compressibility conditions are relaxed in the same way, but the potentials are very different. From a comparison between the results of the two calculations one can estimate the effect of the potential on the moment of inertia. Table V shows that the moments of inertia for a Fermi gas moving in a harmonic-oscillator potential are smaller that those for a bag full of fermions. Finally a comparison between two calculations assuming a harmonic-oscillator potential is made. Here again the difference stems from the difference in the assumed models. The Fermi-gas model predicts slightly higher moments of inertia which increase with angular momentum. The shell model predicts constant and smaller moments of inertia. An attempt was made to study the moments of inertia as one goes from the extreme case of the liquid-drop model to the extreme case of the shell model. In all calculations interactions are neglected except as they show up in the surface energy. Therefore, all calculations can be considered as single-particle calculations. The two last calculations using a harmonic oscillator could become more realistic if the potential would be allowed to deform. Such calculations with the inclusion of electrostatic forces would approximate very well the behavior of real nuclei. Grover[33] has already concluded that the latter results agree well with moments of inertia derived by him from experimental data.

The moments of inertia calculated from the dependence of the density of levels on angular momentum obtained by Bloch[34] and Ericson and Strutinski[35] indicate similar results to those obtained in this calculation.

ACKNOWLEDGMENT

The author is deeply indebted to Professor J. A. Wheeler for suggesting the problem and for his uninterrupted encouragement and interest.

[33] G. R. Grover, Phys. Rev. **127**, 2142 (1962).
[34] C. Bloch, Phys. Rev. **93**, 1054 (1954).
[35] J. Ericson and V. Strutinski, Nucl. Phys. **8**, 1054 (1958).

PHYSICAL REVIEW VOLUME 138, NUMBER 5B 7 JUNE 1965

Axially Asymmetric Regions in the *s-d* Shell*

J. BAR-TOUV
Department of Nuclear Physics, The Weizmann Institute of Science, Rehovoth, Israel
AND
I. KELSON
Nuclear Structure Laboratory, Yale University, New Haven, Connecticut
(Received 22 January 1965)

The Hartree-Fock problem for even-even nuclei in the *s-d* shell is solved without requiring the intrinsic state to be axially symmetric. Two regions of axial asymmetry are found—around Mg^{24} and around S^{32}. The existence of an energy gap between occupied and nonoccupied single-particle states is established for all cases. The moment-of-inertia tensor for the axially asymmetric cases is computed and provides an improvement over the axially symmetric results of former calculations.

I. INTRODUCTION

THE calculational limitations of the shell model in treating nuclei with many nucleons (more than four) outside closed shells led to the development of the collective picture of nuclei. In this treatment, the nucleus is described as performing some kind of collective motion, such as rotations and vibrations.[1] The coordinates of the individual nucleons are replaced by a smaller number of degrees of freedom, of a more macroscopic nature. (In the unified model, which is an extension of this, some individual nucleons are still considered.)[2] A small number of parameters is sufficient to specify exactly the new Hamiltonian of the system. In the case of a pure axially symmetric rotator, only one such parameter—the moment of inertia—is necessary. The initial justification of such a model was the actual occurence of, say, rotational spectra in certain nuclei. Moreover, the values of the parameters were artificially adjusted to fit those observed spectra. From this point of view, the model provides merely a phenomenological presentation of experimental data. Its qualitative and quantitative relations to the actual many-body problem of the nucleus (or, at least, to the shell-model version of it) are therefore of great interest.

The significance of the "intrinsic" state of the nucleus was long recognized in this connection. The "intrinsic"

* This work was supported in part by the U. S. Atomic Energy Commission.
† On leave from the Weizmann Institute of Science and from the Israeli Atomic Energy Commission, Soreq Research Establishment.
[1] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[2] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **30**, No. 1 (1955).

state is not the actual state of the nucleus, but rather, provides a basis from which the model extracts and constructs the physical states. For example, the intrinsic state may not have J as a good quantum number, but may give rise—through proper projection—to a set of states of good J of the nucleus.

The possibility of the existence of deformed nuclei, and deformed intrinsic states, was originally suggested[3] in order to explain abnormally large electric-quadrupole transition probabilities. For closed-shell nuclei, the argument went, a spherical shape is prevalent. However, when more and more external nucleons are added, a growing permanent deformation becomes energetically more stable. The nucleus retains this large deformation, and thus may quantum-mechanically give rise to rotational motion—as opposed to a spherically symmetric body. For axially symmetric rotators, an approximate expression—the cranking formula—relates the moment of inertia and the single-particle structure of the intrinsic state.[4] Similar relations hold for parameters associated with other modes of collective motion. Therefore, a knowledge of that internal structure becomes very important.

The most widely used method of obtaining an axially symmetric deformed single-particle structure—eigenfunctions and eigenenergies—was the Mottelson-Nilsson model.[5] The nucleons are assumed to be moving in an axially symmetric deformed harmonic-oscillator well, and a set of wave functions and energies, characterized by a deformation parameter is obtained by diagonalizing the single-particle Hamiltonian in a spherical shell-model representation. Although this proves to be generally a successful basis, it fails to produce good results when applied to the cranking formula. The predicted moments of inertia are very large compared to experimental values, differing by more than a factor of two in the s-d shell, and actually coincide with the rigid-body values.[6] Since the cranking formula was based on a self-consistent Hartree-Fock treatment, it became worthwhile to approach the whole problem via this treatment. Thus, rather than limiting the intrinsic structure to one generated by a deformed harmonic oscillator, a fully self-consistent program was carried out.[7] A shell-model Hamiltonian with both a single-body and a two-body part gave rise self-consistently to intrinsic structures having the same general properties as those of the Nilsson model, but not limited as they are. Because of the more complicated numerical nature of the problem, only the s-d shell was treated. Indeed, when the self-consistent solutions were substituted into the cranking formula, values much closer to experiment were obtained.[8] The main feature responsible for this correction was the appearance of an energy gap between occupied and unoccupied states. This is connected with the exchange character of the two-body Hamiltonian.

The limitation of axial symmetry in this region may, however, prove to be too severe. It results in having k, the projection of angular momentum along the body z axis, as a good quantum number. Consequently, there are rotational k bands in odd-even nuclei, mixed by a Coriolis force with a $\Delta k=1$ selection rule. Electromagnetic (in particular $E2$) transition probabilities between bands with $\Delta k=2$ should therefore be much smaller than those for transitions within the same band. This is notably not the case in Mg^{25} and Al^{25}, for example.[9] Moreover, in this region even the self-consistent values for the moment of inertia are too large.

It is the purpose of this article to investigate the possibility of nonaxially-symmetric intrinsic states in the s-d shell, and its experimental consequences.

The extended self-consistency problem will be formulated in Sec. II. Section III will give a treatment of the asymmetric rotator and the moment-of-inertia tensor associated with it. Numerical results will be presented and analyzed in Sec. IV.

II. THE SELF-CONSISTENCY PROBLEM

The Hamiltonian of the nuclear system is assumed to be of the following form, in second-quantization notation:

$$H=\sum_{\alpha,\beta}\langle\alpha|K|\beta\rangle a_\alpha^\dagger a_\beta+\tfrac{1}{4}\sum_{\substack{\alpha\beta\\ \gamma\delta}}\langle\alpha\beta|V_A|\gamma\delta\rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma, \quad (1)$$

where, by definition, V_A stands for the two-body interaction between antisymmetrized states,

$$\langle\alpha\beta|V_A|\gamma\delta\rangle\equiv\langle\alpha\beta|V|\gamma\delta\rangle-\langle\alpha\beta|V|\delta\gamma\rangle. \quad (2)$$

The single-body part in the Hamiltonian K consists of the kinetic energy term, a spin-orbit interaction and an $\mathbf{l}^2$ force

$$k=\frac{\hbar^2}{2m}\Delta+\alpha_{1\cdot s}\mathbf{l}\cdot\mathbf{s}+\alpha_{1^2}\mathbf{l}^2. \quad (3)$$

The two-body interaction V is taken as the Rosenfeld mixture[10] used by Elliott and Flowers,[11]

$$V=V_0\times\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{3}(0.3+0.7\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\frac{e^{-r_{12}/a}}{r_{12}\cdot a}. \quad (4)$$

[3] J. Rainwater, Phys. Rev. **79**, 432 (1950).
[4] D. R. Inglis, Phys. Rev. **96**, 1059 (1954).
[5] S. G. Nilsson, Kgl. Danske Videnskab. Selskab. Mat. Fys. Medd. **29**, No. 16 (1955).
[6] R. M. Rockmore, Phys. Rev. **116**, 469 (1959).
[7] I. Kelson, Phys. Rev. **132**, 2189 (1963).
[8] I. Kelson and C. A. Levinson, Phys. Rev. **134**, B269 (1964).
[9] I. Kh. Lemberg, Nucl. Phys. **19**, 400 (1960).
[10] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1958), p. 233.
[11] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

This mixture has the proper saturation properties, and yielded good results when used in shell-model calculations.

The intrinsic single-particle structure of the nucleus is derived from the many-body Hamiltonian by solving the self-consistent Hartree-Fock equations.[12] That is, one finds simultaneously the one-body operator h, its self-energies e_α, and eigenfunctions $\{\phi_\alpha\}$, such that

$$\langle\alpha|h|\beta\rangle=\langle\alpha|k|\beta\rangle+\sum_\lambda\langle\alpha\lambda|V_A|\beta\lambda\rangle \quad (5)$$

and

$$h|\lambda\rangle=e_\lambda|\lambda\rangle. \quad (6)$$

The summation in the first equation runs over all the occupied states. A determinantal wave function made up of these states gives a stationary point of the expectation value of the Hamiltonian. In particular, the minimum of $\langle H\rangle$ with respect to this kind of wave function is found. Thus, knowing the subspace which is spanned by the set of occupied states $\{\lambda\}$ is equivalent to knowing the Hartree-Fock Hamiltonian. In particular, symmetry properties demanded for the Hartree-Fock Hamiltonian may be incorporated into it by properly restricting the set $\{\lambda\}$; if a certain operator Ω leaves the set invariant,

$$\Omega\{\lambda\}=\{\lambda\}, \quad (7)$$

and it also commutes with H,

$$[H,\Omega]=0, \quad (8)$$

it will clearly also commute with h,

$$[h,\Omega]=0. \quad (9)$$

In the case of the axially symmetric treatment, each of the single-particle states occupied was a sum of eigenstates of j_z. As a result, h commuted with j_z. Another property of the set $\{\lambda\}$ was its invariance under time reversal; i.e., along with each function ϕ_λ, its time-reversed function $\bar{\phi}_\lambda$ also belonged to the set.[13] Moreover, with each proton in a certain state, a neutron at the same state is included, and vice versa. The Hartree-Fock spectrum had therefore a fourfold degeneracy built into it. Each level was filled by two protons and two neutrons with their spins up and down.

In the present treatment the same fourfold degeneracy is retained, but the single-particle states are no longer eigenstates of j_z. Each level is chosen as a linear combination of the following form:

$$|\lambda\rangle=\sum_{j,m}a_{jm}{}^\lambda|jm\rangle, \quad (10)$$

with $a_{jm}{}^\lambda$ vanishing unless $m-\frac{1}{2}$ is even. In the s-d shell, to which we restrict the sum, the states $d_{5/2}{}^{5/2}$, $d_{1/2}{}^{5/2}$, $d_{1/2}{}^{3/2}$, $S_{1/2}{}^{1/2}$, $d_{-3/2}{}^{5/2}$, $d_{-3/2}{}^{3/2}$ will appear. The time-reversed[14] state

$$R_t|\lambda\rangle=\sum_{jm}a_{jm}{}^\lambda(-1)^{j-m}|j-m\rangle \quad (11)$$

will be a linear combination of states with $m-\frac{1}{2}$ odd, and will thus be orthogonal to $|\lambda\rangle$. A rotation of θ radians about the z axis will transform

$$R_z(\theta)|jm\rangle=e^{-im\theta}|jm\rangle. \quad (12)$$

Hence, each of the states (10)–(11) is invariant under a rotation through π radians about the z axis:

$$R_z(\pi)|\lambda\rangle=\pm i|\lambda\rangle. \quad (13)$$

The physical meaning of these restrictions on the intrinsic structure is that the nucleus may assume ellipsoidal shapes, with x, y, z being the major axes. This will be more strongly emphasized in the next section.

A further advantage of this particular choice is that it removes the degeneracy of the orientation of the deformed field in space. Since the major axes are fixed, the only possible degenerate intrinsic states are obtained by permutations of those axes. Thus, for example, one may obtain a state which is axially symmetric around the x or the y axis. This will be illustrated in the following sections as well.

The solution of the self-consistency equation is achieved by iterative methods. Various relative minima in the energy surface are obtained by starting the iteration with different initial conditions. For more details, the reader should consult Ref. 7.

III. THE ASYMMETRIC ROTATOR

The energy associated with rotational motion of the intrinsic structure is obtained by applying cranking model ideas to it.[15] The expectation values of all the components of angular momentum in the self-consistent intrinsic state ϕ_0 vanish:

$$\langle\phi_0|J_x|\phi_0\rangle=\langle\phi_0|J_y|\phi_0\rangle=\langle\phi_0|J_z|\phi_0\rangle=0. \quad (14)$$

Now, one is looking for the determinantal wave function which minimizes $\langle H\rangle$ under the subsidiary condition that $\langle J_x\rangle$, $\langle J_y\rangle$, $\langle J_z\rangle$ taken on prescribed values. Clearly, the expectation value of the Hamiltonian will be higher than in the nonrestricted case, and the energy excess will be described as the rotational energy of the system. The solution of the new restricted self-consistency problem is obtained by the Lagrange-multiplier method. Three Lagrange multipliers—ω_x, ω_y, ω_z—are introduced and a determinantal solution $\phi(\omega)$ is found, such that

$$\delta\langle\phi(\omega)|H-\boldsymbol{\omega}\cdot\mathbf{J}|\phi(\omega)\rangle=0. \quad (15)$$

The expectation values of J are related to the

[12] G. E. Brown, *Lecture Notes on the Many Body Problem* (Nordita Publications, Copenhagen, 1961).

[13] It should be noted that in such a case, the variational parameters can be chosen real.

[14] In the s-d shell, where $(-1)^l=1$, the time-reversal operation on a single-particle state is equivalent to rotation of π radians about the x axis.

[15] See, for example, Ref. 12.

Lagrange multipliers by

$$\langle \mathbf{J} \rangle = \mathcal{I} \cdot \boldsymbol{\omega}, \tag{16}$$

where $\mathcal{I}$ is the moment-of-inertia tensor, and the energy increment is given by

$$\langle \phi(\boldsymbol{\omega}) | H | \phi(\boldsymbol{\omega}) \rangle - \langle \phi_0 | H | \phi_0 \rangle = \tfrac{1}{2} \boldsymbol{\omega} \cdot \mathcal{I} \cdot \boldsymbol{\omega}. \tag{17}$$

The self-consistency equations for $h(\boldsymbol{\omega})$ are

$$\langle \alpha | h(\boldsymbol{\omega}) | \beta \rangle = \langle \alpha | k | \beta \rangle + \sum_{\lambda(\omega)} \langle \alpha \lambda(\boldsymbol{\omega}) | V_A | \beta \lambda(\boldsymbol{\omega}) \rangle - \langle \alpha | \boldsymbol{\omega} \cdot \mathbf{J} | \beta \rangle \tag{18}$$

and

$$h(\boldsymbol{\omega}) | \lambda(\boldsymbol{\omega}) \rangle = e_{\lambda(\omega)} | \lambda(\boldsymbol{\omega}) \rangle. \tag{19}$$

The approximation that we make in solving these equations is the one used in deriving the Inglis cranking formula, namely, we neglect the effect of the two-body interactions, and take simply

$$h(\boldsymbol{\omega}) = h(0) - \boldsymbol{\omega} \cdot \mathbf{J}. \tag{20}$$

The occupied states, of which the determinant $\phi(\boldsymbol{\omega})$ is composed, are the lowest eigenstates of $h(\boldsymbol{\omega})$. These are readily obtained from the eigenfunctions of $h(0)$ by perturbation methods. To first order in $\boldsymbol{\omega}$,

$$| \lambda(\boldsymbol{\omega}) \rangle = | \lambda \rangle + \sum_{\sigma} \frac{| \sigma \rangle \langle \sigma | \boldsymbol{\omega} \cdot J | \lambda \rangle}{e_\sigma - e_\lambda}, \tag{21}$$

where the sum runs over the unoccupied states $\{\sigma\}$. The expectation values of $\mathbf{J}$ with respect to each single-particle state $|\lambda(\boldsymbol{\omega})\rangle$ to first order in $\boldsymbol{\omega}$ is

$$\langle \lambda(\boldsymbol{\omega}) | \mathbf{J} | \lambda(\boldsymbol{\omega}) \rangle = \sum_{\sigma} \frac{\langle \lambda | \mathbf{J} | \sigma \rangle \langle \sigma | \boldsymbol{\omega} \cdot \mathbf{J} | \lambda \rangle + \langle \lambda | \boldsymbol{\omega} \cdot \mathbf{J} | \sigma \rangle \langle \sigma | \mathbf{J} | \lambda \rangle}{e_\sigma - e_\lambda}, \tag{22}$$

and

$$\langle \phi(\boldsymbol{\omega}) | \mathbf{J} | \phi(\boldsymbol{\omega}) \rangle = \sum_{\lambda} \langle \lambda(\boldsymbol{\omega}) | \mathbf{J} | \lambda(\boldsymbol{\omega}) \rangle. \tag{23}$$

Going back to the definition of $\mathcal{I}$, we find

$$\mathcal{I}_{\alpha\beta} = \sum_{\sigma,\lambda} \frac{\langle \lambda | J_\alpha | \sigma \rangle \langle \sigma | J_\beta | \lambda \rangle + \langle \lambda | J_\beta | \sigma \rangle \langle \sigma | J_\alpha | \lambda \rangle}{e_\sigma - e_\lambda}, \tag{24}$$

with λ running over occupied, and σ over unoccupied states. This reduces, in the case of axial symmetry about the z axis, to the Inglis cranking formula

$$\mathcal{I}_z = 2 \sum_{\sigma,\lambda} \frac{| \langle \sigma | J_x | \lambda \rangle |^2}{e_\sigma - e_\lambda}. \tag{25}$$

In the present case, the particular choice of the form of the wave functions introduces simplifications in $\mathcal{I}$. Since the matrix elements of J_x and J_y have the selection rule $|\Delta m| = 1$, and those of J_z have $\Delta m = 0$, we immediately have

$$\mathcal{I}_{xz} = \mathcal{I}_{yz} = 0. \tag{26}$$

Moreover, our variational coefficients are all real so that the matrix elements of J_x in the sum are pure real, and of J_y pure imaginary.

Thus

$$\langle \lambda | J_x | \sigma \rangle = (\langle \sigma | J_x | \lambda \rangle)^* = \langle \sigma | J_x | \lambda \rangle,$$
$$\langle \lambda | J_y | \sigma \rangle = (\langle \sigma | J_y | \lambda \rangle)^* = -\langle \sigma | J_y | \lambda \rangle. \tag{27}$$

Each separate term in the sum (24) vanishes, and we have

$$\mathcal{I}_{xy} = 0. \tag{28}$$

We therefore conclude that for our choice of variational treatment, x, y, and z are indeed the principal axes of the system. The rotational energy would therefore be given by the three principal moments of inertia $\mathcal{I}_\alpha = \mathcal{I}_{\alpha\alpha}$ as

$$E_{\text{rot}} = \sum_\alpha (\hbar^2 / 2\mathcal{I}_\alpha) J_\alpha^2, \tag{29}$$

where the J_α's are the components of angular momentum in the body-fixed coordinate system.

In the case of axial symmetry one of the $\mathcal{I}_\alpha$'s vanishes, and the other two are equal. The one that vanishes will not necessarily by $\mathcal{I}_z$, as in previous treatments. This reflects the fact, stated above, that the intrinsic state is degenerate under permutations of the axes.

It is of interest to compare the obtained moments of inertia with those of the irrotational model of Bohr and Mottelson. Their parameters—β and γ—were used by Davydov and Fillipov[16] to get eigenenergies of an even-even asymmetric rotator. There we have

$$\mathcal{I}_\alpha = 4B\beta^2 \sin^2(\gamma - \tfrac{2}{3}\pi\alpha), \tag{30}$$

with $\alpha = 1, 2, 3$, referring to the three axes. The three moments of inertia are not independent in this treatment. In the last section, the extent to which the cranked moments of inertia can be represented by such a parametrization is discussed. This will enable us to judge the applicability of the ideas of irrotational motion which underlie the Davydov-Fillipov treatment in this region.

IV. RESULTS

The program of solving the self-consistency problem in the *s-d* shell was carried out for all even-even nuclei with an equal number of protons and neutrons. Odd-even nuclei were not treated since the main purpose of the work was to find the general trends in the shell.

The extra freedom allowed to the intrinsic state by removing the restriction of axial symmetry results in a rearrangement of the single-particle levels. Usually, more than one stationary intrinsic state exists, and therefore we generally find a competition between the lowest axially symmetric and nonaxially-symmetric in-

[16] A. S. Davydov and G. F. Filippov, Nucl. Phys. 8, 237 (1958).

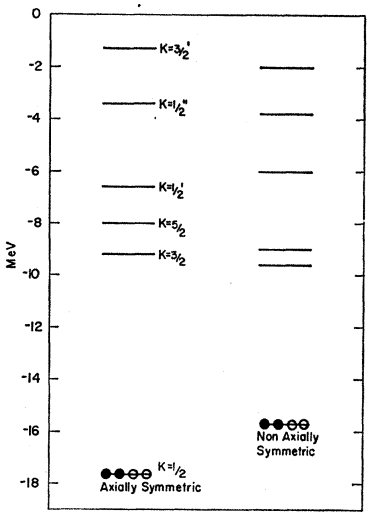

FIG. 1. A comparison between the single-particle self-consistent Hamiltonians for the axially symmetric and nonaxially symmetric cases in Ne^{20}. The force parameters employed are $V_0=50$ MeV, $\alpha_{\mathbf{L}\cdot\mathbf{S}}=2.8$ MeV. Two protons and two neutrons occupy the lowest single-particle level in each case.

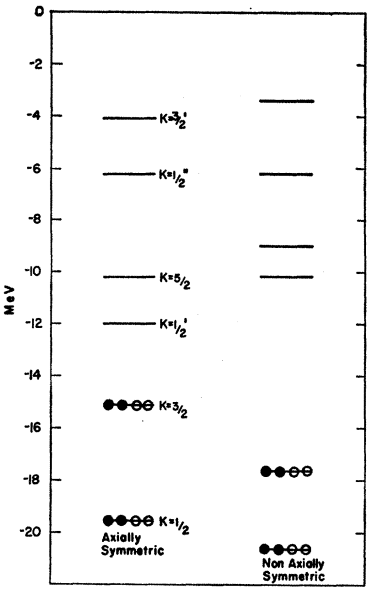

FIG. 2. A comparison between the single-particle self-consistent Hamiltonians for the axially symmetric and nonaxially symmetric cases in Mg^{24}. The force parameters employed are $V_0=50$ MeV, $\alpha_{\mathbf{L}\cdot\mathbf{S}}=2.8$ MeV. protons and two neutrons occupy the lowest two single-particle levels in each case.

trinsic states. Two such typical situations are illustrated in Figs. 1 and 2. In Fig. 1, for Ne^{20}, the axially symmetric state is energetically lower, and the reverse is true for Mg^{24}, shown in Fig. 2. It should be emphasized that the larger energy gap between occupied and nonoccupied states which is associated with the lower expectation value of the Hamiltonian is an essential feature of the Hartree-Fock basis. It increases the stability of the ground-state determinant against two-particle excitations. Such a gap is absent from the Mottelson-Nilsson deformed harmonic-oscillator spectrum.

The complete set of solutions for even-even nuclei is given in Table I.

A very interesting feature of the intrinsic states is the abrupt change of the sign of the nuclear deformation around mass 28 (see Fig. 3). This is most probably also supported by experimental evidence.[17]

Figure 4 gives the difference in expectation values of the Hamiltonian H

$$\Delta E=\langle\phi_{\text{nonax}}|H|\phi_{\text{nonax}}\rangle-\langle\phi_{\text{ax}}|H|\phi_{\text{ax}}\rangle.$$

Negative values of ΔE on the curve indicate that an axially asymmetric intrinsic state is energetically preferable. Two regions of axial asymmetry are clearly seen,

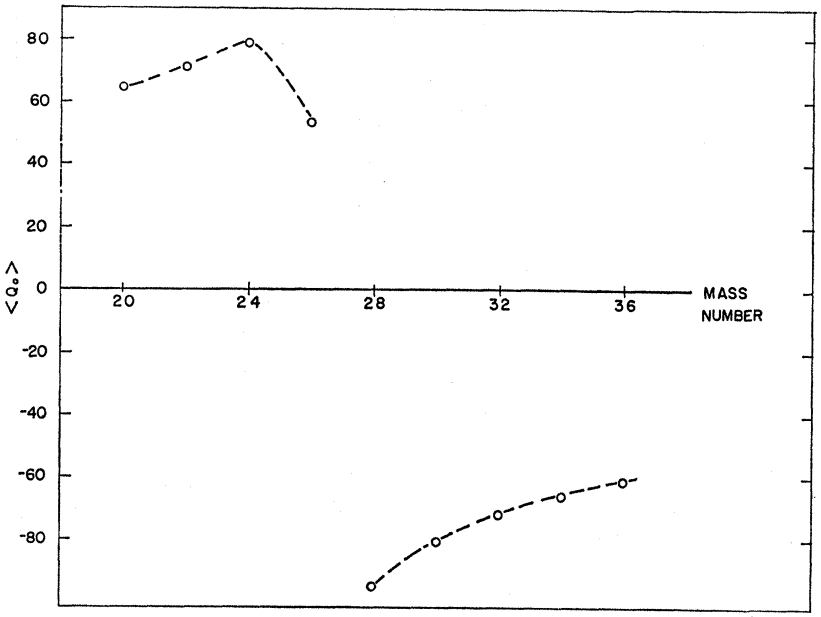

FIG. 3. The dependence of the quadrupole moment $\langle Q_0\rangle$ of the energetically lowest intrinsic state on the mass number. $\langle Q_0\rangle$ is given in arbitrary units (of 6.22 mb).

[17] See: H. E. Gove, in *Proceedings of the International Conference on Nuclear Structure, Kingston*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 438.

TABLE I. Single-particle self-consistent Hamiltonians for the even-even nuclei in the s–d shell. For each nucleus, the lowest axially symmetric and nonaxially-symmetric cases are presented. The italicized numbers give the single-particle energies in MeV, followed by the components of the single-particle wave functions in the representation (in this order): $d_{5/2}{}^{5/2}$, $d_{-3/2}{}^{5/2}$, $d_{-3/2}{}^{3/2}$, $d_{1/2}{}^{5/2}$, $d_{1/2}{}^{3/2}$, $S_{1/2}{}^{1/2}$. The expectation value of the Hamiltonian H (in MeV), the three principal moments of inertia (in MeV^{-1}), and the expectation values of Q_0 and Q_2 (in arbitrary units) are also given for each case. The Hamiltonian H is the Rosenfeld mixture used by Elliott and Flowers, with $V_0=50$ MeV and a spherical single-particle spin-orbit term with $\alpha_{\mathbf{L}\cdot\mathbf{S}}=2.8$ MeV.

	Ne^{20}		Mg^{24}		Si^{28}		S^{32}		Ar^{36}	
	Ax	Nonax	Ax	Nonax	Ax	Nonax	Ax	Nonax	Ax	Nonax
	−17.656	−15.681	−19.499	−20.607	−22.706	−21.811	−22.729	−24.729	−26.761	−26.147
	0	0.836	0	−0.118	1.000	0.284	0	0.947	1.000	−0.727
	0	−0.383	0	0.268	0	−0.458	0	0.036	0	−0.325
	0	0.394	0	0.167	0	−0.093	0	−0.001	0	0.043
	0.819	0	−0.850	0.816	0	−0.811	−0.699	0.075	0	−0.460
	−0.381	0	0.250	−0.118	0	0.021	0.302	−0.014	0	0.025
	0.429	0	−0.465	0.454	0	−0.208	0.648	−0.310	0	0.390
	−9.225	−9.562	−15.106	−17.656	−21.256	−19.120	−22.684	−23.648	−25.569	−25.547
	0	0	0	−0.103	0	0.295	1.000	0.230	0	−0.613
	−0.997	0	0.974	−0.758	0	−0.687	0	−0.581	−0.996	0.476
	−0.080	0	0.226	−0.149	0	0.167	0	0.095	0.094	−0.006
	0	0.944	0	−0.004	−0.628	0.397	0	−0.598	0	0.631
	0	−0.042	0	−0.522	0.299	−0.439	0	−0.053	0	0.012
	0	−0.328	0	0.346	0.719	0.251	0	0.491	0	0.000
	−8.016	−9.030	−12.060	−10.243	−18.141	−18.063	−20.547	−21.179	−24.904	−23.152
	1.000	−0.411	0	−0.819	0	0.793	0	0.168	0	−0.310
	0	−0.912	0	0.304	0.723	0.260	0	0.674	0	−0.139
	0	−0.015	0	0.062	−0.691	0.408	0	−0.152	0	−0.159
	0	0	0.465	−0.390	0	0.126	0.610	−0.228	−0.637	−0.196
	0	0	0.771	−0.126	0	0.393	−0.221	0.420	−0.024	−0.092
	0	0	−0.436	0.255	0	−0.129	0.761	0.516	0.770	−0.901
	−6.569	−6.004	−10.166	−9.004	−10.400	−11.008	−18.721	−18.693	−21.327	−22.697
	0	0	1.000	−0.544	0	0.110	0	0.068	0	−0.017
	0	0	0	−0.418	0.691	0.381	−0.831	−0.077	0	−0.703
	0	0	0	−0.053	0.723	0.201	0.556	0.796	0	0.241
	0.552	0.329	0	0.379	0	−0.390	0	0.447	0.594	0.524
	0.321	0.223	0	0.431	0	−0.398	0	0.264	0.621	−0.416
	−0.769	0.918	0	−0.445	0	0.701	0	0.295	0.511	0.000
	−3.405	−3.835	−6.187	−6.173	−10.298	−9.979	−13.361	−12.024	−19.474	−18.601
	0	−0.365	0	−0.094	0	0.306	0	0.098	0	−0.022
	0	0.149	0	0.260	0	0.322	0.556	−0.348	0.094	−0.009
	0	0.919	0	−0.262	0	−0.444	0.831	−0.575	0.996	0.850
	0.155	0	−0.250	0.189	−0.774	0.069	0	0.599	0	−0.014
	0.867	0	0.586	−0.681	−0.343	−0.648	0	0.146	0	0.491
	0.473	0	0.771	−0.597	−0.533	−0.426	0	0.399	0	−0.188
	−1.297	−1.996	−4.069	−3.400	−5.296	−7.984	−11.120	−8.785	−11.985	−13.922
	0	0	0	−0.002	0	0.311	0	0.091	0	−0.001
	−0.080	0	−0.226	0.139	0	−0.043	0	0.283	0	−0.392
	0.997	0	0.974	−0.935	0	−0.748	0	0.067	0	−0.438
	0	−0.035	0	−0.046	−0.087	0.112	−0.373	0.163	0.491	0.278
	0	0.974	0	0.220	0.891	0.353	−0.928	−0.854	−0.784	0.759
	0	−0.224	0	0.235	−0.446	0.452	0.030	0.388	0.381	0.000
$\langle H\rangle$	−46.233	−43.190	−94.414	−97.986	−155.37	−152.30	−213.47	−215.83	−281.89	−279.45
g_1	1.864	1.042	3.242	2.119	2.352	2.067	2.674	2.374	1.783	1.215
g_2	1.864	1.042	3.242	2.400	2.352	2.116	2.674	2.007	1.783	2.169
g_3	0	3.274	0	0.924	0	2.645	0	1.234	0	1.215
$\langle Q_0\rangle$	64.7	−33.5	77.5	79.3	−95.6	36.0	−24.6	−72.2	−61.4	−16.3
$\langle Q_2\rangle$	0	0	0	−20.1	0	−15.7	0	−21.2	0	−19.9

around Mg^{24} and around S^{32}. The curve displays a striking symmetry about the middle of the shell.

The analysis of the deformation of the lowest intrinsic state, in terms of the asymmetric rotator parameters β and γ is shown on Fig. 5. For the nonaxially-symmetric case, the three moments of inertia cannot be fitted exactly by the two irrotational-flow parameters. The extent to which such a fit can be achieved is indicated by the crosses through the corresponding points. Here, again, an approximate symmetry about the middle of the shell may be noticed. Experimentally, however, no rotational spectrum has been found in the upper half of the shell.

In the case where two of the principal moments of

inertia are approximately equal and the third one is small,

$$\mathcal{I}_1 \approx \mathcal{I}_2, \quad \mathcal{I}_3 < \mathcal{I}_2$$

the ground-state rotational band may still be approximated by a pure rotator with

$$\mathcal{I} = \tfrac{1}{2}(\mathcal{I}_1 + \mathcal{I}_2).$$

The position of the excited rotational bands will then depend on the ratio $\mathcal{I}_3/\mathcal{I}_2$. Since this is the situation for Mg^{24} and S^{32}, we could plot $\hbar^2/2\mathcal{I}$ as a function of mass number throughout the shell. For Mg^{24}, where a rotational spectrum is experimentally clearly observed, the asymmetric result provides an important improvement over the former axially symmetric calculation. The $\hbar^2/2\mathcal{I}$ values calculated using equivalent Nilsson energies and wave functions are considerably smaller.

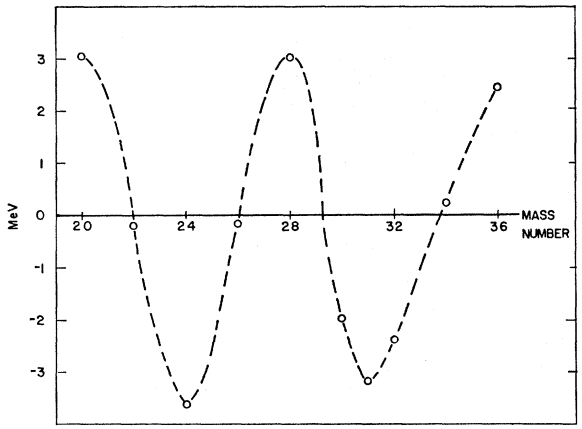

FIG. 4. The difference between the lowest axially symmetric and nonaxially-symmetric intrinsic state energies, $\Delta E = \langle\phi_{\text{nonax}}|H|\phi_{\text{nonax}}\rangle - \langle\phi_{\text{ax}}|H|\phi_{\text{ax}}\rangle$, as a function of mass number.

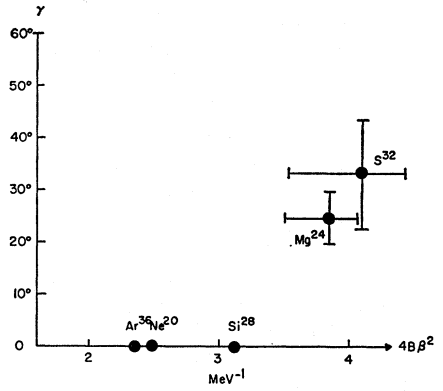

FIG. 5. An analysis of the lowest energy intrinsic states in terms of the irrotational-flow parameters γ and $4B\beta^2$. The axially symmetric cases lie on the $\gamma=0°$ line. For the nonaxially-symmetric cases Mg^{24} and S^{32}, a best fit is given and the fit is indicated by the crosses through those points.

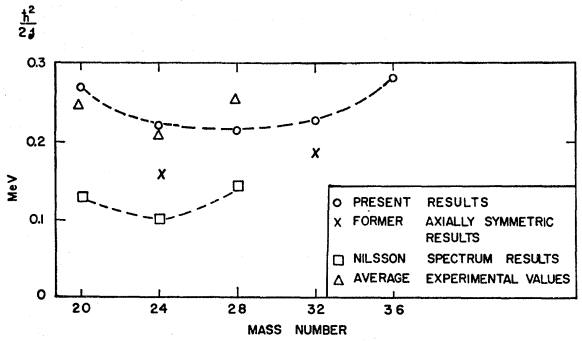

FIG. 6. The moment-of-inertia parameter $\hbar^2/2\mathcal{I}$ as a function of the mass number for the present calculations. For comparison, the Nilsson spectrum values, the previous axially symmetric results for Mg^{24} and S^{32}, and the average experimental values are shown.

The parameters used throughout the calculations were $V_0 = 50$ MeV for the two-body interaction strength, $\alpha = 1.37 \times 10^{-13}$ cm for the range, and $\alpha_{\mathbf{L}\cdot\mathbf{S}} = 2.8$ MeV, $\alpha_1^2 = 0$ in the single-body part of the Hamiltonian. All the calculations were done with oscillator functions of range 1.65×10^{-13} cm. Although there are indications that these values may change over the wide range of nuclei treated, it was found that the results reported above are qualitatively the same for reasonably large deviations from those values.

ACKNOWLEDGMENTS

We thank Professor C. A. Levinson and N. Auerbach of the Weizmann Institute for helpful suggestions and discussions. One of the authors (I. K.) wishes to thank Professor D. A. Bromley and the members of the Nuclear Structure Laboratory at Yale University for their kind hospitality and cooperation during his visit.