

KBr increased more rapidly than that of the 21°K annealing stage. This result suggests that heavier irradiations produce more stable complex forms of interstitial halide ions and that these account for the occurrence of higher temperature annealing stages. It is interesting to note that the concentration of the isolated halide interstitial atoms (the  $H$  centers) saturates as a function of radiation dose but that the growth of the  $H'$  center is a linear function of the  $F$ -center concentration.

<sup>14</sup> B. J. Faraday and W. D. Compton, International Symposium on Color Centers in Alkali Halides, Stuttgart, Germany, 1962 (unpublished).

#### ACKNOWLEDGMENT

The authors would like to express their deep gratitude to Professor A. Smakula of MIT for supplying them with Cl-doped KBr crystals.

*Note added in proof.* The authors are grateful to Dr. F. Lüty for bringing to their attention the work of U. Strähle (Diplom, Stuttgart, 1963) in which similar annealing of the  $\alpha$  center in Cl-doped KBr is discussed together with an experimental determination of the efficiency of vacancy formation between liquid-helium temperature and 100°K.

## Far-Infrared Magnetic Resonance in NiF<sub>2</sub>

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Measurements of the far-infrared transmission of the canted-spin antiferromagnet NiF<sub>2</sub> using the technique of Fourier-transform spectroscopy are described. Resonant absorptions were found at  $\nu_1 = 3.33 \pm 0.05$  cm<sup>-1</sup> and  $\nu_2 = 31.14 \pm 0.1$  cm<sup>-1</sup> with applied field  $H = 0$  and temperature  $T \approx 0$ . Using the resonance relations derived by Joenk and Bozorth from Moriya's model, these give anisotropy and exchange parameters  $E = 1.66$  cm<sup>-1</sup> and  $8JD = 482.2$  cm<sup>-2</sup>. A good fit is obtained to the measured magnetic-field dependence of both modes for  $g_1 = 2.35$  which, along with the measured perpendicular susceptibility, gives  $8J = 125$  cm<sup>-1</sup> and  $D = 3.86$  cm<sup>-1</sup>. The temperature dependence of  $\nu_2$  was measured between 1.2 and 65°K ( $T_N = 73.2$ °K) and is found to be in agreement with the Zener theory when the temperature dependence of the sublattice magnetization is estimated from neutron-diffraction intensity measurements. The linewidth and strength of mode 2 were also measured as a function of  $T$ , and are discussed qualitatively.

### I. INTRODUCTION

THE magnetic properties of the iron-group difluorides MnF<sub>2</sub>, FeF<sub>2</sub>, CoF<sub>2</sub>, and NiF<sub>2</sub> have been studied extensively from both an experimental and a theoretical point of view. All these compounds have the rutile crystal structure shown in Fig. 1, and have Néel temperatures  $T_N$  between 35 and 75°K, below which magnetic ordering takes place. Susceptibility measurements on MnF<sub>2</sub>, FeF<sub>2</sub>, and CoF<sub>2</sub> show that the ordering is antiferromagnetic, and neutron-diffraction experiments indicate that the spins point along the  $\pm z$  axis.<sup>1</sup> Torque measurements by Matarrese and Stout<sup>2</sup> show that NiF<sub>2</sub>, on the other hand, has a weak ferromagnetic moment along a  $[100]$  direction. The existence of this moment was explained by Alikhanov,<sup>3</sup> from neutron-diffraction experiments, and by Moriya,<sup>4</sup> from a theoretical interpretation of the torque measurements, in terms of the canted two-sublattice magnetic structure shown in Fig. 2. As was pointed out by Dzialoshinski,<sup>5</sup>

and calculated quantitatively by Moriya,<sup>6</sup> there are, in general, two interactions which can cause canted-spin systems. Of these, the antisymmetric exchange coupling,

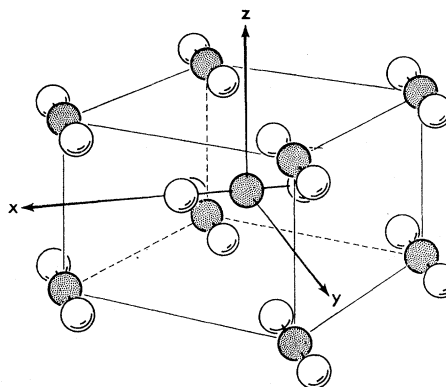


FIG. 1. Rutile-type crystal structure of NiF<sub>2</sub>. The solid and open circles represent Ni<sup>2+</sup> and F<sup>-</sup> ions, respectively. It should be noticed that the body-center sites differ from the corner sites only by a rotation of 90° about the  $z$  axis.

[English transl.: Soviet Phys.—JETP **6**, 1120 (1958)]; J. Phys. Chem. Solids **4**, 241 (1958).

<sup>6</sup> For a recent review, see T. Moriya, in *Magnetism: A Treatise on Modern Theory and Materials*, edited by G. T. Rado and H. Suhl (Academic Press Inc., New York, 1963), Vol. I, Chap. 3.

<sup>1</sup> For a general review, see T. Nagamiya, K. Yoshida, and R. Kubo, *Advan. Phys.* **4**, 1 (1955).

<sup>2</sup> L. M. Matarrese and J. W. Stout, *Phys. Rev.* **94**, 1792 (1954).

<sup>3</sup> A. Alikhanov, *Zh. Eksperim. i Teor. Fiz.* **37**, 1145 (1959) [English transl.: *Soviet Phys.—JETP* **10**, 814 (1960)].

<sup>4</sup> T. Moriya, *Phys. Rev.* **117**, 635 (1960).

<sup>5</sup> I. Dzialoshinski, *Zh. Eksperim. i Teor. Fiz.* **30**, 1454 (1957)

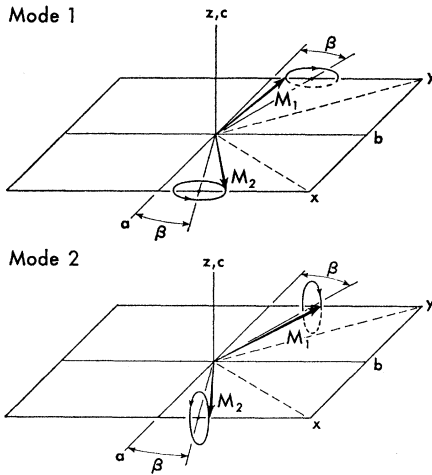


FIG. 2. Zero-field resonant modes 1 and 2. The canting angle  $\beta = \eta E/8J \approx 0.38^\circ$  and the widths of the ellipses are exaggerated. The aspect ratios of the ellipses are  $\hbar\omega_1/16JS \approx 0.01$  and  $\hbar\omega_2/16\sqrt{2}JS \approx 0.088$ , respectively.

or Moriya interaction, vanishes by symmetry in  $\text{NiF}_2$ , but the crystalline field anisotropy in the rutile structure accounts for the canting in a natural way. Two resonant frequencies are expected in such canted-spin systems even in zero external field. Both resonances have been observed in  $\text{MnCO}_3$ , in which canting occurs owing to the Moriya interaction.<sup>7-9</sup> Several authors<sup>4,10,11</sup> have computed the resonant-frequency conditions appropriate for  $\text{NiF}_2$ . The lower frequency  $\omega_1$  is proportional to the anisotropy in the (001) plane and so resembles a ferromagnetic resonance of the net moment. The higher frequency  $\omega_2$  is proportional to the geometric mean of the exchange and out-of-plane anisotropy, so is more clearly antiferromagnetic in nature. As is the case for the other iron-group fluorides, these resonances occur at high-microwave and far-infrared frequencies in  $\text{NiF}_2$ .

In this paper, measurements of the resonant frequencies of  $\text{NiF}_2$  using far-infrared techniques are described. The data include the magnetic-field dependence of both  $\omega_1$  and  $\omega_2$ , and the temperature dependence of the frequency, line width, and line strength of mode 2. These measurements, combined with the susceptibility measurements of Joenk and Bozorth,<sup>12</sup> are compared with theory, and values of anisotropy and exchange parameters deduced. A brief account of this work has been given previously.<sup>9,13</sup>

## II. THEORY

A summary is presented of the calculation of the resonant properties of  $\text{NiF}_2$ .<sup>4,10-12</sup> The ground orbital

<sup>7</sup> M. Date, J. Phys. Soc. Japan **15**, 2251 (1960).

<sup>8</sup> H. J. Fink and S. Shaltiel, Phys. Rev. **130**, 627 (1963).

<sup>9</sup> P. L. Richards, J. Appl. Phys. **35**, 850 (1964).

<sup>10</sup> G. F. Herrmann, J. Phys. Chem. Solids **24**, 597 (1963).

<sup>11</sup> H. J. Fink, Phys. Rev. **133**, A1322 (1964).

<sup>12</sup> R. J. Joenk and R. M. Bozorth, in Proceedings of the 1964 International Magnetism Conference, Nottingham (to be published).

<sup>13</sup> P. L. Richards, J. Appl. Phys. **34**, 1237 (1963).

state of  $\text{Ni}^{2+}$  in  $\text{NiF}_2$  lies well below the first excited state, so a spin Hamiltonian can be written in the form

$$\mathcal{H} = J \sum_{j>k} \mathbf{S}_j \cdot \mathbf{S}_k + \sum_j [DS_{jz}^2 - E(S_{jx}^2 - S_{jy}^2) + \mu_B \mathbf{S}_j \cdot \mathbf{g}_j \cdot \mathbf{H}] + \sum_k [DS_{kz}^2 + E(S_{kx}^2 - S_{ky}^2) + \mu_B \mathbf{S}_k \cdot \mathbf{g}_k \cdot \mathbf{H}], \quad (1)$$

where  $j$  represents a corner site, and  $k$  a body-center site, and the coordinate axes are chosen as shown in Fig. 1. The first term is the superexchange interaction, and the second and third terms are the single-ion anisotropy and Zeeman energies of the ions on the two sublattices.  $g_j$  and  $g_k$  are the  $g$  tensors of the  $\text{Ni}^{2+}$  ions at the corner and body-center sites, respectively. Since  $S=1$ , there are no higher order terms in the single-ion anisotropy. Equation (1) neglects the exchange interactions within one sublattice (which do not affect the resonant frequencies) and also the anisotropic exchange and magnetic dipole interactions. Moriya<sup>6</sup> has estimated these to be  $(\Delta g/g)^2 J \sim 0.02J$  and  $\sim 0.2 \text{ cm}^{-1}$ , respectively.

Using a semiclassical approximation, the free energy of  $N$  spins can be written as

$$F = -\frac{N}{2} [8J \mathbf{S}_j \cdot \mathbf{S}_k + \eta D(S_{jz}^2 + S_{kz}^2) - \eta E(S_{jx}^2 - S_{jy}^2 - S_{kx}^2 + S_{ky}^2) + \mu_B (\mathbf{S}_j \cdot \mathbf{g}_j + \mathbf{S}_k \cdot \mathbf{g}_k) \cdot \mathbf{H}], \quad (2)$$

where  $S_{jz} = S_j \cos\theta$ , etc. The factor  $\eta = (1 - \frac{1}{2}S)$  was introduced by Joenk and Bozorth<sup>12</sup> to correct the classical (large spin) approximation for the quantum-mechanical effects in the anisotropy energy.

The dynamical equations for  $S_i$  can be written in the form

$$\dot{\mathbf{S}}_i = -\gamma \mathbf{S}_i \times \mathbf{H}_{i \text{ eff}}, \quad (3)$$

where  $\mathbf{H}_{i \text{ eff}} = -\nabla_{\mathbf{M}_i} F$ . Here  $\mathbf{M}_i = (N/2)g\mu_B \mathbf{S}_i$  is the moment of the  $i$ th sublattice, and  $\nabla_{\mathbf{M}_i}$  is the gradient with respect to that moment. The effective fields depend, in general, on the directions of both  $\mathbf{M}_1$  and  $\mathbf{M}_2$ .

The equilibrium positions of  $\mathbf{S}_1$  and  $\mathbf{S}_2$ , determined from Eq. (3) with  $\dot{\mathbf{S}}_1 = \dot{\mathbf{S}}_2 = 0$ , lie in the (001) plane as is shown in Fig. 2 with canting angle  $\beta = E\eta/8J$ . The static susceptibilities computed from Eq. (3) are

$$4\chi_{aa} = \chi_{bb} = Ng^2\mu_B^2/16J \quad (4)$$

and

$$\chi_{cc} = (Ng^2\mu_B^2/16J + 2D\eta) \approx \chi_{bb}. \quad (5)$$

In the  $b$  and  $c$  directions, these are the same as  $\chi_1$  for a conventional antiferromagnet.

In order to calculate the resonant frequencies, the dynamical Eq. (3) must be expressed in terms of deviations from the equilibrium spin positions and linearized by assuming that the deviations are small.

The resulting resonant frequencies are

$$(\hbar\omega_1)^2 = (4E\eta S)^2 + 10E\eta S g_{11} \mu_B H_b + 8JS(g_1 - g_2) \mu_B H_b - \frac{(4E\eta S g_{11} \mu_B H_c)^2}{32JD\eta S^2 - 12E^2\eta^2 S^2}, \quad (6)$$

$$(\hbar\omega_2)^2 = 32JD\eta S^2 + 4E^2\eta^2 S^2 + 2E\eta S g_{11} \mu_B H_b + 8JS(g_1 - g_2) \mu_B H_b + \frac{32JD\eta S^2 + 4E^2\eta^2 S^2}{32JD\eta S^2 - 12E^2\eta^2 S^2} (g_{11} \mu_B H_c)^2, \quad (7)$$

where  $H_b$  and  $H_c$  are the static fields along the  $b$  and  $c$  axes. The  $g$  tensors  $g_j$  and  $g_k$  have principal axes  $g_1, g_2, g_{11}$  and  $g_2, g_1, g_{11}$  in the  $x, y, z$  coordinate system, and  $g_{11} = \frac{1}{2}(g_1 + g_2)$ . A perturbation calculation gives the  $g$ -factor anisotropy  $g_1 - g_2 = 4E/|\lambda|$  and  $g_1 - g_{11} = 2D/|\lambda|$  in terms of the spin-orbit coupling parameter  $\lambda$ . This small anisotropy has an important effect on the resonance frequencies since  $(g_1 - g_2)$  enters Eqs. (6) and (7) multiplied by  $8J$ .<sup>12</sup>

A simple procedure<sup>14</sup> for estimating the strengths of modes 1 and 2 is to use the Kramers-Kronig transform,

$$\chi'(\omega') = \frac{2}{\pi} \int_0^\infty \frac{\omega \chi''(\omega) d\omega}{\omega^2 - \omega'^2}, \quad (8)$$

which relates the real and imaginary parts of the susceptibility  $\chi(\omega) = \chi'(\omega) - i\chi''(\omega)$ . Choosing  $\omega' = 0$ , and assuming a narrow resonant peak in  $\chi''$  at  $\omega_0$ , we have

$$\int_0^\infty \chi''(\omega) d\omega = \frac{1}{2} \pi \omega_0 \chi'(0). \quad (9)$$

From the phases of the modes pictured in Fig. 2, we see that for mode  $\omega_1$  there are net oscillating moments along  $a$  and  $c$ , while for  $\omega_2$  this occurs only along the  $b$  axis. Thus, the integrated susceptibilities for rf fields along the  $a, c$ , and  $b$  directions are  $\frac{1}{2}\pi\omega_1\chi_{aa}$ , and  $\frac{1}{2}\pi\omega_1\chi_{cc}$  for mode 1, and  $\frac{1}{2}\pi\omega_2\chi_{bb}$  for mode 2.

If we use the fact that the magnetic absorptions in  $\text{NiF}_2$  are weak compared with the lattice absorptions, it can be shown from the theory of the transmission of magnetic insulators that the integrated magnetic absorption strengths  $S = \int \alpha(\nu) d\nu$  in  $\text{cm}^{-2}$  are  $2\omega_0 n/c^2$  times the integrated susceptibilities.<sup>15</sup> Here  $\alpha$  is the magnetic absorption coefficient,  $\omega_0$  the resonant frequency,  $n$  the index of refraction at  $\omega_0$ , and  $c$  the velocity of light. Thus, the integrated absorption strengths, in units of  $\text{cm}^{-2}$ , for rf fields along the  $a, b$ , and  $c$  directions are

$$\begin{aligned} S_a &= \pi\omega_1^2 n \chi_{aa} / c^2, \\ S_c &= \pi\omega_1^2 n \chi_{cc} / c^2, \end{aligned} \quad (10)$$

<sup>14</sup> M. Tinkham, Phys. Rev. **124**, 311 (1961).

<sup>15</sup> A. J. Sievers, III, thesis, University of California, 1962 (unpublished).

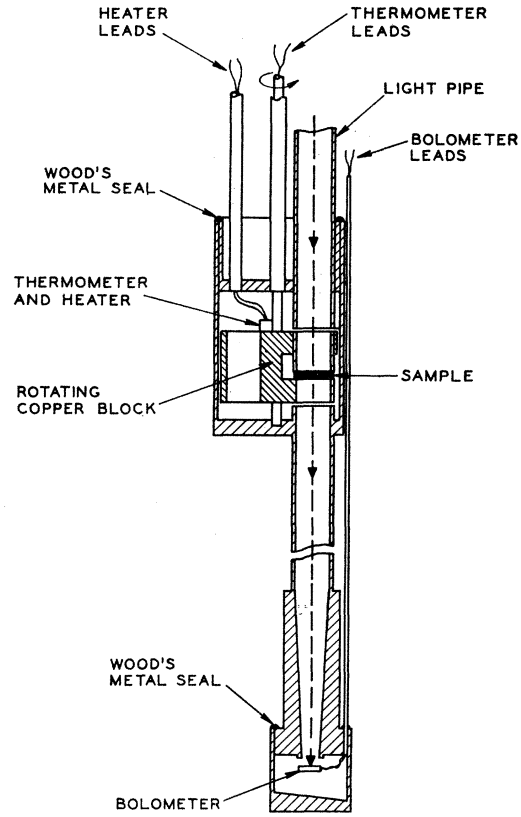


Fig. 3. Sample holder and detector mount for measuring the temperature dependence of sample transmission.

for mode 1, and

$$S_b = \pi\omega_2^2 n \chi_{bb} / c^2 \quad (11)$$

for mode 2.

### III. EXPERIMENTAL METHODS

In order to extend the available resolution and frequency range, and to increase the speed of the measurements, the data presented here were obtained by far-infrared Fourier-transform spectroscopy, rather than by using the more conventional diffraction-grating monochromator. With this technique, the interference pattern from a two-beam interferometer is recorded, and the spectrum is obtained by computing the Fourier cosine transform of the interference pattern on a digital computer. The measurements of mode 1 were made primarily with a lamellar-grating interferometer, and these of mode 2 with a Michelson interferometer. Both interferometers, as well as the method of Fourier-transform spectroscopy, have been described fully elsewhere.<sup>16</sup>

The sample holder assembly shown in Fig. 3 was used for measurements of transmission as a function of temperature over the range  $1.5 < T < 100^\circ\text{K}$ . A  $\frac{1}{2}$ -in.-diam evacuated light pipe conveyed the  $\sim f/1.4$  radiation from the interferometer into the cryostat, through the

<sup>16</sup> P. L. Richards, J. Opt. Soc. Am. **54**, 1474 (1964).

sample, and to the detector. Since the sensitivity of the germanium bolometer detector<sup>16</sup> used for our measurements increased rapidly with decreasing temperature, the whole sample and detector assembly was immersed in liquid helium at  $\sim 1.2^\circ\text{K}$  in a conventional glass cryostat.

The sample, an oriented single crystal slab of  $\text{NiF}_2$ , was glued to a disk of crystalline quartz with rubber cement. Both the sample and the supporting quartz disk were wedge-shaped to reduce interference effects. The quartz disk was in turn cemented to a copper block whose temperature was controlled with a heater and measured with a GaAs diode thermometer.<sup>17</sup> This thermometer was calibrated in baths of liquid He, H, N, and O. Copper-block temperatures were maintained constant to  $\pm 0.3^\circ\text{K}$  by hand adjustment of the heater current. Careful filtering of room-temperature radiation by cooled windows of sooted crystalline quartz maintained the sample temperature within less than  $0.3^\circ\text{K}$  of the block temperature.

The sample holder shown in Fig. 3 was designed to permit rotating the sample in and out of the light beam to obtain normalized spectra. In general, however, more useful information on the magnetic absorptions was obtained by comparing spectra obtained above and below  $T_N$ , or at different values of magnetic field.

A NbZr superconducting solenoid was used in order to measure sample transmission in magnetic fields up to

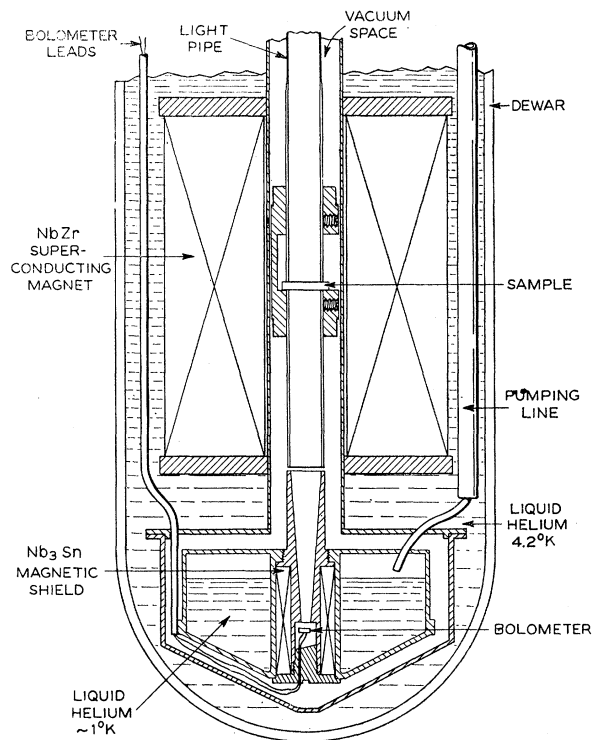


FIG. 4. Sample holder and detector mount for measuring the magnetic-field dependence of sample transmission up to 50 kOe.

<sup>17</sup> B. G. Cohen *et al.*, *Rev. Sci. Instr.* **34**, 1901 (1963).

50 kOe. It did not prove practical to cool the entire solenoid to the temperature required for bolometer operation, so a small  $\sim 1^\circ\text{K}$  helium bath (which was filled by condensation) was used to cool the detector as is shown in Fig. 4. The solenoid and sample were maintained at  $4.2^\circ\text{K}$  by a liquid-helium bath at atmospheric pressure. A sample temperature of  $4.2^\circ\text{K}$  is sufficiently far below  $T_N$  that the field dependence data have essentially their values for  $T=0$ .

#### IV. EXPERIMENTAL RESULTS

##### Field Dependence of $\omega_1$ and $\omega_2$

Equations (5), (10), and (11) show that the strength of the lower frequency mode  $\omega_1$  is reduced by a factor of  $\sim (\omega_1/\omega_2)^2$  from that of  $\omega_2$ . For this reason, the relatively thick (4–7 mm) sample 2 was required to observe  $\omega_1$ .<sup>18</sup> It was not convenient to orient sample 2 exactly, so the data shown in Fig. 5 were obtained with the static field in the (001) plane, but  $12^\circ$  from the [100] axis.

Moriya<sup>4</sup> has predicted that in zero field  $\text{NiF}_2$  is divided into domains parallel to the (001) plane, such that the net moment ( $b$  direction) points equally in the [100],  $[\bar{1}00]$ , [010], and  $[0\bar{1}0]$  directions. Since the sensitivity of our infrared measurements decreased rapidly below  $\sim 4\text{ cm}^{-1}$ , we were unable to detect the expected splitting of  $\omega_1$  in a small [100] field, but did observe a decreased absorption strength for the lowest field point measured ( $H_b \sim 2.5\text{ kOe}$ ). The domains align in a [100] field greater than  $\sim 5\text{ kOe}$ , and all of the spins see  $H_b = H$  (or  $H_b = 0.978H$  because of the mis-orientation). Since the measurements were made with unpolarized radiation propagating nearly parallel to  $H$ , there is a resonance strength  $\sim S_c$  for one polarization and  $\sim S_a = \frac{1}{2}S_c$  for the other when  $H$  is along a [100] direction.

We were unable to observe mode 1 with a field along the [001] direction. In this orientation the signal is weaker (the strength being  $S_a$  for one polarization, and zero for the other) and the field dependence is negative so that the resonance moves out of the useful range of the spectrometer.

We could not measure mode 1 in zero field, so its frequency was obtained by extrapolating the available data to zero field using the empirical function  $\nu_1 = (\alpha + \beta H)^M\text{ cm}^{-1}$  which falls within  $0.03\text{ cm}^{-1}$  of the measured points when  $\alpha = 5.40$ ,  $\beta = 0.508$ ,  $M = 0.715$ , and field  $H$  is in kOe. The resulting value of  $\nu_1(0) = 3.33 \pm 0.05\text{ cm}^{-1}$ , along with Eq. (6) and the spin value  $S = 1$ , gives  $E = 1.66\text{ cm}^{-1}$ . In order to use Eq. (6) to fit the measured field dependence of  $\omega_1$ , it is necessary to choose a value for  $|\lambda|$  and to estimate  $g_1 - g_2 = 4E/|\lambda|$ . Using  $|\lambda| = 250\text{ cm}^{-1}$  and  $g_1 = 2.35$  we obtain the fit shown in Fig. 5. This  $g$  factor is in good agreement with the  $g_1 = 2.33$  obtained from paramagnetic resonance of  $\text{Ni}^{2+}$  in  $\text{ZnF}_2$ ,<sup>19</sup> and the

<sup>18</sup> Crystal grown by Dr. H. Guggenheim and loaned to the author by Dr. J. Ferguson, Jr.

<sup>19</sup> M. Peter and J. B. Mock, *Phys. Rev.* **118**, 137 (1960).

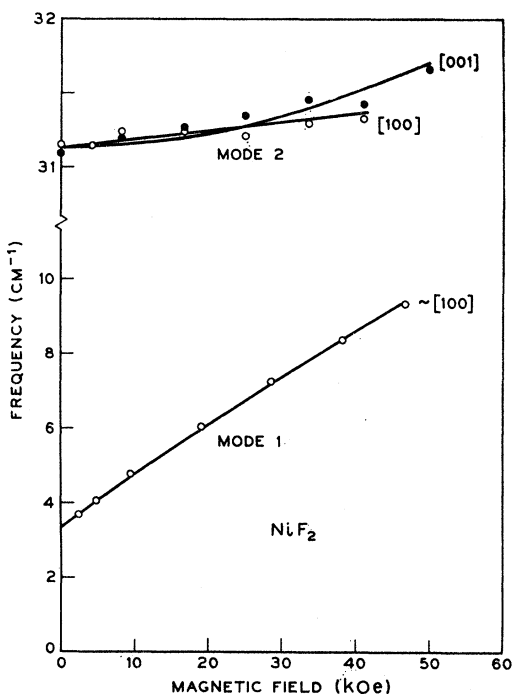


FIG. 5. Magnetic-field dependence of the resonant frequencies  $\omega_1$  and  $\omega_2$ . The frequency scale is different for the two branches. The solid lines are the theoretical predictions with parameters chosen as described in the text.

$g_1 = 2.32$  obtained from the paramagnetic susceptibility of NiF<sub>2</sub>.<sup>12</sup>

Preliminary measurements<sup>13</sup> were made of the higher frequency mode 2 in zero field using an unoriented single crystal plate of irregular shape (sample 1) whose thickness varied between 1.5 and 2.5 mm.<sup>20</sup> Samples 3 and 4, which were in the form of 1–2-mm-thick wedge-shaped plates cut from sample 2 with one face perpendicular to the [001] and [100] axes, respectively, were used for the measurements of the field dependence of mode 2 shown in Fig. 5. For sample 3, with  $H$  and radiation propagation along the [001] axis, each polarization should see  $\omega_2$  with strength  $S_p \approx 100S_c$  from half the spins and no contribution from the other half. The observed resonance was considerably stronger than mode 1. In sample 4, with  $H < 5$  kOe and both field and propagation along [100], the absorption from mode 2 was confined to one polarization, but was otherwise similar to that seen in sample 3. It dropped sharply, however, for higher fields in which the domains were aligned. Only the imperfect collimation of the light and the irregular sample shape, which gave finite components of  $H_r$  along the [100] axis, permitted observation of mode 2 in sample 4 when the field was large enough to align the domains.

Averaging the results on the higher frequency mode from samples 1, 3, and 4, gives  $\nu_2(0) = 31.14 \pm 0.1$  cm<sup>-1</sup>. From Eq. (17) we obtain  $8JD = 482.2$  cm<sup>-2</sup>. By using

<sup>20</sup> Crystal supplied by Dr. R. G. Shulman.

the temperature-dependent portion of susceptibility  $\chi_{cc} = 5.75 \times 10^{-3}$  emu/mole<sup>12</sup> and the resonance value of  $g_1 = 2.35$  in Eq. (5), we find  $8J = 125$  cm<sup>-1</sup> and  $D = 3.86$  cm<sup>-1</sup>. It is instructive to compare the anisotropy parameters  $D$  and  $E = 1.66$  cm<sup>-1</sup> evaluated from the resonant frequencies and susceptibility of the ordered state with the estimates of the spin Hamiltonian parameters  $D = 4.19$  cm<sup>-1</sup> and  $E = 2.67$  cm<sup>-1</sup> obtained from paramagnetic resonance of Ni<sup>2+</sup> in ZnF<sub>2</sub>.<sup>19</sup>

The field dependences of mode 2 computed from Eq. (7) using the values of  $g$ ,  $E$ , and  $8JD$  obtained above, are compared with the experimental points in Fig. 5. The points for a [100] field are fit somewhat better by a line of slope 0.0045 cm<sup>-1</sup>/kOe, than by the predicted value of 0.0054. The points for an [001] field are fit at least as well by a straight line of slope 0.10 cm<sup>-1</sup>/kOe as by the quadratic prediction of the theory. The scatter of the data is sufficiently large, however, that neither theoretical curve lies outside the limits of experimental error.

As has been shown by Bozorth *et al.*,<sup>12</sup> the measured net magnetic moment of 161.5 emu/mole is in excellent agreement with the value  $N\mu_B SE(g_1\eta/8J_1 + 2/|\lambda|) = 162$  emu/mole computed using the parameters determined from resonance and susceptibility data and assuming that  $|\lambda| = 250$  cm<sup>-1</sup>.

#### Temperature Dependence of $\omega_2$

The lower frequency  $\omega_1$  was not measured at elevated temperatures since it moves out of range of our spectrometer. Measurements were made of  $\omega_2(T)$ , however, and the results are shown in Fig. 6. These data have an estimated limit of experimental error of  $\pm 0.1$  cm<sup>-1</sup> at low temperatures, increasing with line width to  $\pm 0.6$

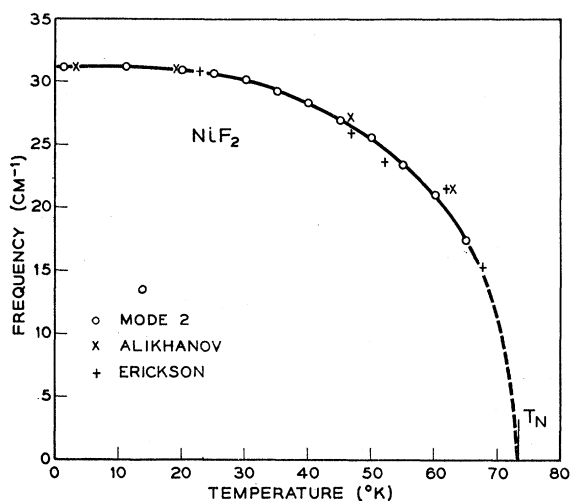


FIG. 6. Temperature dependence of the higher frequency mode 2 in zero magnetic field. The data appear to extrapolate smoothly to zero at  $T_N$ , as is expected for antiferromagnetic resonance. The crosses represent the three-fourths power of the temperature dependence of the neutron-diffraction intensities of Erickson and Alixhanov.

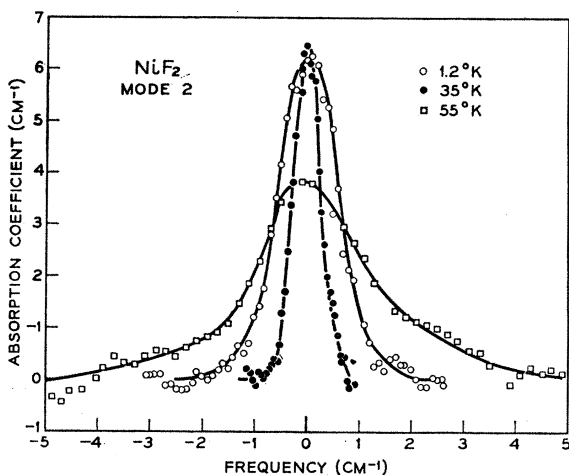


FIG. 7. Temperature dependence of the magnetic absorption coefficient in the neighborhood of the higher frequency mode  $\omega_2$ . The periodic structure in the wings of the resonance lines is attributed to interference effects, due to multiple reflections in the sample, which are not entirely cancelled by the use of ratios of spectra at different temperatures.

$\text{cm}^{-1}$  for the highest temperature point. The negligible slope of  $\omega_2(T)$  at helium temperatures justifies our assumption that measurements at 1.2°K give frequencies corresponding to  $T=0$ . The similar assumption made for  $\omega_1$  should also be valid.

To a good approximation we may write  $\omega_2$  in the form of a conventional antiferromagnetic resonance,  $\omega_2/\gamma = (2K/\chi_1)^{1/2}$ , if we let  $K = NDS^2$  and  $\chi_1 = \chi_{bb} = \chi_{cc}$ . The "perpendicular" susceptibility  $\chi_{cc}$  is known to be approximately independent of temperature below  $T_N$ .<sup>12</sup> According to the Zener theory,<sup>21</sup> the anisotropy constant  $K$  is proportional to the third power of the sublattice magnetization. It has been shown<sup>22</sup> for MnO and NiO, and for the iron-group fluorides,<sup>13</sup> that a good fit for  $\omega(T)$  can be obtained if the square root of neutron-diffraction-intensity measurements is used to an estimate of the temperature dependence of the sublattice magnetization. The three-fourths powers of the neutron-diffraction points of Erickson<sup>23</sup> and Alikhanov<sup>3</sup> are compared with the measured resonant frequencies in Fig. 6. The accuracy of the neutron data is poor, but the agreement appears to be reasonably good.

#### Linewidth and Strength of Mode 2

Measurements of the magnetic absorption near  $\omega_2$  were made as a function of temperature by computing the ratio of the sample transmission below the Néel temperature to that at  $T = 80^\circ\text{K} > T_N$ . Since the magnetic absorptions are weak compared with the lattice absorption, the total reflection from the sample surface is relatively constant near  $\omega_2$ , and the magnetic absorption

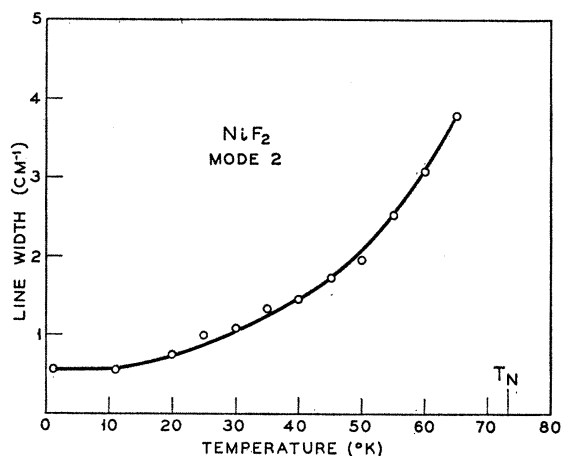


FIG. 8. The temperature dependence of the linewidth of the higher frequency mode.

coefficient  $\alpha$  can be obtained from the negative logarithm of the transmission ratios. Typical data for  $\alpha(T)$  in the neighborhood of  $\omega_2(T)$  are shown in Fig. 7. These measurements were made on sample 3, which had been cut down to a wedge whose thickness varied from 0.79 to 1.24 mm.

The temperature dependence of the linewidth, defined as the full width when  $\alpha = \alpha_{\text{max}}/2$ , is plotted in Fig. 8. These data have an estimated limit of experimental error of  $\pm 0.1 \text{ cm}^{-1}$  for  $T < 55^\circ\text{K}$ . At higher temperatures, because of the difficulty of choosing an accurate base line, the errors may be as large as  $\pm 0.3 \text{ cm}^{-1}$ . The modest accuracy of our data, and the small range of linewidths covered, make it difficult to distinguish between fluctuation theories of the temperature dependence of the antiferromagnetic linewidth<sup>24,25</sup> and the various mechanisms discussed by Pincus.<sup>26</sup> The width of the line at  $T \approx 0$  is not strongly sample-dependent. This result is in agreement with the prediction<sup>26</sup> that impurity and imperfection linewidths in antiferromagnets will be very small compared with  $1 \text{ cm}^{-1}$ . Only those mechanisms<sup>26</sup> which relax the antiferromagnetic resonance through the optical phonons appear to be capable of producing a linewidth of  $0.5 \text{ cm}^{-1}$  at  $T \approx 0$ . This conclusion is supported by the observation that the linewidths of infrared antiferromagnetic resonances at  $T \approx 0$  are much larger than those for microwave frequency resonances.<sup>13,23,24,25</sup>

The absorption strength associated with mode 2 was measured by numerically integrating the data for the magnetic absorption coefficient. As can be seen directly from Fig. 7, the line strength appeared to increase nearly linearly with temperature from  $4.0 \text{ cm}^{-2}$  at  $0^\circ\text{K}$  to  $14.0 \text{ cm}^{-2}$  at  $55^\circ\text{K}$ . These data were obtained with unpolarized radiation propagating along the  $[001]$  axis

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<sup>22</sup> A. J. Sievers, III, and M. Tinkham, Phys. Rev. **129**, 1566 (1963).

<sup>23</sup> R. A. Erickson, Phys. Rev. **90**, 779 (1953).

<sup>24</sup> F. M. Johnson and A. H. Nethercott, Jr., Phys. Rev. **114**, 705 (1959).

<sup>25</sup> R. C. Ohlman and M. Tinkham, Phys. Rev. **123**, 425 (1961).

<sup>26</sup> P. Pincus, J. Phys. Radium **23**, 536 (1962).

of sample 3. Assuming equal numbers of domains of each kind and using the measured index of refraction of  $n \sim 2.9$  at frequency  $\omega_2$ , Eq. (11) predicts a strength of  $47 \text{ cm}^{-2}$  which, like  $\chi_{bb}$ , should be nearly independent of temperature.

This discrepancy may be analogous to the *apparent* violation of Eq. (9) which occurs for the simple easy-axis antiferromagnets like MnF<sub>2</sub>. There  $\chi_{11}$  is approximately proportional to  $T$ , but no antiferromagnetic resonance absorption is seen when the rf magnetic field is along the  $c$  axis. Equation (8) is satisfied (as it must be) by a spread-out region of absorption at higher frequencies which arises from second-order processes.<sup>27</sup>

An alternative explanation of both difficulties could be that the domains were predominantly of one type at low temperatures, but approached a more symmetrical distribution as the temperature increased.

### V. CONCLUSIONS

The theory of magnetism in NiF<sub>2</sub> outlined above, originally due to Moriya<sup>4</sup> and extended by Joenk and Bozorth,<sup>12</sup> is adequate for a quantitative description of

<sup>27</sup> T. Nakamura, *Progr. Theoret. Phys. (Kyoto)* **7**, 539 (1952).

the measured magnetic-resonance modes of NiF<sub>2</sub>. The parameters  $E = 1.66 \text{ cm}^{-1}$  and  $8JD = 482.2 \text{ cm}^{-2}$  are obtained directly from the measured frequencies at  $T \approx H = 0$ . We must choose  $g_1 = 2.35$  in order for the theory to fit  $\omega_1(H)$ . This value is in good agreement with the  $g_1 = 2.32$  and  $2.33$  obtained from other experiments and adequately predicts  $\omega_2(H)$ . One further datum is required to separate the exchange parameter  $8J = 125 \text{ cm}^{-1}$  from the out-of-plane anisotropy  $D = 3.86 \text{ cm}^{-1}$ . This can be either the susceptibility  $\chi_{bb} = \chi_{ee}$  or the net ferromagnetic moment  $M$ . The quantitative agreement with the theory is most clearly indicated by the fact that these two approaches lead to values of  $8J$  and  $D$  which agree within a few percent.

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## Vibrational Spectra of Lithium-Oxygen and Lithium-Boron Complexes in Silicon

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Vibrational spectra of lithium-oxygen and lithium-boron complexes in silicon have been studied using enriched boron and lithium isotopes. The absorption bands in the  $300\text{--}4000 \text{ cm}^{-1}$  region are due to vibrations of interstitial oxygen atoms perturbed by lithium ions, and substitutional boron ions, also perturbed by lithium ions. The  $517\text{-cm}^{-1}$  band of interstitial oxygen is displaced to  $525 \text{ cm}^{-1}$  by Li<sup>7</sup> and to  $537 \text{ cm}^{-1}$  by Li<sup>6</sup>, and the  $1106\text{-cm}^{-1}$  band is displaced to  $1006 \text{ cm}^{-1}$  by both lithium isotopes. (Frequencies are for  $78^\circ\text{K}$  except for the  $1106\text{-}$  and  $1006\text{-cm}^{-1}$  bands which are for  $300^\circ\text{K}$ .) No counterpart of the  $1203\text{-cm}^{-1}$  band of interstitial oxygen is detected for the lithium-oxygen complex, and on this basis the  $1203\text{-cm}^{-1}$  band is reassigned to a combination of the asymmetric SiO stretching vibration with the librational motion of oxygen around the (111) axis rather than to the symmetric SiO stretching fundamental. This new assignment provides a value of  $67 \text{ cm}^{-1}$  for the frequency of the vibrational mode associated with the libration. The  $623\text{-cm}^{-1}$  triply degenerate vibration of isolated substitutional boron ions is split into two bands at  $567$  and  $656 \text{ cm}^{-1}$  by interaction with lithium, indicating axial symmetry for the lithium-oxygen complex. (These frequencies are for the B<sup>11</sup> isotope.) The precipitation of lithium from the lithium-oxygen complex is inhibited in the presence of boron, probably because in the absence of free electrons the precipitation nuclei develop a positive charge which repels the diffusing lithium species (Li<sup>+</sup>).

### I. INTRODUCTION

VIBRATIONAL infrared spectra of light impurity atoms in elemental semiconductors which crystallize with the diamond lattice are of interest for many reasons, not the least of which is that the defects seem simple enough to be understood in considerable detail.

The present study was undertaken to learn more about the defect which is formed when dissolved lithium interacts with interstitial oxygen in silicon.

Oxygen forms an uncharged interstitial defect in

silicon, the spectrum and chemical behavior of which have been the subject of a number of papers.<sup>1-7</sup> This

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