# Electromagnetic Decay of Nuclei with High Angular Momenta According to the Liquid-Drop Model*

DANIEL SPERBER
*Physics Division, IIT Research Institute, Chicago, Illinois*
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The spectrum and the angular distribution of gamma rays from deformed liquid-drop nuclei are calculated. It is shown that the angular distribution changes when the equilibrium shapes change from oblate spheroids rotating about the minor axis to prolate spheroids rotating around one of their minor axes.

## I. INTRODUCTION

A CALCULATION of the spectrum and angular distribution of gamma rays from a nucleus with high angular momentum, according to the liquid-drop model, is presented in this paper.

With the recent availability of heavy-ion accelerators compound nuclear states with high angular momenta can be formed. When the fission channel is closed these compound nuclear states will necessarily decay by neutron emission, if energetically allowed. However, the emission of one or more gamma rays between two successive neutron emissions is not infrequent.[1–8] Furthermore, the neutrons emitted in the decay from the compound nuclear state carry away considerable amounts of energy but only a small fraction of the total angular momentum. Consequently a state with high angular momentum may eventually be reached from which neutron emission is energetically forbidden. Decay from this state can therefore only occur by photon emission.

Previously, Strutinski[9] and Babikov[10] calculated the angular distribution of gamma rays from nuclei with high angular momentum. However, both calculations lacked any specific assumption concerning the decay mechanism, the nuclear matrix element being introduced as a constant parameter. In addition, Babikov neglects transitions other than those which are preferential in the direction of the angular momentum of the emitting system. To be able to extract information about highly excited nuclear states with high angular momentum a model has to be assumed and the calculated spectrum and angular distribution compared with experiment. Mollenaur[7] was the first to measure the angular distribution of gamma rays from nuclei with high angular momentum. His measurements indicate that the gamma rays can be attributed mainly to $E2$ transitions; however, a small fraction has to be attributed to $E1$ transitions. Various nuclear models imply a dominating $E2$ transition. However, the calculated angular distribution may vary from model to model since the strengths of the contributing components of the quadrupole tensor are model-dependent. In the present paper the consequences of assuming the liquid-drop model, as far as the spectrum and angular distribution are concerned, are investigated.

According to the liquid-drop model equilibrium shapes of nuclei are very well approximated by spheroids.[11–18] However, these spheroids are only approximate shapes of equilibrium. But, even for angular momenta much higher than those considered here they are excellent approximations.[11] The quadrupole moments of the shapes discussed by Carlson and Pao Lu[12] differ at most by 1% from the quadrupole moment of the spheroidal shapes discussed by Beringer and Knox[13] for cases relevant to the present discussion. It is therefore reasonable to use the Beringer-Knox[13] shapes for calculating quadrupole moments. These spheroidal shapes have a vanishing dipole moment but a nonvanishing quadrupole moment. Such a rotating charged drop will emit $E2$ radiation. Transitions due to octupole and higher multipolarities are neglected in the present treatment. Even for the highly deformed nuclei considered here octupole transitions are inhibited by a factor of $(1/20)\times(EA^{1/3}/200)^2$. The energy $E$ is measured in MeV and $A$ is the mass number. Transitions due to higher multipolarities than 3 are even less

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probable. For $E2$ contributions, values of $m$ ranging from $-2$ to 2 are included. For nuclei with an angular momentum smaller than a critical value they find that the equilibrium shapes are oblate spheroids with the axis of rotation coinciding with the axis of cylindrical symmetry. However, for nuclei with angular momenta exceeding this critical value the shapes of equilibrium are prolate spheroids rotating around one of their minor axes. At high excitations the angular momentum is entirely associated with rotation of the nucleus as a whole[14,19]; hence, the use of the liquid-drop model is most appropriate.

In the present paper the angular distribution from the two types of spheroids is calculated. This calculation shows that the angular distribution from the two types of spheroids differ from one another. Consequently an experimentally discovered change in angular distribution, in conformity with the present theory, could be considered as an experimental verification of the Beringer-Knox theory[13] of equilibrium shapes.

The main factors contributing to the asymmetry are (a) the values of the contributing components of the quadrupole tensor, (b) the dependence of the density of levels on angular momentum, and (c) the orientation of the angular momentum of the compound system. The difference in the angular distribution from the two types of spheroids is attributed to the first factor. The corresponding components of the quadrupole tensor are different.

First the angular distribution of gamma rays from a nucleus with high angular momentum is calculated with respect to a system of coordinates in which the $z$ axis points along the angular momentum of the emitting system (Sec. II). The angular distribution would appear isotropic as long as the direction of the angular momentum of the compound system is random. In heavy-ion bombardment the direction of the angular momentum of the compound system is far from random; in particular, the angular momenta of the compound system are close to a plane perpendicular to the direction of the heavy-ion beam. For comparison with experiment one would rather have an explicit expression for the angular distribution of the gamma rays with respect to the direction of the heavy-ion beam. Therefore the previously calculated angular distribution is rewritten in a system of coordinates in which the $z$ axis is in the direction of the heavy-ion beam and an average over-all allowed directions of the spin compound nucleus is performed (Sec. III).

## II. THE SPECTRUM AND ANGULAR DISTRIBUTION OF EMITTED GAMMA RAY WITH RESPECT TO THE ANGULAR MOMENTUM OF THE EMITTING SYSTEM

The spectrum and angular distribution of an $E2$ gamma ray emitted from a nucleus with high angular momentum will be calculated in this section using the liquid-drop model. In this calculation the $z$ axis is to be chosen in the direction of the angular-momentum vector of the emitting system. The probability $I(E,\Omega)$ of emitting a photon per second with energy between $E$ and $E+dE$ into the solid angle $d\Omega$ at an angle $\Omega$ is given by the following[20]:

$$I(E,\Omega)dEd\Omega=\frac{4\pi c}{75}\left(\frac{e^2}{\hbar c}\right)\left(\frac{E}{\hbar c}\right)^5 \times\sum_{J_fM_fm}|\langle J_fM_f|Q_m{}^2|J_iM_i=J_i\rangle|^2 \times|\mathbf{Y}_{-m}{}^{2(21)}|^2\rho(E_fJ_fM_f)dEd\Omega. \tag{1}$$

In (1), $E$ is the energy of the emitted gamma ray, the $\langle J_fM_f|Q_m{}^2|J_iM_i\rangle$ are the nuclear matrix elements for the quadrupole transition, $\mathbf{Y}_{-m}{}^{2(21)}$ are the vector spherical harmonics of order 2, and $\rho(E_fJ_fM_f)$ is the density of final states. The summation in (1) is over all allowed values of $J_fM_f$ and over all values of $m$ for which the component of the quadrupole tensor does not vanish. Using the liquid-drop model, the nuclear matrix element $\langle J_fM_f|Q_m{}^2|J_iM_i\rangle$ is replaced by $\langle Q_m{}^2\rangle$. Here

$$\langle Q_m{}^2\rangle=\int\rho_z(\mathbf{r})Y_m{}^2(\mathbf{r})d^3r. \tag{2}$$

In (2) $\rho_z(\mathbf{r})$ is the charge density and $Y_m{}^2(\mathbf{r})$ are the solid spherical harmonics of order 2. For an oblate spheroid rotating around the minor axis a simple calculation shows that

$$\langle Q_0{}^2\rangle=\tfrac{1}{4}(5/\pi)^{1/2}\tfrac{1}{5}ZeR_0{}^2(2/\eta^{2/3})(\eta^2-1), \tag{3a}$$

$$\langle Q_{\pm2}{}^2\rangle=\langle Q_{\pm1}{}^2\rangle=0. \tag{3b}$$

Here $Z$ is, as usual, the nuclear charge, $R_0$ is the radius of the spherical nucleus and $\eta$ is the ratio of the minor to the major axis. The dependence of $\eta$ on angular momentum can be found in Ref. 13.

For a prolate spheroid a similar calculation shows

$$\langle Q_0{}^2\rangle=\tfrac{1}{4}(5/\pi)^{1/2}\tfrac{1}{5}ZeR_0{}^2(1/\eta^{4/3})(\eta^2-1), \tag{4a}$$

$$\langle Q_{\pm1}{}^2\rangle=0, \tag{4b}$$

$$\langle Q_{\pm2}{}^2\rangle=\tfrac{1}{4}(15/2\pi)^{1/2}\tfrac{1}{5}ZeR_0{}^2(1/\eta^{4/3})(\eta^2-1). \tag{4c}$$

In Eq. (4) again $\eta$ is the ratio of the minor to the major axis. For angular momenta lower than the critical value Eqs. (3a) and (3b) have to be used for evaluation of the $\langle Q_m{}^2\rangle$. For angular momenta higher than the critical angular momentum Eqs. (4a), (4b), and (4c) have to be used for the evaluation of the $\langle Q_m{}^2\rangle$.

For the density of levels the usual expression[21–25] was

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used

$$\rho(E,J,M)=N'(2J+1)\rho(0)\exp(-J^2\hbar^2/2\mathcal{I}T). \quad (5)$$

In (5) $N'$ is a normalization factor, $\rho(0)$ is the density of states with zero angular momentum, $\mathcal{I}$ is the nuclear moment of inertia, and $T$ the nuclear temperature.

Introducing (3) and (4) into (2) and using the explicit forms of the vector spherical harmonics, one obtains $I_0(E,\Omega)$ for a rotating oblate spheroid

$$I_0(E,\Omega)=(N'cZ^2/\pi)(e^2/\hbar c)R_0^4\exp(-\hbar^2J_i^2/2\mathcal{I}T)\exp(E_i/T)(E/\hbar c)^5\exp(-E/T)$$
$$\times\{(2J+1)+(2J+3)\exp[-(2J+1)\tau]+(2J+5)\exp[-(4J+4)\tau]\}\cos^2\theta\sin^2\theta. \quad (6)$$

Here

$$\tau=(\hbar^2/2\mathcal{I}T). \quad (7)$$

On the other hand, for a prolate spheroid, one obtains for $I_p(E,\Omega)$

$$I_p(E,\Omega)=(N'cZ^2/\pi)(e^2/\hbar c)R_0^4\exp(-\hbar^2J_i^2/2\mathcal{I}T)\exp(E_i/T)(E/\hbar c)^5\exp(-E/T)$$
$$\times\{(2J+1)+(2J+3)\exp[-(2J+1)\tau]+(2J+5)\exp[-(4J+4)\tau]\}\cos^2\theta\sin^2\theta$$
$$+[2(2J+5)\exp[-(4J+4)\tau]+(2J+3)\exp[-(2J+1)\tau]+(2J+1)$$
$$+(2J-1)\exp(2J-1)\tau+(2J-3)\exp(4J-4)\tau]\sin^4\theta. \quad (8)$$

It is apparent from (6) and (8) that the angular distribution from the two types of spheroids differ from one another.

## III. THE SPECTRUM AND THE ANGULAR DISTRIBUTION WITH RESPECT TO THE DIRECTION OF THE HEAVY-ION BEAM

To be able to compare theory and experiment easily, one has to calculate the angular distribution in (6) and (8) with respect to the direction of the heavy-ion beam. One has to consider all possible directions of the angular-momentum vector of the compound nucleus. This angular momentum can be anywhere in a plane perpendicular to the direction of the heavy-ion beam. One averages of all directions in this plane. The polar and azimuthal angles of the angular momentum of the compound system with respect to a system in which the $z$ axis is along the beam are $\pi/2$ and $\alpha$, where $\alpha$ varies between 0 and $2\pi$. The direction characterized by $\theta$ and $\phi$ in the old system of coordinates is characterized by $\Theta$ and $\Phi$ in the new coordinate system so that[26]

$$\cos^2\theta\sin^2\theta=\frac{8\pi}{15}\sum_{l,m}(2l+1)^{1/2}\begin{pmatrix}2&2&l\\0&0&0\end{pmatrix}\begin{pmatrix}2&2&l\\1&1&0\end{pmatrix}$$
$$\times D_{m,0}{}^l\left(\alpha,\frac{\pi}{2},0\right)Y_m{}^l(\Theta\Phi), \quad (9a)$$

$$\sin^4\theta=\frac{32\pi}{15}\sum_{l,m}(2l+1)^{1/2}\begin{pmatrix}2&2&l\\0&0&0\end{pmatrix}\begin{pmatrix}2&2&l\\2&2&0\end{pmatrix}$$
$$\times D_{m,0}{}^l\left(\alpha,\frac{\pi}{2},0\right)Y_m{}^l(\Theta\Phi). \quad (9b)$$

In (9) $\begin{pmatrix}a&b&c\\\alpha&\beta&\gamma\end{pmatrix}$ are the Wigner $3j$ symbols and $D_{m,m'}{}^l(\alpha,\beta,\gamma)$ are the matrices of the five-dimensional representation of the three-dimensional rotation group. Averaging over $\alpha$ yields

$$\frac{1}{2\pi}\int D_{m,0}{}^l\left(\alpha,\frac{\pi}{2},0\right)d\alpha=d_{0,0}{}^l\left(\frac{\pi}{2}\right)\delta_{m,0}. \quad (10)$$

Using (6), (8), (9a), (9b), and (10) the spectrum and the angular distribution from an oblate and prolate spheriod, respectively, are

$$I_0(E,\Theta,\Phi)=(N'cZ^2/\pi8)(e^2/\hbar c)R_0^4\exp(-J_i^2\hbar^2/2\mathcal{I}T)\exp(E_i/T)(1/\eta^{4/3})(\eta^2-1)^2(E/\hbar c)^5\exp(-E/T)$$
$$\times\{(2J+1)+(2J+3)\exp[-(2J+1)\tau]+(2J+5)-(4J+4)\tau\}[1+2\cos^2\Theta-3\cos^4\Theta]$$
$$=N_0(1+2\cos^2\Theta+-3\cos^4\Theta E^5\exp(-E/T). \quad (11a)$$

and

$$I_p(E,\Theta,\Phi)=N_p[A_0+A_2\cos^2\Theta+A_4\cos^4\Theta]E^5\exp(-E/T). \quad (11b)$$

Here

$$N_0=N'\frac{\pi c}{4625}\left(\frac{e^2}{\hbar c}\right)\left(\exp\left[E-\frac{J_i\hbar^2}{2\mathcal{I}T}\right]\right)\frac{Z^2e^2(\eta^2-1)^2}{\eta^{4/3}}, \quad (12a)\qquad N_p=N'\frac{\pi c}{4625}\left(\frac{e^2}{\hbar c}\right)\left(\exp\left[E-\frac{J_i\hbar^2}{2\mathcal{I}T}\right]\right)\frac{Z^2e^2(\eta^2-1)}{\eta^{8/3}}, \quad (12b)$$

[26] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 61.

and

$$A_0=18(2J+5)\exp[-(4J+4)\tau]+11(2J+3)\exp[-(2J+1)\tau] +11(2J+1)+(2J-1)\exp(2J-3)\tau+7\exp(4J-4)\tau, \quad (13a)$$

$$A_2=28(2J+1)\exp[-(4J+4)\tau]+18(2J+3)\exp[-(2J+1)\tau] +18(2J+1)+10(2J-1)\exp(2J-1)\tau+10\exp(4J-4)\tau, \quad (13b)$$

$$A_4=-30(2J+5)\exp[-(4J+4)\tau]-21(2J+1)\exp(2J21)\tau -21(2J+1)-9(2J-1)\exp(2J-1)\tau-9(2J-3)\exp(4J-4)\tau. \quad (13c)$$

## IV. DISCUSSION

From (11a), (11b), (13a), (13b), and (13c) it is obvious that the angular distribution from oblate and prolate spheroids differ from one another. For the case of $Cu^{66}$ Beringer and Knox[14] calculated that at an angular momentum of 50 $\hbar$ the nucleus changes from an oblate to a prolate spheroid. Using this value of the angular momentum and 1.5 MeV for the nuclear temperature using (11b), (13a), (13b), and (13c) the angular distribution for prolate spheroids $I_p$ can be written

$$I_p(\Theta,\Phi)=N_p[(1+1.55\cos^2\Theta-1.65\cos^4\Theta]E^5 \times\exp(-E/T). \quad (14)$$

This angular distribution differs from the one derived in (11a). Consequently, as states of the compound nucleus with angular momenta higher than the critical become available the angular distribution, at least partially, should follow (14). An interesting experiment would consist of varying the angular momentum of the compound system by varying the energy of the heavy-ion beam and discovering a change in the angular distribution. An observed change in the angular distribution in accord with the present calculation would be an experimental proof to the existence of the two different shapes of equilibrium predicted by Beringer and Knox.[13]

The anisotropy predicted in (11a) and (11b) apply to nuclei all having the same angular momentum. In an actual experiment the angular distribution observed is due to an average over many angular momenta, on account of the following two effects, (a) there are many impact parameters in a heavy-ion bombardment so that the angular momentum of the compound nucleus may vary over a range of angular momenta, and (b) in the de-excitation cascades the decrease in angular momentum may vary from one nucleus to another. An exact knowledge of the distribution of angular momenta due to these two effects would enable one to calculate an average anisotropy by averaging the presently calculated anisotropy over such an angular momentum distribution. However, at the present time these angular momentum distributions have not been obtained. One has therefore, to estimate the importance of such an angular-momentum distribution. For this purpose a comparison between two calculated anisotropies was made. First the anisotropy of gamma rays emitted from a nucleus of $A=200$ and $J=30$ was calculated. Then the gamma-ray anisotropy from the same nucleus was calculated using a Gaussian distribution for the angular momenta such that the mean angular momentum was 30 and half-width at half-height of the Gaussian was 5. The results of the calculated anisotropies differed by five percent. This result indicates that as long as the spread in the angular momentum of the distribution is relatively small the calculated anisotropy using (11a) and (11b) with the mean angular momentum of the distribution is a very good approximation.