Angular-Correlation Studies for the $C^{12}(He^3,\alpha\gamma)C^{11}$, $C^{12}(He^3,p\gamma)N^{14}$, and $O^{18}(p,\alpha\gamma)N^{15}$ Reactions

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The particle-gamma angular correlation method of Litherland and Ferguson was used to gain information on the decay modes and spin assignments of the C^{11} 2.00-MeV level; the N^{14} 5.83- and 7.03-MeV levels; and the N^{15} 5.28-, 6.33-, 8.31-, and 8.57-MeV levels. The $C^{12}(He^3,\alpha)C^{11}$, $C^{12}(He^3,p)N^{14}$, and $O^{18}(p,\alpha)N^{15}$ reactions were used to populate the levels. Protons and α particles were detected in an annular counter at 170° to the beam and gamma rays were detected at angles between 20 and 90° to the beam. The spin of the C^{11} 2.00-MeV level was found to be $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$ with the quadrupole-dipole mixing parameter of the $2.00 \rightarrow 0$ transition given by $x=+(0.27\pm0.07)$, or $|x|>11$ for $\frac{3}{2}$ and $x=-(0.18\pm0.07)$ for $\frac{5}{2}$. The angular distribution of the N^{14} $7.03\rightarrow 0$ transition gave $0.15\leqslant x\leqslant 1.15$ for this $J=2$ to $J=1$ transition. A $7.03\rightarrow 3.95$ transition was observed with a branching ratio of $9\pm5\%$. The results for the N^{14} 5.83-MeV level were consistent with earlier work. The N^{15} 5.28-MeV level was assigned $J^\pi=\frac{5}{2}^{(+)}$ with a value of $-(0.15\pm0.06)$ or $+(6.1\pm1.4)$ for the octupole-quadrupole mixing ratio of the $5.28\rightarrow 0$ transition. The N^{15} 6.33-MeV level was assigned $J^\pi=\frac{3}{2}^{(-)}$ with a value of $+(0.09_{-0.03}^{+0.06})$ or $+(1.4_{-0.4}^{+0.1})$ for the $6.33\rightarrow 0$ mixing ratio. Most probable assignments of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ are made to the N^{15} 8.31- and 8.57-MeV levels on the basis of this and previous work. The results are compared to shell-model predictions.

I. INTRODUCTION

In this report we describe an application of method II of Litherland and Ferguson[1] to the study of some gamma-ray transitions in C^{11}, N^{14}, and N^{15}. These authors pointed out the great theoretical simplification gained by detecting the outgoing particles h_2 in a nuclear reaction $X(h_1,h_2)Y^*$ in an axially symmetrical counter at 0 or 180° to the beam. With this condition, only those magnetic substates of Y^* will be formed which have magnetic quantum numbers α equal to or less than the sum of the spins of X, h_1, h_2. Also, for unpolarized beams, $P(\alpha)=P(-\alpha)$, where $P(\alpha)$ is the population number of the substate with magnetic quantum number α. If only a few magnetic substates of Y^* are populated, then Litherland and Ferguson showed that the analysis of the angular distributions of the subsequent de-excitation gamma rays can be made independently of the reaction mechanism and can quite often lead to determinations of spins and gamma-ray multipole mixing parameters. Previous applications of this method have been made at this laboratory for the $Ca^{40}(p,p')C^{40}$ and $S^{32}(p,p')S^{32}$ reactions[2] and the $O^{16}(He^3,p)F^{18}$ reaction.[3]

In the present work the reactions $C^{12}(He^3,\alpha)C^{11}$ ($Q=1.856$ MeV), $O^{18}(p,\alpha)N^{15}$ ($Q=3.980$ MeV), and $C^{12}(He^3,p)N^{14}$ ($Q=4.779$ MeV) were used to study gamma-ray transitions from the C^{11} 2.00-MeV level, the N^{14} 5.83- and 7.03-MeV levels, and the N^{15} 5.271-, 5.300-, 6.238-, 8.31-, and 8.57-MeV levels. In the first two reactions only the $\alpha=\pm\frac{1}{2}$ magnetic substates of the excited levels in C^{11} and N^{15} are populated while in the $C^{12}(He^3,p)N^{14}$ reaction the $\alpha=0,\pm1$ magnetic substates of the N^{14} levels are populated.

The experimental procedure and method of analysis have been described in some detail in a previous report[3] so that only a brief description is given here (Sec. II). The results are presented in Sec. III and discussed in Sec. IV.

II. PROCEDURE

The He^3 beam used for the $C^{12}(He^3,\alpha)C^{11}$ and $C^{12}(He^3,p)N^{14}$ reaction studies was supplied by the Harwell Van de Graaff. An analyzed beam of about 0.2 μA with an energy of 4.67 MeV was used. The proton beam used for the $O^{18}(p,\alpha)N^{15}$ reaction studies was supplied by the Harwell tandem electrostatic generator. Beam energies between 7.7 and 9.0 MeV were used with analyzed beam currents of about 0.2 μA. The beam was stopped 3.6 m from the target. The particle counter used was one of various annular silicon semiconductor counters placed at 180° to the beam and normally 4.0 cm from the target. The sensitive area was an annular ring, the inner and outer edges of which subtended angles at the target center of 172 and 168°, respectively. The beam passed through a 6-mm-diam hole in the center of the counter.

Gamma rays were detected in a 12.7-cm-diam by 15.2-cm-long NaI(Tl) crystal with its front face 25 cm from the target. The angular correlation measurements were made for angles between the crystal and beam axis of 20 to 90°.

The carbon targets used were self-supporting foils of about 40 $\mu g/cm^2$. The O^{18} targets were prepared[4] by

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‡ NATO Science Fellowship, 1963–4. Present address: Chalk River Nuclear Laboratories, Chalk River, Canada.

§ Shell Oil (New Zealand) Ltd., Post Graduate Scholar.

1 A. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788 (1961).

2 A. R. Poletti and M. A. Grace (to be published).

3 A. R. Poletti and E. K. Warburton, Phys. Rev. **137**, B595 (1965).

4 The targets were kindly prepared by A. H. F. Muggleton of the Atomic Weapons Research Establishment, Aldermaston, Berks., England. See A. H. F. Muggleton and F. A. Howe, Nucl. Instr. Methods **12**, 192 (1961).

evaporation of tungsten oxide, enriched to $\approx 50\%$ in O^{18}, onto thin (10–20 μg/cm^2) carbon foils. The target thickness was about 180 μg/cm^2. Single-channel analyzers selected pulses corresponding to the charged particle groups it was desired to study. Coincidences were formed between these single-channel analyzer pulses and all the pulses from the gamma-ray detector. The resulting gamma-ray coincidence spectra were routed and stored in a 4×256-channel analyzer. Random spectra were also recorded. Either two real and two random or three real and one random spectra were recorded simultaneously.

For each particle group studied, gamma-ray spectra were recorded at five or more angles to the beam and resolved into monoenergetic gamma-ray lines. The angular distributions of the gamma-rays were then fitted by a Legendre polynomial expansion of the form

$$W(\theta)=I_\gamma[1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)], \qquad (1)$$

and the I_γ so determined were used, together with tables of efficiencies and photofractions for gamma rays detected in NaI(Tl) crystals,[5] to obtain branching ratios for the decaying level.

Computer-aided fits to the angular distributions were then made for the theoretical expressions for various assumed spins of the emitting level as described previously.[3]

III. RESULTS

A. The C^{11} 2.00-MeV Level

A partial charged-particle spectrum recorded by a 6500 Ω-cm n-type silicon counter following bombardment of a C^{12} target with a 4.67-MeV He^3 beam is shown in Fig. 1. The elastically scattered He^3 particles ($He^3{}_0$) and the α_1 group populating the first-excited state of C^{11} at 2.00 MeV are broadened and preferentially degraded by a thin gold coating on the face of the semiconductor counter. The counter resolution was adequate to separate all the particle groups expected[6] in the energy region shown in Fig. 1.

Gamma-ray spectra in coincidence with the α-particle group feeding the C^{11} 2.00-MeV level and the proton groups feeding the N^{14} 5.83- and 7.03-MeV levels were recorded simultaneously at angles to the beam of 20, 30, 45, 60, and 90°. Randoms in coincidence with the α_1 group were recorded in the remaining quadrant of the 4×256-channel analyzer.

The 60° gamma-ray spectrum in coincidence with the α_1 group, with the random spectrum subtracted, is shown in Fig. 2. As expected, the spectrum shows annihilation radiation and the gamma-ray peak corresponding to the C^{11} 2.00 → 0 transition. The angular distribution of the 2.00-MeV gamma ray was fitted by the least-squares method to the Legendre polynomial expansion [Eq. (1)] with the result $a_2=+(0.036\pm0.041)$, $a_4=(0.00\pm0.05)$, so that the distribution is isotropic within the uncertainty of the measurement.

The angular distribution was also fitted by the least-squares method to the theoretical expression[3] for a gamma-ray transition between a state of spin J and a $J^\pi=\frac{3}{2}^-$ ground state[6] for assumed values of $\frac{1}{2}$ through $\frac{9}{2}$ for J. The transition was assumed to be a mixture

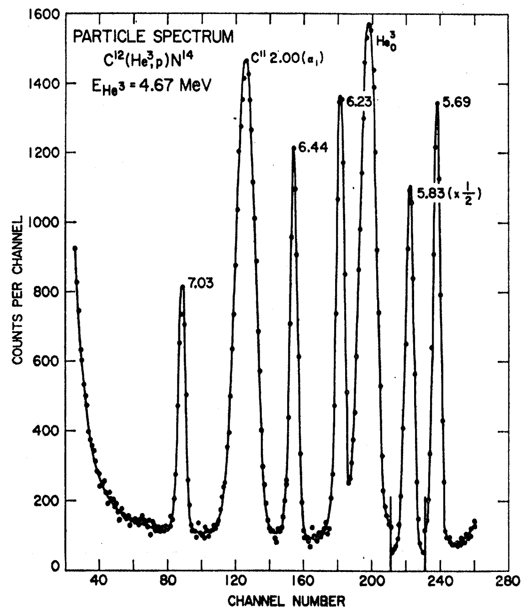

FIG. 1. Particle spectrum in the 180° annular counter from bombardment of a ~40 μg/cm^2 carbon target with a 4.67-MeV He^3 beam. The N^{14} proton peaks are identified by the excitation energies (in MeV) of the levels to which they correspond. The α-particle group from the reaction $C^{12}(He^3,\alpha)C^{11}$ (2.00-MeV level) and the elastic He^3 group are also identified.

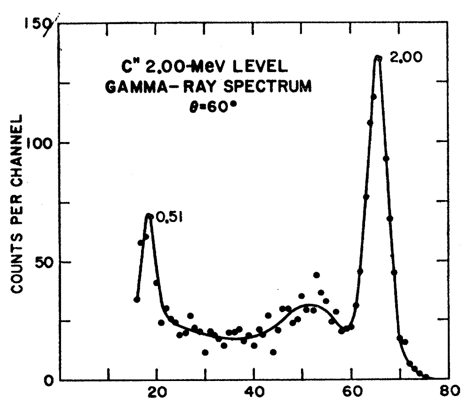

FIG. 2. Spectrum of gamma rays observed at 60° to the beam in coincidence with the α-particle group populating the C^{11} 2.00-MeV level in the $C^{12}(He^3,\alpha)C^{11}$ reaction at a He^3 energy of 4.67 MeV. The randoms have been subtracted.

[5] S. H. Vegors, L. L. Marsden, and R. L. Heath, Phillips Petroleum Company, Atomic Energy Division, IDO-16370 (unpublished).

[6] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959); T. Lauritsen and F. Ajzenberg-Selove, *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D. C., 1962. NRC 61–Sets 5, 6 (339 pp.)).

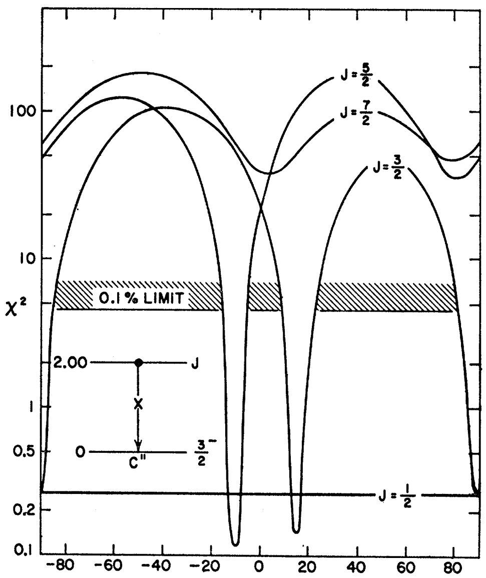

FIG. 3. χ^2 versus arctan x curves for the ground-state decay of the C^{11} 2.00-MeV level and assumed spins of $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ for the 2.00-MeV level. For a correct solution the expectation value of χ^2 is unity and the probability of χ^2 exceeding the value of χ^2 marked as the 0.1% limit is 0.1%.

of L and $L+1$ radiation where L is the lowest allowed multipolarity. The least-squares fit was performed for discrete values of arctan x between -90 and $90°$ where x is the amplitude ratio of the multipolarities $L+1$ and L. The phase convention used for x here and throughout this section is that of Litherland and Ferguson[1] for a $ML,EL+1$ mixture. The result of the least-squares fits is illustrated by the χ^2 versus arctan x curves[3] of Fig. 3. From those curves we conclude that $J=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ are all allowed by the angular distribution but that $J=\frac{7}{2}$ is not. For $J=\frac{9}{2}$ the χ^2 curve (not shown) had a lowest value for χ^2 of 48 so that $J=\frac{9}{2}$ is not allowed either.

The curves shown in Fig. 3 were calculated assuming that the $\alpha=\pm\frac{1}{2}$ magnetic substates of the C^{11} 2.00-MeV level are the only ones populated. However, because of the finite size of the particle counter we expect some ($\lesssim 3\%$) population of the $\alpha=\pm\frac{3}{2}$ magnetic substates also.[1] This finite-size effect (FSE) was estimated in this case and in the analysis of the $O^{18}(p,\alpha)N^{15}$ results by performing the least squares fits with $P(\frac{3}{2})=0.1P(\frac{1}{2})$, and sometimes $P(\frac{3}{2})=0.05P(\frac{1}{2})$ also, as well as with $P(\frac{3}{2})=0$. In the present case the FSE was found in this way to be of no importance.

In conclusion, then, we find $J\leqslant\frac{5}{2}$ for the C^{11} 2.00-MeV level with the quadrupole-dipole mixing ratio x undetermined for $J=\frac{1}{2}$, $x=-(0.18\pm0.07)$ for $J=\frac{5}{2}$, and $x=+(0.27\pm0.07)$ or $|x|>11$ for $J=\frac{3}{2}$.

B. The N^{14} 7.03-MeV Level

The sum of all five gamma-ray spectra in coincidence with the proton group feeding the N^{14} 7.03-MeV level is shown in Fig. 4. The randoms for this spectra, which have been subtracted, and that for the spectra in coincidence with the proton group feeding the 5.83-MeV level were obtained by normalizing to the total number of counts recorded in the proton channels relative to those in the α_1 channel. The peak labeled C^{11} 2.00 in Fig. 4 arises from coincidences with α particles from the tail of the α_1 group which fall into the energy region of the single-channel analyzer channel set on the 7.03-MeV proton group. The other gamma-ray peaks identified in Fig. 4 are assigned to de-excitation of the N^{14} 7.03-MeV level. The intensities of these gamma rays were extracted with the resulting decay scheme shown in the insert of Fig. 4. There is obviously no conclusive evidence for a 3.08-MeV gamma ray but the shape of the spectrum is consistent with a 3.08-MeV gamma ray with the same intensity as the 2.31- and 1.64-MeV gamma rays as is expected for a $7.03\to3.95\to2.31\to0$ cascade. (The 3.95-MeV level branches 96% to the 2.31-MeV level.[6])

The measured angular distribution of the N^{14} $7.03\to0$ transition was characterized by $a_2=-(0.84\pm0.08)$, $a_4=(0.00\pm0.08)$, i.e., an almost pure $\sin^2\theta$ distribution. The χ^2 versus arctan x curves for the $7.03\to0$ transition are shown in Fig. 5 for a $J^\pi=1^+$ assignment to the N^{14} ground state[6] and $J=1$, 2, and 3 for the N^{14} 7.03-MeV level. It is seen that $J=3$ is not allowed but that $J=1$ and 2 are possible for a large range of values of x. The assignment $J=2$ was uniquely determined by investigations[7–9] of the $C^{13}(p,\gamma)N^{14}$ reaction. For $J=2$ we find the 0.1% limits on the quadrupole-dipole mixing ratio to be $0.08\leqslant x\leqslant1.43$ and to one standard deviation: $0.15\leqslant x\leqslant1.15$. These values are in good

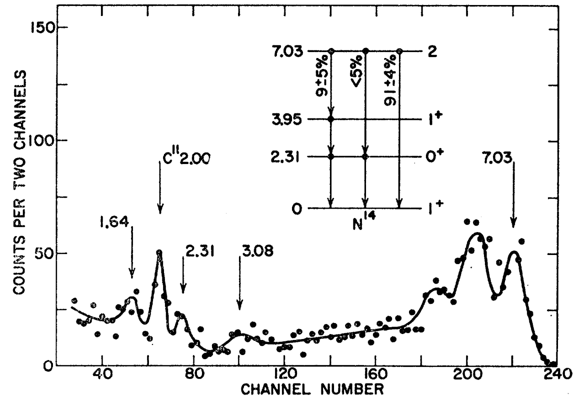

FIG. 4. Spectrum of gamma rays in coincidence with the proton group populating the N^{14} 7.03-MeV level in the $C^{12}(He^3,p)N^{14}$ reaction at a He^3 energy of 4.67 MeV. The spectrum is the sum of five spectra taken at five different angles to the beam. The randoms have been subtracted. All of the indicated gamma rays except the C^{11} 2.00-MeV gamma ray are associated with the decay of the N^{14} 7.03-MeV level as can be inferred from the inserted decay scheme.

[7] H. J. Rose, Nucl. Phys. **19**, 113 (1960).
[8] F. W. Prosser, Jr., R. W. Krone, and J. J. Singh, Phys. Rev. **129**, 1716 (1963).
[9] H. J. Rose, F. Riess, and W. Trost, Nucl. Phys. **52**, 481 (1964).

agreement with previous measurements of the mixing ratio which gave $0.13<x<3.5$,[7] $+(0.6\pm0.1)$,[8] and $0.02<x<3$.[9]

The FSE was estimated by setting $P(2)=0.1P(0)$ and $P(2)=0.1P(1)$ in turn. For $J=3$, the effect was negligible, while for $J=2$ the effect for both cases was to narrow the limits on x. In general, any conceivable effect of the finite counter size can be shown in the present case to narrow the allowed limits on x for the case $J=2$.

C. N^{14} 5.83-MeV Level

The sum of all five spectra, with randoms subtracted, for the gamma rays in coincidence with the proton group feeding the N^{14} 5.83-MeV level is shown in Fig. 6. The assignment of the labeled gamma rays to the transitions resulting from de-excitation of the 5.83-MeV level can be inferred from the inserted decay scheme. The branching ratios extracted from the angular distributions and the relative intensities are indicated in the decay scheme. The branches obtained for the 5.10-MeV level decay are in excellent agreement with the best previous values[10] of $(72\pm3)\%$ and $(28\pm3)\%$ for the $5.10\rightarrow0$ and $5.10\rightarrow2.31$ branches, respectively. However, the branching ratios found for the $5.83\rightarrow0$ and $5.83\rightarrow5.10$ transitions are in rather poor agreement with the previous values[11] of $(16\pm4)\%$ and $(84\pm4)\%$. The average of these previous values with the present ones is $(20\pm4)\%$ and $(80\pm4)\%$, respectively.

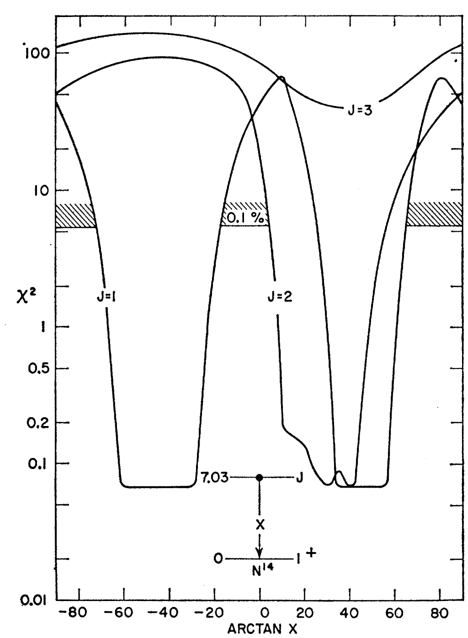

FIG. 5. χ^2 versus arctan x curves for the N^{14} $7.03\rightarrow0$ transition and assumed spins of 1, 2, and 3 for the 7.03-MeV level.

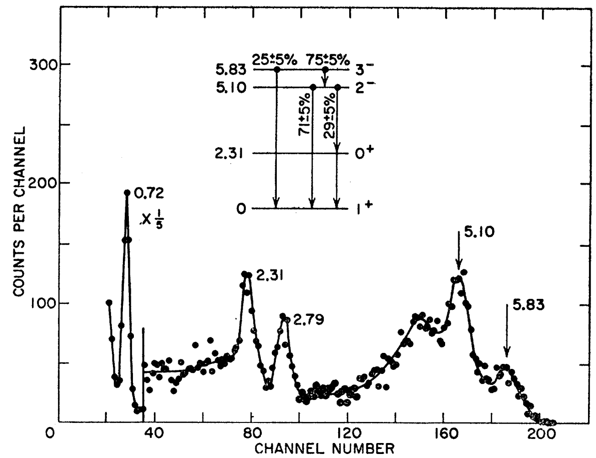

FIG. 6. Spectrum of gamma rays in coincidence with the proton group populating the N^{14} 5.83 MeV level in the $C^{12}(He^3,p)N^{14}$ reaction at a He^3 energy of 4.67 MeV. The spectrum is the sum of five spectra taken at five different angles to the beam. The randoms have been subtracted. The gamma-ray peaks associated with the inserted decay scheme are indicated.

The measured angular distributions for the de-excitation gamma rays from the 5.83-MeV level are characterized by $a_2=+(0.88\pm0.15)$, $a_4=+(0.39\pm0.16)$ for the $5.83\rightarrow0$ transition; $a_2=-(0.26\pm0.05)$, $a_4=-(0.11\pm0.07)$ for the $5.83\rightarrow5.10$ transition; $a_2=+(0.18\pm0.05)$, $a_4=+(0.05\pm0.06)$ for the $5.10\rightarrow0$ transition; and $a_2=+(0.55\pm0.20)$, $a_4=-(0.36\pm0.25)$ for the $5.10\rightarrow2.31$ transition.

The gamma-ray angular distributions were fitted to the theoretical expressions using the χ^2 versus arctan x computer programs assuming the spin-parity assignments[6,12] 3^-, 2^-, and 0^+ for the N^{14} 5.83-, 5.10-, and 2.31-MeV levels. The purpose of this analysis was to confirm previous[10–12] determinations of the mixing parameters for the $5.83\rightarrow5.10$, $5.83\rightarrow0$, and $5.10\rightarrow0$ transitions.

In Fig. 7 is shown a least-squares fit of the theory[3] to the two transitions $5.83\rightarrow5.10$ and $5.10\rightarrow2.31$ with the quadrupole-dipole mixture of the $5.83\rightarrow5.10$ transition variable. The dashed curve is the FSE estimated by $P(2)=0.1P(0)$, the curve for $P(2)=0.1P(1)$ is practically undistinguishable from the solid curve, i.e., from $P(2)=0$. The minimum near arctan $x=0$ gives $x=-(0.03\pm0.05)$. The minimum near arctan $x=75°$ can be excluded with high probability since the χ^2 at the minimum is between the 0.1% limit and the 6% limit for $P(2)$ between 0.0 and 0.1 and it is quite unlikely that $P(2)$ is greater than 0.03.[1,3] Thus, the present result confirms the conclusion made by Becker[13] that the $5.83\rightarrow5.10$ transition is predominantly dipole. The value of $-(0.03\pm0.05)$ obtained for x is in good agreement with the smaller of the two previously allowed values[11] $x=-(0.045\pm0.045)$ and $x=+(4.8\pm0.8)$.

The results of the measurement of the octupole-

[10] E. K. Warburton, J. W. Olness, D. E. Alburger, D. J. Bredin, and L. F. Chase, Jr., Phys. Rev. **134**, B338 (1964).
[11] E. K. Warburton, H. J. Rose, and E. N. Hatch, Phys. Rev. **114**, 214 (1959).
[12] E. K. Warburton, D. E. Alburger, A. Gallmann, P. Wagner, and L. F. Chase, Jr., Phys. Rev. **133**, B42 (1964).
[13] J. A. Becker, Phys. Rev. **131**, 322 (1963).

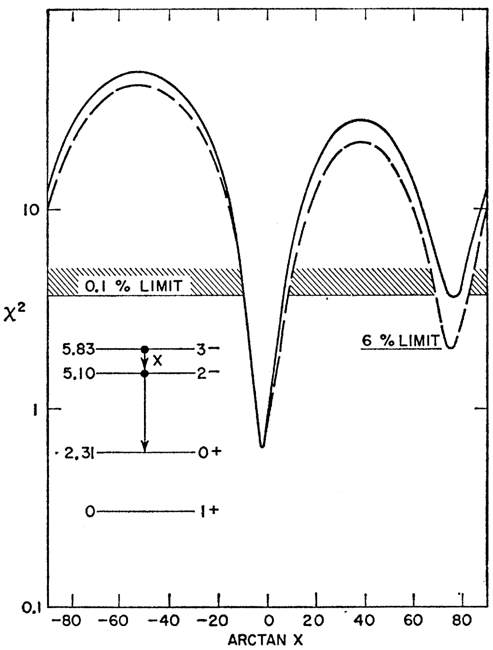

FIG. 7. χ^2 versus arctan x curve for a simultaneous fit to the N^{14} 5.83 → 5.10 and 5.10 → 2.31 transitions. The 5.10 → 2.31 transition is pure quadrupole and the mixing parameter of the 5.83 → 5.10 transition is varied. The broken curve is for $P(2)=0.1P(0)$.

quadrupole mixing ratio for the 5.83 → 0 transition are illustrated in Fig. 8. Figure 8(a) shows the result of a fit to the 5.83 → 0 transition while Fig. 8(b) shows the result of a simultaneous fit to the 5.83 → 0, 5.83 → 5.10, and 5.10 → 2.31 transitions with the 5.83 → 5.10 transition assumed to be pure dipole. The FSE was negligible in both cases. The two results are in good agreement with the single fit to the 5.83 → 0 transition slightly more restrictive on x. The agreement of these two results shows that the three angular distributions used in the fit of Fig. 8(b) are all consistent with the same ratio $P(0)/P(1)$ for the population of the 5.83-MeV level. The allowable limits on x are $-9.5<x<-0.27$ to one standard deviation (34% limit) and $x<0$ or >7 with 99.9% probability. This result is consistent with, but less accurate than, the previous result[11] of $-4.0<x<-0.4$.

The analysis of the angular distribution of the 5.10 → 0 transition is illustrated by Fig. 9 which shows a two-distribution fit to the 5.83 → 5.10 and 5.10 → 0 transitions with the former assumed to be pure dipole. The minimum near arctan $x=12°$ gives $x=-(0.17\pm0.12)$. The FSE has negligible effect on this minimum. This result is in excellent agreement with the best previous value[10] of $-(0.14\pm0.03)$ but again is less accurate. The solution for x allowed by the χ^2 minimum near arctan $x=80°$ in Fig. 9 is excluded by the fact that the N^{14} 5.10 → 0 transition is predominantly dipole.[12]

D. The N^{15} 5.28- and 5.30-MeV Levels

The first and second excited states of N^{15} at 5.271 and 5.300 MeV are most probably $J^\pi=\frac{5}{2}^+$ and $\frac{1}{2}^+$, respectively.[6] However, a survey of the literature shows that these assignments have not been rigorously fixed from *experimental* evidence alone. Our purpose in studying the gamma-ray decay of this doublet was to gain what further information we could on their spin assignments and on the multipole mixing of their ground-state decays. The only information bearing on these two levels that we shall accept without further question is that the N^{15} 5.300-MeV level has $J^\pi=\frac{1}{2}^+$ or $\frac{3}{2}^+$. These possible assignments follow from a clear $l_n=0$ assignment to the $C^{14}(d,p)C^{15}$ (ground-state) angular distribution[14] which fixes the C^{15} ground state as $J^\pi=\frac{1}{2}^+$, and the clearly allowed beta decay (log $ft=4.1$) of C^{15} to the N^{15} 5.30-MeV level.[15,16]

A partial α-particle spectrum obtained in a ≈750 Ω-cm n-type silicon annular counter following bombardment of an O^{18} target with an 8.925-MeV proton beam is shown in Fig. 10. The dimensions of the counter were

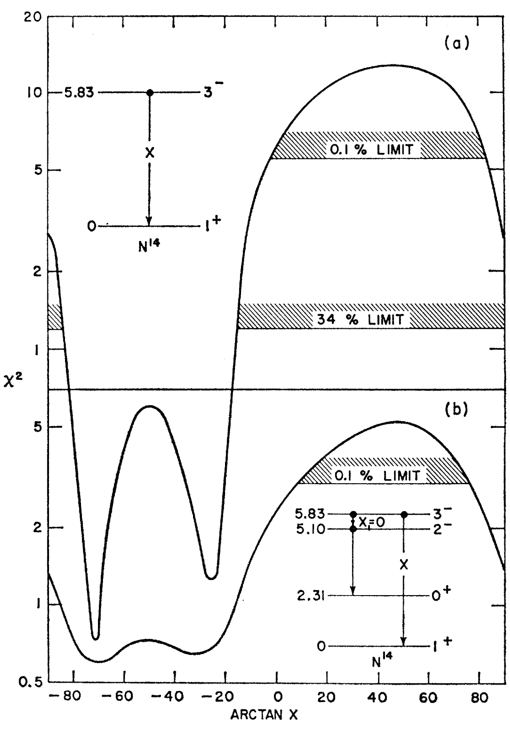

FIG. 8. χ^2 versus arctan x curves for the decay of the N^{14} 5.83-MeV level. The inserted decay schemes indicate the angular distributions included in the least-squares fit. The 34% limit corresponds to one standard deviation.

[14] D. J. Pullen, D. H. Wilkinson, and A. B. Whitehead, *Proceedings of the Rutherford Jubilee International Conference, Manchester*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 565.

[15] D. E. Alburger, A. Gallmann, and D. H. Wilkinson, Phys. Rev. **116**, 939 (1959).

[16] D. E. Alburger, C. Chasman, K. W. Jones, and R. A. Ristinen, Phys. Rev. **136**, B913 (1964).

the same as those of the proton counter. The rapid rise in counts below channel 70 is mostly due to the continuum of counts from protons which were not stopped in the active region of the silicon counter. The resolution of the α peaks in Fig. 10 is about 4% full width half-maximum (FWHM) which was adequate for study of the $\alpha_{1,2}$ doublet and α_3 groups but was not sufficient to resolve lower energy α groups. The real and random gamma-ray spectra in coincidence with the $\alpha_{1,2}$ doublet and the $\alpha_{7,8}$ doublet were recorded simultaneously. Fourteen gamma-ray spectra in coincidence with the $\alpha_{1,2}$ group were taken at 8 angles. The spectra were all characteristic of a 5.3-MeV gamma ray. The sum of nine of the fourteen spectra with the randoms subtracted is shown in Fig. 11. No attempt was made to

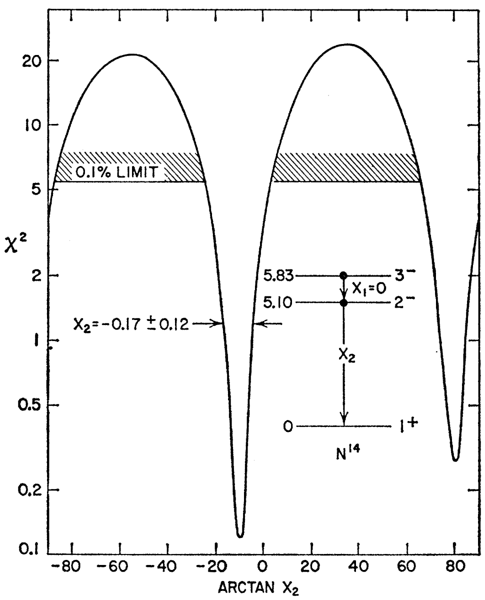

FIG. 9. χ^2 versus arctan x curve for a simultaneous fit to the N^{14} 5.83 → 5.10 and 5.10 → 0 transitions. The 5.83 → 5.10 transition is assumed to be pure dipole and the mixing parameter of the 5.10 → 0 transition is varied.

separate the contributions of α_1 and α_2 to the spectra or to separate the contributions of the 5.28- and 5.30-MeV gamma rays. The angular distribution of the composite gamma-ray doublet was fitted by the Legendre polynomial expansion [Eq. (1)] with even powers of $P_k(\cos\theta)$ up to $k=6$ included. The result was $a_2=+(0.419\pm0.024)$, $a_4=-(0.13\pm0.036)$, and $a_6=+(0.026\pm0.048)$.

The analysis of the angular distribution of the decay of the 5.3-MeV doublet to the N^{15} ground state which has[6] $J^\pi=\frac{1}{2}^-$ was commenced by assuming $J=\frac{1}{2}$ for one member of the doublet and assuming spins between $\frac{1}{2}$ and $\frac{9}{2}$ for the other.

For only the $\alpha=\pm\frac{1}{2}$ magnetic substates populated and $P(\frac{1}{2})=P(-\frac{1}{2})$, the angular distribution of a transition is completely fixed theoretically for a given value of the

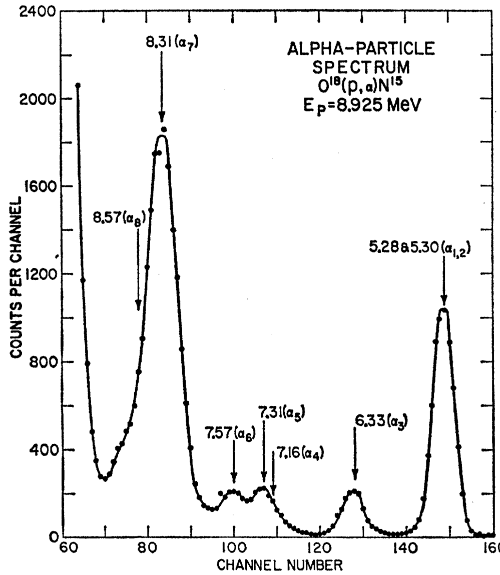

FIG. 10. Alpha-particle spectrum in the 180° annular counter from bombardment of a ≈180 μg/cm², ≈50% O^{18} target with 8.925-MeV protons. The N^{15} α-particle peaks are identified by the excitation energies (in MeV) and sequence of the levels to which they correspond.

mulitpole mixing parameter x. Thus for fixed values of the two mixing parameters the only unknown occurring in the theoretical expression for the composite 5.28 → 0 and 5.30 → 0 angular distribution is the ratio of the intensities for feeding the two levels, i.e., $I(\alpha_1)/I(\alpha_2)$. For $J=\frac{1}{2}$ assumed for one member of the doublet, the least-squares fitting procedure was performed for discrete values of arctan x_1, where x_1 is the mixing parameter of the member of the doublet with J not fixed at $\frac{1}{2}$. For each least-squares fit the computer output gave χ^2 and the value of the intensity ratio $I(\alpha_1)/I(\alpha_2)$ which resulted in the best fit to the theoretical expression.

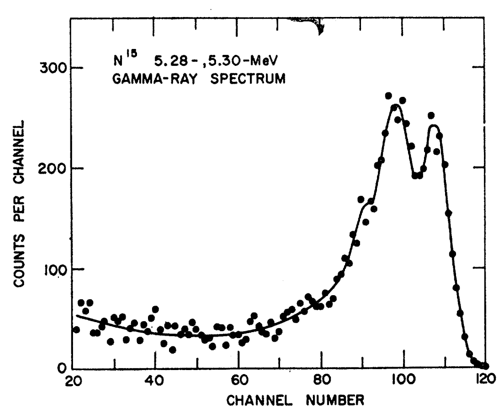

FIG. 11. Spectrum of gamma rays in coincidence with the α-particle groups populating the N^{15} 5.28- and 5.30-MeV levels in the $O^{18}(p,\alpha)N^{15}$ reaction at a proton energy of 8.925 MeV. The spectrum is the sum of nine spectra taken at eight angles to the beam. The randoms have been subtracted.

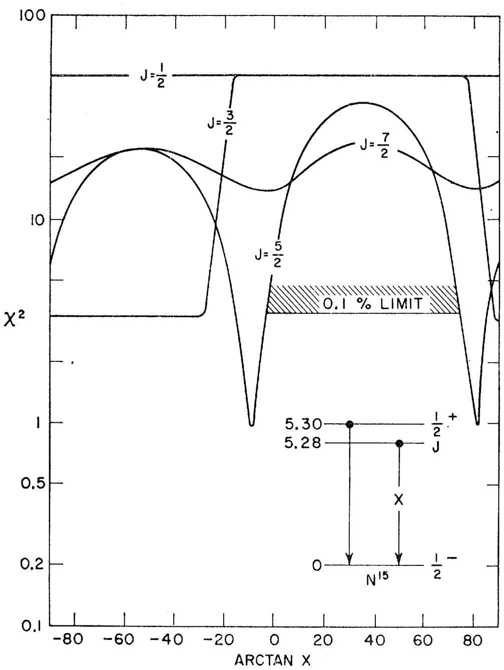

FIG. 12. χ^2 versus arctan x curves for a fit to the sum of the N^{15} 5.28 $\rightarrow$ 0 and 5.30 $\rightarrow$ 0 transitions. The 5.30-MeV level is assumed to be $J^\pi = \frac{1}{2}^+$ with the 5.30 $\rightarrow$ 0 pure dipole and the mixing parameter of the 5.28 $\rightarrow$ 0 transition is varied.

The FSE was estimated in the usual way by repeating with $P(\frac{3}{2}) = 0.1P(\frac{1}{2})$. The result of this procedure is illustrated by Fig. 12 which shows χ^2 versus arctan x curves for an assignment of $\frac{1}{2}$ to one member of the doublet and $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, or $\frac{7}{2}$ to the other. For $J = \frac{1}{2}$ or $\frac{3}{2}$, χ^2 is everywhere above (or nearly so) the 0.1% limit so one member of the doublet must have $J \geqslant \frac{5}{2}$ if the other has $J = \frac{1}{2}$. Furthermore, if one member of the doublet has $J = \frac{3}{2}$, then the other cannot have $J = \frac{1}{2}$ or $J = \frac{3}{2}$ since in the first case the appropriate curve is that labeled $J = \frac{3}{2}$ and in the second case the lowest value of χ^2 cannot be lower than the smallest value for the $J = \frac{3}{2}$ curve of Fig. 2. Thus we conclude that one member of the doublet has $J \geqslant \frac{5}{2}$. (This conclusion also follows from the observation of a nonzero value for a_4 in the angular distribution.) And since the 5.30-MeV level has $J^\pi = \frac{1}{2}^+$ or $\frac{3}{2}^+$, the 5.28-MeV level must have $J \geqslant \frac{5}{2}$.

For an assignment of $J = \frac{1}{2}$ to the 5.30-MeV level the lowest value of χ^2 for a $J = \frac{9}{2}$ assignment to the 5.28-MeV level is 12. Also, it is clear from Fig. 12 that the upper and lower members of the doublet cannot have $J = \frac{1}{2}$ and $\frac{7}{2}$, respectively. The FSE has no effect on these results thus we conclude that the 5.28-MeV level has $J = \frac{5}{2}$ if the 5.30-MeV level has $J = \frac{1}{2}$. In this case the octupole-quadrupole mixing parameter x for the 5.28 $\rightarrow$ 0 transition as determined by the $J = \frac{5}{2}$ curve of Fig. 12 can be either $x = -(0.15 \pm 0.06)$ or $x = +(6.1 \pm 1.4)$ where the uncertainties include the FSE. The intensity ratio $I(\alpha_1)/(\alpha_2)$ which gave the best fit to the measured angular distribution was 1.4 ± 0.2 at both values of x.

We now consider the case where the 5.30-MeV level has $J^\pi = \frac{3}{2}^+$ and the 5.28-MeV level has $J = \frac{5}{2}, \frac{7}{2}$, or $\frac{9}{2}$. In this case the 5.30 $\rightarrow$ 0 transition will be a mixture of $E1$ and $M2$. The procedure used was to assume a value for the $E1,M2$ mixing parameter (x_2) of this transition and then to proceed in the normal way with the mixing parameter (x_1) of the 5.28 $\rightarrow$ 0 transition variable. The results for $x_2 = 0$ are shown in Fig. 13. It is clear from this figure that $J = \frac{7}{2}$ and $\frac{9}{2}$ assignments to the 5.28-MeV level are not allowed if the 5.30-MeV level has $J = \frac{3}{2}$ and the 5.30 $\rightarrow$ 0 transition is pure dipole.

The possibility of an assignment of $J = \frac{7}{2}$ or $\frac{9}{2}$ to the 5.28-MeV level was investigated further by repeating this procedure for various values of x_2. In this way the complete x_1, x_2 map was explored. It was found that for all values of x_1 and x_2, χ^2 was larger than the value of the 0.1% limit for either $J = \frac{7}{2}$ or $\frac{9}{2}$. The FSE shifted the values of x_1 at the χ^2 minima but did not lower the values of χ^2 at the minima. Thus the N^{15} 5.28-MeV level has been shown to have $J = \frac{5}{2}$ regardless of whether the 5.30-MeV level has $J = \frac{1}{2}$ or $\frac{3}{2}$.

Next the positions of the χ^2 versus x_1 minima for $J = \frac{5}{2}$ were investigated as a function of x_2 and it was found that as x_2 varied from -0.3 to 7.0 the values of x_1 at the minima varied from the values for $x_2 = 0$ by less than $\pm 10\%$. Solutions with $x_2 < -0.7$ or $x_2 > 10$ were not allowed. Since the possibility of admixutres of $M2$ radiation larger than 10% (i.e., $x_2^2 = 0.1$) in the

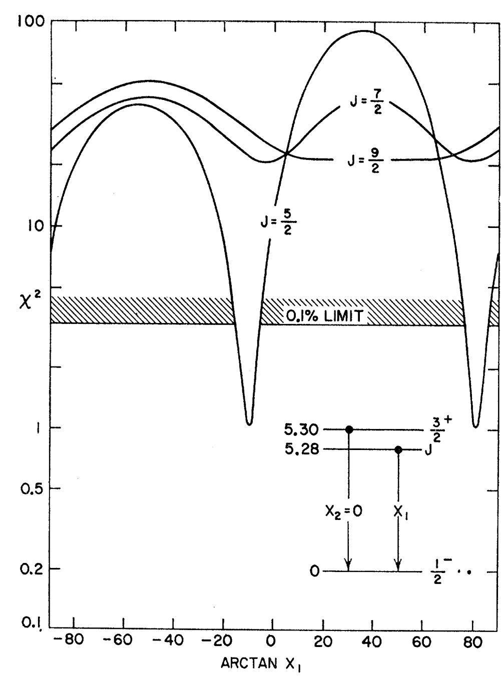

FIG. 13. χ^2 versus arctan x curves for a fit to the sum of the N^{15} 5.28 $\rightarrow$ 0 and 5.30 $\rightarrow$ 0 transitions. The 5.30-MeV level is assumed to be $J^\pi = \frac{3}{2}^+$ with the 5.30 $\rightarrow$ 0 transition pure dipole, and the mixing parameter of the 5.28 $\rightarrow$ 0 transition is varied.

$5.30 \to 0$ transition is exceedingly remote we conclude that the allowed values of x_1 are given to $\pm 10\%$ by the χ^2 minima of Fig. 13. These minima are at the same values of x_1 as those for a $J=\frac{1}{2}$ assignment to the 5.30-MeV level[17] (compare Figs. 12 and 13). Thus, we conclude that the octupole-quadrupole mixing ratio of the $5.28 \to 0$ transition is $-(0.15 \pm 0.06)$ or $+(6.1 \pm 1.4)$ regardless of whether the 5.30-MeV level has $J=\frac{1}{2}$ or $\frac{3}{2}$. A value for this mixing ratio significantly different from zero is reasonable for a $M2,E3$ mixture but would be quite surprising for an $E2,M3$ mixture. Thus, the present results give a strong preference for an assignment to the 5.28-MeV level of $\frac{5}{2}^+$ rather than $\frac{5}{2}^-$.

E. The N^{15} 6.33-MeV Level

The angular distribution of the N^{15} $6.33 \to 0$ transition in coincidence with the α-particle group (α_3) feeding the 6.33-MeV level was measured at $E_p=8.625$ MeV and at $E_p=7.70$ MeV. At $E_p=8.625$ MeV the distribution was measured with the annular counter at 2 cm from the target as well as at the normal distance of 4 cm. In all cases the NaI(Tl) crystal was at a distance of 25 cm from the target.

The measured angular distributions were characterized by $a_2=-(0.57 \pm 0.05)$ for $E_p=7.70$ MeV and by $a_2=-(0.62 \pm 0.04)$ and $a_2=-(0.70 \pm 0.06)$ for $E_p=8.625$ MeV and the annular counter at 4.0 and 2.0 cm, respectively. In all three cases there was no evidence for terms in a_4. The χ^2 versus arctan x fits to the angular distribution measured at $E_p=8.625$ MeV with the annular counter at 4.0 cm are shown in Fig. 14 for assumed spins of $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ to the 6.33-MeV level. A spin assignment of $\frac{1}{2}$ is excluded by the large anisotropy of the angular distribution; while it can easily be inferred from the theoretical angular distributions[3] that the limits on coefficients a_k with $k>2$ rule out $J=\frac{9}{2}$ or $\frac{11}{2}$. From Fig. 14 we see that $J=\frac{5}{2}$ and $\frac{7}{2}$ are not allowed. No conceivable FSE can change this conclusion. Thus, we find that the N^{15} 6.33-MeV level has $J=\frac{3}{2}$ in agreement with previous[6,18] work.

The solid curve for $J=\frac{3}{2}$ in Fig. 14 is for $P(\frac{3}{2})=0$, while the dashed curve is for $P(\frac{3}{2})=0.1P(\frac{1}{2})$. The former gives $x=+(0.09 \pm 0.025)$ or $x=+(1.42 \pm 0.08)$ while the latter gives $x=+(0.22 \pm 0.035)$ or $x=+(1.0 \pm 0.10)$. The distribution measured at $E_p=7.7$ MeV gives results consistent with those shown in Fig. 14.

If the distribution fitted in Fig. 14 were the only one measured then we would include the possibility of the FSE effect and quote $0.055 \leqslant x \leqslant 0.26$ or $1 \leqslant x \leqslant 1.5$. However, the agreement of the distributions measured at the two energies $E_p=7.7$ and 8.625 MeV decreases the likelihood of a FSE as large as that corresponding

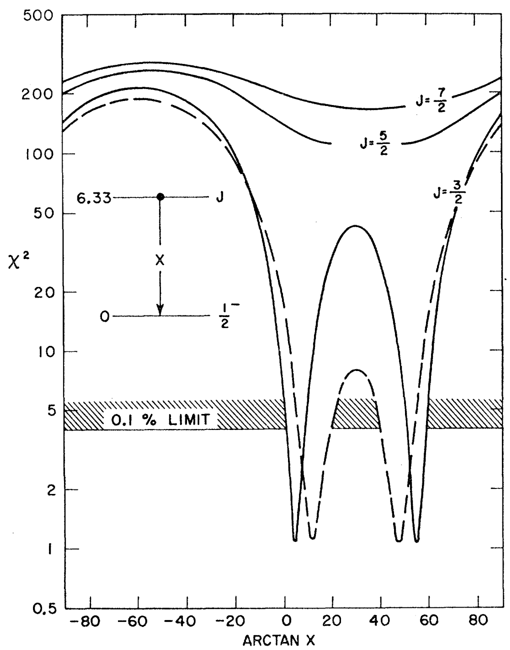

FIG. 14. χ^2 versus arctan x curves for the ground-state decay of the N^{15} 6.33-MeV level and assumed spins of $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ for the 6.33-MeV level. The broken curve is for $J=\frac{3}{2}$ and $P(\frac{3}{2})=0.1P(\frac{1}{2})$.

to $P(\frac{3}{2})=0.1P(\frac{1}{2})$ and, more importantly, the measurement of the angular distribution at two different distances for the annular counter at a proton energy of 8.625 MeV allows us to set a quantitative limit on the FSE. The theoretical formula for the a_2 coefficient in a $\frac{3}{2}$ to $\frac{1}{2}$ angular distribution can be put in the form

$$-[P(\tfrac{1}{2})-P(\tfrac{3}{2})](1+3.464x-x^2)/(1+x^2), \qquad (2)$$

where $2P(\frac{1}{2})+2P(\frac{3}{2})=1$ and x is the quadrupole-dipole mixing ratio. We note that any conceivable FSE will decrease $[P(\frac{1}{2})-P(\frac{3}{2})]$ and thus the magnitude of a_2. It can be shown[1] that for small departures of the particle detection from 180°, the contribution of $P(\frac{3}{2})$ to the formation of the 6.33-MeV level will be proportional to δ^2, where δ is the departure of the particle detection angle from 180°. Thus, in the present case the population $P(\frac{3}{2})$ should be four times as great for the distribution measured with the annular counter at 2.0 cm than for the distribution measured with it at 4.0 cm. Actually the former distribution gave a larger magnitude for a_2 than the latter, contrary to the expected effect. However, the two distributions did not agree too well: $a_2=-(0.70 \pm 0.06)$ and $-(0.62 \pm 0.04)$, respectively. If we increase the uncertainties on these values for a_2 so that the first overlaps with the second we find $P(\frac{3}{2}) \leqslant 0.03P(\frac{1}{2})$ for one standard deviation from each measured a_2, and we adopt this as the limit on $P(\frac{3}{2})$. With this limit on the FSE we find $x=+0.09_{-0.03}^{+0.06}$ (with the 0.1% limit $x>0.02$) or $x=+1.4_{-0.4}^{+0.1}$ for the quadrupole-dipole mixing ratio of the N^{15} $6.33 \to 0$ transition. This result, which is of special interest, will

[17] This is not surprising since the solutions for x_1 are strongly dependent on the measured value of a_4 and the angular distribution of the $5.30 \to 0$ transition contains no a_4 terms if the 5.30-MeV level has $J=\frac{1}{2}$ or $\frac{3}{2}$.

[18] S. Gorodetzky, P. Fintz, G. Bassompierre, and A. Gallmann, Compt. Rend. 252, 713 (1961).

be discussed in the next section. It is in agreement with the only previous determination,[18] which gave $-0.16 < x < +0.25$ or $1.03 < x < 2.64$. The nonzero value obtained for x favors an $M1,E2$ mixture for the $6.33 \to 0$ transition rather than an $E1,M2$ mixture; thus we find $J^\pi = \frac{3}{2}^{(-)}$ in agreement with previous results[6,18] which demand $J^\pi = \frac{3}{2}^-$.

F. The 8.31- and 8.57-MeV Levels

The α-particle groups leading to the N^{15} 8.31- and 8.57-MeV levels were not resolved (see Fig. 10). The gamma-ray transitions from these two states were studied by measuring gamma-ray angular distributions in coincidence with the high- and low-energy halves of the unresolved α-particle doublet separately. Spectra were recorded at five angles in both cases.

The N^{15} 8.31-MeV level is reported to decay to the ground state, the 5.3-MeV doublet, and 6.33-MeV level with relative intensities of 70:15:15, and the 8.57-MeV level is reported to decay to the same levels with relative intensities of 30:60:10, respectively.[19]

Because of the position of the single-channel analyzer gate, the gamma-ray spectra in coincidence with the high-energy side of the α-particle doublet contain negligible contributions from the decay of the 8.57-MeV level. The observed spectra were consistent with the reported decay of the 8.31-MeV level.[19] The angular distribution of the 8.31-MeV gamma ray was characterized by $a_2 = -(0.28 \pm 0.17)$, $a_4 = +(0.22 \pm 0.20)$. The large uncertainties are due to poor statistics which were necessitated by time limitations. The χ^2 versus arctan x curves for assumed assignments of $J = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ to the 8.31-MeV level are shown in Fig. 15. It is seen that $J = \frac{1}{2}$ or $\frac{3}{2}$ are possible but that $J = \frac{5}{2}$ is not. A spin assignment of $J = \frac{7}{2}$ is also forbidden, The FSE does not affect these conclusions. Assignments of $J \geqslant \frac{9}{2}$ are eliminated by the decay scheme. For $J = \frac{3}{2}$ the possible values of x are $-(0.16 \pm 0.09)$ and $+(2.9 \pm 0.9)$, where the uncertainties include the FSE.

The spectra in coincidence with the low-energy half of the α-particle doublet $\alpha_{7,8}$ indicated approximately equal contributions from the decays of the 8.57- and 8.31-MeV levels as was expected from the position of the analyzer gate. The angular distributions of the 5.3-MeV gamma ray and the unresolved 8.31- and 8.57-MeV gamma rays were obtained from these spectra. The angular distribution of the 5.3-MeV gamma ray was characterized by $a_2 = +(0.42 \pm 0.10)$ with no significant evidence for a term in $P_4(\cos\theta)$. This result demands $J > \frac{1}{2}$ for the 8.57-MeV level and also demands that the cascade from the 8.57-MeV level is to a level at 5.3 MeV with $J > \frac{1}{2}$. If the 5.30-MeV level has $J^\pi = \frac{1}{2}^+$, the decay is predominantly to the $J = \frac{5}{2}$, 5.28-MeV level. Furthermore, if the decay is to the 5.30-MeV level with $J^\pi = \frac{3}{2}^+$ for that state, then examination of the theoretical angular distribution formulas shows that, if we restrict the $M2,E1$ mixing ratio of the $5.30 \to 0$ transition to $|x| < 0.5$, $|x| > 2$ is demanded for the $8.57 \to 5.30$ transition for assignments of $J = \frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$ to the 8.57-MeV level in order to obtain $a_2 = 0.42 \pm 0.10$ for the $5.30 \to 0$ transition. Since $|x| > 0.5$ for the $5.30 \to 0$ transition and $|x| > 2$ for the $8.57 \to 5.3$ transition are not very likely we conclude that the 8.57-MeV level most probably decays predominantly to the $J = \frac{5}{2}$, 5.28-MeV level. The measured angular distribution of the $5.3 \to 0$ transition is consistent with that expected for the cascade $J \to \frac{5}{2} \to \frac{1}{2}$ with the first transitions pure dipole, the second pure quadrupole, and $J = \frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$.

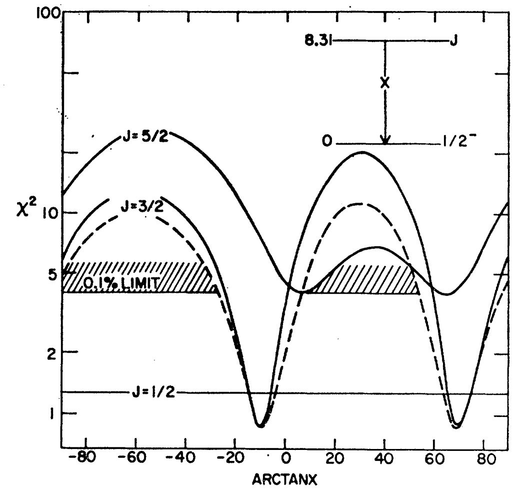

FIG. 15. χ^2 versus arctan x curves for the ground-state decay of the N^{15} 8.31-MeV level and assumed spins of $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ for the 8.31-MeV level. The broken curve is for $J = \frac{3}{2}$ and $P(\frac{3}{2}) = 0.1P(\frac{1}{2})$.

The angular distribution of the $8.57 \to 0$ transition can be extracted from the measurements using the known branching ratios[19] of the $8.57 \to 0$ and $8.57 \to 5.3$ transitions to estimate the relative intensities of the unresolved 8.31- and 8.57-MeV gamma rays and by subtracting the measured angular distribution of the 8.31-MeV gamma ray, properly normalized, from the angular distribution of the unresolved 8.31- and 8.57-MeV gamma rays. However, the 8.31-MeV angular distribution measurement was quite inaccurate, and the result of this procedure gives no meaningful information. On the other hand, if we assume that the 8.31-MeV level has $J = \frac{1}{2}$ then the $8.31 \to 0$ transition is rigorously isotropic, and this procedure gives $a_2 = -(0.35 \pm 0.16)$, $a_4 = +(0.10 \pm 0.23)$ for the $8.57 \to 0$ transition. The χ^2 versus arctan x curves for this distribution are shown in Fig. 16 for spin assignments to the 8.57-MeV level of $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. We regard $J \geqslant \frac{9}{2}$ to be eliminated by the presence of a strong ground-state transition. From Fig. 16 it is seen that $J = \frac{3}{2}$ is allowed, but both $\frac{5}{2}$ and $\frac{7}{2}$ are extremely unlikely. The FSE does not change this conclusion. Thus, if the N^{15} 8.31-MeV level has $J = \frac{1}{2}$, the 8.57-MeV level has $J = \frac{3}{2}$ with $x = -(0.13 \pm 0.25)$ or $+(2.7 \pm 1.6)$, where the uncertainties

[19] D. Pelte, B. Povh, and W. Scholz, International Congress of Nuclear Physics, Paris, July 1964 (unpublished); and B. Povh (private communication).

include our estimate of the FSE. This argument can also be reversed to state that if the 8.57-MeV level has $J=\frac{3}{2}$ and the $8.57 \rightarrow 0$ and $8.31 \rightarrow 0$ transitions are pure dipole, or nearly so, then the 8.31-MeV level must have $J=\frac{1}{2}$.

To summarize our results, we find that the N^{15} 8.31-MeV level has $J=\frac{1}{2}$ or $\frac{3}{2}$ with $x=-(0.16\pm0.09)$ or $+(2.9\pm0.9)$ for the latter case and that the 8.57-MeV level has $J>\frac{1}{2}$. Also, if the 8.31-MeV level has $J=\frac{1}{2}$, then the 8.57-MeV level has $J=\frac{3}{2}$ with $x=-(0.13\pm0.25)$ or $+(2.7\pm1.6)$ or, conversely, if the 8.57-MeV level has $J=\frac{3}{2}$ and the ground-state decays of the 8.57- and 8.31-MeV levels are pure dipole, then the 8.31-MeV level has $J=\frac{1}{2}$.

IV. DISCUSSION

A. The C^{11} 2.00-MeV Level

Negative parity is indicated for the C^{11} 2.00-MeV level by results from the $B^{10}(d,n)C^{11}$ stripping reaction and the $Be^9(He^3,n)C^{11}$ double stripping reaction,[6] and also by recent work[20] which shows that the transitions[5] from the C^{11} 7.50-MeV level to the C^{11} ground state and 2.00-MeV level are both predominantly $E1$. Thus the C^{11} $2.00 \rightarrow 0$ transition is an $M1,E2$ mixture for any of the possible spin assignments of $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$ to the C^{11} 2.00-MeV level, in which case the possible values of x obtained for $J=\frac{3}{2}$ and $\frac{5}{2}$ are quite reasonable. The mirror of this level in B^{11} at an excitation energy of 2.13 MeV has been assigned[6] $J^\pi=\frac{1}{2}^-$ so that $J^\pi=\frac{1}{2}^-$ is indicated for the C^{11} 2.00-MeV level also. However, the results illustrated in Fig. 3 show the impossibility of obtaining a direct proof of this from directional distribution measurements on the $2.00 \rightarrow 0$ transition alone without performing ancilliary experiments. This is a special case of a general ambiguity that exists in gamma-ray angular distribution measurements whenever the angular distribution is limited to terms in $P_2(\theta)$ only.

B. The N^{15} 5.28-MeV Level

The study of the N^{15} 5.3-MeV doublet gave no information on the upper member of the 5.28–5.30 doublet but fixed the spin of the 5.28-MeV level as $\frac{5}{2}$ with parity most probably even. The even parity assignment is in agreement with $N^{14}(d,p)N^{15}$ stripping results[6] which allow $J^\pi \leqslant \frac{7}{2}^+$.

The octupole-quadrupole mixing parameter of the N^{15} $5.28 \rightarrow 0$ transition has been recently obtained by Pelte *et al.*[19] from analysis of the angular distributions of the N^{15} cascades: $7.57 \rightarrow 5.28 \rightarrow 0$ and $7.16 \rightarrow 5.28 \rightarrow 0$. They obtained $x=-(0.07\pm0.02)$ or $+(3.7\pm0.4)$ in slight disagreement with our values of $-(0.15\pm0.06)$ or $+(6.1\pm1.4)$.

The work of Hebbard and Dunbar[21] on the $C^{14}(p,p)C^{14}$ reaction and of Hebbard[22] on the $C^{14}(p,\gamma)N^{15}$ reaction shows that the N^{15} 5.28-MeV level most probably has a spin of $\frac{5}{2}$ with $\frac{3}{2}$ and $\frac{7}{2}$ allowed with small probability. The present results confirm this suspected assignment of $J=\frac{5}{2}$ which was obtained from a study of the decay of the N^{15} 10.543-MeV level. The results of Hebbard[22] for the $10.543 \rightarrow 5.28 \rightarrow 0$ cascade are consistent with $J=\frac{5}{2}$ or $\frac{3}{2}$ for the 10.543-MeV level if the 5.28-MeV level has $J=\frac{5}{2}$. However, the value for the mixing parameter of the $5.28 \rightarrow 0$ transition which was demanded if the 10.543-MeV level has $J=\frac{3}{2}$ was ≈ 1.4, which is in strong disagreement with the present results. Thus, the present results support the assignment $J=\frac{5}{2}$ given by Hebbard to the N^{15} 10.543-MeV level.

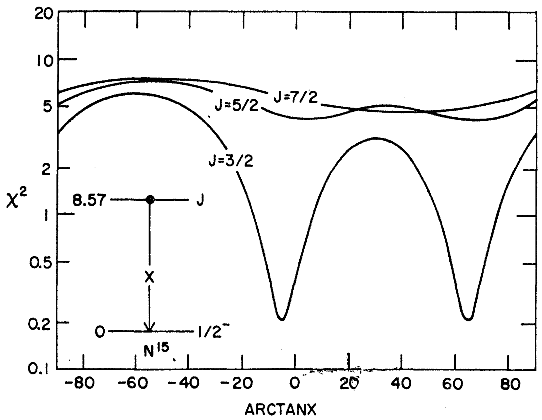

FIG. 16. χ^2 versus arctan x curves for the ground-state decay of the N^{15} 8.57-MeV level and assumed spins of $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ for the 8.57-MeV level.

C. The N^{15} 6.33-MeV Level

The available experimental evidence[6] strongly suggets that the ground state and 6.33-MeV level of N^{15} are well described by the shell-model configuration s^4p^{11}, i.e., a proton hole in the otherwise filled p shell. The N^{15} ground state is then $p_{1/2}^{-1}$ and the 6.33-MeV level is $p_{3/2}^{-1}$. The mirror of the 6.33-MeV level in O^{15} is almost certainly the O^{15} 6.16-MeV level which has been assigned[23] $J^\pi=\frac{3}{2}^-$. The dipole-quadrupole mixing parameter of the O^{15} $6.16 \rightarrow 0$ transition has been reported[23] to be $x=-(0.12\pm0.03)$ or $+(2.3\pm0.2)$.[24] It is of current interest to compare the values of the mixing parameters obtained for the N^{15} and O^{15} $p_{3/2}^{-1} \rightarrow p_{1/2}^{-1}$ transitions with each other and with theory.

The $M1$ and $E2$ rates of these two transitions are

[20] J. W. Olness, E. K. Warburton, D. E. Alburger, and J. A. Becker (to be published).

[21] D. F. Hebbard and D. N. F. Dunbar, Phys. Rev. **115**, 624 (1959).

[22] D. F. Hebbard, Nucl. Phys. **19**, 511 (1960).

[23] B. Povh and D. F. Hebbard, Phys. Rev. **115**, 608 (1959).

[24] The phase convention used by Povh and Hebbard (Ref. 23) is opposite to that of Litherland and Ferguson (Ref. 1) for $E2,M1$ mixtures. Thus, Povh and Hebbard quoted opposite phases from those given here for these two possibilities.

easily calculated. We find

$$\Gamma(M1)=4.91 \text{ eV},$$
$$\Gamma(E2)=3.25\times10^{-2}(1+\beta)^2 \text{ eV},$$
$$x(\text{N}^{15})=[\Gamma(E2)/\Gamma(M1)]^{1/2}=+0.081(1+\beta), \quad (3)$$

for the N^{15} 6.33 $\rightarrow$ 0 transition, and

$$\Gamma(M1)=3.15 \text{ eV},$$
$$\Gamma(E2)=2.83\times10^{-2}\beta^2 \text{ eV},$$
$$x(\text{O}^{15})=[\Gamma(E2)/\Gamma(M1)]^{1/2}=-0.095\beta, \quad (4)$$

for the O^{15} 6.16 $\rightarrow$ 0 transition. The quantity β which appears in the expressions for $\Gamma(E2)$ and x is the effective charge parameter of the weak coupling approximation such that the proton, or proton hole (N^{15}), and the neutron, or neutron hole (O^{15}), have charge $(1+\beta)e$ and βe, respectively. The $E2$ rates of N^{15} and O^{15} differ only by the $E_\gamma{}^5$ factor and the difference in effective charge. The $M1$ rates differ by the $E_\gamma{}^3$ factor and by the different effective magnetic moments which are $(\mu_p-\frac{1}{2})$ for N^{15} and μ_n for O^{15}. The difference in sign of $x(\text{N}^{15})$ and $x(0^{15})$ is due to the difference in sign of $(\mu_p-\frac{1}{2})$ and μ_n.

The $\Gamma(E2)$ were evaluated using 7.06×10^{-26} cm² for the mean-square radius, $\langle r^2\rangle_p$, of the p-shell protons. This value for $\langle r^2\rangle_p$ is in excellent agreement with the average of the values for N^{14} and O^{16} extracted[25] from total charge distributions as determined[26] by electron scattering measurements. It was actually evaluated using harmonic oscillator radial wave functions which have radial falloffs of the form $\exp(-\frac{1}{2}\gamma r^2)$ and which give $\langle r^2\rangle_p=\frac{5}{2}\gamma^{-1}$. Following Visscher and Ferrell[27] a value of 1.68 fermi was used for $\gamma^{-1/2}$ which results in $\langle r^2\rangle_p=7.06\times10^{-26}$ cm².

The lifetimes of the first excited states of N^{16}, O^{17}, and F^{17} and the second-excited states of F^{19} and Ne^{19}—all of which decay by $E2$ transitions—can all be explained by the shell model if it is modified by the weak coupling approximation with an effective charge of $\beta\simeq0.5$. This has recently been discussed by Raz.[28] The same is true for the lifetimes of the first-excited states of B^{10} and Be^{10} which also are in disagreement with pure shell-model calculations but are consistent with the weak coupling approximation with[25] $\beta\simeq0.5$. Thus, we expect $\beta\simeq0.5$ for the $p_{3/2}{}^{-1}\rightarrow p_{1/2}{}^{-1}$ transitions of N^{15} and O^{15} also. We could, of course, use one of the refined theories of collective enhancement of $E2$ transitions such as the approach of Barton.[29] Essentially this would mean taking different values of β for protons and neutrons. However, the experimental data for mass 15 are not accurate enough at the present time to distinguish between this approach and that of the weak coupling approximation —which in any case will reproduce the gross features of the effect of the polarization of the core.

The predictions for these transition strengths should not be very sensitive to admixtures of higher configurations. This is so because the calculated $M1$ transitions are above average strength. For instance, the N^{15} 6.33 $\rightarrow$ 0 transition has a strength of 0.94 Weisskopf units compared to the average[30] of about 0.15 Weisskopf units, while the calculated $E2$ transitions have about average strength—4 Weisskopf units for the N^{15} transition if $\beta=0.5$. The $M1$ transitions should be particularly insensitive to such admixtures not only because they are stronger than average but also because the $M1$ operator cannot connect wave functions belonging to different shell-model configurations. In any case we expect admixtures of higher configurations to be small for the two states of p^{-1}.

The experimental alternatives for $x(\text{N}^{15})$ are $+(0.09_{-0.03}{}^{+0.06})$ or $+(1.4_{-0.4}{}^{+0.1})$. The smaller value agrees quite well with the theoretical prediction, $x=+0.081(1+\beta)$, and gives $\beta\leqslant0.85$ to one standard deviation if no error is assumed in the theoretical prediction. The experimental alternatives for $x(\text{O}^{15})$ are $-(0.12\pm0.03)$ or $+(2.3\pm0.2)$ and again the theoretical prediction, -0.095β, favors the smaller admixture of $E2$ radiation. It is pleasing that the theory predicts the difference in phase of $x(\text{N}^{15})$ and $x(\text{O}^{15})$; however, the value of β extracted for O^{15} is 1.26 ± 0.3 which is in poor agreement with the limit obtained for N^{15} and with the expected value of ~0.5.

If we now take the viewpoint that the theoretical predictions for N^{15} and O^{15} are liable to the same systematic errors then the predicted ratio $x(\text{O}^{15})/x(\text{N}^{15})$ is more reliable than predicted values for $x(\text{O}^{15})$ or $x(\text{N}^{15})$ alone. The theory predicts $x(\text{O}^{15})/x(\text{N}^{15})=-1.17\beta/(1+\beta)\simeq-0.4$, whereas experimentally we find approximately -1.4 ± 0.7 for this ratio. It is clear that both the experimentally determined amplitude admixture of $E2$ radiation in the O^{15} 6.16 $\rightarrow$ 0 transition and the ratio of $x(\text{O}^{15})$ to $x(\text{N}^{15})$ are larger by a factor of about 2 or 3 than our theoretical prediction.

The strengths of the $E2$ transition strengths in mass 15 are important to an understanding of the collective enhancement of $E2$ transitions near $A=16$. Thus, a remeasurement of both $x(\text{N}^{15})$ and $x(\text{O}^{15})$ would be quite worthwhile. Also, it would be of great advantage to have a lifetime measurement for either or both of the two states in question so that a comparison of the $E2$ strengths could be made independently of the theoretical predictions for the $M1$ transition strengths.

The information which can be gained by comparing the measured phases of electromagnetic mixing parameters with theory has been almost universally ignored in spite of the fact that such information is, in principle,

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quite valuable. The reason for this is the great difficulty of tracing back to a common starting point the phase associated with angular correlation theory on the one hand and model calculations of electromagnetic transition amplitudes on the other hand. Until a careful appraisal of the relative phase of these two theories is available, informative comparisons between theoretical and experimental phases of mixing parameters can be made in some cases with the phase of a reliable mixing parameter retained as a standard. We feel that the N^{15} $6.33 \rightarrow 0$ and O^{15} $6.16 \rightarrow 0$ transitions are excellent ones upon which to base a phase convention. They are both simple transitions, theoretically, and, as noted above, the two $M1$ transitions are quite strong while the $E2$ transitions are of average strength; thus the theoretical predictions for the phase of the mixing ratios should be quite reliable. In Sec. IVC we use the phase convention obtained by accepting $x(N^{15})$ and $x(O^{15})$ as standard in a comparison of the experimental and theoretical values of x for the N^{14} $7.03 \rightarrow 0$ transition.

D. The N^{15} 8.31- and 8.57-MeV Levels

The results obtained for the decay of these two levels were limited by poor statistics and the poor resolution of the α-particle counter. However, the simplicity of analysis gained because the magnetic substates populated were limited to $\alpha = \pm\frac{1}{2}$ compensates somewhat for the poor quality of the experimental measurements.

The analysis of $N^{14}(d,p)N^{15}$ angular distributions gives an unambiguous assignment of $l_n = 0$ and thus $J^\pi = \frac{1}{2}^+$ or $\frac{3}{2}^+$ to the 8.31-MeV level.[6] Since the $8.31 \rightarrow 0$ transition is an $E1,M2$ mixture in either case, it is most probable that this transition is nearly pure dipole so that we can assign the 8.31-MeV level $J^\pi = (\frac{1}{2})^+$, in which case the 8.57-MeV level is most probably $J = \frac{3}{2}$ (see Sec. IIIF).

The angular distribution in the $N^{14}(d,p)N^{15}$ reaction leading to the 8.57-MeV level is best fitted with a mixture of $l_n = 0$ and $l_n = 2$ with the possibility of $l_n = 2$ only.[6] This indicates that the level has $J^\pi \leqslant \frac{7}{2}^+$ if $l_n = 2$ and $J^\pi = \frac{3}{2}^+$ if $l_n = 0+2$. The $J^\pi = \frac{3}{2}^+$ assignment is consistent with our assignment of $J = (\frac{3}{2})$ for the 8.57-MeV level.

The assignments $J^\pi = \frac{1}{2}^+$ and $\frac{3}{2}^+$ for the 8.31- and 8.57-MeV levels of N^{15} are in agreement with the shell-model calculations of Halbert and French.[31] They are also in agreement with the spin-parity assignments[6] of the O^{15} 7.50- and 8.28-MeV levels which are most probably the mirrors of the N^{15} 8.31- and 8.57-MeV levels, respectively.

E. The N^{14} 7.03-MeV Level

The N^{14} 7.03-MeV level has $(J,T) = (2,0)$,[6–9] and recent electron scattering measurements[32] give a strong preference for even parity. The available experimental evidence strongly favors a configurational assignment of s^4p^{10} to this level.[33] If it is the $(J^\pi,T) = (2^+,0)$ level of this configuration, as we assume, then it has the two-hole wave function $p_{1/2}^{-1}p_{3/2}^{-1}$. The N^{14} ground state is predominantly $p_{1/2}^{-2}$ from both experimental and theoretical evidence, and so the N^{14} $7.03 \rightarrow 0$ transition is essentially a $p_{1/2}^{-1}p_{3/2}^{-1} \rightarrow p_{1/2}^{-2}$ transition. The similarity of this transition to the mass 15 $p_{3/2}^{-1} \rightarrow p_{1/2}^{-1}$ transitions makes a calculation of the relative phase of the $M1,E2$ mixing parameters of these two transitions especially easy.

Using the same prescription (i.e., the same phase convention) to calculate the $M1$ and $E2$ rates for the $p_{1/2}^{-1}p_{3/2}^{-1} \rightarrow p_{1/2}^{-2}$ transition as we used for the $p_{3/2}^{-1} \rightarrow p_{1/2}^{-1}$ transition, we obtain $\Lambda(M1) = 0.096$, $\Lambda(E2) = 11.95(1+2\beta)^2$, and $[\Lambda(E2)/\Lambda(M1)]^{1/2} = +11.15(1+2\beta)$ where $\Lambda(M1)$ and $\Lambda(E2)$ are the transition strengths defined by Warburton and Pinkston,[33] and the mixing parameter is given by $x = 5.38\times10^{-3}E_\gamma [\Lambda(E2)/\Lambda(M1)]^{1/2}$. For a 7.03-MeV transition, then, $x = +0.42(1+2\beta)$ on this model. This is the same result as was obtained by Warburton and Pinkston[33] and, in general, it turns out that the phase of the results given by these authors for the $M1,E2$, $\Delta T = 0$, s^4p^{10} transitions is the same as that adopted in the present work (see Sec. IVC).

Warburton and Pinkston also give values of $\Lambda(M1)$, $\Lambda(E2)$ and $[\Lambda(E2)/\Lambda(M1)]^{1/2}$ calculated using the wave functions of Visscher and Ferrell[27] and of Elliott[34] for the N^{14} ground state. These wave functions mix $p_{1/2}^{-1}p_{3/2}^{-1}$ and $p_{3/2}^{-2}$ into the predominantly $p_{1/2}^{-2}$ wave function and are therefore considerably more realistic. The results given for x by these wave functions for the 7.03-MeV s^4p^{10} transition are $+0.296(1+2\beta)$ and $+0.251(1+2\beta)$, for the wave functions of Elliott[34] and Visscher and Ferrell,[27] respectively. Thus the phase of the N^{14} $7.03 \rightarrow 0$ $M1,E2$ mixing parameter is predicted theoretically to be positive (with the N^{15} $6.33 \rightarrow 0$ transition as a standard) in all three cases considered, which is in agreement with experiment (see Sec. IIIB). If we take the experimentally determined value of x with the smallest uncertainty, $x = +(0.6\pm0.1)$,[8] we can estimate the effective charge-parameter β by comparison with the theoretical predictions. This procedure gives values of β equal to 0.21 ± 0.12, 0.51 ± 0.17, and 0.70 ± 0.20 for the N^{14} ground-state wave functions given by $p_{1/2}^{-2}$, Elliott,[34] and Visscher and Ferrell,[27] respectively. These results, especially the last two, are in excellent agreement with our expectation that $\beta \simeq 0.5$.

The $E2$ transition strength of the N^{14} $7.03 \rightarrow 0$ transition has recently been measured by inelastic electron scattering.[32] The result, expressed in our notation, is $\Lambda(E2) = 33.3\pm2.4$ which gives values of β, to be compared to those given above, of 0.34 ± 0.05, 0.56 ± 0.09,

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and 0.82±0.12. The excellent agreement between these values of β and those obtained from the mixing parameter means, of course, that the theoretical prediction for the $M1$ transition strength is in good agreement with experiment. Thus the theoretical predictions agree with the experimentally determined $M1$ transition strength (if $\beta \simeq 0.5$) and the relative phase of the $M1$ and $E2$ transition strengths.

In the present work we found that the N^{14} 7.03-MeV level decays mostly to the ground state with a (9±5)% branch to the 3.95-MeV level and an upper limit of 5% for the 7.03 → 2.31 -MeV branch. The transition strengths of both of these cascades were calculated by Warburton and Pinkston[33] assuming $\beta=0.64$ for the 7.03 → 3.95 transition and $\beta=0$ for the 7.03 → 2.31 transition (since it has $\Delta T=1$). These calculations give predictions of ~1% and ~0.5% for these two cascades for all three sets of wave functions assumed. The latter result is consistent with experiment, whereas the former is in poor agreement. However, we note that the 7.03 → 3.95 $M1$ transiton strength is predicted to be extremely weak for the wave functions of Elliott[34] or of Visscher and Ferrell[27]—about 200–300 times weaker than the $M1$ 7.03 → 0 transition. Thus, small admixtures of higher configurations or of $T=1$ wave functions would be expected to dominate the 7.03 → 3.95 $M1$ rate even though they are expected to have a small effect on the 7.03 → 0 $M1$ rate. Thus, the theoretical prediction for the branching ratio of the 7.03 → 3.95 transition is not considered reliable except as a prediction that this branching ratio is small ($\lesssim$5%).

F. The N^{14} 5.83-MeV Level

The N^{14} 5.83 → 0 and 5.83 → 5.10 transitions have been considered before,[13,33,35] and we have nothing further to add at this point.

The angular distribution of the 5.10 → 0 transition was analyzed, as in earlier work,[10,11] assuming that it is a mixture of dipole and quadrupole radiation only. However, as was pointed out previously[33] the 5.10 → 0 transition should have a significant contribution of $E3$ radiation also. This has been confirmed by recent electron scattering results[32] which are interpreted to give $\Gamma(E3)=(3.5\pm0.7)\times10^{-6}$ eV for the predominantly $E1$ N^{14} 5.10 → 0 transition. Combining this result with the measured lifetime limit[11] and branching ratios for the 5.10-MeV decay gives $|x_3|>0.04$, where we use x_2 for the amplitude ratio of quadrupole to dipole radiation and x_3 for the amplitude ratio of octupole to dipole radiation. Other information on x_2 and x_3 comes from studies of the internal pairs emitted by the 5.10-MeV level, this is $x_2^2<0.2$, $x_3^2<0.25$.[12]

The angular distribution of the decay of the 2^- 5.10-MeV level to the 1^+N^{14} ground state can be expressed as[3]

$$W(\theta)=1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta) \tag{5}$$

with

$$a_k=\rho_k(2)F_k(21)Q_k\,, \tag{6}$$

where the Q_k are the attenuation coefficients for the gamma-ray detector,[1] the $\rho_k(J)$ are statistical tensors,[3] and the $F_k(21)$ are given by

$$F_2(21)=\frac{0.4183-0.2988x_2^2-0.7171x_3^2-1.8708x_2-0.4781x_3+1.0690x_2x_3}{1+x_2^2+x_3^2}, \tag{7a}$$

$$F_4(21)=\frac{0.7127x_2^2+0.0891x_3^2-1.6036x_3-1.9920x_2x_3}{1+x_2^2+x_3^2}. \tag{7b}$$

The phase convention used in Eq. (7) is that of Litherland and Ferguson[1] for an $E1,M2,E3$ mixture. (The phase of x_2 is therefore opposite to that used in Sec. IIIC.) It can be seen from Eq. (7b) that the value of a_4 is sensitive to small admixtures of $E3$ radiation. Unfortunately in the present work and in previous work[10,11] the formation of the 5.10-MeV level was such that the statistical tensor $\rho_4(2)$ was small so that a meaningful measurement of $F_4(21)$ was not obtained. In this work and the previous work, a value of x_2 was essentially extracted (with x_3 assumed zero) from a measurement of a_2; for instance, the most accurate determination of x_2 was obtained from the measurement[10] $F_2(21)=+(0.18\pm0.06)$ which gives $x_2=+(0.12\pm0.03)$ if x_3 is zero. The constraints on x_2 and x_3 implicit by this measurement of $F_2(21)$, rather than the value of x_2 obtained with x_3 neglected, should be compared to further studies of this transition. In the meantime, we can combine this measurement with the constraints mentioned above, $|x_3|>0.04$, $x_2^2<0.2$, $x_3^2<0.5$ to obtain the new conditions $|x_2|<0.2$, $0.04<|x_3|<0.5$.

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