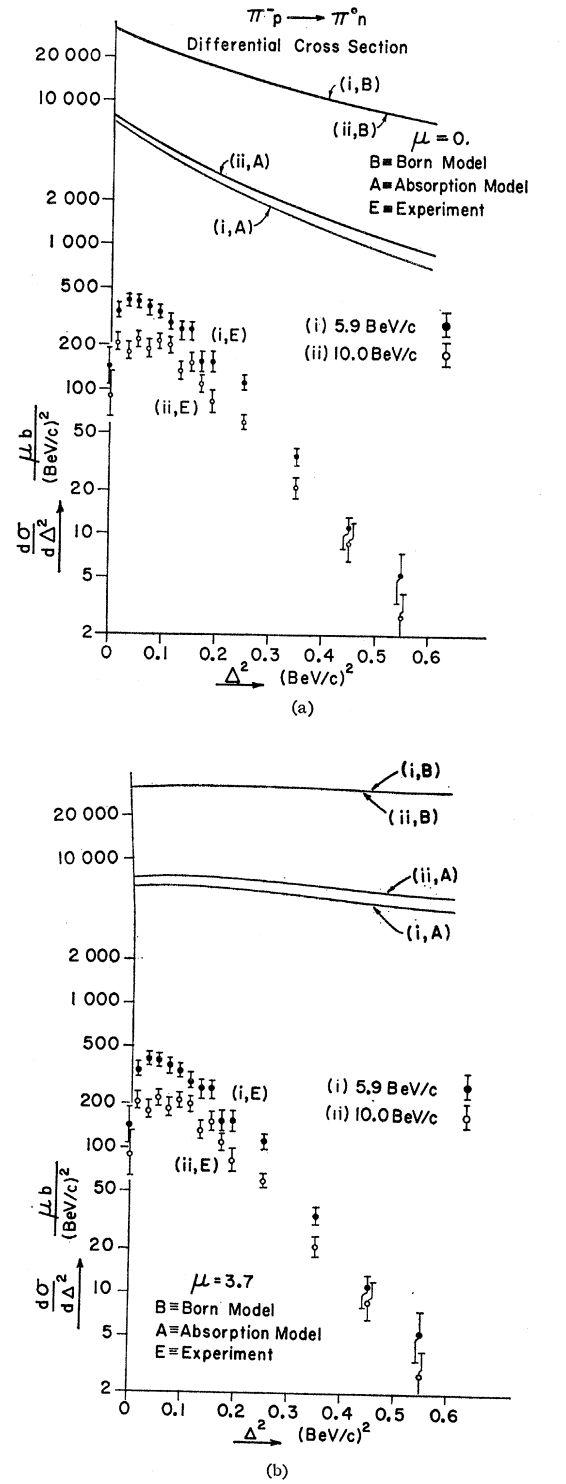

FIG. 1. Differential π^-p charge-exchange cross sections at laboratory momenta of 5.9 and 10.0 BeV/c. For comparison the experimental data of Ref. 14 and the Born-model cross section are illustrated, as well as that of the absorption model. The Born cross sections at the two energies differ by only about 3% and hence do not appear separated on this scale. (a) for $\mu_{\rho NN}=0$; (b) for $\mu_{\rho NN}=3.7$.

TABLE I. Comparison of experiment and predictions of various models (with $\mu_{\rho NN}=0$) for magnitude of differential cross section at $\Delta^2=0.05$.

$\pi^-p\to\pi^0n$	$(d\sigma/d\Delta^2)\,(\Delta^2=0.05)$ $[\mu b/(\text{BeV}/c)^2]$			
Beam momentum	Experiment	Born model	Absorption model	Unitarized absorption model ($C=1$)
5.9 BeV/c	400	26 260	5290	1390
10.0 BeV/c	220	25 840	6010	1370

in detail the case $\mu_{\rho NN}=0$. The selection of $\mu_{\rho NN}=3.7$ makes worse the already unsatisfactory comparison with experiment. In Table I the magnitude of the cross section produced by the absorption model described here and also by the Born amplitude is compared with experiment at $\Delta^2=0.05$ for both incident energies. The point 0.05 has been chosen to lie well within the assumed range of validity of the model, and yet away from the turnover of the experimental points in the forward direction. It is seen that the inclusion of absorption reduces the Born result by a factor of about 4–5, but that the cross section is still too large by more than an order of magnitude. Also indicated in the table are the values which would be obtained if C were set equal to unity, corresponding to essentially complete absorption of the lowest partial waves, but with the width of the πp elastic diffraction peak held at its measured value. This is referred to as the "unitarized" absorption model in the tables. Various authors have made this choice of C for the processes considered previously. One reason advanced for this assumption is that the final-state absorption has not been known in these processes, in contrast to the situation for pion-nucleon charge exchange scattering. A second justification has been that the model is not believed to be valid for low partial waves, which should be strongly absorbed. This absorption has then been put arbitrarily equal to its maximum. The result of this *ad hoc* assumption is to enhance the absorption, but not sufficiently to bring $d\sigma/d\Delta^2$ close to the experimental value. A calculation with $C=1$ indicates that the width of the elastic diffraction peak would have to be increased by a factor of ~ 3 at 5.9 BeV/c and ~ 5 at 10.0 BeV/c in order to get sufficient absorption to bring the magnitude of the forward differential cross section in agreement with experiments. We, of course, could not justify this value of C on the first of the grounds listed above, since in our case the absorption in both the initial and final state is known. Moreover, since our partial-wave amplitude satisfies the admittedly weak unitarity criterion for essentially all partial waves (cf. Figs. 2 and 3), we have not chosen to increase C to its maximum value except as a test of the sensitivity of the model to this parameter.

Table II provides a comparison of the observed and predicted angular distributions for the various models.

TABLE II. Comparison of experiment and prediction of various models (with $\mu_{\rho NN}=0$) for the angular dependence of the cross section.

$\pi^- p \to \pi^0 n$	$\left[\frac{d\sigma}{d\Delta^2}(\Delta^2=0.1) \Big/ \frac{d\sigma}{d\Delta^2}(\Delta^2=0.35)\right]$			
Beam momentum	Experiment	Born model	Absorption model	Unitarized absorption model ($C=1$)
5.9 BeV/c	9.1	1.9	2.8	8.5
10.0 BeV/c	9.8	1.9	2.7	8.4

Here the effect of absorption is to increase the falloff of the cross section at higher momentum transfers, although again not sufficiently to explain the observations. In this case, however, taking $C=1$ does bring agreement of the angular distribution with experiment. This may be understood since the higher momentum transfers appear to be dominated by the low partial waves. The sensitivity to the value of C follows from the fact that the Born amplitude for these waves is large. There is thus a considerable difference between absorbing them out completely and merely reducing them by an appreciable factor.

Table III presents the energy dependence of the cross section at a typical momentum transfer. The Born cross section is essentially energy independent in this range, and the expected slight decrease in absorption with energy causes the resulting cross section to rise slightly with energy, in contradiction to the appreciable falloff observed. The selection of $C=1$ removes the decrease in absorption with energy and duplicates the Born energy dependence.

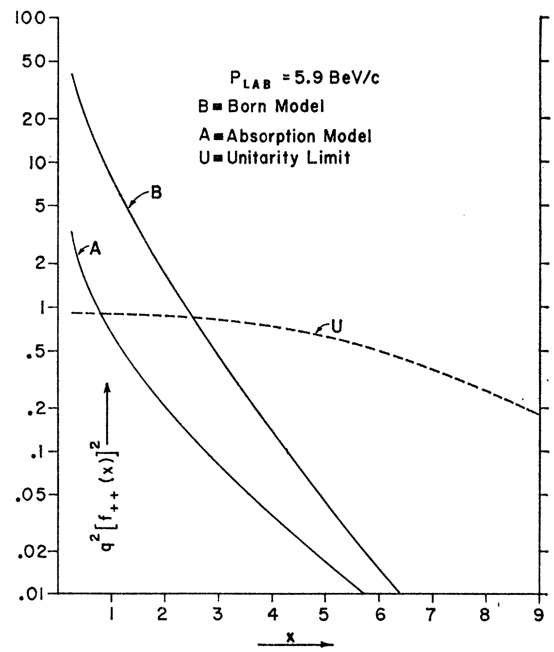

FIG. 2. Partial-wave Born and absorption amplitudes (with $\mu_{\rho NN}=0$) at 5.9 BeV/c as a function of $x=J+\frac{1}{2}$. The amplitudes are normalized so that the unitarity limit (dashed curve) at $x=0$ is $C(2-C)$.

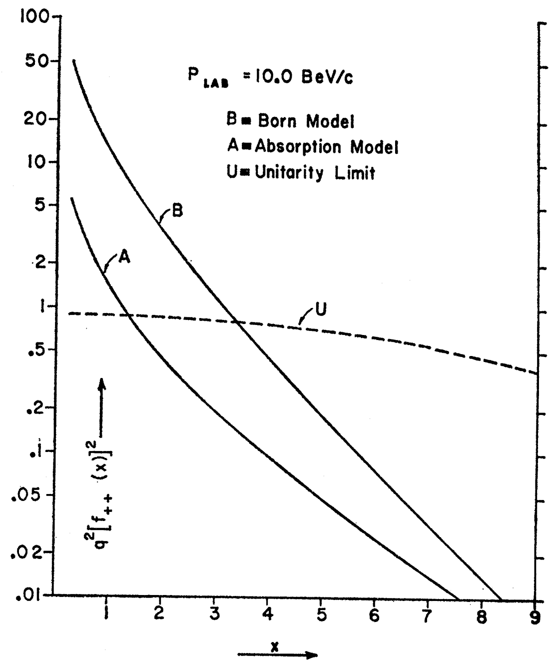

FIG. 3. Partial-wave Born and absorption amplitudes (with $\mu_{\rho NN}=0$) at 10.0 BeV/c as a function of $x=J+\frac{1}{2}$. The amplitudes are normalized so that the unitarity limit (dashed curve) at $x=0$ is $C(2-C)$.

In summary, the absorption model with parameters determined by experiment fails to explain the data in any quantitative sense. Increasing the absorption by blocking out the low partial waves does give a fit to the angular distribution but still fails to explain the magnitude and energy dependence of the cross section. This increased absorption brings the individual partial-wave amplitudes far below the unitarity limit (about 2.5% of the unitarity limit at $x=3$ or 4, where the partial-wave amplitude is a maximum). Accordingly, the absorption is too strong to be justified in any quantitative sense as an application of the requirement of unitarity, but must be regarded as essentially an upper limit to the absorption which can reasonably be postulated.

Several conclusions seem to present themselves at

TABLE III. Comparison of experiment and predictions of various models (with $\mu_{\rho NN}=0$) for the energy dependence of the cross section at a fixed momentum transfer.

$\left[\frac{d\sigma}{d\Delta^2}(P_{\rm lab}=5.9\text{ BeV}/c) \Big/ \frac{d\sigma}{d\Delta^2}(P_{\rm lab}=10.0\text{ BeV}/c)\right]$, $\Delta^2=0.25$			
Experiment	Born model	Absorption model	Unitarized absorption model ($C=1$)
1.92	1.01	0.84	1.01

this stage. The first of these is that an absorption model which is based upon modifications of the Born term does not provide an adequate explanation of the data on $\pi^- p$ charge exchange reaction, at least in the energy range considered. The above considerations strongly suggest that the fault lies in starting with the Born form itself. It obviously has the wrong energy dependence, probably does not fall off sufficiently with increasing momentum transfer, and is certainly too large in absolute magnitude. Two possibilities present themselves. The first is to introduce a form factor at either the $\rho\pi\pi$ or ρNN vertices, or both. This (together with absorption) could give a cross section of the proper magnitude, but it is extremely difficult to understand how it could help with the energy dependence. One of the striking conclusions from the analysis above is that the energy dependence of the differential cross section at fixed Δ^2 is wrong. This problem would seem to remain even with the introduction of a form factor.

A second, and more promising possibility, is to start with a Reggeized amplitude instead of the Born amplitude, and then to introduce absorption. The Regge amplitude provides an imaginary contribution at $\Delta^2=0$. One might expect that such a model would modify the energy dependence and angular distribution, as well as the absolute magnitude of the cross section in the proper direction. Only a detailed calculation can indicate whether such modification would be mutually consistent with the data. The principal objection to this procedure seems to be that it may introduce too much arbitrariness into the model, and thus the significance of a fit to the data might be considerably reduced. It remains an interesting possibility, however.

The second major conclusion that is indicated by our analysis is that the results seem to be quite sensitive to the value chosen for the parameter C, the degree of absorption of the lowest partial wave. This is in contrast to the result obtained by Gottfried and Jackson[5] who found a change in C of a similar amount to introduce only moderate changes in the $\pi^- p \to \rho^- p$ cross section. The difference here is presumably connected with the fact that their process involved the exchange of a π meson, spinless, and the present process the exchange of the spin-one ρ meson. (The expected Regge parameters of the pion trajectory are such that one might not expect an appreciable difference of the Regge amplitude from the Born amplitude for small Δ^2.) This sensitivity to the value of C in turn places a much greater burden upon the model in the case of the exchange of particles with spin. Quite apart from the question of the suitability of the Born term as a starting point, what is obviously required is some more careful treatment of the low partial waves, where the absorption model is admittedly the weakest. This treatment is the more crucial the more the amplitude is dominated by low partial waves. It may be reasonable to expect that such a treatment would suppress the low partial waves more strongly than has been done here, on the basis of the DWBA with experimental elastic parameters, but it would be much more satisfactory to see this suppression appearing as the result of a calculation rather than as a reasonable assumption.

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