Determination of the Coherent Neutron Scattering Amplitudes of Boron, Nitrogen, and Oxygen by Mirror Reflection*

R. E. DONALDSON, L. PASSELL,† W. BARTOLINI, AND D. GROVES
Lawrence Radiation Laboratory, University of California, Livermore, California
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The coherent neutron scattering amplitudes of boron, nitrogen, and oxygen have been measured by the mirror-reflection method. The measured values are $a_B=(5.40\pm0.04)\times10^{-13}$ cm, $a_N=(9.19\pm0.11)\times10^{-13}$ cm, and $a_O=(5.80\pm0.05)\times10^{-13}$ cm. The nitrogen and oxygen values are in agreement with previous measurements. The boron measurement is discussed with regard to the amplitudes of the separate isotopes, and evidence for a resonance in the B^{10} scattering cross section is given.

INTRODUCTION

UNDER certain circumstances neutron reflection from mirror surfaces has definite advantages over the diffraction method of measuring nuclear coherent scattering amplitudes. Reflection from mirrors, for example, is usually a better method when absorption or incoherent scattering obscures the normal Bragg reflections.[1] Also, it is sometimes preferred when only limited amounts of material are available. Thus, the mirror method was selected for the most recent measurements of the neutron-proton scattering amplitude[2,3] and it has also been applied to the measurement of the scattering amplitudes of cadmium[4] and a number of tellurium isotopes.[5]

This paper describes a mirror-reflection measurement of the boron scattering amplitude. Here the large absorption cross section of B^{10} makes it difficult to apply the normal diffraction methods. We also measured the scattering amplitudes of nitrogen and oxygen by the same technique. The scattering amplitude of oxygen is of particular interest because oxygen was used for the most precise measurement of the neutron-electron interaction[6] and we felt it would be of value to measure its scattering amplitude by a completely independent method.

THEORY OF MEASUREMENT

The variation of the reflection coefficient R with the grazing angle ϕ can be expressed in terms of n, the index of refraction

$$R(\phi)=\left|\frac{1-[1-2(1-n)/\phi^2]^{1/2}}{1+[1-2(1-n)/\phi^2]^{1/2}}\right|^2, \quad (1)$$

where n is related to the coherent scattering amplitude a by the formula[7]

$$1-n^2=(4\pi N/k^2)a. \quad (2)$$

Here k is the neutron wave number and N is the number of scattering centers per unit volume. Since n is very nearly equal to unity, the reflection coefficient can also be written

$$R(\phi)=\left|\frac{1-[1-4\pi Na/k^2\phi^2]^{1/2}}{1+[1-4\pi Na/k^2\phi^2]^{1/2}}\right|^2. \quad (3)$$

In general a will contain contributions from both resonance and potential scattering and should therefore be expressed in terms of the Breit-Wigner resonance parameters. Limiting the discussion to s-wave neutrons, the scattering amplitude for a resonant spin state is[8]

$$a=-\frac{1}{2ik}\left[1-\exp(-2ikR')+\frac{i\Gamma_n\exp(-2ikR')}{E-E_0+i\Gamma/2}\right], \quad (4)$$

where E and E_0 are the incident neutron and resonance energies, respectively, Γ_n and Γ are the neutron and total level widths, and R' is the effective nuclear radius. For $kR'\ll1$ this can be written

$$a_{res}=R'+\frac{\Gamma_n(E-E_0)}{2k[(E-E_0)^2+\Gamma^2/4]}-\frac{R'\Gamma_n\Gamma}{2[(E-E_0)^2+\Gamma^2/4]}-i\left[\frac{\Gamma_n\Gamma}{4k[(E-E_0)^2+\Gamma^2/4]}+kR'^2+\frac{R'\Gamma_n(E-E_0)}{(E-E_0)^2+\Gamma^2/4}\right], \quad (5)$$

and for a nonresonant spin state

$$a_{nonres}=R'-ikR'^2. \quad (6)$$

We note that for both resonant and nonresonant states

$$\sigma_t=\mathrm{Im}(4\pi a/k), \quad (7)$$

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† Present address: Brookhaven National Laboratory, Upton, Long Island, New York.
[1] M. Hamermesh, Phys. Rev. **77**, 140 (1950).
[2] D. J. Hughes, M. T. Burgy, and G. R. Ringo, Phys. Rev. **77**, 291 (1950).
[3] W. C. Dickinson, L. Passell, and O. Halpern, Phys. Rev. **126**, 632 (1962).
[4] D. Bally, S. Todiereanu, S. Ripeanu, and M. G. Belloni, Rev. Sci. Instr. **33**, 916 (1962).
[5] C. J. Heindl, I. W. Ruderman, J. M. Ostrowski, J. R. Liguenza, and D. M. Gardner, Rev. Sci. Instr. **27**, 620 (1956).
[6] D. J. Hughes, J. A. Harvey, M. D. Goldberg, and M. J. Stafne, Phys. Rev. **90**, 497 (1953).
[7] D. J. Hughes, *Neutron Optics* (Interscience Publishers, Inc., New York, 1954), Chap. I.
[8] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).

in which Im means "imaginary part of." Usually there is a resonance contribution to the amplitude from only only one spin state. When this is the case

$$a \cong R' + \frac{g\Gamma_n(E-E_0)/2k}{(E-E_0)^2+\Gamma^2/4} - i\frac{g\Gamma_n\Gamma/4k}{(E-E_0)^2+\Gamma^2/4}, \quad (8)$$

assuming $|E-E_0|/\Gamma \gg kR' \cong 10^{-5}$. Here g is taken to be the statistical weighting factor for the resonant state.

METHOD OF MEASUREMENT

Mirror measurements are most accurate when the scattering amplitudes are determined relative to a known standard, as has been discussed in Ref. 3 in which details of the experimental arrangement can also be found. It is convenient to use liquid surfaces as mirrors whenever possible; hydrocarbons of low vapor pressure are preferred because the ratio of the carbon to hydrogen scattering amplitudes is known with accuracy. The procedure is simply to find the angles ϕ at which the same intensity is reflected from a standard liquid and a liquid containing the unknown. When the intensities are equal it follows from Eq. (3), independently of the spectral distribution of the neutron beam, that

$$(1/\phi_i^2)\sum_i N_i a_i = (1/\phi_j^2)\sum_j N_j a_j. \quad (9)$$

The subscripts i and j refer to the sample liquid and the standard liquid, respectively.

The scattering amplitudes are, in principle, always complex, but when the neutron energy is far from resonance the imaginary term is only that due to potential scattering and is on the order of $kR' \cong 10^{-5}$ times smaller than the real term; therefore it can be neglected. When there is an appreciable resonance contribution to the cross section, however, the imaginary part, which is [from Eq. (7)] $k(\sigma_s+\sigma_a)/4\pi$, must be considered. Actually Halpern[9] has pointed out that only the incoherent part of σ_s removes neutrons from the reflected beam and, therefore, he gives the imaginary term as $k(\sigma_{\text{inc}}+\sigma_a)/4\pi$. For the cases we will consider, however, this distinction is not important. Generally $\sigma_a \gg \sigma_{\text{inc}}$ in the neighborhood of a resonance and, in fact, the imaginary term is often given simply as $k\sigma_a/4\pi$.[10]

The boron data required correction for both absorption and incoherent scattering. This was done by numerically integrating Eq. (3) over the measured incident neutron spectrum and computing a correction factor to adjust the intensity to what it would have been in the absence of the imaginary term. The application of Eq. (9) then gave us directly the real part of the scattering amplitude. The same correction was made for the nitrogen and oxygen measurements, but only for incoherent scattering.

In addition to considering the effects of absorption and incoherent scattering, corrections were made to the data for temperature drift, scattering in the vapor above the liquid surface, variation of the incident beam intensity with angle, and the finite angular divergence of the beam. These corrections were small and tended to compensate to some extent.

We used a beryllium-filtered neutron beam for all measurements since the angular range was such that neutrons with energies above the cutoff would contribute little to the reflected intensity, but would raise the background considerably.

Boron

We compared the intensity reflected from isopropylcarborane ($C_5H_{18}B_{10}$) with phenyl-n-hexane ($C_{12}H_{18}$). Figure 1 shows the reflected intensities before and after correction for absorption and incoherent scattering. Using the value $(6.64\times0.02)\times10^{-13}$ cm for the carbon-scattering amplitude, and 1.775 ± 0.004 for the ratio of the carbon to hydrogen-scattering amplitudes,[3] the value obtained for the bound boron amplitude is $a_B = (5.40\pm0.04)\times10^{-13}$ cm.

The isotopic abundance of B^{10} in the sample was measured and found to be 20.4%. This value was used in computing the absorption correction to the reflected intensity.

Nitrogen and Oxygen

Nitrogen was measured by comparing the reflected intensity from formamide (CH_3ON) with dimethylnapthalene ($C_{12}H_{12}$). For oxygen we used ethylene glycol ($C_2H_6O_2$) and *d*-limonene ($C_{10}H_{16}$). The corrected reflected intensities are shown in Fig. 2. We obtained for the bound-nitrogen amplitude $a_N = (9.19\pm0.11)\times10^{-13}$ cm, and for the bound-oxygen amplitude $a_O = (5.80\pm0.05)\times10^{-13}$ cm.

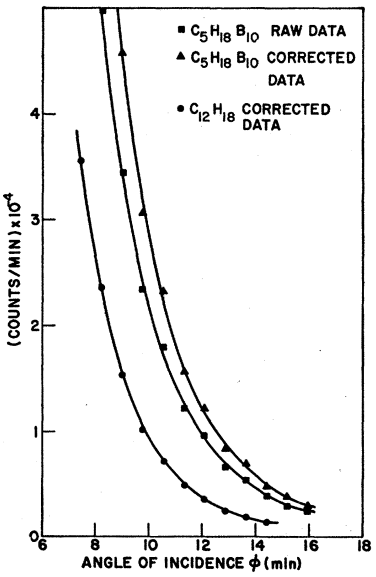

FIG. 1. Reflected intensities for the boron measurements as a function of the angle of reflection before and after corrections.

[9] O. Halpern, Phys. Rev. 88, 1003 (1952).
[10] M. L. Goldberger and F. Seitz, Phys. Rev. 71, 294 (1947).

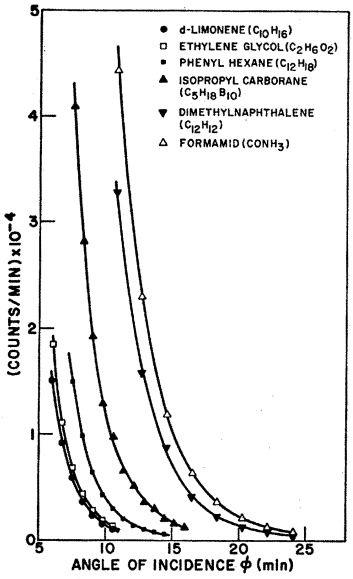

FIG. 2. Corrected reflected intensities for the boron, nitrogen, and oxygen measurements as a function of the angle of reflection.

DISCUSSION OF RESULTS

Boron

To our knowledge no other measurement of the boron-scattering amplitude has been reported. If we knew the amplitudes of the separate isotopes, however, there are some comparisons we could make with other data on boron. It is possible to derive these amplitudes by combining our result with the measured value of the total scattering cross section. This gives us two equations[11]:

$$a_B = C_{B^{10}} a_{B^{10}} + C_{B^{11}} a_{B^{11}} = (5.40 \pm 0.04) \times 10^{-13} \text{ cm}, \quad (10)$$

$$\sigma_S(B) = 4\pi (C_{B^{10}} a_{B^{10}}^2 + C_{B^{11}} a_{B^{11}}^2) = 4.4 \pm 0.2 \text{ b}, \quad (11)$$

which are satisfied by two pairs of values of the amplitudes of the separate isotopes,

$$a_{B^{10}} = (10 \pm 1) \times 10^{-13} \text{ cm}, \quad a_{B^{11}} = (4.1 \pm 0.3) \times 10^{-13} \text{ cm}, \quad (12)$$

or

$$a_{B^{10}} = (0.8 \pm 1) \times 10^{-13} \text{ cm}, \quad a_{B^{11}} = (6.6 \pm 0.3) \times 10^{-13} \text{ cm}. \quad (13)$$

To choose between them we note that if $a_{B^{10}} = 10 \times 10^{-13}$ cm, then the coherent cross section alone is 12.6 b, i.e., more than three times larger than the measured value of the total scattering cross section $[\sigma_S(B^{10}) = 4.0 \pm 0.5 \text{ b}]$.[11] In addition, beam-broadening experiments in progress at Livermore, using a B_2O_3 powder enriched in B^{11}, indicate that the second pair of numbers is the most likely choice.

Willard *et al.*[12] have measured the angular distribution of neutrons elastically scattered from both B^{10} and B^{11} between 0.4 and 1.5 MeV and have derived from their data the s-wave potential scattering phase shifts. From these phase shifts we can estimate the potential scattering contributions to the scattering amplitudes. For B^{10} and B^{11} we obtain, respectively, 5.0×10^{-13} and 4.6×10^{-13} cm. Assuming the second pair of values is the correct choice, then the small magnitude of $a_{B^{10}}$ must be an indication of a resonance contribution to the low-energy scattering cross section from a level or levels above the neutron binding energy. This agrees with the conclusions of Bergman and Shapiro[13] based on their study of the departure of the $B^{10}(n,\alpha)$ cross section from $1/V$ at energies up to 30 keV. It is also consistent with Bichsel and Bonner's[14] assignment of the thermal cross section of B^{10} to levels[15] at 11.46 and 11.68 MeV in B^{11} and is in accord with Inglis'[16] theoretical argument that the (n,α) cross section is due to levels above the neutron binding energy.

Turning now to B^{11}, Bockelman *et al.*[17] have observed that its total cross section is rising at low energies, indicating a level in B^{12} that can be reached by s-wave neutrons. Recently, Mooring and Segal[18] reported a neutron resonance in B^{11} that appears to correspond to the 3.38-MeV level[15] in B^{12} and may account for the rise of the cross section at low energies. If so, since it lies approximately 20 keV above the neutron binding energy, we would expect the scattering amplitude of B^{11} to be smaller than the estimated value for potential scattering alone. Actually, our derived value of the B^{11} amplitude, 6.6×10^{-13} cm, is larger rather than smaller than the amplitude for purely potential scattering. It could, of course, be argued that there may be compensating contributions from a level or levels lying below the neutron binding energy, however, we also note that if the amplitude is as large as 6.6×10^{-13} cm, then the coherent scattering cross section of B^{11} is 5.5 ± 0.5 b, which is larger than the measured value of the total scattering cross section $[\sigma_S(B^{11}) = 4.4 \pm 0.3 \text{ b}]$.[11]

The derived values of the scattering amplitudes of the separated isotopes appear to be only partly consistent with other available data. This is probably an indication of a systematic error in one or more of the scattering cross section measurements—hardly surprising in view of the difficulties of making such measurements in the presence of a large absorption cross section.

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[11] D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report No. BNL-325, 2nd ed., 1958 (unpublished).

[12] H. B. Willard, J. K. Bair, and J. D. Kington, Phys. Rev. **98**, 669 (1955).

[13] A. A. Bergman and F. L. Shapiro, Zh. Eksperim. i Teor. Fiz. **40**, 1270 (1961) [English transl.: Soviet Phys.—JETP **13**, 895 (1961)].

[14] H. Bichsel and T. W. Bonner, Phys. Rev. **108**, 1025 (1957).

[15] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).

[16] D. R. Inglis, Phys. Rev. **81**, 914 (1951).

[17] C. K. Bockelman, D. W. Miller, R. K. Adair, and H. H. Barschall, Phys. Rev. **84**, 69 (1951).

[18] F. P. Mooring and R. E. Segal, Bull. Am. Phys. Soc. **9**, 434 (1964).

Nitrogen and Oxygen

The measured values of the coherent amplitudes of nitrogen and oxygen, $a_N=(9.19\pm0.11)\times10^{-13}$ cm and $a_O=(5.80\pm0.05)\times10^{-13}$ cm, are in good agreement with the presently accepted values: $a_N=9.40\times10^{-13}$ cm and $a_O=5.77\times10^{-13}$ cm.[19]

[19] G. E. Bacon, *Neutron Diffraction* (Oxford University Press, New York, 1962), 2nd ed., p. 31.

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Production Amplitudes. I. Construction of Invariant Amplitudes

A. O. BARUT AND Y. C. LEUNG
International Atomic Energy Agency, International Centre for Theoretical Physics, Trieste, Italy
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Lorentz-covariant scattering amplitudes for production processes involving both massive and massless particles are systematically discussed. Explicit forms of both the spin and the isospin amplitudes for production processes involving pions, nucleons, photons, and neutrinos, as well as the crossing relations in both spin and isospin variables, are given.

I. INTRODUCTION

SCATTERING amplitudes in a relativistic-particle theory are Lorentz invariant, which means that the amplitudes constructed out of sets of particles remain invariant when each of the particles transforms according to some irreducible unitary representation of the inhomogeneous Lorentz group. A basic simplification in the theory of strong interactions is that the scattering amplitudes are further invariant (or "isotropic") under isospin rotations. It is, therefore, of practical interest to exhibit explicitly the invariance structure of the amplitude, and in achieving this it is also possible to isolate from the amplitude features which are of a kinematical nature before they complicate the essence of the dynamical theory. This aspect of the problem has recently received particular attention in the analytic S-matrix theory where one tries to extend the analytic postulates of scattering theories in a unified way to include particles with arbitrary spin. It is well known that invariant amplitudes of this type can be expressed as linear combinations of spin or isospin basis functions with scalar coefficients. Several papers[1–6] on this subject dealing with the two-body scattering amplitudes have appeared. The present paper is an extension of these results to include production amplitudes and to furnish some explicit results for the more common processes.

[1] A. O. Barut, I. Muzinich, and D. N. Williams, Phys. Rev. **130**, 442 (1963).
[2] A. O. Barut and B. C. Unal, Nuovo Cimento **28**,112 (1963).
[3] D. Zwanziger, Phys. Rev. **133**, B1036 (1964).
[4] D. N. Williams, Lawrence Radiation Laboratory UCRL-11113, Berkeley, 1963 (unpublished).
[5] K. Hepp, Helv. Phys. Acta **36**, 355 (1963); **37**, 55 (1964).
[6] H. Joos, Fortschr. Physik **10**, 65 (1962).

In Sec. II, the method for finding isospin bases for production amplitudes is discussed, and bases for some pion-production processes are given. The method for establishing crossing relations among different total isospin amplitudes is presented in Subsec. D, where the crossing matrices in compact form are given. In Sec. III, some of the problems peculiar to constructing spin bases for production amplitudes involving massive particles are discussed. Crossing relations among amplitudes involving particles with spin are discussed. In Sec. IV, the problem of constructing spin bases for amplitudes involving massless particles is presented.

II. ISOSPIN AMPLITUDES AND THE CROSSING MATRICES

The isospin indices of a scattering amplitude transform according to the direct product of irreducible representations of the rotation group, one for each individual particle. Such a direct product is, in general, reducible and may be expressed as the sum of the irreducible parts by means of the Clebsch-Gordan series. Since the scattering amplitudes are assumed to be invariant or isotropic under rotations in the isospin space, only the one-dimensional representations of the irreducible parts are to be retained. These are often called the isotropic tensors. The construction of invariant isospin amplitudes for a particular scattering process is therefore equivalent to finding all the independent isotropic tensors from a direct-product representation of these particles. Each independent isotropic tensor forms a basis component for the isospin amplitude. The scattering amplitude is then a linear combination of such tensors with coefficients which are independent of the