the double summation by an integral to get

$$\Sigma_3=\pi\frac{kT}{\Delta}\left(\frac{kT}{\delta}\right)^{1/2}\int_{6\Delta/kT}^{\infty}d\eta\int_0^{\infty}\frac{\xi^{-1/2}d\xi}{\exp(\xi+\eta)-1}-\frac{\pi kT}{\Delta}\int_{6\Delta/kT}^{\infty}\frac{d\eta}{\exp(\eta)-1}$$

$$=\pi\frac{kT}{\Delta}\left(\frac{kT}{\delta}\right)^{1/2}\int_0^{\infty}\xi^{-1/2}d\xi\int_0^{\infty}\frac{dx}{\exp[x+\xi+6\Delta/kT]-1}+\frac{\pi kT}{\Delta}\ln\left[1-\exp\left(-\frac{6\Delta}{kT}\right)\right]$$

$$=\pi\frac{kT}{\Delta}\left(\frac{kT}{\delta}\right)^{1/2}\int_0^{\infty}\xi^{-1/2}\left[-\ln\left(1-\exp\left[-\xi-\frac{6\Delta}{kT}\right]\right)\right]d\xi+\frac{\pi kT}{\Delta}\ln\left[1-\exp\left(-\frac{6\Delta}{kT}\right)\right]$$

$$=\pi\frac{kT}{\Delta}\left(\frac{kT}{\delta}\right)^{1/2}\int_0^{\infty}\xi^{-1/2}d\xi\sum_{p=1}^{\infty}\frac{\exp[-p(\xi+6\Delta/kT)]}{p}-\frac{\pi kT}{\Delta}\sum_{p=1}^{\infty}\frac{\exp(-p6\Delta/kT)}{p}$$

$$=\pi^{3/2}\frac{kT}{\Delta}\left(\frac{kT}{\delta}\right)^{1/2}\sum_{p=1}^{\infty}\frac{\exp[-p6\Delta/kT]}{p^{3/2}}-\frac{\pi kT}{\Delta}\sum_{p=1}^{\infty}\frac{\exp(-p6\Delta/kT)}{p}\leqq\pi^{3/2}S\frac{kT}{\Delta}\left(\frac{kT}{\delta}\right)^{1/2},\tag{A8}$$

where

$$S=\sum_{p=1}^{\infty}\frac{\exp(-p6\Delta/kT)}{p^{3/2}}.\tag{A9}$$

If $6\Delta/kT\ll1$,

$$S\approx\zeta(\tfrac{3}{2})=2.61.\tag{A10}$$

On the other hand, when $(3\Delta/kT)\gg1$, appropriate series expansions[12] yield

$$\Sigma_2=2\left(\frac{kT}{\delta}\right)^{1/2}\pi^{1/2}\sum_{p=1}^{\infty}\frac{\exp(-p3\Delta/kT)}{p^{1/2}}\approx2\left(\frac{kT}{\delta}\right)\sqrt{\pi}\left(\frac{\delta}{kT}\right)^{1/2}\exp\left(-\frac{3\Delta}{kT}\right)\tag{A11}$$

and

$$\Sigma_3=2\left(\frac{kT}{\delta}\right)\left[\frac{\pi^{3/2}}{2}\left(\frac{\delta}{kT}\right)^{1/2}\frac{kT}{\Delta}\exp\left(-\frac{6\Delta}{kT}\right)\right].\tag{A12}$$

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Electron Interactions in Cryogenic Helium Plasmas*

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Microwave-propagation and microwave-interaction techniques have been used to determine the electron collision frequency for momentum transfer in helium for electron energies in the vicinity of 0.001 eV. Measurements of the complex microwave conductivity and electron-energy relaxation rates have been performed in the afterglow of a pulsed discharge in helium in the pressure range 0.1 to 5 Torr submerged in a bath of liquid helium at 4.2°K. Electron-radiation temperature measurements during plasma decay have demonstrated monotonically decreasing electron temperatures as a function of time. For times when an extrapolation of the electron temperature decay indicated near thermal equilibrium with the parent gas, a momentum-transfer cross section in the range 10×10^{-16} to 19×10^{-16} cm^2 was determined. Measurements of electron-energy relaxation rates for atomic densities exceeding 2.3×10^{18} cm^{-3}, where the electron de Broglie wavelength is becoming long in comparison to the average inter-scatterer spacing, indicate the limit of validity of binary-collision concepts.

I. INTRODUCTION

THE elastic scattering of low-energy electrons by helium atoms in the ground state has received considerable attention in the past with a number of authors attempting to predict the behavior of the scattering cross section in the limit of zero electron energy. The calculations of Morse and Allis,[1] using an exchange approximation, have been shown to be in good agreement with the experimentally determined cross section over the energy range 1 to 40 eV. Their calculations indicate a cross section rising with decreasing electron

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[1] P. M. Morse and W. P. Allis, Phys. Rev. 44, 269 (1933).

energy and approaching a value of 8.2×10^{-16} cm^2 at zero energy. A calculation by Moiseiwitsch[2] based upon a variation technique yielded an anticipated zero-energy cross section of 7.5×10^{-16} cm^2, that of Hashino Matsuda[3] 8×10^{-16} cm^2, while a modified effective range analysis of available experimental data by O'Malley[4] indicates a value of 5×10^{-16} cm^2 at zero energy. Finally, a calculation by Kivel,[5] in which the relatively long-range attractive polarization force assumes a dominant role, yields a value of 0.44×10^{-16} cm^2 for the cross section in question.

This spread in predicted values for the zero-energy elastic-scattering cross section led us to believe that it might be useful to attempt a determination using microwave techniques under cryogenic conditions where electron energies considerably lower than those attained in other swarm measurements at room temperature ($\approx$0.04 eV) might be achieved. The use of microwave techniques on afterglow helium plasmas, created in a parent gas in thermal contact with a bath at 4.2°K, allowed the determination of electron collision frequencies for momentum transfer for electron energies as low as 0.001 eV. Since the scattering is expected to be isotropic at such low energies, the momentum-transfer cross section should be identical with the total elastic-scattering cross section.

II. THEORY

The theory of the interpretation of plasma characteristics through the use of guided microwave propagation techniques has been adequately described in the literature[6,7] and will only be briefly recapitulated here.

The complex microwave conductivity of a partially ionized gas may, on the basis of *binary* elastic collisions, be related to the scattering cross section of the various constituents.[8]

$$\sigma_{\text{complex}}=\frac{8}{3\sqrt{\pi}}\left[\int_0^\infty \frac{\{\nu(u)-i\omega\}u^4\exp(-u^2)du}{\omega^2+\nu^2(u)}\right]$$
$$=\frac{n_e e^2}{m}\left[\frac{\nu_{\text{eff}}-i\omega}{\nu_{\text{eff}}^2+\omega^2}\right], \quad (1)$$

where $u=[m/(2kT_e)]^{\frac{1}{2}}v$, $\nu(u)=\sum_j N_jQ_j(v)v$, N_j is the jth plasma constituent, $Q_j(v)$ its momentum-transfer cross section for the elastic scattering of electrons of velocity v, and a Maxwellian velocity distribution has been assumed for the electrons. The effective collision frequency for momentum transfer, ν_{eff}, may be considered to be defined by Eq. (1).

Under conditions of sufficiently high neutral particle densities (or sufficiently low electron, and therefore ion, densities) the electrons will interact predominantly with the neutral particles. In the case where only one neutral species is present, a measurement of ν_{eff} can provide information about $Q_{em}(v)$ under the assumptions stated.

For the case where such a plasma uniformly fills a section of waveguide, the electron number density n_e and collision frequency ν_{eff} may be deduced from the directly measured microwave absorption and relative phase shift caused by the presence of the plasma.[7]

$$\nu_{\text{eff}}=\alpha\omega\lambda_g(0)[1-\varphi]/\pi[2-\varphi], \quad (2)$$

$$n_e=m\epsilon_0 e^{-2}\lambda_0^2\lambda_g^{-2}(0)[2\varphi-\varphi^2][\nu_{\text{eff}}^2+\omega^2], \quad (3)$$

$$\varphi=\Delta\theta\lambda_g(0)/360l, \quad (4)$$

where α is the plasma attenuation constant, $\lambda_g(0)$ is the guide wavelength in the absence of the plasma, l is the length of the plasma, $\Delta\theta$ is the relative phase shift in degrees and the other symbols have their usual significance.

For the present experiments, the predominant mechanism for dissipation of the afterglow plasma is volume recombination rather than diffusion. This, together with the fact that the discharge tube very nearly filled the waveguide, makes the above assumption of a plasma uniformly filling a section of waveguide quite realistic and little error is introduced by the use of Eqs. (2) and (3).

In the foregoing it has been tacitly assumed that the microwave signal is of sufficiently low power that the plasma is not significantly disturbed by its presence. When the power absorbed from the microwave field becomes sufficiently large, the electron temperature, T_e is raised with a consequent alteration in the electrical conductivity. The use of a second microwave signal of power low enough so that no detectable effects result from its presence, allows this alteration to be observed as a so-called cross modulation upon the low-level "sensing" signal.[7] If a pulsed "heating" wave is applied, a transient cross modulation is observed; the microwave conductivity relaxing back to its unperturbed level upon removal of the "heating" field at a rate which is characteristic of the plasma medium.

It has been shown that the assumption of an energy-independent electron atom cross section leads to an anticipated exponential decay of the transient cross modulation for arbitrarily small perturbations at a rate $\tau_{em}=1/G\nu_0$, where $G=2m_e/M_A$ and ν_0 is the unperturbed value of ν_{eff}.[7] The restriction that Q_{em} be independent of electron energy is not an especially stringent one for helium, as the technique requires small energy deviations and Q_{em} in helium has been observed to be relatively constant over the energy range 0.04 to 2 eV.

[2] B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A219**, 102 (1953).
[3] T. Hashino and H. Matsuda, Progr. Theoret. Phys. (Kyoto) **29**, 370 (1963).
[4] T. F. O'Malley, Phys. Rev. **130**, 1020 (1963).
[5] B. Kivel, Phys. Rev. **116**, 1484 (1959).
[6] L. Goldstein, M. A. Lampert, and R. H. Geiger, Elec. Commun. **29**, 243 (1952).
[7] J. M. Anderson and L. Goldstein, Phys. Rev. **100**, 1037 (1955).
[8] H. Margenau, Phys. Rev. **69**, 508 (1946).

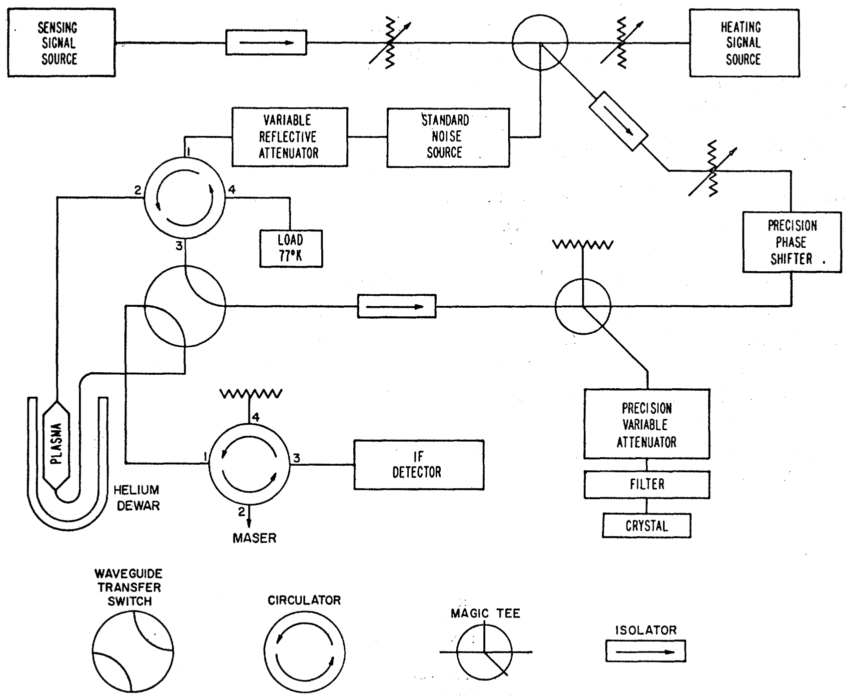

FIG. 1. Simplified block diagram of microwave bridge and radiometer circuit.

Moreover, the theoretically predicted values of the zero-energy cross section lead one to expect the above results to be valid to that limit providing the elastic scattering of electrons in binary collisions remains a valid concept.

III. EXPERIMENTAL APPARATUS AND PROCEDURE

In order to obtain sufficiently low temperatures for the present experiments, the section of waveguide containing the discharge tube was immersed in a bath of liquid helium at 4.2°K contained in a suitable Dewar. A simplified block diagram of the microwave circuit used is shown in Fig. 1. The afterglow plasma, which was formed by the application of short (1 to 2μ sec) high-voltage (5 to 10 kV) dc pulses at a repetition rate of 10 pulse/sec, was contained within a 24-cm-long Pyrex tube of rectangular cross section which very nearly filled the x-band waveguide used.[9] For the investigated pressure range of 0.1 to 6 Torr, corresponding to neutral densities of 2.3×10^{17} and 1.4×10^{19} atoms per cc, respectively, the measurable afterglow had a duration of 1 to 2 msec.

Pulsed microwave heating at approximately 9.8 Gc is accomplished at appropriate times after cessation of active discharge by the use of rapid rise and fall ($\leq 0.06\,\mu$sec) pulses at power levels variable up to 50 mW. Extremely short duration "heating" pulses (≈ 0.4 μsec) have been used to minimize the problems inherent in the retention and/or production of ionic collisional partners during the application of the heating pulse.[10] Such short "heating" pulses, although not allowing the electron temperature to reach a steady state, produce a sufficient perturbation of T_e for the conductivity relaxation rate to be measured. The electron-electron collision frequency was sufficiently high ($\sim 10^9$ sec^{-1}) to insure Maxwellization of the electron energy distribution in times short in comparison to the "heating" pulse length and τ_{em}.

Microwave "sensing" at approximately 8500 Mc/sec was accomplished at power levels less than 100 μW; the higher powered "heating" wave being excluded from the detection system by the use of microwave filters.

In order to determine the magnitude of discharge heating of the neutral gas, measurements of the increase in average system pressure upon application of the 10-pulse/sec-dc-breakdown pulse have been made. Average gas temperature increases were never observed to exceed 0.6°K. As a further safeguard, breakdown conditions were adjusted so that the upper bound on joule heating, as indicated by total breakdown pulse power, was <3°K—small enough to be neglected for the purposes of these experiments. In addition, the initiation of dc breakdown was found, under a wide range of conditions, to set up a weakly damped sound wave which reflected between the walls of the discharge tube.[9] Measurement of the sound propagation velocity allowed an independent check of the gas temperature which was found to be within the bounds set by the above-described techniques.

[9] J. A. Berlande, P. D. Goldan, and L. Goldstein, Appl. Phys. Letters **5**, 51 (1964).

[10] P. D. Goldan, J. A. Berlande, and L. Goldstein, Phys. Rev. Letters **13**, 182 (1964).

Measurements of the electron gas "radiation temperature" have been made utilizing a ruby cavity maser operating at 8530 Mc/sec. A waveguide standard noise tube generating 15.3±0.2 db of excess noise served as a temperature standard, the video oscilloscopic display of the pulsed noise tube signal being directly compared to the plasma radiation output as a function of time in the afterglow after appropriate correction for plasma absorptivity and microwave mismatch. The equivalent radiation temperature of the standard noise tube was controlled by means of a variable reflective attenuator which, in conjunction with a circulator and waveguide load at 77°K (see Fig. 1), allowed a reduction of the system background temperature from 300 to approximately 100°K. Consequently electron temperatures considerably below 300°K could be measured with some confidence.

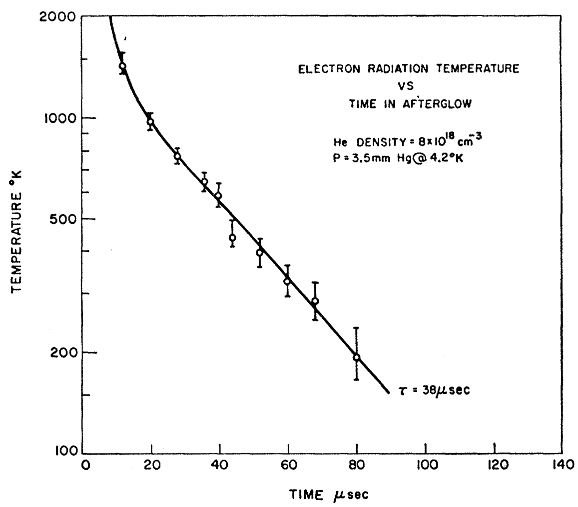

FIG. 2. Electron-radiation temperature versus time in the afterglow: Helium density $=8\times10^{18}$ cm^{-3} corresponding to a pressure of 3.5 Torr at 4.2°K.

IV. EXPERIMENTAL RESULTS

Electron-radiation temperature measurements have been made as a function of time after cessation of active discharge for afterglow plasmas created in helium at 4.2°K and in the pressure range 0.3 to 4 Torr. Figure 2 shows the result of a typical experiment after the measured radiation power has been corrected for plasma absorptivity and plasma-glass-air interface mismatch. It may be seen that, after an initial transient period, the electron temperature decay becomes fairly exponential with, in this case, a time constant of 38 μsec. The thermal decay constant τ_T determined in this manner is found to be some two orders of magnitude greater than that which would be expected solely on the basis of energy relaxation through elastic collisions. The implications of this and the fact that $1/\tau_T$ is found to have an apparently linear pressure dependence (see

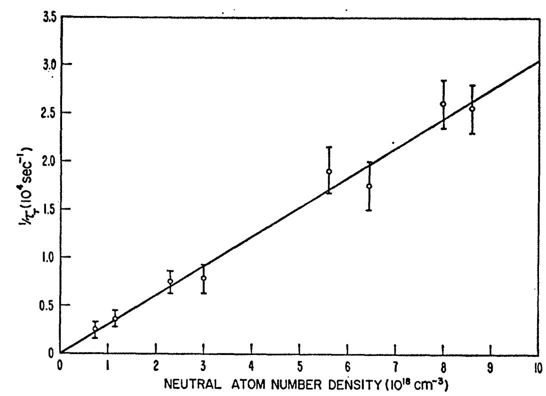

FIG. 3. Inverse electron-radiation temperature relaxation time $(1/\tau_T)$ in helium versus neutral atom number density.

Fig. 3) have been discussed elsewhere.[11] For the purposes of the present discussion, it is sufficient to note that extrapolation of such radiation measurements of the electron energy to lower temperatures indicates that, in the atomic density range considered, the electron gas is expected to be in or very near thermal equilibrium with the parent gas for times in the afterglow exceeding 500 μsec after cessation of the breakdown pulse. Measurements of electron-atom collision frequency and electron-energy relaxation rates (cross modulation) discussed subsequently, have been made at times sufficiently late in the afterglow that near thermal equilibrium is to be expected.

A direct experimental check of the accuracy of the radiometric measurements is available and has been made. When the indicated electron temperature is 300°K, a measurement of the electron neutral collision frequency for momentum transfer yields a value of the cross section for this process. Measurements were performed at 0.51 and 1.3 Torr at 4.2°K corresponding to neutral atom densities of 1.17×10^{18} and 3.0×10^{18} cm^{-3},

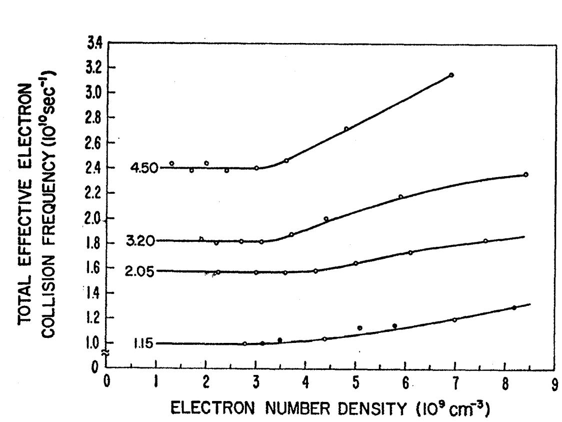

FIG. 4. Total effective electron collision frequency versus electron number density; typical experiments at 4.50, 3.20, 2.05, and 1.15 Torr at 4.2°K.

[11] P. D. Goldan, J. H. Cahn, and L. Goldstein, Sixteenth Gaseous Electronics Conference, 1963 (unpublished).

respectively. The results are shown below:

Gas pressure	ν_{em} (T_e=300°K)	Q_{em}
0.51 Torr	9.7×10^9	5.7×10^{-16} cm^{-2}
1.30 Torr	2.3×10^{10}	5.3×10^{-16} cm^{-2}

The cross section thus determined is in good agreement with the results of other workers using different techniques.[12,13]

In order to establish a value for the "effective" momentum-transfer collision frequency for electrons with helium atoms, ν_{em} in an isothermal plasma in the vicinity of 5°K, measurements were made of the relative phase shift and relative absorptive loss suffered by a low-level microwave during propagation through the plasma. The variables in such measurements are the neutral gas pressure and time of measurement in the afterglow delayed from the time of cessation of active discharge. The electron number density and total "effective" collision frequency are determined from Eqs. (2) and (3). The results of several typical measurements are given in Fig. 4. It will be noted that

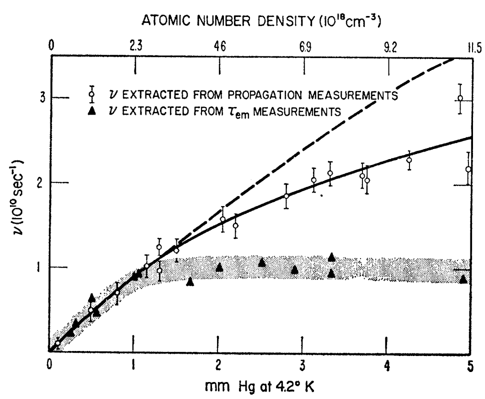

FIG. 5. Electron neutral collision frequency versus helium pressure at 4.2°K; comparison of results of propagation and cross modulation techniques.

for sufficiently low-electron number density, at each pressure the measured electron collision frequency becomes independent of the electron density but is dependent upon pressure. This terminal value of ν is interpreted as being due to electron-molecule elastic collisions, and values of ν_{em} so obtained are plotted as a function of pressure (or atomic density) in Fig. 5.

Since, in the present experiments, $\nu^2/\omega^2<1$ but not $\ll1$, a linear dependence of ν_{em} on p was not to be anticipated. The dashed line in Fig. 5 represents a solution of Eq. (1) for ν_{eff} under the assumption of a constant cross section equal to 1.9×10^{-15} cm^2. As may be seen, although the calculated curve is a good fit at the lower pressures and shows some curvature, the experimental data saturates much more rapidly for

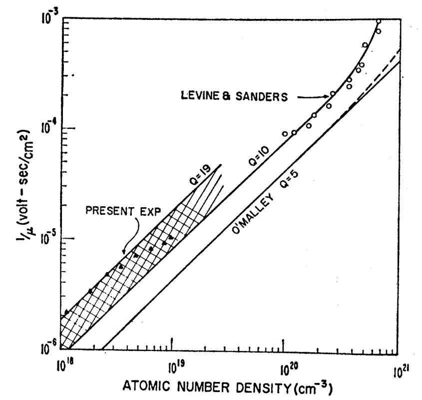

FIG. 6. Electron mobility versus helium atom density at 4.2°K. Cross sections given in square angstroms.

pressures exceeding 1.5 Torr. Nor does it appear possible that any other energy dependence of the collision cross section will more accurately reproduce the experimentally observed pressure dependence of the collision frequency within the framework of Eq. (1). (The collision cross section must, of course, approach the measured value of approximately 5.6×10^{-16} cm^2 at 0.04 eV.)

A further check of the value of ν_{em} should be obtainable from measurements of the time constant $\tau_{em}=1/G\nu_{em}$ associated with the relaxation of the disturbed plasma in the vicinity of 5°K. A comparison of this measure of ν with that previously determined is also shown in Fig. 5. For atomic number densities N_A below 2.3×10^{18} cm^{-3}, the two measurement techniques are in agreement whereas at densities above this value τ_{em} becomes independent of N_A.

V. CONCLUSION

Electron-helium-atom momentum-transfer collision frequencies determined by microwave propagation and microwave interaction techniques were found to be in agreement for neutral densities below 2.3×10^{18} cm^{-3}

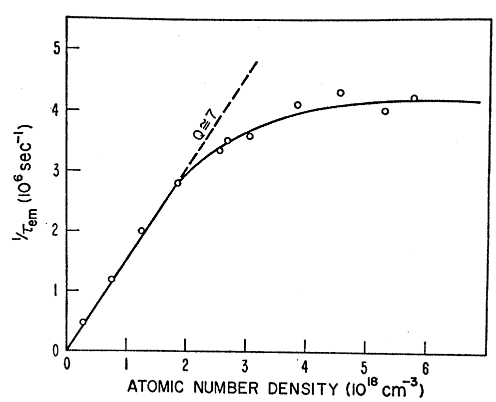

FIG. 7. Inverse cross-modulation relaxation time ($1/\tau_{em}$) versus helium atom density at 77°K. Cross section given in square angstroms.

[12] L. Gould and S. C. Brown, Phys. Rev. **95**, 897 (1954).
[13] A. V. Phelps, J. L. Pack, and L. S. Frost, Phys. Rev. **117**, 470 (1960).

at 4.2°K. Since the analysis of these techniques is based upon the assumption of elastic binary collisions, this agreement may be taken as a demonstration of the applicability of that concept for the above conditions. For helium in this density range at 4.2°K, a cross section of 1.9×10^{-15} cm^2 (some $3\frac{1}{2}$ times larger than the momentum-transfer cross section at 300°K) has been found to satisfactorily fit the data. Owing to the present impossibility of making an accurate determination of the equilibrium electron temperature under the conditions of these experiments, however, this value may be subject to a downward revision by as much as a factor of 2 to approximately 10^{-15} cm^2. In the temperature extremes encountered in the present experiments, a slight pressure dependence of electron steady-state mean thermal energy could account for the pressure saturation of ν_{em} observed (Fig. 5).

Some corroborative evidence has been found by Levine and Sanders[14] from electron dc mobility measurements in helium at 4.2°K and below for atomic densities in the range 10^{20} to 10^{22} cm^{-3}. Some of their data, along with the dc mobility computed from the microwave propagation data of the present experiments, are reproduced in Fig. 6. For comparison, the dc mobility anticipated from the zero-energy cross section of 5×10^{16} cm^2 calculated by O'Malley[4] is shown as the solid line together with a pressure correction for the second virial coefficient at high densities (dashed portion). The cross-hatched area indicates the range of possible zero-energy cross sections anticipated from the present experiments. It may be seen that a cross section of approximately 10^{-15} cm^2 would seem a best fit to the available data.

Perhaps even more significant is the disagreement between the interpretation of the microwave propagation and microwave interaction techniques for helium densities above 2.3×10^{18} cm^{-3} at 4.2°K as shown in Fig. 5. One possible basis for such a phenomenon lies in the excitation, by the "heating" wave, of collective plasma oscillations which, when weakly damped, could serve as a continuing source of electron energy in the wake of the heating perturbation. All attempts to detect such an excitation both at 4.2 and 77°K have been unsuccessful.

A more fruitful interpretation seems to lie in a fundamental disagreement with the classical concept of collisions in terms of binary encounters embodied in Eq. (1) and the interpretation of τ_{em}. For electron temperatures in the vicinity of 10°K, the number of neutral atoms in a sphere whose diameter is equal to the electron de Broglie wavelength, λ_e, may be as high as 50 for neutral gas pressures as low as 1 Torr at 4.2°K. Under such conditions, the concept of electron *binary* collisions might be expected to have little validity. Levine[14] has found that at sufficiently high densities ($N_A>10^{21}$ cm^{-3}) the electron multiatom interaction results in the formation of a "bubble" around the electron, producing a very low-electron mobility. He estimates that this system breaks up below $N_A\sim10^{21}$ cm^{-3}, but the details of the process are not clear and it seems possible that such multiparticle interactions could significantly affect the electron transport properties for N_A as low as 2×10^{18} cm^{-3}.

If such a process is responsible for the observed saturation in τ_{em} at 4.2°K for $N_A>2.3\times10^{18}$ cm^{-3}, a similar effect should take place at higher temperatures when $N_A{}^{1/3}\lambda_e>1$. Measurements of $1/\tau_{em}$ have been made in helium at 77°K and are shown in Fig. 7. Again it may be seen that for $N_A>2.5\times10^{18}$ cm^{-3} a saturation is achieved. (The larger values of $1/\tau_{em}$ here observed demonstrate that the indicated saturation in $1/\tau_{em}$ at 4.2°K was not of instrumental origin.) The slope of the linear portion of this curve ($N_A<2\times10^{18}$ cm^{-3}) yields a value of 7×10^{-16} cm^2 for the averaged momentum-transfer cross section at 77°K in agreement with the results of Dougal.[15]

The above discussion is, of course, only indicative, and other explanations cannot be ruled out at the present time. The development of a transport theory capable of treating the continuous scattering of matter waves in a field of densely packed randomly distributed scatterers would undoubtedly clarify the present position.

Note added in proof. In view of the uncertainties involved in the extrapolation of the electron temperature in the present experiments, the possibility exists that the observed large values of ν_{em} are the result of sustained high values of T_e in the late afterglow rather than a large scattering cross section. This possibility does not however invalidate the comments concerning electron multiatom interactions.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Dr. J. H. Cahn for his helpful discussions and suggestions and to Dr. J. A. Berlande for his assistance in the latter phases of this work.

[14] J. Levine and T. M. Sanders (private communication).

[15] A. A. Dougal (private communication).