Levels in ^{43}Ca and ^{46}Ca Studied by the ^{43}Ca(d,d'), ^{46}Ca(d,d'), and ^{46}Ca(p,p') Reactions*

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The ^{43}Ca(d,d') and ^{46}Ca(d,d') reactions were studied at bombarding energies of 8.522 and 10.005 MeV, respectively. The deuterons were recorded on photographic emulsions in a multiple-gap heavy-particle spectrograph with an energy resolution of 15 keV. The measured angular distributions were analyzed using the distorted-wave theory, and probable values of the angular-momentum change involved in the excitations were determined. The absolute cross sections are compared with the predictions obtained by using shell-model wave functions as well as collective vibrational wave functions. A simple model for the deuteron-nucleus interaction is introduced. In addition to the (d,d') experiments, the ^{46}Ca(p,p') reaction was studied at 3.8- and 7.00-MeV bombarding energies. The energies (in MeV) of the states observed in ^{46}Ca and their assigned spins and parities are 1.347 (2^+), 2.423, 2.575, 3.023 ($2, 3^+$), 3.614 (3^-), 3.645, 3.780, and 4.434 (3^-). In ^{43}Ca(d,d') we observe levels at 0.369 ($\frac{5}{2}^-$), 0.595 ($\frac{3}{2}^-$), 1.675 (11/2 ?$^-$), 1.932 (?$^-$), 2.051 ($\frac{3}{2}^-$), 2.070 (15/2 ?$^-$), 2.098 ($\frac{9}{2}$?$^-$), and 2.252 (?$^-$) MeV.

I. INTRODUCTION

In a previous report[1] on the ^{43}Ca(d,t) reaction, the structure of the low-lying ^{42}Ca levels and of the ^{43}Ca ground state were studied. The present paper reports on results from inelastic-deuteron scattering from ^{43}Ca and ^{46}Ca at bombarding energies of 8.522 and 10.005 MeV, respectively, together with some measurements of the ^{46}Ca(p,p') reaction.

At these energies, the (d,d') reaction proceeds through direct mechanisms (both Coulombic and nuclear) and a distorted-waves (DW) analysis may be used to obtain information on spins and parities of excited states and on the magnitudes of the multipole-transition matrix elements involved. The (p,p') reaction at 3- to 7-MeV proton energy includes strong compound-nucleus contributions and is used mainly to provide information on excitation energies.

With a few exceptions, previous analyses of inelastic scattering at low and medium energies have used the collective-model interaction, namely, a nonspherical deformation of the optical-model potential which describes the elastic scattering. This has been particularly successful in accounting for the excitation of the lowest 2^+ and 3^- states in even nuclei, interpreted as quadrupole and octupole vibrations. Adjacent odd nuclei may be treated by regarding the odd nucleon as weakly coupled to the even core; when the core is excited we obtain a narrow multiplet of levels associated with the various total angular momenta of the excited core and odd nucleon. In the case of ^{43}Ca with an odd $f_{7/2}$ neutron, quadrupole excitation of the core would give a quintuplet with spins $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, $\frac{11}{2}$, and odd parity. Such a model has been applied with some success to proton scattering[2] from ^{63}Cu.

However, there are a considerable number of reasons[3] for believing the low states of nuclei in this region are well described within the shell model by excitations of $(1f_{7/2})^n$ configurations. Because of this, we present here an analysis of the experimental results on ^{43}Ca and ^{46}Ca in terms of shell-model excitations. To do this, we also develop a simple model for the deuteron-nucleus interaction.

II. EXPERIMENTAL TECHNIQUES AND RESULTS

1. Targets

The ^{43}Ca and ^{46}Ca targets were prepared in the Copenhagen isotope separator, as described in Refs. 1 and 4, respectively. The final enrichments obtained were better than 99% in both cases.

2. Elastic Deuteron Scattering

Elastic-scattering data were measured at the Copenhagen Tandem laboratory. The results for ^{43}Ca were

* Research sponsored in part by the U. S. Atomic Energy Commission under contracts with the Union Carbide Corporation and the Massachusetts Institute of Technology.

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[1] J. H. Bjerregaard, H. R. Blieden, O. Hansen, G. Sidenius, and G. R. Satchler, Phys. Rev. **136**, B1348 (1964).

[2] F. G. Perey, R. J. Silva, and G. R. Satchler, Phys. Letters **4**, 25 (1963).

[3] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964); J. N. Ginnochio and J. B. French, Phys. Letters **7**, 137 (1963).

[4] J. H. Bjerregaard, O. Hansen, and G. Sidenius, Phys. Rev. **138**, B1097 (1965).

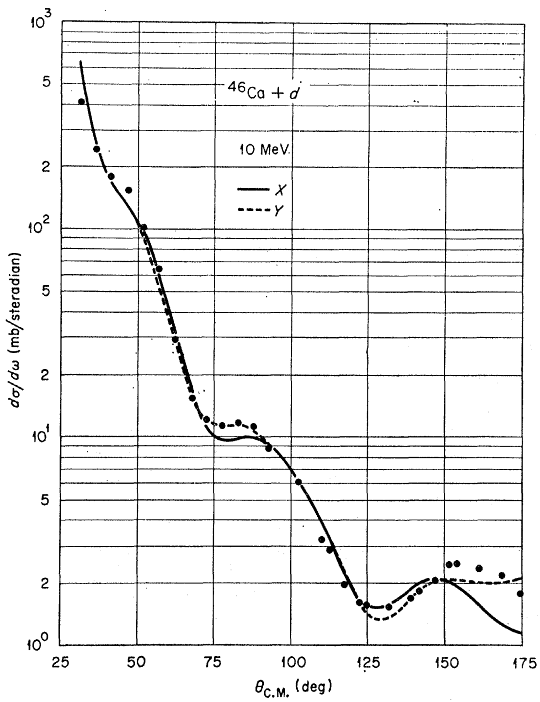

FIG. 1. Elastic scattering of deuterons from ^{46}Ca at 10 MeV. The full curve is the optical-model prediction using the set X parameters (see Table III); the dashed curve is the optimum fit obtained by varying all parameters. The experimental errors are discussed in the text.

given in Ref. 1. The ^{46}Ca results are presented in Fig. 1, together with optical-model predictions.

The experimental setup used in these measurements employed a movable semiconductor detector as well as a monitoring detector fixed at 135-deg scattering angle. The beam was integrated in a Faraday cup after having passed through the target. Because of the rather inhomogeneous targets, special care had to be taken that the beam geometry did not change during the measurements. Frequent remeasurements at selected angles resulted in the assignment of a $\pm 10\%$ error to all measured yields, irrespective of the fact that the statistics always were better than $\pm 5\%$. An absolute cross-section scale was established by measurements at 4-MeV bombarding energy and 60° to 75° scattering angle, at which energy the elastic (d,d) cross sections at 60° equal 0.78 ± 0.02 times the Rutherford cross section (see the discussion in Ref. 1). During the measurements of the excitation functions connecting the 4-MeV data to the data at higher energies, it was not possible to keep a well-defined beam geometry. A slightly defocused beam was found to give the best reproducibility (again of $\pm 10\%$ for a single measurement).

3. Inelastic Deuteron Scattering

The (d,d') experiments were made at the tandem electrostatic generator of the Atomic Weapons Research Establishment (AWRE), Aldermaston, England. The inelastically scattered deuterons were detected on 50-μ-thick photographic emulsions in the 24-gap heavy-particle spectrograph of Middleton and Hinds.[5] The plates were not covered by any stopping foils, and the deuterons were selected in the scanning by the length and grain density of their tracks in the emulsions.

The ^{43}Ca(d,d') results derive from the same exposure that gave information on the ^{43}Ca(d,t) reaction.[1] The bombarding energy of 8.522 MeV was chosen so that the triton groups leading to the ground state of ^{42}Ca in the ^{43}Ca(d,t) reaction could be recorded on the photographic emulsions, using the highest available magnetic field in the spectrograph. In the ^{46}Ca experiment the bombarding energy was 10.005 MeV.

The (d,d') excitation energies given in Tables I (^{43}Ca) and II (^{46}Ca) derive from position measurements in the 35-deg gap, the gap in which the magnetometer probe is positioned; if a group was obscured at this angle, measurements from the neighboring gaps (27.5 or 42.5 deg) were used. The deuteron groups were assigned to the mass of the residual nucleus from the observed energy shift with angle.

The energy resolution varies from angle to angle from 10 keV in the best cases up to 15 keV. The exposures were 3000 μC for ^{43}Ca and 2360 μC for ^{46}Ca.

Part of a deuteron spectrum from the ^{46}Ca bombardment is shown in Fig. 2. Besides mass 46 (^{46}Ca), d groups were observed corresponding to mass 37 (Cl),

TABLE I. ^{43}Ca(d,d') excitation energies. Column two lists the presently obtained excitation energies. The next column gives the value of the dominating angular momentum transfer as discussed in Sec. IV. The spins and parities of column 4 are those tentatively assigned from shell-model analysis of the (d,d') cross sections. The last two columns list the level-scheme information from Ref. 6.

	Present experiment			Endt and van der Leun[a]	
Level	E_x(keV)	L	$J_f\pi$	E_x(keV)	$J_f\pi$
1	369±5	2		373.7±0.4	$\frac{5}{2}-$
2	595±5	2		594 ±2	$\frac{3}{2}-$
3	...			992.6±0.7	$\frac{5}{2}+$
4	...			1389 ±3	$\frac{3}{2}+$
5	1675±5	2	(11/2)$^-$	1678 ±4	
6	...			1904 ±4	
7	1932±5	(2)	$-$	1932 ±5	
8	...			1957 ±4	$\frac{1}{2}+$
9	...			1985 ±5	
10	2051±5	(2)		2048 ±5	$\frac{3}{2}$, $\frac{1}{2}-$
11	2070±5	4	(15/2)$^-$	2069 ±5	
12	2098±8	2	(9/2)$^-$	2095 ±5	
13	...			(2107 ±5)	
14	...			2225 ±5	
15	2252±5	2		2250 ±5	
16	...			2273 ±5	
17	...			2409 ±5	
18	...			2607 ±5	$\frac{3}{2}$, $\frac{1}{2}-$

[a] Reference 6.

[5] R. Middleton and S. Hinds, Nucl. Phys. 34, 404 (1962).

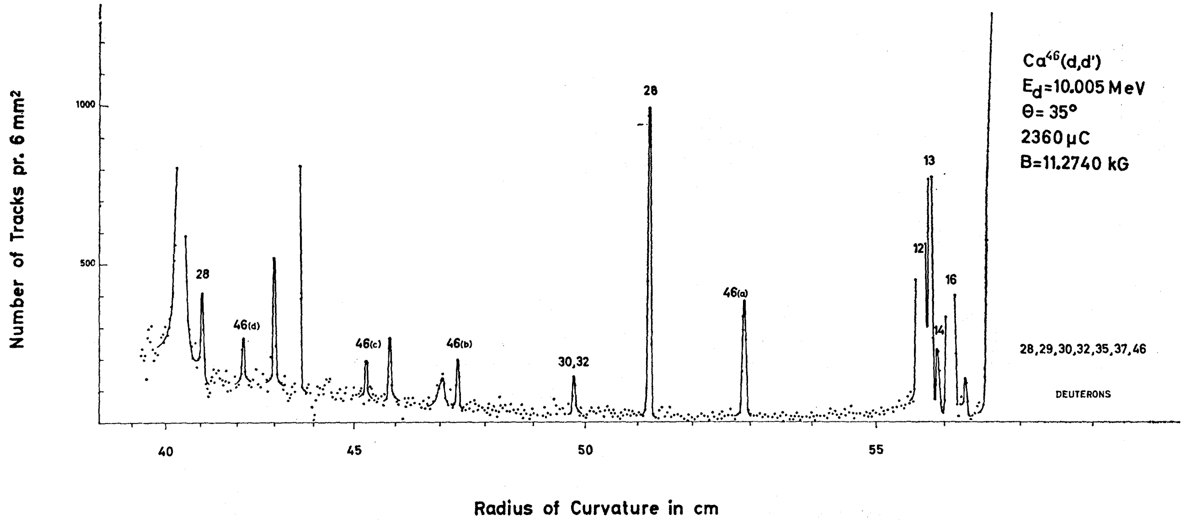

FIG. 2. Deuteron spectrum from the ^{46}Ca bombardment at 10.005 MeV and 35-deg laboratory system. The upper region on the plates could not be scanned at this angle because of the strong elastic deuteron scattering. The masses quoted on the figure were identified from the energy shift with angle of the elastically scattered deuterons at somewhat more backward angles. The ^{46}Ca states are assigned (a), (b), (c), and (d), corresponding to levels (1), (4), (5), and (8), respectively. The (d,d') groups belonging to masses 28, 30, and 32 are marked on the figure. The unassigned groups all belong to lighter contaminants.

35 (Cl), 32 (S), 30 (Si) 29 (Si), 28 (Si), 16 (O), 14 (N), 13 (C), and 12 (C) (see also Fig. 3).

Excitation energies for low-lying states in these nuclei are known with high precision,[6] and these values thus provided a convenient check on the energy calibration of the present experiment. The impurities in the ^{43}Ca target were the same as above.

The complete deuteron spectra from the elastic Ca group and down to the lowest energies accepted were scanned at eight angles. At all other angles only those parts of the plates that contained deuteron groups belonging to the target mass of interest were examined. Angles more forward than 27.5 deg (laboratory system) could not be scanned because of intense deuteron background. [Excepted is the ^{46}Ca(1) state which was also observed at 20-deg laboratory system.]

The absolute cross-section scale was established by scanning the elastic Ca peaks at back angles and normalizing the observed intensities to the results of the Ca(d,d) experiments of Sec. II.2.

Disregarding possible differences in the Copenhagen and Aldermaston scales of bombarding energy, the errors involved in the absolute cross-section determinations accumulate to $\pm 25\%$. The relative errors on the inelastic scattering yields are displayed in Figs. 5 through 11, which present the (d,d') angular distributions in comparison with DW predictions.

4. Inelastic Proton Scattering

Because of its direct mechanism the (d,d') reaction is a rather selective process. Since nothing has been published previously on the ^{46}Ca level scheme it was thought worthwhile to investigate ^{46}Ca by means of a less selective reaction like the (p,p') reaction at low energies (<10 MeV). The ^{46}Ca(p,p') reaction was investigated at 3.8 MeV at the Copenhagen 4-MV electrostatic generator and at 7.000 MeV at the M.I.T.–O.N.R. electrostatic generator.

The 4-MeV experiment employed a single-gap broad-range spectrograph for analyzing the protons scattered from the ^{46}Ca target through an angle of 143°. The experimental setup and the spectrograph calibration were identical to those used by Bjerregaard *et al.*[7] One inelastic proton group belonging to mass 46 was identified (see Table II).

In the 7-MeV experiment the scattered protons were analyzed in the multiple-gap spectrograph of Enge and Buechner[8] and were detected in 50-μ Kodak NTA nuclear emulsion track plates. The complete spectra

TABLE II. ^{46}Ca level scheme.

Level number	(p,p') 3.8 MeV	(p,p') 7.0 MeV	(d,d')	Accepted value (MeV)	$J_f\pi$[a]
1	1344±4	1349±3	1344±5	1347±3	2+
2		2423±5	···	2423±5	
3		2575±5	···	2575±5	
4		3025±5	3019±5	3023±5	(2,3)+
5		3618±6	3605±7	3614±5	3−
6		3645±6	···	3645±6	
7		3780±10	···	3780±10	
8			4434±8	4434±8	3−

[a] Suggested from the discussion of Sec. IV.1.

[6] P. M. Endt and C. van der Leun, Nucl. Phys. 34, 1 (1962).

[7] J. H. Bjerregaard, P. F. Dahl, O. Hansen, and G. Sidenius, Nucl. Phys. **51**, 641 (1964).

[8] H. A. Enge and W. W. Buechner, Rev. Sci. Instr. 34, 155 (1963).

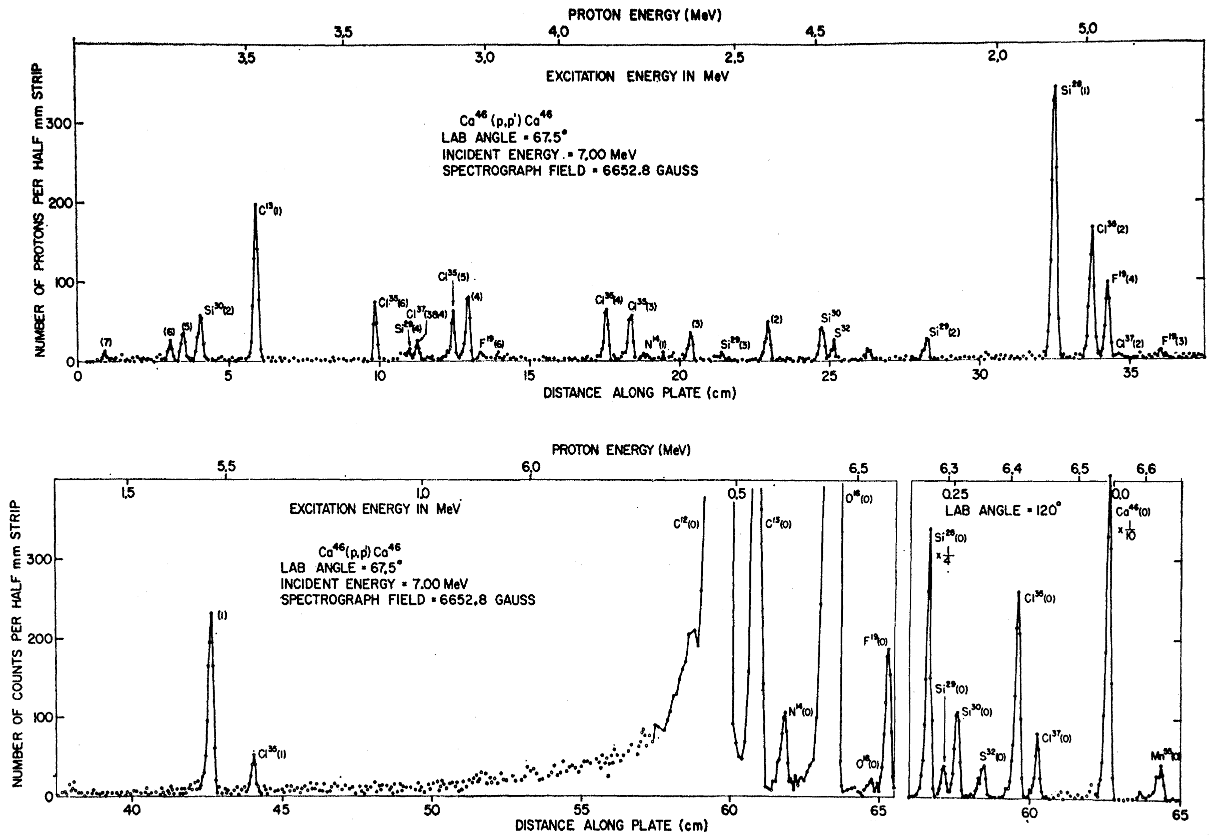

FIG. 3. The observed inelastic proton spectrum at $\theta_{\mathrm{lab}}=67.5°$, and the spectrum near the elastic scattering peak at 120°. The groups corresponding to ^{46}Ca states are identified by a number to the right of each group. The elastic and inelastic contaminant groups are also identified.

were scanned for the 67.5-, 90-, 120-, and 150-deg gaps. Part of the 67.5-deg spectrum of scattered protons, along with the elastically scattered proton groups from the 120-deg gap are shown in Fig. 3. The over-all energy resolution was 9 keV; the exposure was 2000 μC.

The seven reaction proton groups ascribed to mass 46 from their energy shift with angle are numbered 1 through 7 in Fig. 3. Further, a number of impurity groups are marked on the figure.

The (p,p') excitation energies given in column 2 of Table II represent averages from position determinations at the 67.5-, 90-, and 120-deg gaps. (For the three groups around 3.7-MeV excitation additional data from the 30-, 37.5-, and 52.5-deg gaps were used.) The multiple-gap spectrograph was calibrated with ^{210}Po α particles taking their energy to be 5.3042 MeV.

III. THEORETICAL ANALYSIS

As already mentioned, it seems more reasonable to discuss the excitation of the low-lying states in ^{43}Ca and ^{46}Ca in terms of shell-model wave functions rather than applying the collective model. In order to do this, we need a model for the interaction between the deuteron and target nucleons. We choose a simple central force, assuming it is two-body in nature and dependent only on the position of the center of mass of the deuteron. Such a force can be obtained by taking a two-body interaction between the neutron and proton of the deuteron and each target nucleon, and averaging over the internal motion of the deuteron. At first sight this seems a very questionable procedure since it neglects contributions which involve excitation of the deuteron. It may be that such excitation does not contribute significantly because of the many break-up channels available, except insofar as it leads to the damping of the deuteron waves which is described by the optical-model absorption. In any case, we introduce the concept here in a phenomenological way. We shall see it relatively successful, and the parameters required are consistent with those required to describe nucleon scattering in an analogous way.[9] It might be mentioned that alpha-particle scattering has been studied with a similar model.[10]

The formal derivation of this effective deuteron-nucleon interaction is given in the Appendix. A more detailed description of the nuclear-matrix elements will

[9] H. O. Funsten, N. R. Roberson, and E. Rost, Phys. Rev. **134**, B117 (1964).

[10] E. Sanderson and N. S. Wall, Phys. Letters **2**, 173 (1962).

be given elsewhere. A Gaussian radial form with a range of 2 F was assumed for the interaction, while the single-particle wave functions for the bound nucleons were computed as eigenfunctions of a Woods-Saxon potential of radius 1.25 $A^{1/3}$ F and diffuseness 0.65 F with a spin-orbit coupling of 25 times the Thomas term. The $1f_{7/2}$ neutron was assumed to be bound by 8 MeV, and the $2p_{3/2}$ neutron by 6 MeV. These required well depths of 49.4 and 51.4 MeV, respectively.[11]

An analysis of proton scattering[9] from nuclei in this region using a similar model required a nucleon-nucleon spin-independent Gaussian interaction of strength about 45 MeV and range 1.85 F; our simple model would then give a strength of about 70 MeV. Currently we do not have any indication of the strength of the "spin-flip" interaction; a Serber exchange mixture, for example, would give it $\frac{1}{3}$ the strength of the spin-independent term. The spin-flip term cannot contribute to excitations within a j^n configuration when n is even and the ground-state spin is zero. It is also forbidden for n odd if seniority is a good quantum number (as in the present case), but it can contribute to interconfigurational transitions such as j^n to $j^{n-1}j'$. (Since the interaction is a one-body operator in the space of the target nucleons, there are no matrix elements between levels differing in the state of more than one particle.) In principle, deuteron spin-flip could transfer two units of angular momentum to the target nucleus, but the interaction assumed here only allows the transfer of one unit. A tensor force component would be required to allow two units.

1. Elastic Scattering

The optical-model analysis of the ^{43}Ca(d,d) data has been described previously.[1] A method similar to that used for the ^{43}Ca data was employed in the ^{46}Ca case. Two sets of optical-model parameters were derived. The set called X was obtained by fixing the radius parameter for the real part at $r_0=1.0$ F, the value obtained for the "average Z" potential for ^{40}Ca in Ref. 12, and varying the other parameters for an optimum fit. The set Y was found by varying all parameters. No volume absorption was used. A least-squares criterion was used in obtaining the fits.[13] The results are shown in Fig. 1 and the parameters of the two potentials are given in Table III.

2. Inelastic Scattering

The scattering induced by the effective deuteron-nucleus interaction was calculated using the distorted-wave method which has been described in detail elsewhere.[14,15] As has been found in other calculations,[1,12] more satisfactory angular distributions were obtained when using the "average," or X, optical potential rather than the "best," or Y, potential which gives an absolute best fit to the elastic data. For this reason, only calculations using these average potentials will be discussed below.

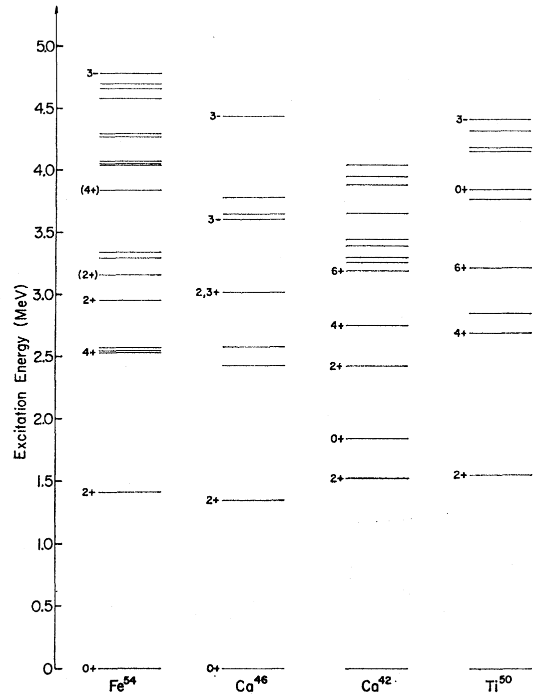

FIG. 4. Comparison of the level schemes of the four $(f_{7/2})^{\pm 2}$ nuclei. The ^{54}Fe data are taken from Refs. 9 and 17, although the 4^+ assignment of the 3.89-MeV level is made by the present authors from the results of Ref. 9. The ^{46}Ca level scheme is taken from Table II. The ^{42}Ca results are from Ref. 6 and from Ref. 1 and references given there. The ^{50}Ti scheme is taken from Ref. 16.

Calculations were also made using the surface-coupling collective model.[14] Qualitatively the angular distributions obtained are very similar to those given by the shell model, provided only the real part of the optical potential is allowed to vibrate. Allowing the imaginary part to vibrate also (complex coupling) worsens the fit to the angular distributions for the "shell-model" states, but somewhat improves the fit to the two groups from ^{46}Ca which are interpreted as corresponding to excitation of 3^- vibrations.

TABLE III. ^{46}Ca optical-model parameters.[a]

Potential	V (MeV)	r_0 (F)	a (F)	W_D (MeV)	r_0' (F)	a' (F)	r_c (F)
X	115.0	1.0	0.804	13.0	1.419	0.660	1.30
Y	160.1	0.769	0.933	12.4	1.488	0.648	1.30

[a] The potential form and the notation are identical to those of Ref. 1.

[11] The radial form factors were computed using the code ATHENA written by M. B. Johnson and L. W. Owen at Oak Ridge National Laboratory.

[12] R. H. Bassel, R. M. Drisko, G. R. Satchler, L. L. Lee, J. P. Schiffer, and B. Zeidman, Phys. Rev. **136**, B960 (1964).

[13] R. M. Drisko (unpublished).

[14] R. H. Bassel, R. M. Drisko, G. R. Satchler, and E. Rost, Phys. Rev. **128**, 2693 (1962).

[15] G. R. Satchler, Nucl. Phys. **55**, 1 (1964).

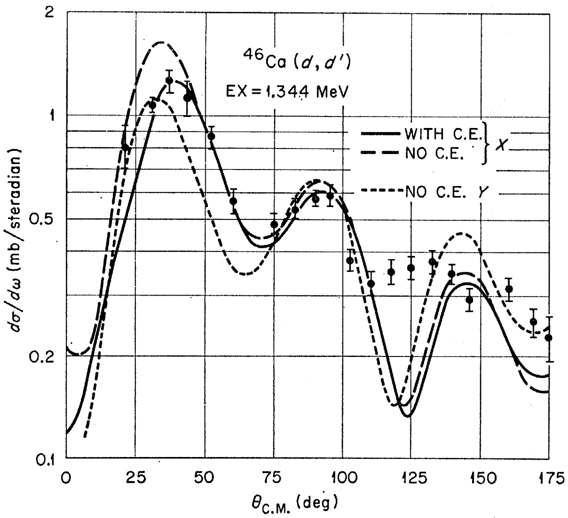

FIG. 5. Differential cross sections for exciting the 1.344-MeV 2^+ level in ^{46}Ca with 10-MeV deuterons. The theoretical curves use the model described in the text, and are calculated using potentials X and Y of Table III. C. E. means Coulomb excitation.

IV. RESULTS OF THE DW ANALYSIS AND DISCUSSION

This section covers the result of the analysis of the observed (d,d') differential cross sections employing vibrational wave functions as well as shell-model wave functions for the excited states. The discussion is mostly confined to the consistency of this analysis, while a discussion of the nuclear structure information obtained from the present experiment is deferred to Sec. V.

1. ^{46}Ca

^{46}Ca in the simple shell model is described by a $(f_{7/2})^{-2}$ configuration, i.e., a 0^+ ground state and excited states of spin and parity 2^+, 4^+, and 6^+. The excitation spectrum of these states should resemble that of ^{54}Fe (two $f_{7/2}$ proton holes) and those of ^{42}Ca and ^{50}Ti with two $f_{7/2}$ particles outside of closed shells. The level schemes of these four nuclei are shown in Fig. 4 (see also Refs. 16 and 17). It is seen that in all cases more levels appear than expected for a $(f_{7/2})^{\pm 2}$ configuration.

The first excited state of ^{46}Ca has an excitation energy of 1.347 MeV; the (d,d') angular distribution for this state is shown in Fig. 5 in comparison with distributions calculated from the shell-model excitation picture (see below).

Very similar distributions are predicted from the collective-vibrational model, taking ^{46}Ca(1) to be a quadrupole vibration, provided real coupling is used. The quadrupole deformability β_2 for ^{46}Ca(1) needed to fit the presently observed cross sections equals 0.28; this figure may be compared to the value of 0.22 ± 0.04 obtained from Coulomb excitation[18] of ^{44}Ca(1), under the assumption of a charge radius of $1.2A^{1/3}$ F. From the vibrational systematics one would expect ^{46}Ca(1) to have a lower β_2 than ^{44}Ca(1), since ^{46}Ca is closer to the doubly magic ^{48}Ca. In view of the $\pm25\%$ uncertainty on our absolute cross sections, which corresponds to a $\pm13\%$ error on β_2, we cannot say that the β_2 for ^{46}Ca(1) is unreasonable. The use of complex coupling gives a much poorer fit to experiment; it requires $\beta_2\approx0.15$. This is in contrast to the analyses of deuteron scattering from heavier nuclei, for which complex coupling is essential in order to obtain good agreement with experiment.

Using shell-model wave functions of a two-hole $(1f_{7/2})^{-2}$ neutron configuration and the interaction described in the previous section, we obtain the theoretical curves shown in Fig. 5. An interaction strength [defined by Eq. (A6) of the Appendix] of $V_0=80$ MeV was used. The figure shows a comparison of the predictions using the two optical potentials of Table III; the potential X gives a somewhat better account of the data. Also shown is the effect of including Coulomb excitation and its interference with the main nuclear amplitudes; the principal change is a reduction in the cross section at forward angles. The magnitude of the Coulomb-excitation term was estimated by assuming the 0^+ to 2^+

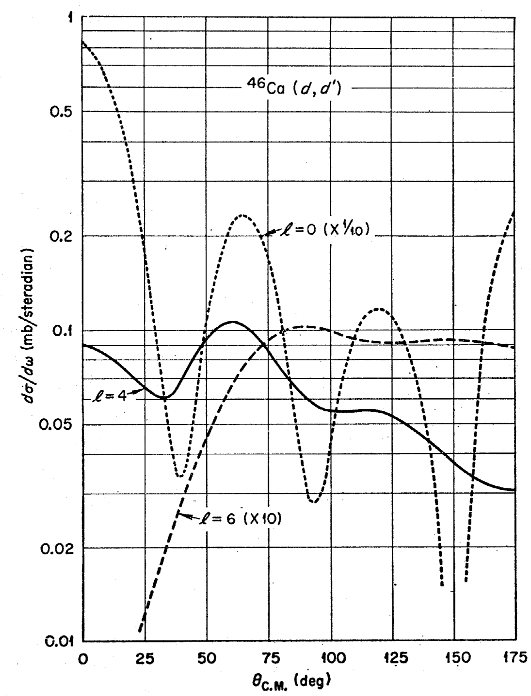

FIG. 6. Predicted differential cross sections for the inelastic scattering of 10-MeV deuterons using shell-model wave functions and an interaction strength $V_0=80$ MeV.

[16] P. D. Barnes, C. R. Bockelman, O. Hansen, and A. Sperduto, (to be published).

[17] A. Sperduto and W. W. Buechner, Phys. Rev. 134, B142 (1964); A. Aspinall, G. Brown, and S. E. Warren, Nucl. Phys. 46, 33 (1963).

[18] D. S. Andreev, V. D. Vasil'ev, G. M. Gusinski, K. I. Erokhina, and I. Kh. Lemberg, Bull. Acad. Sci. USSR, Phys. Ser. 25, 842 (1961).

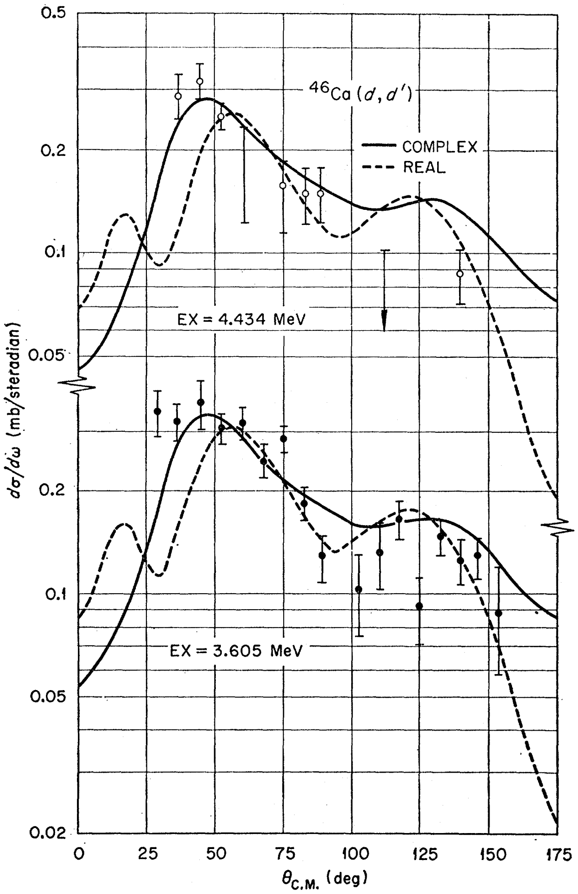

FIG. 7. Differential cross sections for exciting the 4.434- and 3.605-MeV levels with 10-MeV deuterons. The theoretical curves assume an octupole 3^- collective vibration and include Coulomb excitation. The low-energy state has a deformability $\beta_3=0.16$ and the upper has $\beta_3=0.15$ when complex coupling is used. With real coupling, these become 0.13 and 0.12, respectively.

transition in ^{44}Ca occurred between $(f_{7/2})^4$ states and giving the neutrons an effective charge, e'. The observed Coulomb excitation[18] probability then requires $e'\langle r^2\rangle=26.3e\mathrm{F}^2$, where $\langle r^2\rangle$ is the mean square radius of the $f_{7/2}$ orbit and e is the charge on a proton. The neutron wave functions used in the present calculations have $\langle r^2\rangle=17.0\ \mathrm{F}^2$, which implies $e'=1.55e$. It was then assumed that the same value of $e'\langle r^2\rangle$ was correct for the $(f_{7/2})^6$ transition in ^{46}Ca. The fact that e' differs from zero implies some form of core polarization by the neutrons[19] and suggests the possibility of the *nuclear* excitation of the 2^+ level also being enhanced.

[19] Effective charges arising from core polarization have been discussed by R. D. Amado and R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **70A**, 532 (1957); Phys. Rev. **108**, 1462 (1957); B. R. Mottelson in *The Many-Body Problem*, edited by C. Dewitt (Dunod Cie., Paris, 1959) p. 286; A. de-Shalit in *Selected Topics in Nuclear Theory*, edited by F. Janouch (International Atomic Energy Agency, Vienna, 1963), p. 209; and by O. Nathan and S. G. Nilsson, in *Alpha-, Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1964).

By comparison with similar nuclei (Fig. 4), one would expect the 4^+ and 6^+ levels of this configuration to appear at about 2.5 and 3.0 MeV, respectively. The calculated cross sections for these, using $V_0=80$ MeV, are shown in Fig. 6. Since the 2^+ excitation from which V_0 was obtained may be enhanced, these may be overestimates. Their smallness explains why these two states are not seen in the present experiment. Also shown in Fig. 6 for completeness is the cross section for an $L=0$ excitation. In practice this could arise through the mixing of the predominantly $(f_{7/2})^{-2}$ ground-state wave function with a higher 0^+ state obtained by exciting two particles. Considerable cancellation would occur in the nuclear-matrix element, and such a transition would be strongly inhibited.

Two states with similar angular distributions are excited at 3.605 and 4.434 MeV, and both are well described by $L=3$ theoretical curves, as shown in Fig. 7. We interpret these as components of the collective-octupole vibration. It is not unusual in this mass region to find two such states strongly excited; two have been identified[20] in ^{48}Ti at 3.36 and 4.58 MeV with deformabilities β_3 of approximately 0.19 and 0.15, respectively. In ^{46}Ca also, the lower state has a somewhat larger strength [$\beta_3\approx0.16$ for state (5) and 0.15 for state (8)]. We note too that a slightly better fit to experiment in this case is obtained using complex coupling. Coulomb excitation is included in both cases, assuming the charge distribution has the same deformability as the optical potential.

The other state which is seen in this experiment is at 3.019 MeV, and the corresponding angular distribution is shown in Fig. 8. The distribution has an $L=2$ character, indicating that this level is 1^+, 2^+ or 3^+. The simplest interpretation is that it is a state of the

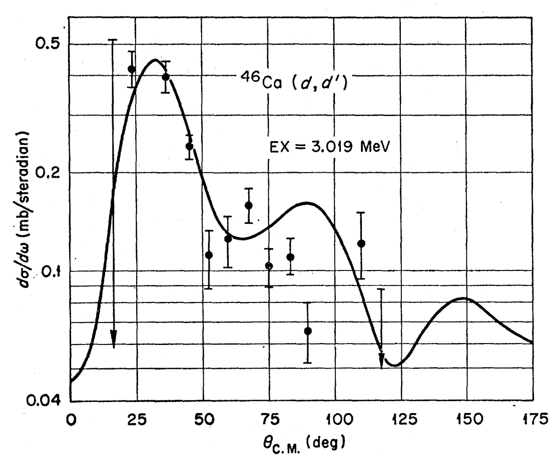

FIG. 8. Differential cross section for exciting the 3.019-MeV level with 10-MeV deuterons. The theoretical curve assumes an $f_{7/2}$ to $p_{3/2}$ excitation with $L=2$; the strength required is discussed in the text.

[20] G. R. Satchler, R. H. Bassel, and R. M. Drisko, Phys. Letters **5**, 256 (1963).

$(f_{7/2}^{-3}p_{3/2})$ configuration. If we assume a unique parent spin of $\frac{7}{2}$ for the f group, only 2^+ or 3^+ are allowed. Since we have neglected spin-orbit coupling in the deuteron-optical potential, the angular distribution depends only on L and is the same for $J=2$ or 3 [see Eq. (A7) of the Appendix]. For the assignment 2^+ we have (in the notation of the Appendix) $M_2^2=0.041$ and $N_{22}^2=0.027$ for the nuclear-matrix elements. Then the curve in Fig. 8 implies $V_0^2+0.445V_1^2\approx 1600$ (MeV^2); thus $V_0\lesssim 40$ MeV. If the spin is 3^+, the state can only be excited with spin flip; since $N_{23}^2=0.020$ the fit in Fig. 8 requires $V_1\approx 59$ MeV. The value of V_0 required for spin 2 is rather low compared to that required for transitions to the $(f_{7/2})^2$ states, but this could be due to to an incorrect choice of parameters for the bound-state wave functions or the deuteron-nucleon interaction, or it might be caused by destructive interference from an admixture of $(f_{7/2})^2$ or $(p_{3/2})^2$ configurations (c.f., the second 2^+ state[1] in ^{42}Ca). Figure 9 shows a comparison of the predictions for an f to p transition compared to those for recoupling f neutrons.

2. ^{43}Ca

Eight levels were excited in the ^{43}Ca(d,d') reaction; the results are shown in Figs. 10 and 11. The level scheme obtained from this and other experiments is shown in Fig. 12. The levels at 0.369, 0.595, 1.675 and 2.252 MeV show principally $L=2$ angular distributions, that at 2.070 MeV is largely $L=4$. The limited data for

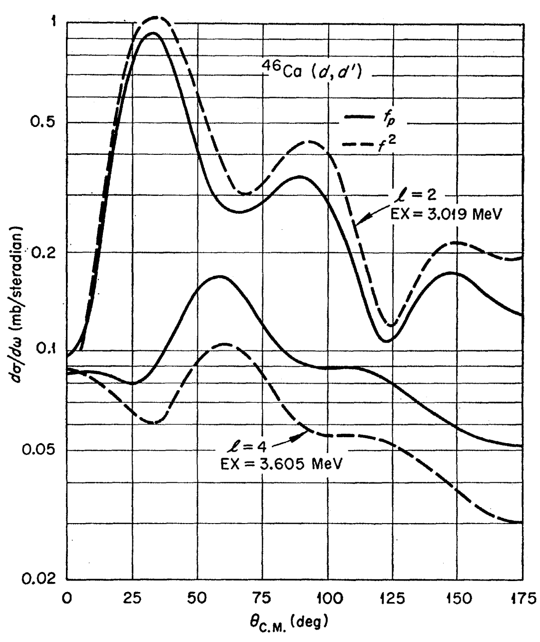

FIG. 9. Comparison of the $L=2$ and $L=4$ theoretical predictions for 10-MeV deuterons when recoupling two $f_{7/2}$ neutrons with those for the transition $(f_{7/2})^2$ to $(f_{7/2}p_{3/2})$. Coulomb excitation is not included and strengths $V_0=80$ MeV, $V_1=0$ MeV were used. (For ease of comparison, the cross section for the f to p, $L=2$, transition has been reduced by a factor of 2.)

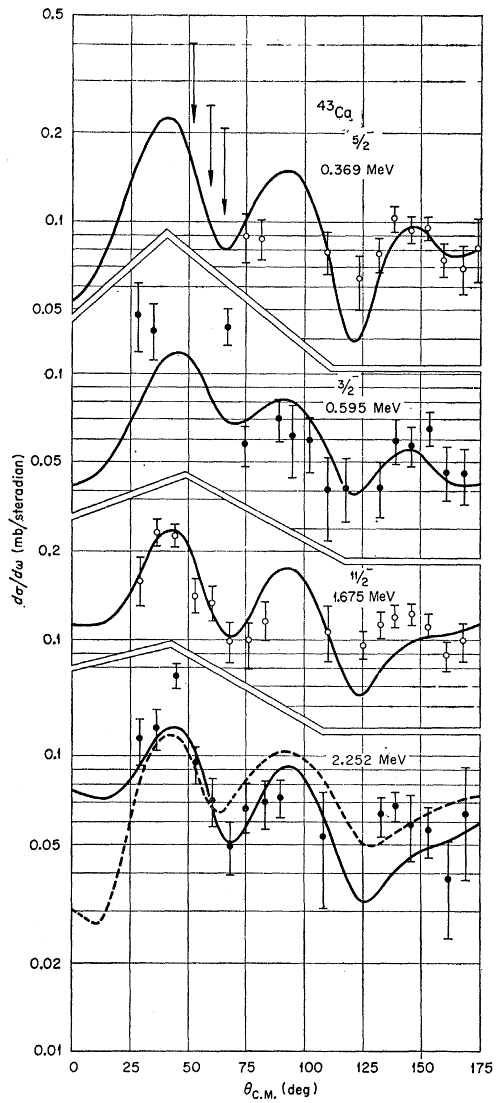

FIG. 10. Differential cross sections for exciting several states in ^{43}Ca with 8.5-MeV deuterons. The full theoretical curves are calculated assuming the recoupling of $(f_{7/2})^3$ neutrons; that for 2.252 MeV uses $L=2$ only. The dashed curve is for an $(f_{7/2}^2p_{3/2})$ configuration. The strengths are discussed in the text.

the other states make identification less certain; however, if there is no change of parity we can say all three transitions [to states (7), (10), and (12)] involve $L=2$.

As already mentioned, a vibrational description of this nucleus predicts that five levels of spins $J_f=\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, $\frac{11}{2}$ will be excited in the (d,d') reaction. In the approximation of weak coupling between particle and vibrational degrees of freedom, each should show an $L=2$ angular distribution with a shape very similar to that for excitation of the 2^+ vibrational state in the adjacent even nucleus.[2] Further, the cross-section magnitudes should be $(2J_f+1)/5(2J_i+1)$ times those for the even nucleus, where $J_i=\frac{7}{2}$ is the spin of the odd target. Any departures from the simple model would appear as departures from these rules. One particularly expects the $\frac{7}{2}^-$ excited state to have a smaller cross section than predicted because of mixing with the $\frac{7}{2}^-$ ground state.[2]

Of the eight levels seen in the (d,d') experiment, one has $L=4$ and one presumably corresponds to the $p_{3/2}$ "single-particle" state at 2.05 MeV which is strongly excited[21] in ^{42}Ca(d,p) (see Fig. 12). This leaves 6 levels instead of 5, and these appear at energies from 0.37 to 2.25 MeV, a spread which suggests that the weak-coupling model is a poor approximation. Nonetheless, it might be of some interest to see whether any inadequacy of the model shows up in the DW analysis of the (d,d') data. We may define a "partial" deformability

$$\beta_2{}^P=[(2J_f+1)/5(2J_i+1)]^{1/2}\beta_2,$$

which can be extracted from experiment without knowledge of the nuclear spins. If the model is valid, β_2 is the deformability of the even core, and the sum over $(\beta_2{}^P)^2$ for the odd nucleus should equal $\beta_2{}^2$. DW calculations using the "average" potential and the real part only of the collective model interaction yielded $\beta_2{}^P$ values of 0.114, 0.087, 0.13, 0.06, 0.08, 0.09, and 0.114 for the states at 0.369, 0.595, 1.675, 1.932, 2.051,

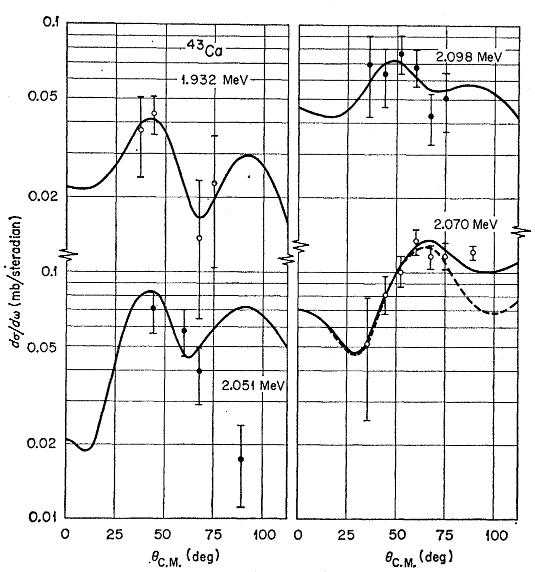

FIG. 11. Differential cross sections for exciting the remaining states observed in ^{43}Ca with 8.5-MeV deuterons. The theoretical curve for the 1.932-MeV level assumes the recoupling of an $f_{7/2}$ neutron, but with $L=2$ only; the 2.098-MeV group is compared to the predictions for the $\frac{9}{2}^-$ member of the $(f_{7/2})^3$ configuration. The 2.051-MeV curve assumes an $(f_{7/2}{}^2p_{3/2})$ configuration for this level. The full curve for the 2.07-MeV state is the prediction for the 15/2$^-$ member of the $(f_{7/2})^3$ configuration, while the dashed curve is for a pure $L=4$ transition obtained by recoupling an $f_{7/2}$ neutron. The strengths are discussed in the text.

[21] C. K. Bockelman, C. M. Braams, C. P. Browne, W. W. Buechner, R. D. Sharp, and A. Sperduto, Phys. Rev. **106**, 176 (1957).

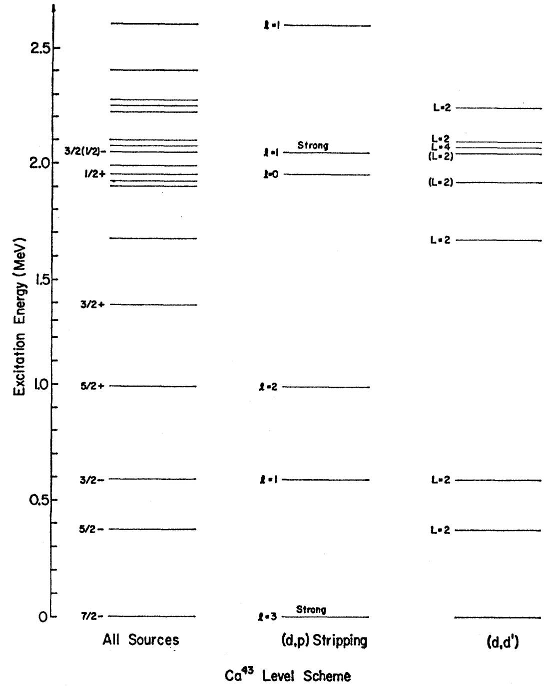

FIG. 12. Level scheme of ^{43}Ca. The scheme drawn to the left was taken from Ref. 6. The spin assignments for the two states close to 2 MeV follow from the results of Ref. 21. The scheme in the middle shows levels which are observed (Ref. 21) to be populated by (d,p) stripping on ^{42}Ca. The word "strong" means that the corresponding (d,p) transitions carry a sizable fraction of the available strength. The right-hand scheme shows those levels observed in the (d,d') reaction of the present experiment. Values of the orbital angular momentum of the transferred neutron (marked l) are shown in the (d,p) case; values of the dominant multipolarity of the (d,d') transitions are given (marked L) in the right-hand scheme.

2.098, and 2.252 MeV, respectively. Of course, the interpretation of the 1.932- and 2.051-MeV groups as $L=2$ is open to question. Only the spins of the two lowest states are known; these yield $\beta_2=0.28$ and 0.29. If we are guided[3,9] by the similar nucleus ^{51}V, we might interpret the 1.675-MeV level as $\frac{11}{2}$ and the 1.932-MeV level as $\frac{9}{2}$, giving $\beta_2=0.24$ and 0.12, respectively. If this interpretation is correct, the $\frac{9}{2}$ excitation is severely inhibited compared to the model predictions. Another candidate for $\frac{9}{2}$ is then the 2.252-MeV state, which would give $\beta_2=0.23$. Either of the weak states at 1.932 or 2.098 MeV could then be assigned $\frac{7}{2}$, the inhibition being attributed to mixing with the ground state. The root-mean-square $\beta_2{}^P$ obtained is then 0.26 or 0.23, according to whether we include the three weak states or not. Quadrupole contributions to the elastic scattering would increase these figures slightly. The corresponding value for the even ^{46}Ca is 0.28 (Sec. IV.1), and for the adjacent even ^{44}Ca it is 0.22 (Ref. 18).

Thus the ^{43}Ca(d,d') results may be in qualitative agreement with the excited-core vibrational model. However, this agreement was obtained partially by assuming spins for levels without corroborative evidence. Further, the angular distributions are fitted by including

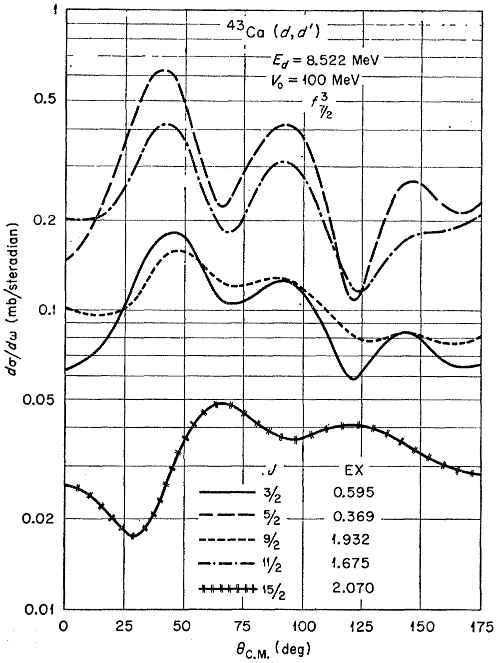

FIG. 13. Theoretical predictions for exciting the states of the $(f_{7/2})^3$ configuration with 8.5-MeV deuterons. A strength $V_0=100$ MeV is used to plot the curves, but the cross sections are proportional to V_0^2.

only the real part of the optical-model interaction. The use of complex coupling gives poorer fits.

The shell model for this nucleus assumes an $(f_{7/2})^3$ configuration for the three neutrons, with five excited states[3] of spins $\frac{3}{2}$, $\frac{5}{2}$, $\frac{9}{2}$, $\frac{11}{2}$, and $\frac{15}{2}$ with negative parity. (Compared to the vibrational model, spin $\frac{7}{2}$ is removed and spin $\frac{15}{2}$ introduced.) The level scheme should then be very similar to that of ^{51}V, where these spins appear at 0.93, 0.32, 1.82, 1.61, and 2.70 MeV, respectively.[3] The position of the $\frac{5}{2}^-$ state in ^{43}Ca at 0.369 MeV does agree well, but the $\frac{3}{2}^-$ has been depressed by about 300 keV relative to ^{51}V. This might be due to differences in the interaction of the $(f_{7/2})^3$ configuration with the $(f_{7/2}{}^2p_{3/2})$ configuration for the two spins; a small $p_{3/2}$ admixture in the $\frac{3}{2}^-$ state has been observed in the ^{42}Ca(d,p) reaction.[21]

Distorted-wave calculations were made using the shell-model wave functions and the interaction described in Sec. III and the Appendix. There is no spin-flip, $s=0$ and $L=J$ (see the Appendix) only, since we are concerned with transitions between states of j^n, and the ground state has seniority 1 while the excited states have seniority 3. The shell model allows mixtures of $L=2$, 4, and 6, weighted by the nuclear matrix elements M_L^2. Figure 13 shows the predicted differential cross sections for a strength $V_0=100$ MeV; the cross-section magnitudes are proportional to V_0^2. Again Coulomb excitation was included in the $L=2$ amplitudes, using the same value of $e'\langle r^2\rangle$ as for ^{44}Ca; the effects of Coulomb excitation for the higher multipoles are negligible.

The calculated $\frac{3}{2}^-$ and $\frac{5}{2}^-$ angular distributions fit the experimental distributions rather well. However, while the $\frac{3}{2}^-$ needs $V_0\approx 80$ MeV in agreement with ^{46}Ca, the $\frac{5}{2}^-$ needs only $V_0\approx 60$ MeV. The energy of the 1.675-MeV level is close to that for the $\frac{11}{2}$ state in ^{51}V, and its angular distribution is in good agreement with the spin $\frac{11}{2}$ predictions. The measured cross section implies $V_0\approx 75$ MeV, which is in reasonable agreement with the other states.

If the trend is for the ^{43}Ca states to be about 10% higher in energy than in ^{51}V, we would expect the 1.932-MeV level to have spin $\frac{9}{2}$. Its angular distribution is not well defined, but it is not in very good agreement with the spin-$\frac{9}{2}$ predictions; also it would require a strength V_0 of only 47 MeV. The angular distribution suggests mainly $L=2$, which if interpreted as due to the recoupling of an $f_{7/2}$ neutron has a strength

$$(2J_f+1)(V_0M_2)^2\approx 1200\ \text{MeV}^2.$$

On the other hand, the 2.098-MeV group is consistent with the $\frac{9}{2}^-$ assignment if $V_0\approx 67$ MeV, a strength closer to that required for the lower states.

The strong $L=4$ character of the 2.070-MeV state suggests its spin is $\frac{1}{2}$ or greater than $\frac{11}{2}$, unless an $L=2$ transition is forbidden for some other reason. This makes it a candidate for the $\frac{15}{2}^-$ assignment, even though its energy is 600 keV lower than the $\frac{15}{2}^-$ state in ^{51}V. However, while the predicted angular distribution is in good agreement with the measured one, a V_0 of 165 MeV is required; the cross section is at least four times larger than expected by comparison with the other, predominantly $L=2$, transitions. It is possible this discrepancy is due to a poor choice of parameters for the neutron bound state and for the range of the nucleon-deuteron interaction; the ratio of the $L=4$ and $L=2$ cross sections is particularly sensitive to this choice, while the angular distribution shapes are much less affected. If the excitation of the 2.070-MeV state is assumed to be pure $L=4$, and if it corresponds to the recoupling of an $f_{7/2}$ neutron with the parameters chosen here, the measured cross section requires

$$(2J_f+1)(V_0M_4)^2\approx 16\,000\ \text{MeV}^2.$$

The level at 2.051 MeV is presumably that one which is strongly excited by $l_n=1$ deuteron stripping,[21] suggesting that its structure is $(f_{7/2}{}^2p_{3/2})$ with the two f neutrons coupled to zero angular momentum, so its spin is $\frac{3}{2}^-$. This transition has matrix elements $M_2^2=0.154$, $M_4^2=0.085$, and the curve in Fig. 11 is drawn assuming $V_0=67$ MeV. Spin-flip is also allowed in this transition; for example, with $L=2$ we have $N_{22}^2=0.102$, $N_{23}^2=0.107$. Since V_1 is not known, we did not include these contributions. Due to energy-resolution difficulties the measured angular distribution of this group is not very well defined; however, the measured value at

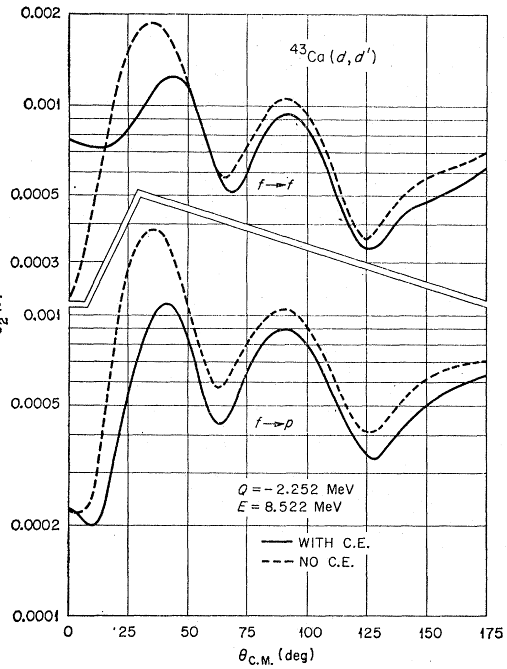

FIG. 14. Comparison of $L=2$ cross sections for an $f_{7/2}$ to $f_{7/2}$ transition with that for an $f_{7/2}$ to $p_{3/2}$ transition. The reduced cross sections of Eq. (A7) are plotted. C.E. stands for Coulomb excitation.

90° is a factor of 3 or 4 smaller than predicted, relative to the smaller angles.

The highest excited state studied, at 2.252 MeV, is strong and as shown by Fig. 10 exhibits an angular distribution with predominantly $L=2$ character. Two theoretical curves are shown in Fig. 12. One assumes an $(f_{7/2}{}^2p_{3/2})$ configuration with $J_f=\frac{3}{2}$ and $V_0=80$ MeV (again neglecting spin-flip), the other assumes an $L=2$ transition caused by the recoupling of an $f_{7/2}$ neutron with strength

$$(2J_f+1)(V_0M_2)^2\approx 4070\ \mathrm{MeV}^2.$$

If the latter were interpreted as due to an $(f_{7/2})^3$ component in a state with spin $\frac{5}{2}$ (for which the $L=4$ and 6 contributions are negligible), this would mean $V_0\approx 61$ MeV. It is interesting to note that if this strength is added to that for the $\frac{5}{2}$ state at 0.369 MeV, the root-mean-square V_0 is about 85 MeV. However, the 2-MeV separation of these two states makes such strong mixing seem unlikely.

Figure 14 compares the $L=2$ cross section for recoupling $(f_{7/2})^3$ neutrons with that for exciting the $(f_{7/2}{}^2p_{3/2})$ configuration. The "reduced" cross section of Eq. (A7) is plotted; the full curves include Coulomb excitation, whereas the dashed curves do not.

V. SUMMARY AND DISCUSSION

1. ^{46}Ca

The first excited state of ^{46}Ca at 1.347 MeV is identified as 2^+; if it is due to the recoupling of $f_{7/2}{}^{-2}$ neutrons, the model of the deuteron-nucleon interaction introduced here requires a strength $V_0\approx 80$ MeV. The 4^+ and 6^+ levels of this configuration are then predicted to be too weakly excited to be seen in the present measurement of the scattering of 10-MeV deuterons. Two states are seen in the 7-MeV ^{46}Ca(p,p') experiment, with energies of 2.42 and 2.58 MeV. The lower one bears the same energy ratio to the known 4^+ level in ^{42}Ca as does the 2^+ state to the corresponding state in ^{42}Ca. On this basis we would expect the 6^+ in ^{46}Ca to appear at 2.8 MeV. It is our experience[22] with the (p,p') reaction at $\approx$7-MeV bombarding energy on nuclei of $Z\approx 20$, that levels needing an angular-momentum transfer of six units are very weakly populated; thus a 6^+ state in ^{46}Ca may easily have been missed.

In further analogy to ^{42}Ca it may be speculated that one member of the 2.5-MeV doublet in ^{46}Ca is the analog of the 1.84-MeV 0^+ state in the former. It has been argued earlier that if such a state involves a mixture of $(f_{7/2})^2$ and higher two-particle configurations it would have a low (d,d') strength. However, the structure of the excited 0^+ state in ^{42}Ca is still an open question, so it is not possible as yet to draw any definite conclusions about the expected direct inelastic-scattering strength from such a state.

The level in ^{46}Ca at 3.023 MeV is excited in a manner consistent with a spin 2^+ or 3^+ of an $(f_{7/2}{}^{-3}p_{3/2})$ configuration. Its angular distribution is similar to that recently observed (and to be reported elsewhere) for the known 2^+ state at 2.4 MeV in ^{42}Ca.

Finally, the two levels at 3.614 and 4.434 MeV in ^{46}Ca are interpreted as 3^- "vibrations" with deformabilities of $\beta_3=0.16$ and 0.15, respectively. Since a conventional single-particle transition[20] has $\beta_3\approx 0.08$, these both represent enhancements of about a factor 4. In fact these two states fit very well into the energy systematics as well as into the strength systematics for $T=0$ octupole vibrations (e.g., see Refs. 20 and 23).

2. ^{43}Ca and the Reaction Model

It was shown that the ^{43}Ca(d,d') results could be in qualitative agreement with the excited-core collective model (provided only real coupling were used), although further information about the spins of some of the levels is needed before a final decision can be made. Comparison of these measurements with the shell model predictions for an $(f_{7/2}{}^3)$ configuration again gives qualitative agreement. The ratio of cross sections for the known $\frac{3}{2}^-$ and $\frac{5}{2}^-$ levels is in error by almost a factor of 2; the $\frac{3}{2}^-$ is also shifted by 335 keV in energy compared to the $\frac{3}{2}^-$ level in ^{51}V. The 1.675-MeV level is interpreted as $\frac{11}{2}^-$, and the ratio of its cross section

[22] T. A. Belote W. E. Dorenbusch, O. Hansen, and A. Sperduto, Phys. Letters 14, 323 (1965).

[23] O. Hansen and O. Nathan, Nucl. Phys. 42, 197 (1963); O. Nathan, *Nuclear Quadrupole and Octupole Vibrations* (Ejnar Munksgaard, Copenhagen, 1964).

to that for the $\frac{3}{2}^-$ level is in reasonable agreement with theory.

An $L=4$ transition is seen to the 2.070-MeV states. Because the cross section is proportional to $(2J_f+1)$, its great strength argues against a spin of $\frac{1}{2}$, and therefore a spin greater than $\frac{11}{2}$ is implied. It probably is the $\frac{15}{2}^-$ level, although its cross section is nearly 4 times that expected on the basis of the model used here.

The remaining $\frac{9}{2}^-$ member of the quintuplet could not be identified with any certainty. The most likely candidate is the level at 2.098 MeV, although the 1.932-MeV state could not be ruled out.

Measurements on the 2.05-MeV state were incomplete because of difficulties in resolving the levels in this region, but the results are consistent with its being the $(f_{7/2}{}^2p_{3/2})$ state strongly excited[21] in ^{42}Ca(d,p). The remaining, strong, excitation to 2.252 MeV is predominantly $L=2$ and, if assumed $\frac{5}{2}^-$, could account for the strength missing from the lower $\frac{5}{2}^-$ state. In any case, $L=2$ restricts its spin to values between $\frac{3}{2}$ and $\frac{11}{2}$ if spin-flip is unimportant. (Spin-flip would allow $\frac{1}{2}$ and $\frac{13}{2}$ also.)

In summary, we can say that the model introduced here for the deuteron-nucleon interaction gives results which are in over-all agreement with the measurements. In particular, the interaction strength required is close to that inferred from studies of inelastic proton scattering on similar nuclei.[9] In detail, the relative magnitudes for exciting the various states of ^{43}Ca do not seem to be in good agreement with shell-model predictions. However, before this discrepancy is taken as evidence that the $(f_{7/2})^3$ assumption is invalid, we should examine more closely the model used for the reaction mechanism. Ignoring spin-flip for the moment, if the effective interaction is assumed to be static it can always be written in the form[14,15]

$$V_L(r)Y_L{}^M(\theta,\phi).$$

We have taken a particular model for $V_L(r)$. However, all but one of the ^{43}Ca transitions are predominantly $L=2$, and we would expect the $V_2(r)$ to be the same for all transitions within the $(f_{7/2})^3$ configuration. Further, spin flip is not allowed for these transitions. Thus discrepancies found for the $L=2$ relative cross sections should be blamed on the nuclear wave functions. Similar conclusions would follow for moderately nonstatic (that is, nonlocal) interactions, because usually these can be well represented by an equivalent local potential. Another possibility not considered so far is the existence of a strong tensor component in the effective nucleon-deuteron interaction. This could arise both from the D component of the internal state of the deuteron and from the tensor part of the nucleon-nucleon force, as well as from polarization of the deuteron in the field of the nucleus. Such a tensor force would modify the predictions made in this paper for the relative excitations of the ^{43}Ca quintuplet (see Fig. 13).

Considerably more uncertainty attends the predictions for transitions to other configurations, or for other L values. The effective interaction $V_4(r)$ for $L=4$ relative to that $V_2(r)$ for $L=2$ is dependent upon the model chosen, and upon the parameter values for a given model. The same is true for a given L for transitions to, say, f^2p compared to f^3. For example, the $V_L(r)$ for f^2p transitions contain a node, while those for f^3 do not; the corresponding predicted cross sections are compared in Figs. 9 and 14. Studies of the sensitivity to parameter variations have not been made in the present case. (It should also be added that the variation of cross section with L and with the excitation energy is somewhat dependent upon the optical potential parameters also, particularly at the low energy used in the ^{43}Ca experiment.) It would be valuable to have accurate (d,d') data for a nucleus where the nuclear wave functions are believed to be well known so that the validity of the model used here could be explored in more detail.

ACKNOWLEDGMENTS

The assistance of Dr. H. R. Blieden in the early stages of this experiment is gratefully acknowledged. The ^{46}Ca(d,d') exposure at Aldermaston was kindly arranged by Dr. S. Hinds. The plates were carefully scanned by Miss Sus Vilmann, Miss Mette Nevald, Miss Vibeke Østerberg, and Miss Norma Hynne. We are indebted to R. M. Drisko for making available the optical-model search code HUNTER and the DW code JULIE, and to M. B. Johnson and L. W. Owen for the interaction form-factor code ATHENA. The assistance of Mrs. Patricia Crowther in the analysis of the (p,p') part of the work is appreciated.

APPENDIX

We assume that the interaction between the deuteron and the ith nucleon in the target is given by

$$v_{id}=\langle m'_d|v_{in}+v_{ip}|m_d\rangle, \qquad \text{(A1)}$$

where v_{in} and v_{ip} are the interactions with the component neutron and proton, respectively. The z component of spin of the deuteron is m_d; the possibility of $m'_d \neq m_d$ corresponds to spin-flip. For the nucleon-nucleon potential we take

$$v_{ij}=(a_0+a_1\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)g(r_{ij}), \qquad \text{(A2)}$$

where

$$a_s=a_{s\alpha}+a_{s\beta}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j. \qquad \text{(A3)}$$

Because the deuteron has zero isobaric spin, only the isospin-independent term $a_{s\alpha}$ will contribute to v_{id}. We soon find that

$$v_{id}=2\bar{g}(r_{id})(a_{0\alpha}+a_{1\alpha}\mathbf{s}_d\cdot\boldsymbol{\sigma}_i), \qquad \text{(A4)}$$

where $\mathbf{s}_d$ is the spin operator of the deuteron and

$$\bar{g}(r_{id})=\int u^2(\rho)g(|\mathbf{r}_{id}+\tfrac{1}{2}\boldsymbol{\rho}|)d\boldsymbol{\rho}. \tag{A5}$$

Here $u(\rho)$ is the deuteron-internal wave function, and $\boldsymbol{\rho}=\mathbf{r}_n-\mathbf{r}_p$; we have neglected the d-state part of $u(\rho)$. The shape of the function $\bar{g}$ will tend to be similar to that of g, with a somewhat increased range and decreased depth. For example, if we represent g by a Gaussian of range 1.8 F and approximate the deuteron wave function u by a $1s$ oscillator function of range 2.5 F, $\bar{g}$ is also a Gaussian, of range 2 F and with its strength reduced by a factor 0.72. In the calculations reported here, we used

$$2a_{s\alpha}\bar{g}(r)=-V_s\exp[-(r/R_G)^2], \tag{A6}$$

with $R_G=2$ F. Proton scattering from $1f$-shell nuclei implies that $a_{0\alpha}\approx-45$ MeV, which would give $V_0\approx65$ MeV; little is known about the effective strength $a_{1\alpha}$ or V_1.

The calculation proceeds by making a multipole expansion of the radial function $\bar{g}(r_{id})$ in the manner described in Refs. 9, 14, and 15. For excitations of a j^n configuration, $s=1$ is forbidden if seniority is a good quantum number and the ground state has the lowest seniority. The transition amplitude for $s=0$ is proportional to a nuclear matrix element M_L; values of M_L^2 for $(f_{7/2})^n$ have been given in Ref. 9. Transitions of the type $j^n\to(j^{n-1}j')$ can be described in the same way; the corresponding values of M_L^2 are quoted in the main text. For these latter transitions, the spin-flip with transfer of angular momentum $s=1$ is also allowed. The cross section for spin-flip is given by a similar expression except that M_L^2 is replaced by $\frac{2}{3}N_{LJ}^2$. Here N_{LJ} is a nuclear matrix like M_L except that the spherical harmonic Y_L^M is replaced by the spin-angle tensor[15] T_{L1J}. The total angular-momentum transfer can now be $J=L$, $L\pm1$, while the parity change is still given by $(-)^L$. There is no interference between contributions from different values of L and S if the optical potential which generates the distorted waves does not contain spin-orbit coupling, while any sum over J is always incoherent.[15] The cross section then has the form

$$\frac{d\sigma}{d\omega}=\frac{2J_f+1}{2J_i+1}\sum_{LJ}(V_0^2M_L^2\delta_{L,J}+\tfrac{2}{3}V_1^2N_{LJ}^2)\sigma_L(\theta), \tag{A7}$$

where $\sigma_L(\theta)$ is a "reduced" cross section which is independent of J in the present model. Of course, only the values

$$|J_f-J_i|\leq J\leq J_f+J_i$$

are allowed, where J_f and J_i are the final and initial nuclear spins. For the "single-particle" transition from an $s_{1/2}$ orbit to an orbit with $J=L+\frac{1}{2}$, M_L^2 has the value $(4\pi)^{-1}$, so that $(L+1)\sigma_L(\theta)/4\pi$ could be regarded conventionally as the "single-particle" cross section. A more detailed discussion of these matters and the properties of the nuclear matrix elements will be given elsewhere.