

dominate over the former. In this case, the peak will again shift uniformly with photon energy.

If the preceding viewpoint is correct and direct transitions predominate, the relative strength of the two photoemission peaks at a given  $h\nu$  is expected to be comparable. Indeed, this can be estimated from a consideration of transitions near  $W$  between bands 5 and 6, and 4 and 6, respectively. For  $h\nu=7$  and 9 eV, we find that  $\delta\epsilon_2^b$  for the two kinds of transitions lie within a factor of 2 of each other. It should be noted, however, that such comparable magnitudes could also be obtained if the interaction determining the indirect transition were strongly coupled.

In attempting to account for the apparent failure of  $k$  conservation, one might be tempted to invoke the fact that since photoemission effects take place within a short distance of the surface, surface effects are in some

way disrupting this selection rule. However, the same could be said of the optical constants as determined by the present reflectance techniques, which are known to be well correlated with bulk properties.

In conclusion, the optical structure for the noble metals in the low-frequency range can be explained quite satisfactorily on the basis of direct interband transitions. While indirect transitions appear to contribute, as suggested by some of the photoemission data, there is no clear experimental evidence indicating that they must be the dominant factor in determining the interband optical properties for the noble metals.

#### ACKNOWLEDGMENTS

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### Magnetostriction of Tb Single Crystals\*

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The magnetostriction of Tb single crystals has been measured from 4 to 350°K in applied fields up to 30 kOe by strain-gage methods. The temperature dependence of the four magnetostriction constants has been determined under the assumption that the magnetic moment remains in the basal plane. Two of the constants were evaluated by strain measurements as a function of applied-field direction in the basal plane, and the remaining two by isofield magnetostrain measurements and subtraction of the thermal strain extrapolated from the paramagnetic region. The theoretical dependence of the anisotropic basal plane constants on the magnetic moment has been evaluated.

#### INTRODUCTION

THE magnetostriction of a material is a result of the interaction between the magnetic anisotropy and exchange energies and the elastic energy. In the process of magnetization, if it becomes energetically favorable, the crystal lattice will distort, producing the observed magnetostrain. This strain is normally on the order of  $10^{-5}$  in./in. in ferromagnets such as iron, nickel, and cobalt. In some of the magnetic rare earths however, owing to the large anisotropy energies, the observed magnetostriction is larger by almost 2 orders of magnitude. Most work in the past on the rare earths has been on polycrystalline material because of the lack of single crystals of sufficient size. Single crystals offer considerable advantage in that they permit a direct measurement of crystalline and magnetic anisotropies so that direct correlations with theoretical results can be made. Magnetostriction of the polycrystalline rare

earths has been studied by Belov *et al.*,<sup>1</sup> by Nikitin,<sup>2</sup> and by Lee and Alpert.<sup>3</sup> More recent single-crystal studies have been made at this laboratory on Ho,<sup>4</sup> Dy,<sup>4</sup> and Gd.<sup>5</sup> Studies elsewhere have been made by Clark *et al.* on Dy,<sup>6,7</sup> and by Bozorth *et al.* on Gd.<sup>8</sup> The principal strain measurements have been made by the use of strain gages (initially by Goldman and Smoluchowski<sup>9</sup>)

<sup>1</sup> K. P. Belov, R. Z. Levitin, S. A. Nikitin, and A. V. Ped'ko, Zh. Eksperim. i Teor. Fiz. **40**, 1562 (1961) [English transl.: Soviet Phys.—JETP **13**, 1096 (1961)].

<sup>2</sup> S. A. Nikitin, Zh. Eksperim. i Teor. Fiz. **43**, 31 (1962) [English transl.: Soviet Phys.—JETP **16**, 21 (1963)].

<sup>3</sup> E. W. Lee and L. Alpert, Proc. Phys. Soc. (London) **79**, 977 (1962).

<sup>4</sup> S. Legvold, J. Alstad, and J. Rhyne, Phys. Rev. Letters **10**, 509 (1963).

<sup>5</sup> J. K. Alstad and S. Legvold, J. Appl. Phys. **35**, 1752 (1964).

<sup>6</sup> A. E. Clark, R. M. Bozorth, and B. F. DeSavage, Phys. Letters **5**, 100 (1963).

<sup>7</sup> A. E. Clark, B. F. DeSavage, and E. R. Callen, J. Appl. Phys. **35**, 1028 (1964).

<sup>8</sup> R. M. Bozorth and T. Wakiyama, J. Phys. Soc. Japan **18**, 97 (1963).

<sup>9</sup> J. R. Goldman and R. Smoluchowski, Phys. Rev. **75**, 140 (1949).

\* Contribution No. 1609. Work was performed in the Ames Laboratory of the U. S. Atomic Energy Commission.

and by capacitive and interferometric techniques which measure the macroscopic crystal strain in an applied magnetic field. Darnell<sup>10</sup> has determined the magnetostriction of Dy and Tb by x-ray examination of the lattice parameters.

In this paper we report recent work on the magnetostriction of Tb single crystals. Strains produced by the application or rotation of a magnetic field were measured by strain gages as described later. The results were used to obtain the magnetostriction constants as a function of temperature from 10 to 300°K.

### THEORY

The calculation of the magnetostriction of Tb is complicated by the 6-fold magnetic anisotropy of the basal plane. Mason<sup>11</sup> has derived an expression for the thermodynamic enthalpy in terms of the crystal stresses and the elastic compliances, the first- and second-order magnetostrictive terms, the anisotropy energy terms to third order, and the components of the magnetization along orthogonal axes. When the terms up to 6th-rank tensors are evaluated an expression is obtained characteristic of hexagonal symmetry. From this result the magnetostrictive strain components  $S_{ij}$  corresponding to energy terms linear in the stresses may be obtained. The magnetostriction in a direction characterized by direction cosines  $\beta_1\beta_2\beta_3$  may then be written  $\Delta l/l = -S_{ij}\beta_i\beta_j$  with summation over repeated indices. Mason<sup>12</sup> obtains the following expression for the magnetostriction in a hexagonal material:

$$\begin{aligned} \Delta l/l = & A[2\alpha_1\alpha_2\beta_1 + (\alpha_1^2 - \alpha_2^2)\beta_2]^2 \\ & + B\alpha_3^2[(\alpha_1\beta_1 + \alpha_2\beta_2)^2 - (\alpha_1\beta_2 - \alpha_2\beta_1)^2] \\ & + C[(\alpha_1\beta_1 + \alpha_2\beta_2)^2 - (\alpha_1\beta_2 - \alpha_2\beta_1)^2] \\ & + D(1 - \alpha_3^2)(1 - \beta_3^2) + E\alpha_3^2\beta_3^2(1 - \alpha_3^2) \\ & + F\alpha_3^2(1 - \alpha_3^2) + G\beta_3^2(1 - \alpha_3^2) + H\alpha_3\beta_3(\alpha_1\beta_1 + \alpha_2\beta_2) \\ & + I\alpha_3^2\beta_3(\alpha_1\beta_1 + \alpha_2\beta_2) + J\alpha_3^2(1 - \beta_3^2) + K\alpha_3^2\beta_3^2. \quad (1) \end{aligned}$$

The  $\alpha$ 's are direction cosines of the magnetization. Mason's equation was originally derived for Co which has the 3 axis as the direction of the spontaneous magnetization. Since Tb has an easy direction in the basal plane ( $b$  axis), it is necessary to retain the  $J$  and  $K$  terms which were dropped by Mason in his expression for Co. The eleven magnetostriction constants,  $A$  through  $K$ , are expressible in terms of the components of the first- and second-order magnetostriction energy tensors and the saturation magnetization (see Mason<sup>12</sup>).

The coordinate system used in this discussion for the hcp Tb structure is orthogonal with the 1 axis along the  $a(11\bar{2}0)$  crystal axis, the 2 axis along the  $b(10\bar{1}0)$  direction, and the 3 axis along the  $c(0001)$  crystal axis.

Magnetization<sup>13,14</sup> studies on Tb have shown that magnetic fields considerably in excess of normal laboratory fields are necessary to produce significant rotation of the magnetic moment out of the basal plane; we therefore make the assumption  $\alpha_3=0$ . Equation (1) then reduces to

$$\begin{aligned} [\Delta l/l]_{\alpha_3=0} = & A[2\alpha_1\alpha_2\beta_1 + (\alpha_1^2 - \alpha_2^2)\beta_2]^2 \\ & + C[(\alpha_1\beta_1 + \alpha_2\beta_2)^2 - (\alpha_1\beta_2 - \alpha_2\beta_1)^2] \\ & + D(1 - \beta_3^2) + G\beta_3^2. \quad (2) \end{aligned}$$

The  $A$  term is the only one characteristic of hexagonal symmetry; the other three correspond to cylindrical symmetry. In determining the magnetostriction constants it is desirable to eliminate the demagnetized state as a reference, since the strain in this state is a function of the magnetic history. The domain configuration and hence the demagnetized strain resulting when the crystal is cooled through the ordering temperature, or when the applied field is removed, is not closely reproducible.

Constants  $A$  and  $C$  may be determined without reference to a demagnetized state as follows: a basal-plane sample is rotated in a saturating field applied parallel to the sample plane, and the linear strain along the  $a$  or  $b$  axis is measured as a function of the angle between the magnetization and the easy ( $b$ ) axis direction. For the  $b$ -axis strain, setting  $\beta_1=\beta_3=0$ ,  $\beta_2=1$  in Eq. (2) gives

$$[\Delta l/l]_b = A[\alpha_1^2 - \alpha_2^2]^2 + C(\alpha_2^2 - \alpha_1^2) + D. \quad (3)$$

The reference state with magnetic moment along the  $b$  axis ( $\alpha_1=0$ ,  $\alpha_2=1$ ) has strain

$$[\Delta l/l]_b^b = A + C + D. \quad (4)$$

The  $b$ -axis strain between this reference state and any other state where the magnetic moment makes an angle  $\theta$  with the  $b$  axis may be written ( $\alpha_1 = \sin\theta$ ,  $\alpha_2 = \cos\theta$ )

$$\left[ \frac{\Delta l}{l} \right]_b^\theta - \left[ \frac{\Delta l}{l} \right]_b^{\theta=0} \equiv \left[ \frac{\Delta l}{l} \right]_b^{\theta'} = -A \sin^2 2\theta' - 2C \sin^2 \theta'. \quad (5)$$

Here  $\theta'$  indicates the strain of the reference state has been subtracted. The expression for the  $a$ -axis strain is similarly

$$[\Delta l/l]_a^{\theta'} = A \sin^2 2\theta' + 2C \sin^2 \theta' \quad (6)$$

defining  $\theta'$  the same as in Eq. (5).  $A$  and  $C$  may be conveniently determined from (5) or (6) by measuring the strain at  $\theta' = 60$  and  $90^\circ$  where respectively:

$$[\Delta l/l]_{b,a}^{\theta'=60^\circ} = \mp \frac{3}{4}(A + 2C), \quad (7a)$$

$$[\Delta l/l]_{b,a}^{\theta'=90^\circ} = \mp 2C. \quad (7b)$$

Owing to basal-plane anisotropy, the magnetic moment will not line up exactly with the field in the

<sup>10</sup> F. J. Darnell, Phys. Rev. **132**, 128 (1963).

<sup>11</sup> W. P. Mason, Phys. Rev. **82**, 715 (1951).

<sup>12</sup> W. P. Mason, Phys. Rev. **96**, 302 (1954).

<sup>13</sup> D. E. Hegland, S. Legvold, and F. H. Spedding, Phys. Rev. **131**, 158 (1963).

<sup>14</sup> R. B. Flippin, J. Appl. Phys. **7**, 2026 (1963).

hard magnetic direction ( $\theta' = 90^\circ$ ). From the results of magnetic-moment measurements,<sup>13</sup> it is estimated that this angle error is less than  $6^\circ$  down to about  $80^\circ\text{K}$  with a 30-kOe applied field. An error in  $\theta'$  as large as  $8^\circ$  will produce less than a 1% change in the value of  $C$  calculated from Eq. (7b). At  $\theta' = 60^\circ$  the moment will be in alignment with the field, as this is also an easy magnetic direction. To obtain the spontaneous magnetostriction constants  $A_0, C_0$ , it is necessary to extrapolate values of  $C$  and  $A+2C$  to zero field from their field dependence at high fields.

The remaining constants  $D$  and  $G$  are determined by measuring the strain as a function of temperature in a saturating applied field and subtracting the thermal expansion extrapolated from above the Néel temperature. The difference is the magnetostriction contribution. It is to be noted that the thermal expansion cannot be measured below the Néel temperature ( $230^\circ\text{K}$ ) as here the magnetostrain is the dominant effect and cannot be isolated from the thermal strain. Measurements of the strain below the Curie temperature in the absence of an applied field are subject to the same hysteresis effects and to the lack of reproducibility mentioned above with respect to the demagnetized state.

If the magnetic field is applied along the  $b$  axis (easy axis) the strain Eq. (2) becomes: ( $\alpha_1 = 0, \alpha_2 = 1$ )

$$[\Delta l/l]^b = A\beta_2^2 + C[\beta_2^2 - \beta_1^2] + D(1 - \beta_3^2) + G\beta_3^2. \quad (8)$$

The strains found by subtraction of the thermal expansion from the  $a$ -,  $b$ -, and  $c$ -axis strains measured in an applied field satisfy the following relations:

$$a \text{ axis } (\beta_1 = 1, \beta_2 = \beta_3 = 0) \left[ \frac{\Delta l}{l} \right]_a^b - \left[ \frac{\Delta l}{l} \right]_a^0 = -C + D, \quad (9)$$

$$b \text{ axis } (\beta_1 = 0, \beta_2 = 1, \beta_3 = 0) \left[ \frac{\Delta l}{l} \right]_b^b - \left[ \frac{\Delta l}{l} \right]_b^0 = A + C + D, \quad (10)$$

$$c \text{ axis } (\beta_1 = \beta_2 = 0, \beta_3 = 1) \left[ \frac{\Delta l}{l} \right]_c^b - \left[ \frac{\Delta l}{l} \right]_c^0 = G. \quad (11)$$

Here the superscripts indicate the field direction and the subscripts the measurement direction. From Eq. (9) and values of  $C$  obtained from rotation measurements the constant  $D$  may be calculated. Equation (11) gives the remaining constant  $G$  directly. To obtain the spontaneous values of  $D$  and  $G$  these results must be corrected for the forced magnetostriction.

It is not possible to obtain all four constants by linear strain measurements as outlined above with the magnetic moment remaining in the  $b$  direction. This is seen by assuming a fourth measuring direction with cosines  $u, v, w$  and inspecting the determinant of the coefficients of  $A, C, D, G$  from Eqs. (9), (10), (11) and the fourth-direction equation. This determinant is seen to be 0. The constant  $A$  which represents departures from cylindrical symmetry in the magnetoelastic energy can-

not be isolated unless the magnetic moment is rotated out of the easy magnetic direction.

### EXPERIMENTAL PROCEDURE

The Tb single crystals used in this work were grown from arc-melted buttons of Tb metal using a method described by Nigh.<sup>15</sup> The crystals were aligned by Laue back-reflection x-ray techniques and disk-shaped specimens were cut using an electrospark cutter. Two disks were prepared, one having the  $a, b$  axes in the plane and the other containing the  $b, c$  axes. The dimensions of the specimens were approximately 9 mm in diameter and 2 mm thick.

Strains were measured using Budd Instruments resistance strain gages made of Budd alloy with an epoxy-phenolic-resin backing. The gages, approximately  $\frac{1}{8}$  in.  $\times$   $\frac{1}{8}$  in., were affixed to the single-crystal specimens using Budd GA-5 heat-curing epoxy. A similar gage was affixed to a piece of fused quartz and placed in thermal contact with the sample to act as a strain reference. The active and reference gages were connected in opposite arms of a sensitive Wheatstone bridge. This arrangement effectively canceled out any apparent thermal strain due to the gages and most of the effects of magnetoresistance in the gages. At temperatures below about  $30^\circ\text{K}$  some differential magnetoresistance was encountered, amounting to approximately  $30 \mu\text{-in./in.}$  at  $4^\circ\text{K}$ . The magnitude was essentially independent of the direction of the magnetic field relative to the gage.

The specimens were affixed to a cardboard spacer using a thin strip of GA-5 epoxy under about  $\frac{1}{3}$  of the disk. In tests on Cu disks negligible errors in strain due to this type of mounting were observed. The specimen assembly was in turn attached to a servo-controlled rotating holder and placed in a temperature-controlled chamber and Dewar. This allowed rotation of the crystal plane parallel to the magnetic field in a temperature environment controlled to  $\pm 0.1^\circ$  from 4 to  $350^\circ\text{K}$ . Temperatures were determined by a Cu-constantan thermocouple above  $20^\circ\text{K}$  and by a Au-Fe versus Cu thermocouple below  $20^\circ\text{K}$ . Relative errors in temperature are about  $\pm 0.1^\circ\text{K}$  and the absolute accuracy of the values is approximately  $\pm 0.5^\circ\text{K}$ . No appreciable effect on the thermocouple emf was observed on application of the magnetic field. The magnet used was a Harvey Wells L-158 electromagnet with a maximum field of 30 kOe.

Strains were computed from fractional changes in resistance of the active gage from the relation

$$\Delta l/l = (1/G)(\Delta R/R), \quad (12)$$

where  $G$  is the gage factor. The Wheatstone bridge had a sensitivity ( $\Delta R/R$ ) of better than  $0.6 \mu\Omega/\Omega$  corresponding to a strain of  $0.3 \mu\text{-in./in.}$  at room temperature. All measurements were taken with the bridge in

<sup>15</sup> H. E. Nigh, J. Appl. Phys. **34**, 3323 (1963).

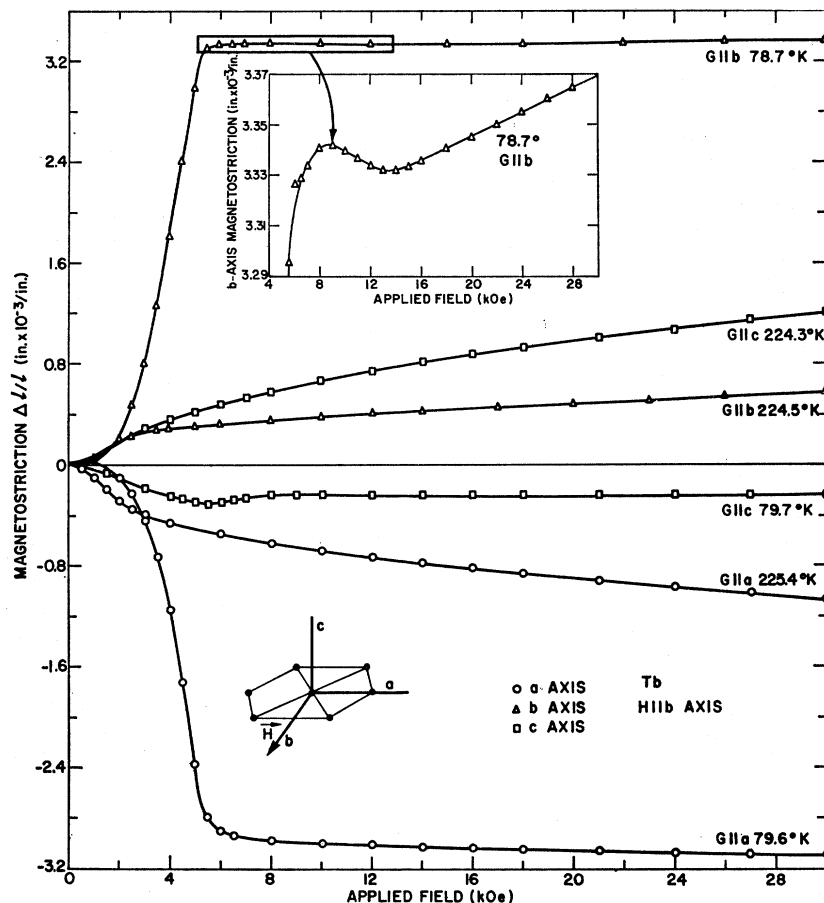


FIG. 1. The *a*-, *b*-, and *c*-axis strain as a function of field magnitude applied along the *b* direction. Curves in the ferromagnetic and antiferromagnetic region are given. The inset shows the forced magnetostriction above saturation. *G* indicates the strain-gage direction.

a balanced condition. The temperature dependence of the gage factor was determined using a cantilever-beam-type strain-gage calibrator manufactured by Cryresco. From room temperature to 78°K the gage factor increased by 9%. The calibration was extended down to

1.4°K although some uncertainty in the gage factor was apparent below 20°K. For strain measurements as a function of temperature the mean value of the gage factor between that temperature and the reference temperature (usually 300°K) was used in Eq. (12). No correction was made for the thermal expansion of the quartz disk holding the "dummy" gage since the thermal expansion of quartz is within the experimental error of the *T<sub>b</sub>* strain measurements.

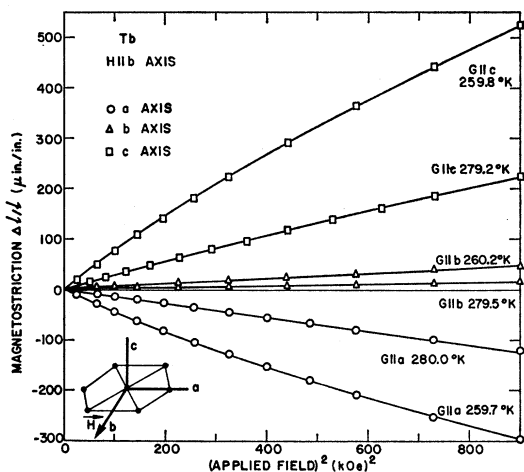


FIG. 2. The *a*-, *b*-, and *c*-axis strain as a function of  $H^2$  applied along the *b* direction in the paramagnetic region. Some ordering is indicated at 260° by the slight curvature at the higher fields. *G* indicates the strain-gage direction.

### RESULTS

Figures 1 and 2 show the *a*-, *b*-, and *c*-axis linear strains as a function of magnetic field applied along the *b* direction. Curves are shown for each magnetic region (paramagnetic,  $T > 230^\circ\text{K}$ ; antiferromagnetic,  $221^\circ\text{K} < T < 230^\circ\text{K}$ ; and ferromagnetic,  $T < 221^\circ\text{K}$ ). At temperatures well above the antiferromagnetic-paramagnetic transition no magnetic ordering is induced by the applied field and the magnetostriction is proportional to  $H^2$  as found from thermodynamic considerations. Examination of the strain curves plotted in Fig. 2 as a function of  $H^2$  shows that some ordering is induced by high fields at temperatures above the Néel point as shown by the slight curvature of the 260°K plot. At 280°K and above no ordering is observed.

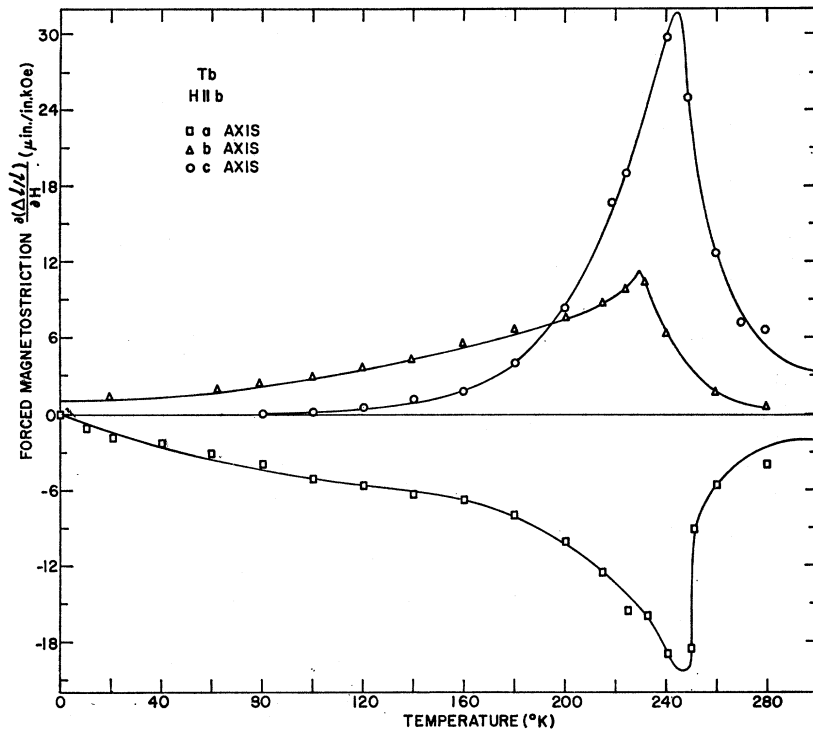


FIG. 3. Forced magnetostriction in the  $a$ -,  $b$ -, and  $c$ -axis directions. The data points are the slopes of the high field region of the isothermal magnetostriction curves. The field was applied along the  $b$  direction. Values near the Curie temperature are field-dependent and do not represent true saturation forced magnetostriction.

In the antiferromagnetic temperature region Tb has a helical spin structure similar to that in Dy;<sup>16</sup> however, the antiferromagnetic interaction is much weaker in Tb and a very small applied field will remove the helical state and produce ferromagnetic ordering along the field direction. The 224°K  $c$ -axis curve of Fig. 1 shows that the resulting strain is much smaller than in Dy.<sup>4</sup> Below the Curie temperature the  $c$ -axis strain is small as expected for a magnetic moment constrained to the basal plane. The  $a$ - and  $b$ -axis strains are of opposite sign and of approximately the same magnitude.

No attempt was made to determine the magnetostriction constants from these isothermal data due to the uncertainty in the zero-field strain state mentioned above. In an effort to obtain as reproducible an initial strain state as possible, the specimens were uniformly annealed to a temperature above the Néel point between each run.

A change in sign of the slope occurred close to saturation at low temperatures in the  $c$ -axis strain field curves and to a lesser extent in the  $a$ - and  $b$ -axis curves (see inset in Fig. 1). This change is presumed to arise in in the magnetization process as a result of domain rotation in which the magnetic moment must cross the hard magnetic direction giving rise to the strains observed.

The demagnetizing field for the specimens used (assuming the disk with an axis ratio of 4.2 to approximate an oblate ellipsoid<sup>17</sup>) is  $14.9\sigma G$  where  $\sigma$  is the

magnetic moment per gram. This amounts to a maximum demagnetizing field of about 5.9 kG.

At fields above technical saturation at low temperatures the linear magnetostriction shows a small field dependence known as the forced magnetostriction. This quantity is normally expressed in units of the slope,  $\partial(\Delta l/l)/\partial H$  of the linear magnetostriction. Figure 3 shows the forced magnetostriction in the  $a$ ,  $b$ , and  $c$  directions as obtained from the slope above saturation of the strain versus field curves. The values quoted for temperatures near the Curie temperature cannot be literally ascribed to forced magnetostriction as complete magnetic saturation cannot be obtained in the applied fields used here. The values given in this region are field-dependent and are used only for correcting the magnetostriction constants to zero applied field. The volume forced magnetostriction,  $\omega = \Delta V/V$ , may be related to the pressure dependence of the magnetization  $I$  through the relation,

$$|\partial\omega/\partial H|_P = -|\partial I/\partial P|_H. \quad (13)$$

Using the values from Fig. 3, at 80°K,  $\partial\omega/\partial H$  is negative indicating an increase in pressure will increase the magnetization while at 200°K,  $\partial\omega/\partial H$  is positive with the result that a lower magnetization results for an increase in pressure. This change is due to the large positive contribution of the  $c$ -axis forced magnetostriction near the Curie temperature.

Figure 4 shows representative curves of the  $b$ -axis strain versus angle of the applied field relative to the  $b$  direction. The 30-kOe field was applied parallel to the

<sup>16</sup> W. C. Koehler, H. R. Child, E. O. Wollan, and J. W. Cable, *J. Appl. Phys.* **34**, 1335 (1963).

<sup>17</sup> R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1951), p. 849.

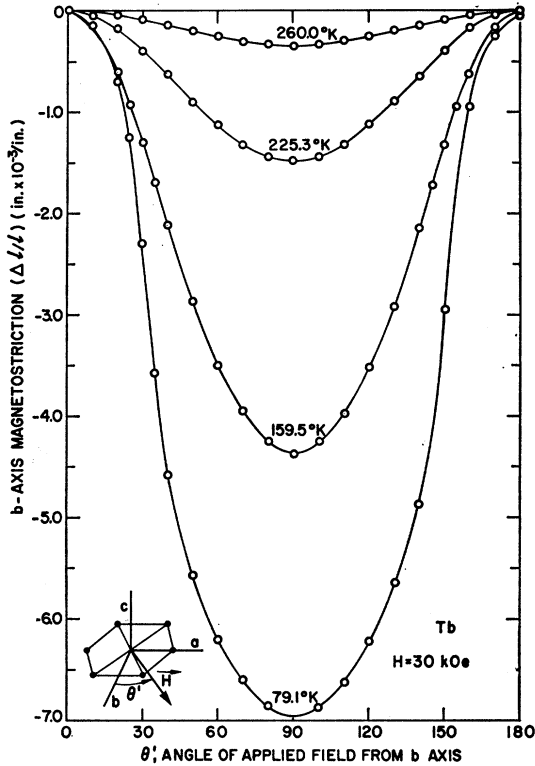


FIG. 4. The *b*-axis strain as a function of angle of the applied field relative to the *b* axis. The 30-kOe field was applied parallel to the basal plane.

basal plane. The angle accuracy is within  $\pm 1^\circ$  including errors from crystal orientation and strain-gage alignment. The strain measured along the *a* axis is opposite in sign to the *b* axis and of the same order of magnitude.

From the strain at angles of 60 and 90° the constants *A* and *C* were determined for 30-kOe fields as described earlier. The temperature dependence of the values found is shown in Fig. 5. Values of *A* and *C* obtained from both *a*- and *b*-axis measurements are given. The discrepancy in the values of *C* from the two determinations is ascribed to nonlinear behavior of the strain gages at low temperature and high strains.

The temperature dependence of the constants *A* and *C* can be correlated with a theory by Callen and Callen<sup>18,19</sup> relating the magnetostriction constants to the magnetic moment. Using a classical internal field Hamiltonian they have found for a one-ion interaction that the temperature dependence of the related magnetostriction constant is represented by

$$\frac{\Delta l}{l}(T) = \frac{\Delta l}{l}(0) \frac{I_{l+1/2}[\mathcal{L}^{-1}(m_n)]}{I_{1/2}[\mathcal{L}^{-1}(m_n)]}, \quad (14)$$

where *I* is a modified Bessel function of the first kind,  $\mathcal{L}^{-1}$  is the inverse of the Langevin function and  $m_n = M(T)/M(0)$  is the reduced magnetization. The subscript *l* denotes the degree of the symmetry polynomial of the ion spin appearing in the Hamiltonian and corresponds to the degree of the magnetization direction cosines in the magnetostriction expression. The magnetostriction constant *C* represents terms of second degree (*l*=2) while *A* represents terms of fourth degree (*l*=4). The theoretical curves for *C* and *A* obtained from Eq. (14) and the magnetic moment data of Hegland *et al.*<sup>13</sup> are shown in Fig. 5. The value of *C*(*T*=0) used was  $4.3 \times 10^{-3}$  in./in., and the value of *A*(*T*=0) was  $2.14 \times 10^{-3}$  in./in. At low temperatures expression (14) predicts a cubic and a tenth-power de-

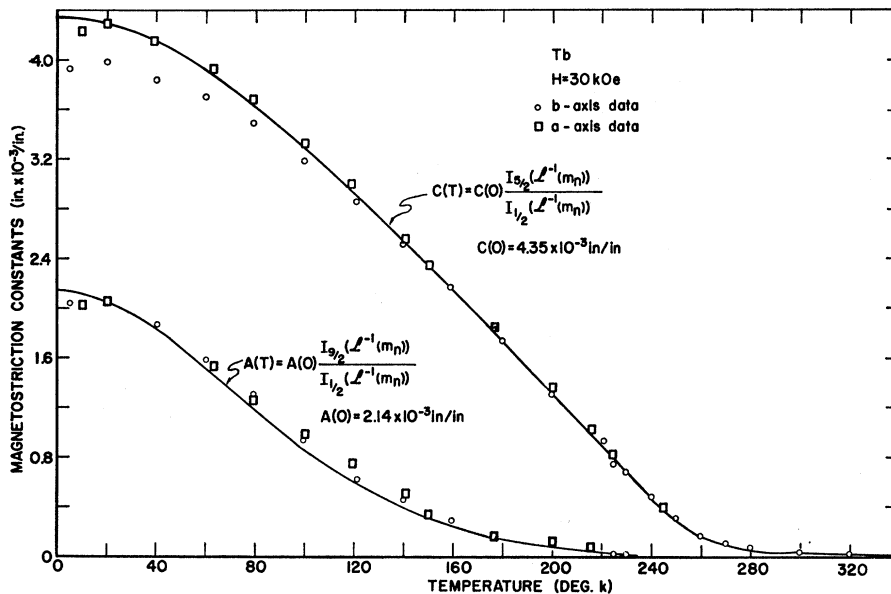


FIG. 5. Experimental values of the magnetostriction constants *A* and *C* at 30-kOe applied field. Data obtained from both *a*- and *b*-axis strain versus field-angle curves are shown. The solid lines represent a theoretical expression for the temperature dependence of the constants (see text).

<sup>18</sup> E. R. Callen and H. B. Callen, Phys. Rev. **129**, 578 (1963).

<sup>19</sup> E. R. Callen, A. E. Clark, B. DeSavage, and W. Coleman, Phys. Rev. **130**, 1735 (1963).

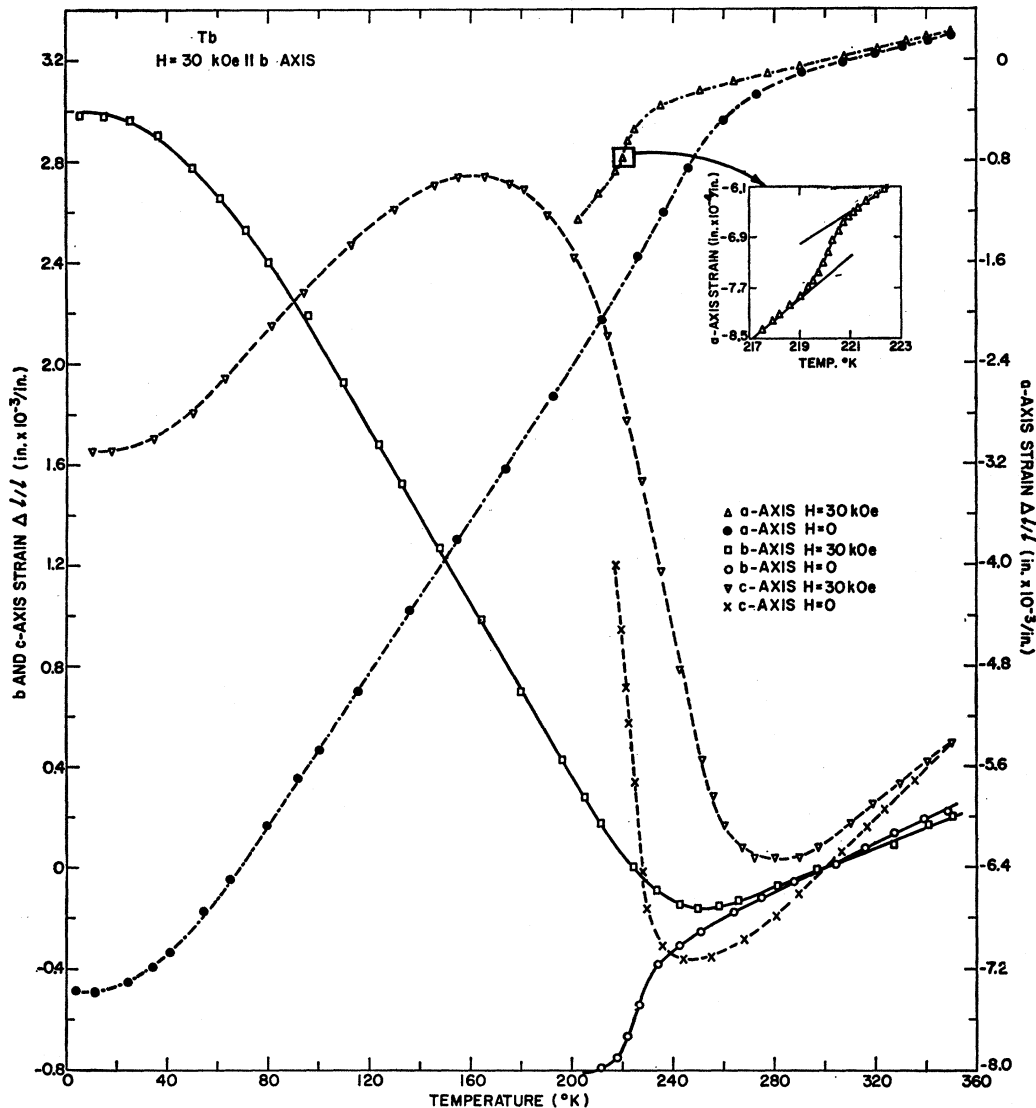


FIG. 6. The  $a$ -,  $b$ -, and  $c$ -axis strain as a function of temperature in zero field and in a 30-kOe field applied along the  $b$  axis. The inset shows the anomalous slope change of the  $a$ -axis zero-field strain at 220°K. This corresponds to a structure change from hexagonal to orthorhombic at the ordering temperature.

pendence on  $m_n$  for  $C$  and  $A$ , respectively, in accord with the work of Kittel and Van Vleck.<sup>20</sup> At high temperatures  $C$  is proportional to  $m_n^2$  as expected. The agreement between theory and experiment evident in Fig. 5 indicates that the one-ion interaction in the Callen theory is sufficient to describe the relation between the basal plane constants and the magnetization without the inclusion of higher order interactions. The theory has been applied previously to the second degree ( $l=2$ ) constant<sup>7</sup> of Dy in the rare-earth group with good success.

Figure 6 shows the temperature dependence of the  $a$ -,  $b$ -, and  $c$ -axis strains in both zero field and in a 30-kOe field applied along the  $b$  direction. In a field the

$b$  and  $c$  axes are observed to expand with decreasing temperature; the  $c$  axis showing a magnetic saturation effect with a decreasing strain at lower temperatures characteristic of the thermal expansion. This is in accord with the negligible  $c$ -axis magnetostriction expected well below the Curie temperature. The  $a$ -axis magnetic strain shows a very large contraction with decreasing temperatures, amounting to 0.6% at 20° after subtracting an extrapolated value of the thermal expansion. The  $b$ -axis magnetostrain is positive and smaller in magnitude than that for the  $a$  axis.

Close examination of the  $a$ -axis strain in zero applied magnetic field near the Curie temperature discloses an anomalous change in strain of 80  $\mu$ -in./in. over a 2° range about 220°K. This effect, shown in the inset of

<sup>20</sup> C. Kittel and J. H. Van Vleck, Phys. Rev. 118, 1231 (1960).

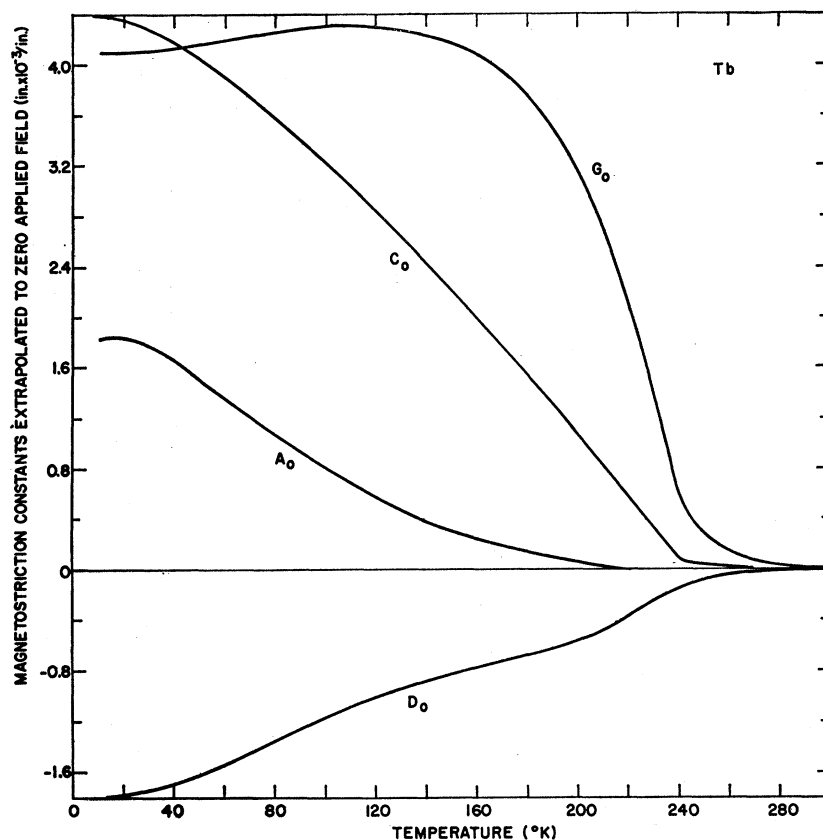


FIG. 7. Magnetostriction constants  $A_0$ ,  $C_0$ ,  $D_0$ , and  $G_0$ . The subscript 0 indicates the values have been extrapolated to zero applied field.

Fig. 6, is in agreement with the observed change in crystal structure from hexagonal to orthorhombic at the ordering temperature as observed by Darnell<sup>9</sup> in x-ray studies on Tb. The zero-field measurements were not made below 200°K due to hysteresis effects encountered below the Curie temperature as mentioned above.

Figure 7 shows the temperature dependence of the constants  $A_0$ ,  $C_0$ ,  $D_0$ , and  $G_0$ .  $D_0$  and  $G_0$  were obtained by subtracting from the magnetostriction curves of Fig. 6 an extrapolation of the linear region of the thermal expansion curve from above the Néel temperature. This extrapolation was done by using the Grüneisen expression

$$\partial V/\partial T = K\gamma C_V(T)$$

to obtain the temperature dependence of the linear thermal expansion coefficient,  $\alpha_T$ .  $C_V$  is the specific heat from the Debye theory and  $K$  and  $\gamma$  are, respectively, the isothermal compressibility and the Grüneisen constant, both of which were assumed constant over the temperature range used. The Debye temperature

for<sup>21</sup> Tb is 175°K. The values indicated for the constants have also been corrected to zero applied field by use of the "forced" magnetostriction curves of Fig. 3 in the case of  $D$  and  $G$ , and by direct measurement and extrapolation of the field dependence of the values for  $A$  and  $C$ . Above 240°K, the forced magnetostriction correction becomes large and difficult to evaluate; hence there is some uncertainty in the values for  $D_0$  and  $G_0$  in this region.

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<sup>21</sup> F. H. Spedding, S. Legvold, A. H. Daane, and L. D. Jennings, *Progr. Low Temperature Phys.* 2, 368 (1957).