translational invariance. Summing over k yields

$$dM/dt = -(M - M_q)/\tau_i\,,$$
$$1/\tau_i = \lambda(2 - \beta \textstyle\sum_k B_{kj})\,; \qquad (18)$$
$$M_0 = \beta\gamma H/(2 - \beta \textstyle\sum_k B_{kj})$$

for the magnetic polarization $\sum_k q_k$. Equation (18) confirms that at high temperatures an Ising system relaxes qualitatively like a paramagnet, with small quantitative differences. In particular this means that if a system is for $t<0$ below the Curie point at equilibrium, and is suddenly brought in contact with a reservoir at $T \ll \omega_j/k$, the magnetization relaxes without any "sudden" or discontinuous behavior. The equations for the r_{jk} become with (15)

$$dr_{jk}/dt = -4\lambda r_{jk} + \lambda\beta \textstyle\sum_l (B_{jl} r_{lk} + B_{kl} r_{lj}) - \lambda\beta\gamma H(q_k + q_j)\,, \quad j \neq k. \qquad (19)$$

The equations do not depend on higher moments. In particular, if translational invariance is assumed:

$$r_{jk} = r(\mathbf{j} - \mathbf{k})\,, \quad q_k = q_j$$

and nearest-neighbor interaction only, (19) becomes

$$dr(f,g,h)/d\tau + 4r(f,g,h) + 2\beta H q = 2\beta B \textstyle\sum_\delta r(f+\delta_f,\, g+\delta_g,\, h+\delta_h)\,, \qquad (20)$$

where

$$dr(0,0,0)/dt = 0\,, \quad r(0,0,0) = 1\,,$$

where $\tau = \lambda t$, and f,g,h are the components of $\mathbf{j} - \mathbf{k}$. The vector $\boldsymbol{\delta} = (\delta_f, \delta_g, \delta_h)$ is a vector to any of the nearest neighbors. The solution to the one-dimensional version of (20) is given by Glauber. The homogeneous parts in the two- and three-dimensional cases are very similar to the equations of coupled harmonic oscillators in a lattice with one defect. It is consequently to be expected that they can be solved by the same methods, in particular by the use of Green functions,[11] or generating functions.[12]

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I would like to thank David L. Falkoff for some helpful comments.

[11] A. A. Maradudin, *Brandeis University Summer Institute Lectures, 1962.* (W. A. Benjamin, Inc., New York, 1963), Vol. 2.
[12] P. M. Mathews, M. S. Seshardri, and R. Vijayalakshni, Z. Angew. Math. Phys. (to be published).

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Microwave Hot-Electron Conduction in Many-Valley Semiconductors

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Hot-carrier conduction in many-valley semiconductors in the presence of microwave signals has been considered. The time-dependent Boltzmann equation applicable for different cases of applied steady electric and microwave fields in different directions is solved to obtain the distribution of carriers. The microwave conductivity and change in dielectric constant to be observed in different cases are also deduced from the distribution function. Two cases are considered: (a) small microwave field in the presence of large steady electric and magnetic field, and (b) large microwave field in the presence of steady magnetic field. For case (b), an expression for the efficiency of third-harmonic generation is also obtained. A microwave experiment for the measurement of the Sasaki-Shibuya voltage is also proposed. The approximate numerical calculation for the efficiency agrees with that of the reported experimental value.

I. INTRODUCTION

Hot-electron conduction in semiconductors has been studied[1–6] experimentally using microwave fields. In some of these studies, the microwave fields have been used to eliminate the problem of carrier injection and in others to detect whether the conductivity characteristics are frequency-dependent or not. Seeger[6] also reported significant amounts of third-harmonic generation using high-microwave pulsed fields in n-type germanium samples. Results of these microwave experiments cannot be explained by applying the theories of dc hot-electron conduction directly. Some work relating to the theory of hot-electron microwave conduction has been reported by Paranjape,[7] Gibson *et al.*,[2] and Nag and Das.[8]

Paranjape and Gibson *et al.* assumed the distribution function of the carriers to be Maxwellian, which is valid only for highly doped samples. Gibson *et al.* considered only the case where a small microwave field and a large

[1] J. B. Arthur, A. F. Gibson, and J. W. Granville, J. Electron. 2, 145 (1956).
[2] A. F. Gibson, J. W. Granville, and E. G. S. Paige, J. Phys. Chem. Solids 19, p. 198 (1961).
[3] J. Zucker, V. J. Fowler, and E. M. Conwell, J. Appl. Phys. 32, 2606 (1961).
[4] S. Kobayashi and M. Aoki, J. Phys. Soc. Japan 17, 1066 (1962).
[5] S. Kobayashi, S. Yabuki, and M. Aoki, Jap. J. Appl. Phys. 34, 1608 (1963).
[6] K. Seeger, J. Appl. Phys. 34, 1608 (1963).
[7] B. V. Paranjape, Phys. Rev. 122, 1372 (1961).
[8] B. R. Nag and P. Das, Phys. Rev. 132, 2514 (1963).

dc field are applied in the same direction. Paranjape developed the theory of harmonic generation considering acoustic-phonon scattering only, and numerical values calculated from his expression show an order-of-magnitude discrepancy with Seeger's experimental values. Nag and Das also considered the case of large dc and small microwave fields, and by solving the time-dependent Boltzmann equation found the values of microwave conductivity and change in dielectric constant. The effects of acoustic- and optical-phonon scattering were considered but the effective mass of the carriers was assumed to be isotropic. For actual semiconductors like n-type germanium, however, it is necessary to include the many-valley band structure of the material. The use of such a band structure is well established and its inclusion has revealed some interesting results, such as the Shibuya-Sasaki effect.[9] The purpose of the present communication is to incorporate the effects of many-valley band structure of semiconductors in different cases of hot-electron microwave field conduction.

In Sec. II, the results of Ref. 9 are extended to include the effects of many-valley band structure and the presence of a steady magnetic field. The microwave conductivity and change in dielectric constant are deduced from the distribution function of the carriers obtained by solving the time-dependent Boltzmann equation. Section III deals with high-microwave-field conduction. The generation of a third-harmonic component is considered in solving the Boltzmann equation. The efficiency of third-harmonic generation and the occurrence of a microwave Shibuya-Sasaki voltage are also discussed in this section with special reference to n-type germanium.

II. MICROWAVE CONDUCTION IN THE PRESENCE OF HIGH STEADY ELECTRIC, SMALL MICROWAVE, AND STEADY MAGNETIC FIELD

1. Distribution Function of the Carriers

The semiconductor is assumed to have n valleys with the effective masses in the principal directions of a valley given by m_1, m_2, and m_3. The energy E of a carrier is given by

$$E=\hbar^2/2[K_1^2/m_1+K_2^2/m_2+K_3^2/m_3], \tag{1}$$

where K_1, K_2, and K_3 are the components of the wave vector $\mathbf{K}$ along the principal directions. For the jth valley, Eq. (1) can also be written in terms of K_x, K_y, and K_z, the components of wave vector $\mathbf{K}$ in the x-y-z coordinate system, as[10]

$$\begin{aligned}E&=(\hbar^2/2m)[M_{xx}^jK_x^2+M_{yy}^jK_y^2+M_{zz}^jK_z^2\\&\quad+2M_{xy}^jK_xK_y+2M_{yz}^jK_yK_z+2M_{xz}^jK_xK_z]\\&=\hbar^2/(2m)\mathbf{K}\mathbf{M}^j\mathbf{K},\end{aligned} \tag{2}$$

where $\mathbf{M}^j$ is a tensor defined by

$$\mathbf{M}^j=\begin{bmatrix}M_{xx}^j & M_{xy}^j & M_{xz}^j\\ M_{yx}^j & M_{yy}^j & M_{yz}\\ M_{zx}^j & M_{zy}^j & M_{zz}^j\end{bmatrix}, \tag{3}$$

where $M_{\alpha\beta}=M_{\beta\alpha}$ for $\beta\neq\alpha$; $\alpha, \beta=x, y, z$

$$\frac{3}{m}=\sum_{i=1}^{3} m_i^{-1};\quad M_{xx}^j=m\sum_{i=1}^{3}\frac{\xi_i^2}{m_i};\quad M_{xy}^j=m\sum_{i=1}^{3}\frac{\xi_i\zeta_i}{m_i};$$

$$M_{xz}^j=m\sum_{i=1}^{3}\frac{\xi_i\eta_i}{m_i};\quad M_{yy}^j=m\sum_{i=1}^{3}\frac{\zeta_i^2}{m_i};\quad M_{yz}=m\sum_{i=1}^{3}\frac{\zeta_i\eta_i}{m_i};$$

and

$$M_{zz}=m\sum_{i=1}^{3}\frac{\eta_i^2}{m_i}.$$

(ξ_1,ξ_2,ξ_3), $(\zeta_1,\zeta_2,\zeta_3)$, and (η_1,η_2,η_3) are, respectively, the direction cosines of the principal axes 1, 2, 3 of the jth valley with respect to the x, y, and z axes.

The distribution function of the carriers in the jth valley, $f^j(\mathbf{K})$, at a time t, satisfies the Boltzmann equation

$$\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{field}}+\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{coll}}=\frac{\partial f^j(\mathbf{K})}{\partial t}. \tag{4}$$

Solution of Eq. (4) may be obtained assuming that $f^j(\mathbf{K})$ can be expanded as

$$f^j(\mathbf{K})=f^j(E)+\mathbf{K}\cdot\mathbf{g}(E)=f^j+\mathbf{K}\cdot\mathbf{g}, \tag{5}$$

where $\mathbf{g}$ has components g_x, g_y, and g_z in the x, y, and z directions. The first term on the left-hand side of Eq. (4) is given by[10]

$$\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{field}}=\frac{e}{\hbar}\left[\mathbf{F}+\frac{1}{\hbar}(\nabla_{\mathbf{K}}E)\times\mathbf{B}\right]\cdot\nabla_{\mathbf{K}}f^j(\mathbf{K}), \tag{6}$$

where $\mathbf{F}$ and $\mathbf{B}$ are the total applied electric and magnetic fields, respectively, e the charge of the current carriers, and $2\pi\hbar$ Planck's constant. Using Eq. (5) one obtains

$$\begin{aligned}\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{field}}&=\frac{e}{\hbar}\left[\tfrac{3}{2}(E)^{1/2}\frac{\partial}{\partial E}\{E^{-3/2}(\mathbf{F}\cdot\mathbf{g}^j)\}\right.\\&\quad\left.+\mathbf{K}\cdot\frac{\hbar^2}{2m}\mathbf{M}\left\{\mathbf{F}\cdot\frac{\partial f^j}{\partial E}+\frac{1}{\hbar}(\mathbf{g}^j\times\mathbf{B})\right\}\right].\end{aligned} \tag{7}$$

In the case of many-valley semiconductors,

$$\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{coll}}=\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{ac}}+\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{op}}+\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{int-v}}, \tag{8}$$

where the terms on the right-hand side of Eq. (8) are due to acoustic, optical, and intervalley phonon scat-

[9] W. Sasaki, M. Shibuya, K. Miziguchi, and G. M. Hatoyama, J. Phys. Chem. Solids 8, 250 (1959).

[10] P. Das and B. R. Nag, Proc. Phys. Soc. (London) 82, 923 (1963).

tering. These terms also can be written as[10]

$$\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\mathrm{ac}}=\frac{A}{\sqrt{E}}\frac{\partial}{\partial E}\left[E^2\frac{\partial f^j}{\partial E}+\frac{E^2}{kT}f^j\right]-\frac{\mathbf{K}\cdot\mathbf{g}^j}{\tau_{\mathrm{ac}}},\tag{9}$$

$$\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\mathrm{op}}=\frac{B}{2\sqrt{E}}\frac{\partial}{\partial E}\left[\hbar\omega_0(e^{S_0}+1)\left(E\frac{\partial f^j}{\partial E}\right)+2(e^{S_0}-1)(Ef^j)\right]-\frac{\mathbf{K}\cdot\mathbf{g}^j}{\tau_{\mathrm{op}}},\tag{10}$$

$$\left.\frac{\partial f^j(\mathbf{K})}{\partial t}\right|_{\text{int-v}}=\sum_{l\neq j}\frac{B_i}{\hbar\omega_i\sqrt{E}}\Big[\{E(E+\hbar\omega_i)\}^{1/2}\{f_0^l(E+\hbar\omega_i)\exp\left(\frac{\hbar\omega_i}{kT}\right)-f_0^{\alpha}(E)$$

$$+\{E(E-\hbar\omega_i)\}^{1/2}\{f_0^l(E-\hbar\omega_i)-f_0^j(E)\exp(s_i)\}\Big]-\mathbf{K}\cdot\mathbf{g}^j/\tau_{\text{int-v}}=X-\mathbf{K}\cdot\mathbf{g}^j/\tau_{\text{int-v}},\tag{11}$$

where

$$\frac{1}{\tau_{\mathrm{ac}}}=A\sqrt{E}/2mc^2;\quad \frac{1}{\tau_{\mathrm{op}}}=\frac{B}{\hbar\omega_i}(e^{S_0}+1)\sqrt{E};\quad \frac{1}{\tau_{\text{int-v}}}=\frac{(n-1)B_i}{\hbar\omega_i}(e^{s_i}+1)\sqrt{E},$$

$$A=\frac{8ec^2}{3(\pi kT)^{1/2}}\frac{1}{\mu_a};\quad s_0=\frac{\hbar\omega_0}{kT},\quad s_i=\frac{\hbar\omega_i}{kT};\qquad B=\frac{9}{16}\cdot\frac{AD^2}{C^2}\frac{\hbar\zeta^2}{2mkT}\frac{1}{e^{s_0}-1};$$

and k is Boltzmann's constant, T the lattice temperature, $\hbar\omega_0$ the characteristic energy of an optical phonon, $\hbar\omega_i$ the characteristic energy of an intervalley phonon, c the velocity of sound in the semiconductor, D the coupling constant between a conduction electron and an optical mode of vibration, C the coupling constant between a conduction electron and an acoustical mode of vibration, ζ the first nonvanishing reciprocal vector of the lattice, μ_a the low-field mobility considering only acoustic-phonon scattering, and B_i a constant for the intervalley scattering similar to B in optical-phonon scattering.

Putting Eqs. (7)–(11) in Eq. (4), one obtains

$$L(f^j)=\frac{\partial}{\partial E}\left[E^2\frac{\partial f^j}{\partial E}+\frac{E^2}{kT}f^j\right]+\frac{\partial}{\partial E}\left[\frac{BkT}{2A}s_0(e^{s_0}+1)\left(E\frac{\partial f^j}{\partial E}\right)+B(e^{s_0}-1)(Ef^j)\right]$$

$$=-\tfrac{2}{3}A\frac{e}{\hbar}\frac{\partial}{\partial E}[E^{3/2}\mathbf{F}\cdot\mathbf{g}^j]+\frac{A}{\sqrt{E}}\frac{\partial f^j}{\partial t}+X(f^j)\tag{12}$$

$$\frac{\hbar^2}{m}\frac{e}{\hbar}\left[\mathbf{F}\frac{\partial f^j}{\partial E}+\frac{1}{\hbar}(\mathbf{g}^j\times\mathbf{B})\right]=\mathbf{M}^{j-1}\cdot\left[\frac{\mathbf{g}^j}{\tau}-\frac{\partial\mathbf{g}^j}{\partial t}\right],\tag{13}$$

where

$$\frac{1}{\tau}=\frac{1}{\tau_{\mathrm{ac}}}+\frac{1}{\tau_{\mathrm{op}}}+\frac{1}{\tau_{\text{int-v}}}.\tag{14}$$

The total electric field for the case considered here is given by

$$\mathbf{F}=\mathrm{Re}\mathbf{F}_0(1+\lambda e^{i\omega t}),\tag{15}$$

where $\mathbf{F}_0$= the steady electric field applied, $\lambda\mathbf{F}_0$= the small microwave signal applied parallel to $\mathbf{F}_0$, and ω= the microwave frequency. The effect of the microwave field would be to perturb both f^j and $\mathbf{g}^j$. Since λ is a small quantity, this perturbation is considered small and f^j and $\mathbf{g}^j$ may be written as

$$f^j=f_0^j+\lambda f_1^je^{i\omega t},\tag{16}$$

$$\mathbf{g}^j=\mathbf{g}_0^j+\lambda\mathbf{g}_1^je^{i\omega t}.\tag{17}$$

Using the above two equations in Eqs. (12) and (13), one obtains, collecting the first-order terms only and solving for $\mathbf{g}_0{}^j$ and $\mathbf{g}_1{}^j$,[11]

$$L(f_0{}^j)=-\frac{2}{3}\frac{e}{\hbar}A\frac{d}{dE}[E^{3/2}\mathbf{F}\cdot\mathbf{g}_0{}^j]+X(f_0{}^j), \tag{18}$$

$$\mathbf{g}_0{}^j=\frac{eh}{m}\tau\frac{df_0{}^j}{dE}\frac{(e\tau/m)(\det\mathbf{M})\mathbf{F}_0X(\mathbf{M}^{-1}\cdot\mathbf{B})+(e\tau/m)^2\mathbf{B}(\mathbf{F}_0\cdot\mathbf{B})\det\mathbf{M}+\mathbf{M}\cdot\mathbf{F}_0}{1+(e\tau/m)^2(\mathbf{B}\cdot\mathbf{M}^{-1}\cdot\mathbf{B})\det\mathbf{M}}=\frac{eh}{m}\tau\frac{df_0{}^j}{dE}\cdot\mathbf{\Theta}_{\mathrm{dc}}, \tag{19}$$

$$L(f_1{}^j)=-\frac{2}{3}\frac{e}{\hbar}A\frac{d}{dE}[E^{3/2}F_0(\mathbf{g}_1{}^j+\mathbf{g}_0{}^j)]+j\omega\frac{\sqrt{E}}{A}f_1{}^j+X(f_1{}^j), \tag{20}$$

$$\mathbf{g}_1{}^j=\frac{eh}{m}\tau\left(\frac{df_0{}^j}{dE}+\frac{df_1{}^j}{dE}\right)\cdot\mathbf{\Theta}_{\mathrm{ac}}, \tag{21}$$

where

$$\mathbf{\Theta}_{\mathrm{ac}}=\frac{(e\tau/m)(\det\mathbf{M})\mathbf{F}_0X(\mathbf{M}^{-1}\cdot\mathbf{B})+(e\tau/m)^2(\det\mathbf{M})(1+i\omega\tau)^{-1}\mathbf{B}(\mathbf{F}_0\cdot\mathbf{B})+\mathbf{M}\cdot\mathbf{F}_0(1+i\omega\tau)}{(1+i\omega\tau)^2+(e\tau/m)^2(\mathbf{B}\cdot\mathbf{M}^{-1}\cdot\mathbf{B})\det\mathbf{M}}.$$

Putting

$$\tau_e=(kT)^{1/2}/A, \qquad (B/2A)(e^{s_0}+1)s_0=q,$$

$$\tau_m=\tau X(kT)^{1/2}/\sqrt{E}, \qquad (B/A)(e^{s_0}-1)=r,$$

$$p^*=\frac{2}{3}\frac{e}{\hbar}\frac{1}{m}\frac{eh}{A^2}\frac{2mc^2}{\Omega}\mathbf{F}_0\cdot\mathbf{M}\cdot\mathbf{F}_0, \qquad p_1{}^*=p^*/\Omega+q$$

and

$$z^2=E/kT.$$

and eliminating $\mathbf{g}_0{}^j$ and $\mathbf{g}_1{}^j$ from (18) through (21) one obtains

$$\frac{d}{dz}\left[\left\{z^3+\frac{zp^*}{\Omega}\Theta_{H\,\mathrm{dc}}+qz\right\}\frac{df_0{}^j}{dz}+(2z^4+2rz^2)f_0{}^j\right]=X(f_0{}^j) \tag{22}$$

and

$$\frac{d}{dz}\left[\left\{z^3+\frac{zp^*}{\Omega}\Theta_{H\,\mathrm{ac}}+qz\right\}\frac{df_1{}^j}{dz}+(2z^4+2rz^2)f_1{}^j\right]=i4\omega\tau_ez^2f_1{}^j-\frac{d}{dz}\left[\frac{zp^*}{\Omega}\frac{df_0{}^j}{dz}\Theta_{H\,\mathrm{ac}}\right]-\frac{d}{dz}\left[\frac{zp^*}{\Omega}\frac{df_0{}^j}{dz}\Theta_{H\,\mathrm{ac}}\right]+X(f_1{}^j), \tag{23}$$

where

$$\Theta_{H\,\mathrm{dc}}=[\mathbf{F}_0\cdot\mathbf{\Theta}_{\mathrm{dc}}]/[\mathbf{F}_0\cdot\mathbf{M}\cdot\mathbf{F}_0]=\frac{1+(e\tau/m)^2(\det\mathbf{M})(\mathbf{F}_0\cdot\mathbf{B})^2/\mathbf{F}_0\cdot\mathbf{M}\cdot\mathbf{F}_0}{1+(e\tau/m)^2(\det\mathbf{M})(\mathbf{B}\cdot\mathbf{M}^{-1}\cdot\mathbf{B})} \tag{24}$$

and

$$\Theta_{H\,\mathrm{ac}}=[\mathbf{F}_0\cdot\mathbf{\Theta}_{\mathrm{ac}}]/[\mathbf{F}_0\cdot\mathbf{M}\cdot\mathbf{F}_0]. \tag{25}$$

Generally, the value of $\omega\tau_m(=\omega\tau z)$ for the range of microwave frequency of experimental interest assumes a negligible value with respect to 1. With this approximation one has

$$\Theta_{H\,\mathrm{dc}}=\Theta_{H\,\mathrm{ac}},$$

and Eq. (23) simplifies to

$$\frac{d}{dz}\left[\left\{z^3+\frac{zp^*}{\Omega}\Theta_{\mathrm{Hdc}}+qz\right\}\frac{df_1{}^j}{dz}+(2z^4+2rz^2)f_1{}^j\right]=X(f_1{}^j)+i4\omega\tau_ez^2f_1{}^j-\frac{2p^*}{\Omega}\frac{d}{dz}\left[z\frac{df_0{}^j}{dz}\Theta_{H\,\mathrm{dc}}\right]. \tag{26}$$

The distribution function for the carriers of the jth valley may now be obtained by solving Eqs. (22) and (26) considering all the valleys together. But as discussed in Ref. 10, in place of using the complicated equations, one can simplify for two particular cases.

Case (A) Independent-Valley Model

In this case the terms contributed by intervalley scattering are completely neglected. Then $f_0{}^j$ can be written

[11] B. Lax and J. G. Mavroides in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1960), Vol. 11, p. 285.

as

$$f_0^j \propto \exp\left[-\int\left\{2z^3+2rz\right\}\Big/\left\{z^2+\frac{p^*}{\Omega}\Theta_{H\ \mathrm{dc}}+qz\right\}\right], \quad (27)$$

which, for $\mathbf{B}=0$, reduces to

$$f_0^j \propto \exp\left[-\int\left\{(2z^3+2rz)dz\right\}\Big/\left\{z^2+p_1^*\right\}\right]. \quad (27a)$$

It is evident the Eq. (27a) is similar to that for f_0^j derived in Ref. 8 considering isotropic effective mass. Only the term p^*, which is dependent on the direction in which the electric field is applied, is different and introduces the anisotropy in conduction. For a magnetic field applied transverse to the applied electric field

$$\Theta_{H\ \mathrm{dc}} \propto 1/\{1+(M_{xx}M_{yy}-M_{xy}^2)\omega_c^2\},$$

where

$$\omega_c=(eB\tau/m), \quad (28)$$

and the factor $\Theta_{H\ \mathrm{dc}}$ effectively decreases the term p^*. Thus the function of a transverse magnetic field is to reduce the effect of high-steady electric field on the semiconductor as far as microwave conductivity and change in dielectric constant are concerned.

Case (B) Intervalley Scattering Considered

The intervalley term[12–14] X in Eqs. (22) and (26) produces two effects: redistribution of number density of carriers in different valleys and equalization of the average energy of the carriers. But generally, consideration of the first effect only gives a result consistent with dc experimental results. Here this effect only will be considered, i.e.,[15]

$$\frac{n_j}{N}=T_j^{1/2}\Big/\sum_{i=1}^{n} T_i^{1/2}, \quad (29)$$

where

$$T_j \approx T[p^*\Theta_{H\ \mathrm{dc}}/r\Omega], \quad (30)$$

n_j is the carrier density of the jth valley, and T_j is the effective temperature of the jth valley. Equation (30) is not strictly valid for acoustic-phonon scattering. In this approximation the effect of the term X on the individual valley distribution function is neglected.

Using the above equations for f_0^j and n^j in different cases, one obtains the complete equation f_1^j which must be solved to obtain the values of microwave conductivity and change in dielectric constant.

2. Microwave Conductivity and Change in Dielectric Constant

Let the applied field be in the x direction. Owing to the presence of the Hall and Shibuya-Sasaki effects, fields F_y and F_z will be developed in the y and z directions. These fields can be evaluated by equating the currents in the y and z directions to zero and solving for F_y and F_z.

The current density in any direction is given by

$$J_i=\sum_{=1}^{n} n_j e\int \mathbf{K}_i\cdot(\mathbf{g}_{0i}+\lambda\mathbf{g}_{1i}e^{i\omega t})(\nabla_{\mathbf{K}}E)_i d^3\mathbf{K}, \quad (31)$$

where $i=x, y, z$. Equation (31) for current density consists of two parts: (i) the dc part and (ii) the microwave part. The dc part is given by

$$\mathbf{F}=\mathbf{Q}_0^{-1}\mathbf{J}, \quad (32)$$

$$Q_{0\alpha\beta}=\sum_{n=1}^{j}\Xi_{\alpha\beta}^j\frac{\int_0^\alpha z^2\tau f_0^j dz}{\int_0^\alpha z^2 f_0^j} \quad (33)$$

for $\mathbf{B}=0$ and

$$\Xi_{\alpha\beta}^j=\frac{2}{3}\frac{n_j}{n}\frac{e}{m}M_{\alpha\beta}^j\frac{1}{\tau_m(kT)^{i/2}}.$$

Whence we get

$$\sigma_{\mathrm{dc}}=\frac{\det\mathbf{Q}_0}{Q_{yy}Q_{zz}-Q_y^2}. \quad (34)$$

Similarly for the microwave component of current one obtains

$$\mathbf{F}=\mathbf{Q}_{\mathrm{tot}}^{-1}\mathbf{J}, \quad (35)$$

where

$$\mathbf{Q}_{\mathrm{tot}}=\mathbf{Q}_0+\mathbf{Q}_{1r}+i\mathbf{Q}_{1i},$$

$$Q_{1r\alpha,\beta}=\sum_{j=1}^{n}\Xi_{\alpha\beta}^j\frac{\int z^2dz(f_{1r}^j\tau+\frac{1}{2}\omega\tau_m^2 f_{1i}^j)}{\int z^2dz f_0^j}, \quad (36)$$

$$Q_{1i\alpha,\beta}=\sum_{j=1}^{n}\Xi_{\alpha\beta}^j\frac{\int z^2dz(f_{1i}\tau-\frac{1}{2}\omega\tau_m^2(f_{1r}+f_0))}{\int z^2dz f_0^j}. \quad (37)$$

For the microwave susceptance also one obtains an expression similar to that of Eq. (34).

For cubic semiconductors, the above terms become much simplified and one obtains approximately (neglecting terms of smaller order due to Shibuya-Sasaki effect):

$$\sigma_m=\text{microwave conductivity}\approx\sigma_{\mathrm{dc}}\left[1+\frac{Q_{1rxx}}{Q_{0xx}}\right] \quad (38)$$

$$\text{change in dieleotric constant}=\Delta\epsilon\approx\frac{dc}{\omega\epsilon_0}\frac{Q_{1ixx}}{Q_{0xx}}. \quad (39)$$

[12] J. Yamashita and K. Inoue, J. Phys. Chem. Solids **12**, 1 (1959).
[13] E. G. S. Paige, Proc. Phys. Soc. (London) **75**, 174 (1960).
[14] H. G. Reik and H. Risken, Phys. Rev. **126**, 1737 (1962).
[15] H. G. Reik and H. Risken, Phys. Rev. **124**, 777 (1961).

To evaluate the constants $Q_{1\alpha,\beta}$, Eq. (26) is solved for f_1^j; once f_0^j is known, this can be done by expanding f_1^j in terms of z in a power series. The coefficients of the expansion can be determined by solving a set of linear algebraic equations, as discussed in Ref. 8.

It is worth mentioning that the above equation simplifies to that of the isotropic-mass case for n-type germanium if the applied electric field is parallel to either of the crystallographic axes. But for other directions of the applied electric field, the different valleys have different effective temperatures and one must solve f_1^j for each valley, since p^* is different for different valleys. Also it is to be noted that some amount of microwave power is excited in the transverse direction because of the Shibuya-Sasaki effect.

III. MICROWAVE CONDUCTION IN THE PRESENCE OF A HIGH-MICROWAVE FIELD

1. Energy Distribution of Carriers

In the presence of a high-microwave field $\mathrm{Re}\mathbf{F}_1 e^{i\omega t}$, one expands f^j and $\mathbf{g}^j$ of Eq. (5) in terms of Fourier series:

$$f^j = f_0^j + f_1^j e^{i\omega t} + f_2 e^{i\omega t} + \cdots, \tag{40}$$

$$\mathbf{g}^j = \mathbf{g}_0^j + \mathbf{g}_1^j e^{i\omega t} + \mathbf{g}_2^j j^{i\omega t} + \cdots, \tag{41}$$

where f_0^j, $f_1^j \cdots$ and $\mathbf{g}_0^j$, $\mathbf{g}_1^j \cdots$ are functions of energy only.

Putting the above two equations in Eqs. (13) and (14) one obtains

$$f_1^j = f_3^j = \cdots = f_{2n+1} = 0,$$

and

$$\mathbf{g}_0^j = \mathbf{g}_2^j = \cdots \mathbf{g}_{2n}^j = 0.$$

The nonvanishing terms f_2^j, $f_4^j, \cdots$ and $\mathbf{g}_3^j$, $\mathbf{g}_5^j, \cdots$ are successively smaller and, neglecting terms higher than f_2^j in the expansion of f^j and higher than $\mathbf{g}_3^j$ in the expansion of $\mathbf{g}_0^j$, the following equations are obtained:

$$L(f_0^j) = -\frac{2}{3}\frac{e}{\hbar}\frac{1}{A}\frac{d}{dE}\left[\tfrac{1}{2}E^{3/2}\mathbf{F}\cdot\mathbf{g}_1\right] + X(f_0^j), \tag{42}$$

$$\mathbf{g}_1^j = \frac{eh}{m}\tau\left[\frac{df_0}{dE} + \frac{1}{2}\frac{df_2}{dE}\right]\mathbf{\Theta}_{\mathrm{ac},\,\omega\tau}, \tag{43}$$

$$L(f_2^j) = -\frac{2}{3}\frac{e}{\hbar}\frac{1}{A}\frac{d}{dE}\left[\tfrac{1}{2}E^{3/2}\mathbf{F}\cdot(\mathbf{g}_1^j + \mathbf{g}_3^j)\right] + \frac{\sqrt{E}}{A}2i\omega f_2 + X(f_2^j), \tag{44}$$

$$\mathbf{g}_3 = \frac{eh}{m}\tau\left(\frac{1}{2}\frac{df_2^j}{dF}\right)\mathbf{\Theta}_{\mathrm{ac},\,3\omega\tau}, \tag{45}$$

where $\mathbf{\Theta}_{\mathrm{ac},\,3\omega t}$ is given by the same expression as Θ_{ac} but with $\omega\tau$ replaced by $3\omega\tau$ wherever it occurs.

By eliminating $\mathbf{g}_1^j$ and $\mathbf{g}_3^j$ from the above equations and simplifying, one obtains

$$\frac{d}{dz}\left[(z^3 + \tfrac{1}{2}zp^*\Theta_{H\,\mathrm{ac}} + qz)\frac{df_0^j}{dz} + 2(z^4 + 2rz^2)f_0^j\right] = \frac{d}{dz}\left[\frac{zp^*}{4\Omega}\Theta_{H\,\mathrm{ac}}\frac{df_2^j}{dz}\right] + X(f_0^j) \tag{46}$$

and

$$\frac{d}{dz}\left[\left\{z^3 + \frac{zp^*}{4\Omega}\Theta_{H\,\mathrm{ac}} + \frac{zp^*}{4\Omega}\Theta_{H\mathrm{ac},\,3\omega} + qz\right\}\frac{df_1^j}{dz} + (2z^j + 2rz^2)f_1^j\right] = i8\omega\tau_e f_1^j - \frac{p^*}{2\Omega}\frac{d}{dz}\left[z\frac{df_0^j}{dz}\Theta_{H\,\mathrm{ac}}\right] + X(f_1^j). \tag{47}$$

The term due to f_2^j on the right-hand side of Eq. (46) is negligible in comparison to other terms. On neglecting this term and also considering ω small, the value of f_0^j reduces to Eq. (27) in the independent-valley model, and the same applies to the case where intervalley scattering is considered. Only in the calculation of p^* must one consider the rms value of the applied microwave field and not the peak value. The equation for f_2^j also simplifies to

$$\frac{d}{dz}\left[\left(z^3 + \frac{zp^*}{2\Omega}\Theta_{H\,\mathrm{dc}} + qz\right)\frac{df_2^j}{dz} + (2z^4 + 2r_1z^2)f_2^j\right] = i8\omega\tau_e f_2^j - \frac{p^*}{2\Omega}\left[z\frac{df_0^j}{dz}\Theta_{H\,\mathrm{dc}}\right] + X(f_2^j). \tag{48}$$

The above equation is similar to that of f_1^j in Eq. (26), and the time-independent parts of the symmetrical component of the distribution function of the carriers are identical.

2. Microwave Conductivity, Change in Dielectric Constant and Conversion Efficiency of Third-Harmonic Generation

The fundamental and the third-harmonic component of current may be written, respectively, as

$$J_{1i} = \mathrm{Re}\,e^{i\omega t}\sum_{j=1}^{n} n_j e\int K_i \mathbf{g}_{1i}^j \frac{1}{h}(\nabla_{\mathbf{K}}E)_i d^3\mathbf{K}. \tag{49}$$

$$J_{3i} = \mathrm{Re}\,e^{3i\omega t}\sum_{j=1}^{n} n_j e\int K_i \mathbf{g}_{3i}^j \frac{1}{h}(\nabla_{\mathbf{K}}E)_i d^3\mathbf{K}. \tag{50}$$

Proceeding as in Sec. II.2, one obtains approximate values of σ_m and $\Delta\epsilon$ for cubic semiconductors given by

$$\sigma_m \approx \sigma_{\text{dc}}\left[1+\frac{Q_{2rxx}}{Q_{0xx}}\right], \quad (51)$$

$$\Delta\epsilon \sim \frac{\sigma_{\text{dc}}}{\omega\epsilon_0}\left[\frac{Q_{2ixx}}{Q_{0xx}}\right], \quad (52)$$

when the applied microwave field is in the x direction.

The conversion efficiency for third-harmonic generation, defined as the fraction of total input power converted into the third-harmonic component, can be written as

$$\eta=\frac{(Q_{3rxx})^2+(Q_{3ixx})^2}{(Q_{0xx}+Q_{2rxx})^2+(Q_{2ixx})^2+(Q_{3rxx})^2+(Q_{3ixx})^2}, \quad (53)$$

where

$$Q_{2r\alpha\beta}=\tfrac{1}{2}\sum_{j=1}^{n}\Xi_{\alpha\beta}{}^{j}\frac{\int(f_{2r}{}^{j}\tau+\frac{1}{2}\omega\tau_m{}^2f_{2i}{}^{j})z^2dz}{\int z^2f_0{}^{j}dz}, \quad (54)$$

$$Q_{2i\alpha\beta}=\tfrac{1}{2}\sum_{j=1}^{n}\Xi_{\alpha\beta}{}^{j}\frac{\int\{f_{2i}{}^{j}\tau+\frac{1}{2}\omega\tau_m{}^2(f_{2r}{}^{j}+f_0{}^{j})\}z^2dz}{\int z^2f_0{}^{j}dz}, \quad (55)$$

$$Q_{3r\alpha\beta}=\tfrac{1}{2}\sum_{j=1}^{n}\Xi_{\alpha\beta}{}^{j}\frac{\int z^2dz\{f_{2i}{}^{j}\tau+\frac{1}{2}\omega\tau_m{}^2f_{2r}{}^{j}\}}{\int z^2f_0{}^{j}dz}, \quad (56)$$

$$Q_{3i\alpha\beta}=\tfrac{1}{2}\sum_{j=1}^{n}\Xi_{\alpha\beta}{}^{j}\frac{\int z^2dz\{f_{2i}{}^{j}\tau-\frac{1}{2}\omega\tau_m{}^2f_{2r}\}}{\int z^2f_0{}^{j}dz}. \quad (57)$$

Since the components of the new tensor defined above depend on the direction in which the electric field is applied, the conversion efficiency will depend on the orientation of the sample. The effect of the magnetic field will be similar to that discussed in Sec. II, i.e., to reduce the effective temperatures of different valleys. Though we shall make no further calculations taking **B** into account, it is expected that the effect of a steady magnetic field will be to reduce the conversion efficiency of third-harmonic generation.

Solving for F_y, the electric field developed due to the Shibuya-Sasaki effect, one finds with the above approximations

$$F_y=(\text{applied field})\times\frac{Q_{2rxx}+iQ_{2ixx}}{Q_{2rxy}+iQ_{2ixy}}; \quad (58)$$

so if one places a semiconductor slab at the junction of the two crossed wave guides as shown in Fig. 1, then for low electric fields, when there is no transverse voltage F_y, no power is transmitted in the other waveguide.[16,17] But for large pulsed microwave power, hot-carrier conditions generate a transverse voltage F_y, which excites some power P_{out} in the other waveguide. F_y can be experimentally determined from the relation

$$\tan\epsilon=F_y/F_x=(P_{\text{out}}/\mathrm{P}_{\text{in}})^{1/2}. \quad (59)$$

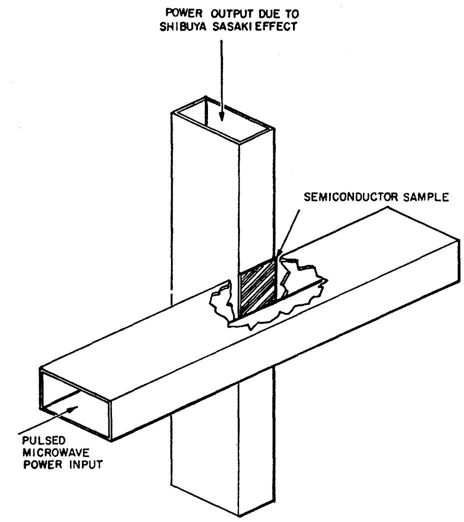

FIG. 1. Crossed-waveguide arrangement for measurement of the microwave Shibuya-Sasaki effect.

This experiment, the microwave counterpart of the Shibuya-Sasaki experiment, will throw much light on the microwave interaction in many-valley semiconductors and will be crucial test of the theory developed here.

3. Numerical Results

In this section a calculation of the values of σ_m, $\Delta\epsilon$, and η for a 4.68-Ω-cm n-type germanium sample will be presented considering isotropic effective mass. The reason for doing this is that using the solution of $f_1{}^j$ in Ref. 8, one can obtain the results very easily without any further solution of $f_2{}^j$ by computers. Expanding the second-harmonic component of the symmetric part of the distribution function f_2 in the form

$$f_2=\tfrac{1}{2}(a_0+a_1z+\cdots)f_0, \quad (60)$$

[16] G. E. Hamilton and W. W. Gartner, J. Phys. Chem. Solids 8, 329 (1959).

[17] T. Stubb, Acta Polytech. Scand. Phys. Nucl. Ser. 11, 18 (1961).

where $a_0, a_1, \cdots$ are constants, one finds that these constants are the same as those tabulated in Ref. 8 for two cases; predominantly acoustic-phonon scattering and optical-phonon scattering for $\omega=\frac{1}{2}[2.18\times10^{11}$ rad/sec] and $F_1=2\sqrt{2}$kV/cm.

Using the above values of the constants one obtains the following results:

Case A—Acoustic-phonon scattering only

$$\text{change in conductivity}=\frac{\sigma_{\rm dc}-\sigma_m}{\sigma_{\rm dc}}=\frac{\Delta\sigma}{\sigma_{\rm dc}}=0.000517\,,$$

$$\text{change in dielectric constant}=\Delta\epsilon=0.149$$

$$=0.018\%\,.$$

Case B—Optical-phonon scattering

$$\Delta\sigma/\sigma_{\rm dc}=0.05572\,,$$

$$\Delta\epsilon=0.262\,,$$

$$\eta=0.35\%\,.$$

Paranjape's formula gives, for acoustic-phonon scattering only

$$\Delta\sigma/\sigma_{\rm dc}=0.0005\%\,,$$

$$\Delta\epsilon=0.09\%\,,$$

$$\eta=0.006\%\,,\quad 0.02\%\quad \text{(at 9.378 kMc/sec)}.$$

Experimentally Seeger obtained, at $2\sqrt{2}$ kV/cm, a conversion efficiency of about 0.45%. From an examination of the above table, it is evident that considering acoustic phonon scattering only (Case A) and from Paranjape's formula, one obtains a negligible difference between the dc and microwave conductivities, but the theoretical efficiency so obtained is an order of magnitude lower than the experimental value. Consideration of optical phonon scattering (Case B) shows that though the result predicts a negligible frequency effect, nevertheless, the conversion efficiency at 17.38 kMc/sec is of the same order as obtained experimentally at the frequency of 9.378 kMc/sec.

4. DISCUSSION

In the treatment above, two configurations of the applied electric field were considered. Another case, where the steady high electric field and small microwave fields are applied at right angles, is of special interest since Erlbach[18] has shown recently from dc incremental calculations that negative resistance is expected for certain favorable conditions of carrier population in the valleys of many-valley semiconductors. The calculations for this case can be made easily using the technique developed in Sec. II.1.

At low temperatures, the effects of ionized-impurity scattering become equally important. One can easily include them in the theories by considering[19]

$$\left.\frac{\partial f^j}{\partial t}\right|_{im}=-\frac{\mathbf{K}\cdot\mathbf{g}_i{}^j}{\tau_{im}}\,, \tag{61}$$

where

$$\frac{1}{\tau_{im}}=-\frac{4e}{3m(\pi k_0T)^{1/2}}\frac{6(k_0T)^2}{\mu_iE\sqrt{E}}\,. \tag{62}$$

Consideration of ionized-impurity scattering is also important in connection with negative resistance.[20]

Few numerical results are presented in this report. The author is at present working on the numerical evaluation of microwave conductivity, change in dielectric constant, and conversion efficiency of third-harmonic generation. The results of these numerical calculations and also that of the microwave Shibuya-Sasaki-effect experiment will be reported at a later date.

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[18] E. Erlbach, Phys. Rev. **132**, 1976 (1963).
[19] A. Hasegawa and J. Yamashita, J. Phys. Soc. Japan **17**, 1751 (1962).
[20] I. Adawii, J. Appl. Phys. **32**, 1101 (1961).